Large-Scale Numerical Investigations into the Dynamics of Nonlinear Classical Systems

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Outline

Acknowledgements

- I would like to thank A.I. Nicolin, and V. Băran for helping and motivating me.
- The author has been supported by PN-III-P4-ID-PCE-2016-0792.
- All numerical simulations were performed on the computing cluster of Department of Computational Physics and Information Technologies, “Horia Hulubei” National Institute for Physics and Nuclear Engineering.

Introduction

The model

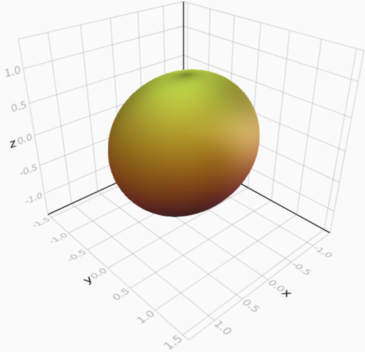
- The physical system that we model is the surface of heavy nuclei.
- The Hamiltonian describes the constrained motion of the vibrational quadrupole degrees of freedom of nuclear surface.

The model

The Hamiltonian of the system

$$H = \frac{A}{2}\left(p_0^2 + p_2^2\right) + \frac{A}{2}\left(q_0^2 + q_2^2\right) + \frac{B}{\sqrt{2}} q_0 \left(3q_2^2 - q_0^2\right) + \frac{D}{4}\left(q_0^2 + q_2^2\right)^2$$

- Harmonic oscillator part
- Integrable part
- Non-integrable term



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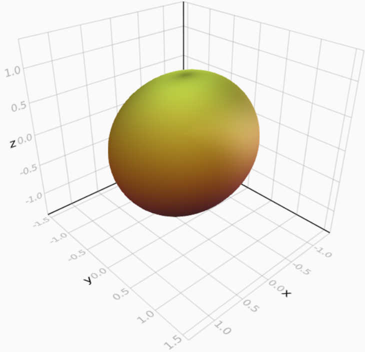

The model

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$$H = \frac{A}{2}\left(p_0^2 + p_2^2\right) + \frac{A}{2}\left(q_0^2 + q_2^2\right) + \frac{B}{\sqrt{2}} q_0 \left(3q_2^2 - q_0^2\right) + \frac{D}{4}\left(q_0^2 + q_2^2\right)^2$$

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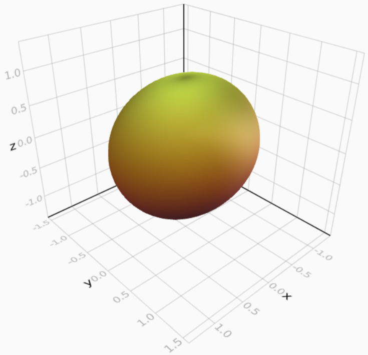
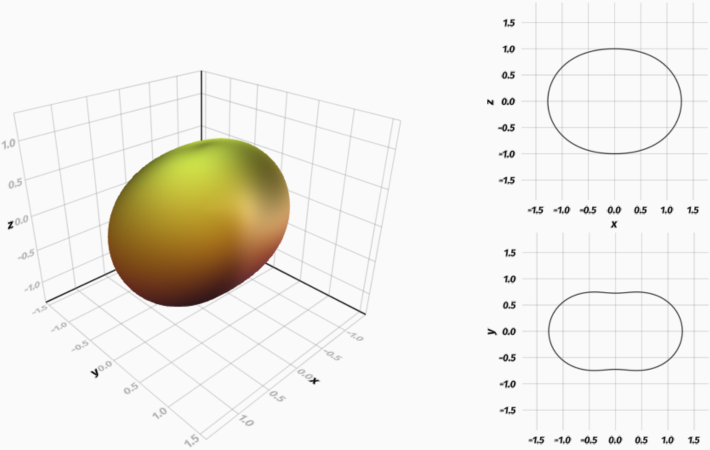
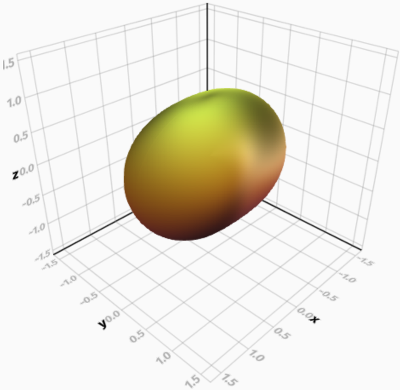
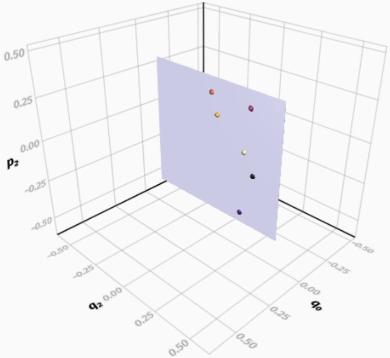


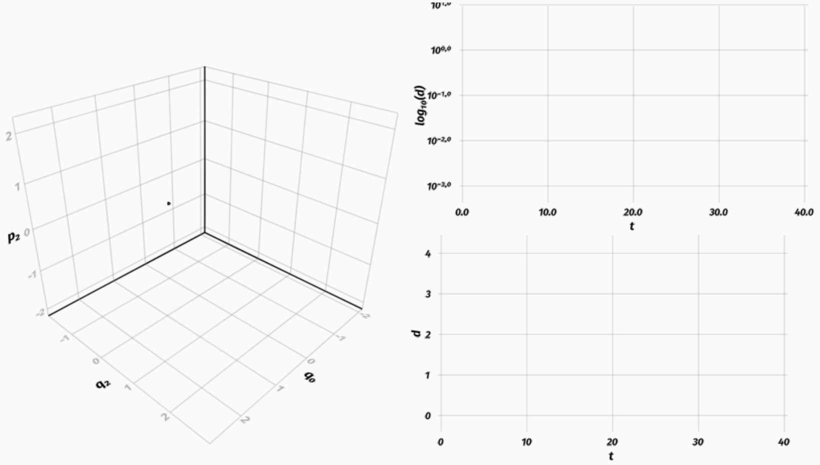


z
y
x
1.0
0.5
0.0
-0.5
-1.0
-1.5
1.5

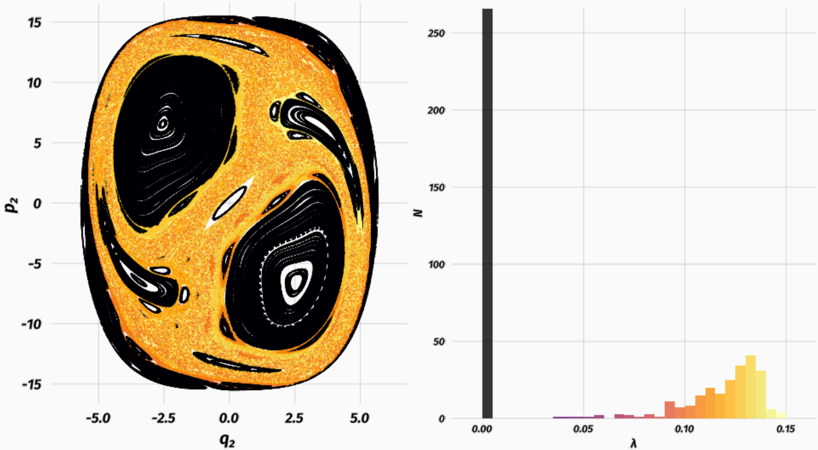

z
y
x
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5


p₂
q₂
q₀
0.50
0.25
0.00
-0.25
-0.50

Numerical simulations



p₂
q₂
q₀
log₁₀(d)
t
d
t



15
10
5
0
-5
-10
-15
p_2
-5.0
-2.5
0.0
2.5
5.0
q_2
250
200
150
100
50
0
N
0.00
0.05
0.10
0.15
λ

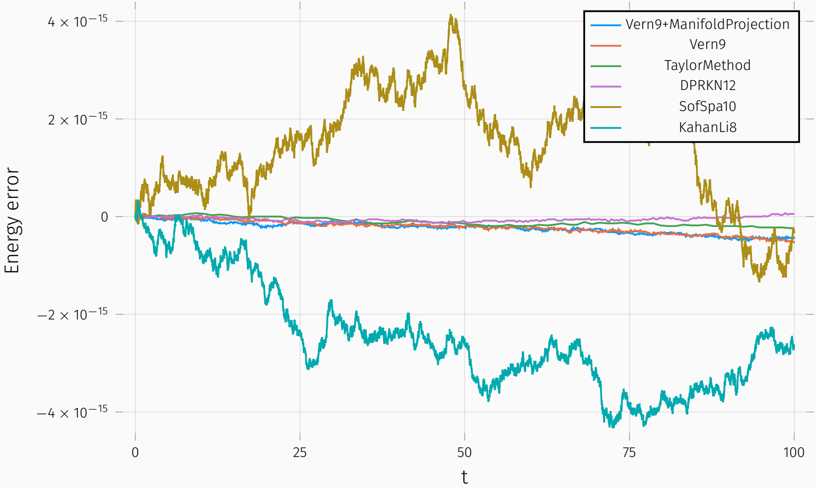


Figure 1: Energy error benchmark for short integration time

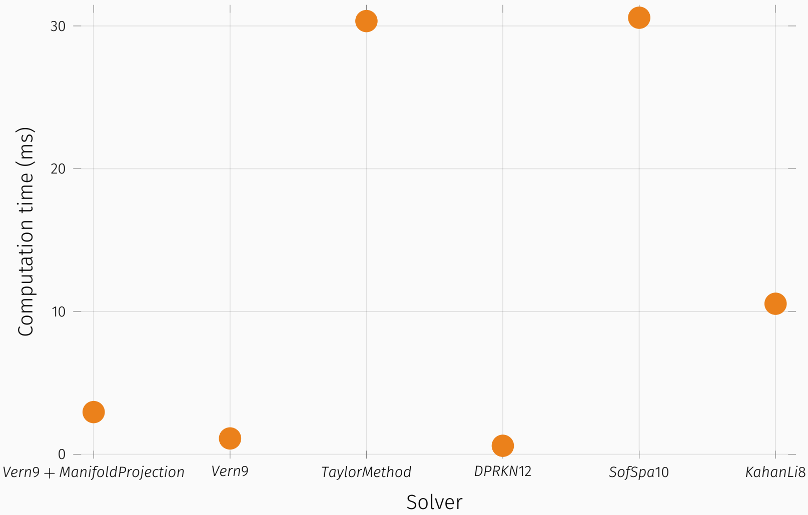


Figure 2: Computational time benchmark for short integration time

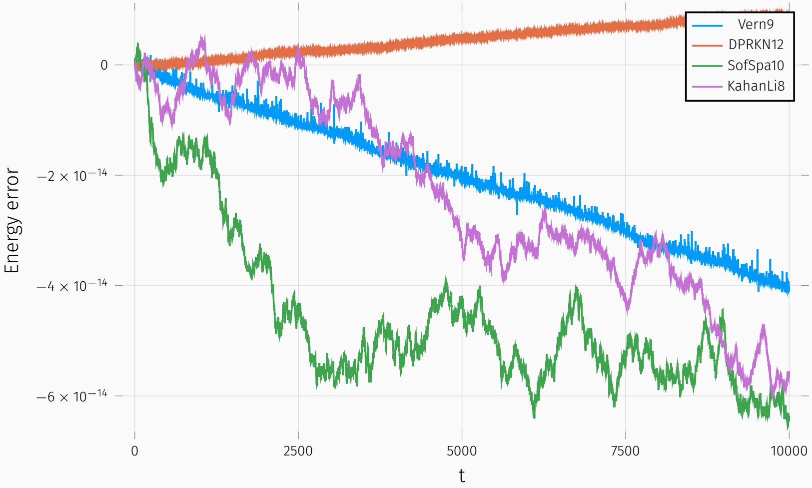


Figure 3: Energy error benchmark for long integration time

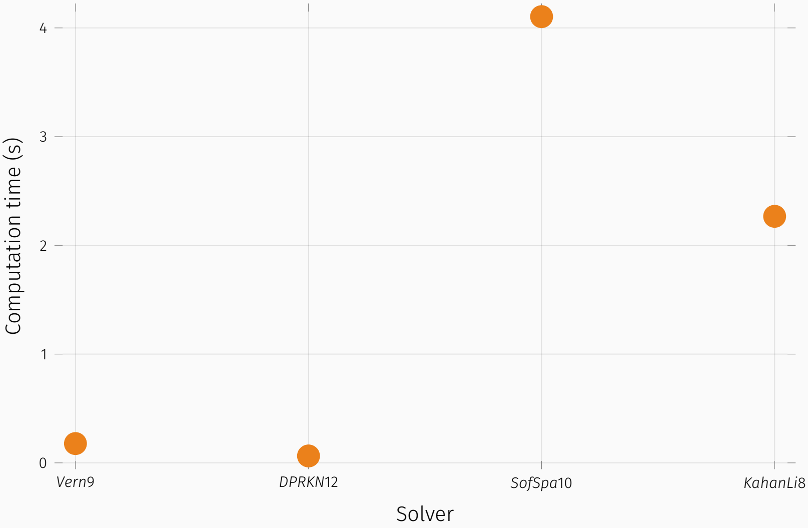


Figure 4: Computational time benchmark for long integration time

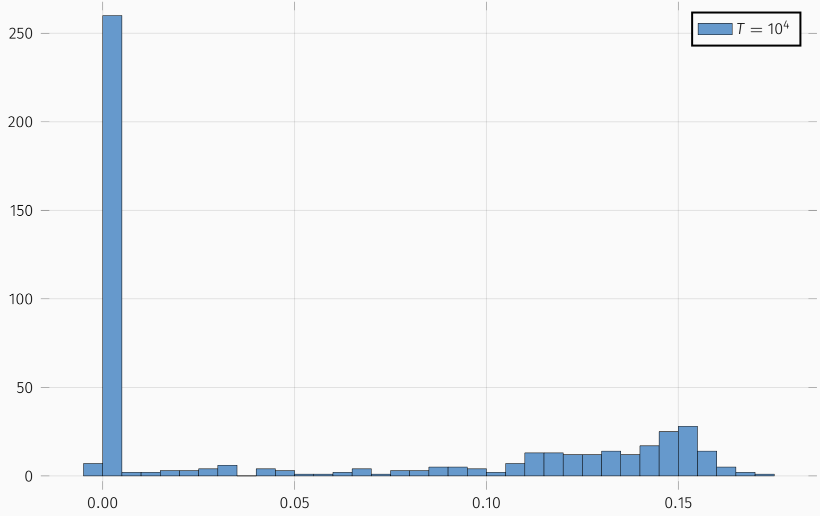


Figure 5: Maximal Lyapunov coefficient histogram for $B = 0.55, E = 120$.

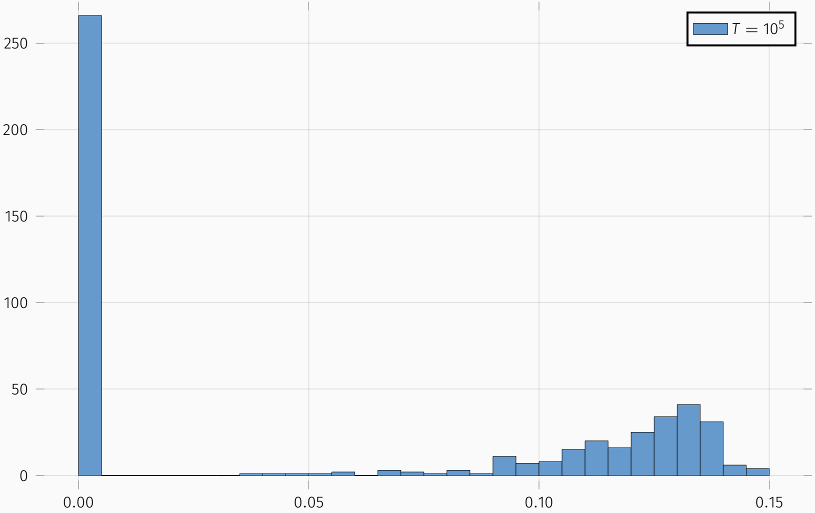


Figure 6: Maximal Lyapunov coefficient histogram for $B = 0.55, E = 120$.

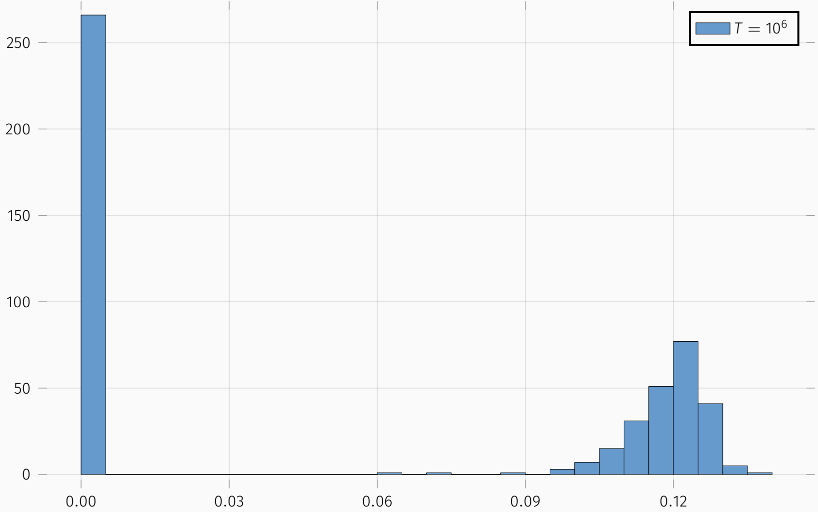


Figure 7: Maximal Lyapunov coefficient histogram for $B = 0.55, E = 120$.

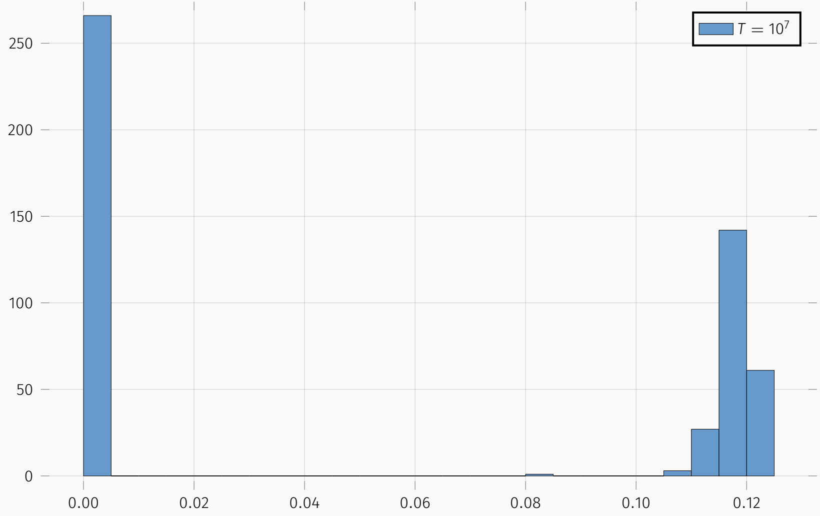


Figure 8: Maximal Lyapunov coefficient histogram for $B = 0.55, E = 120$.

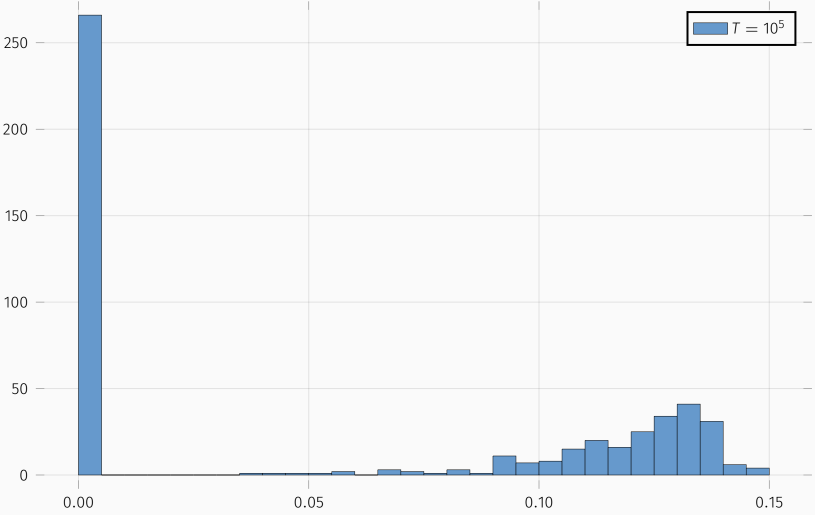


Figure 9: Selecting the chaotic trajectories for $B = 0.55$, $E = 120$.

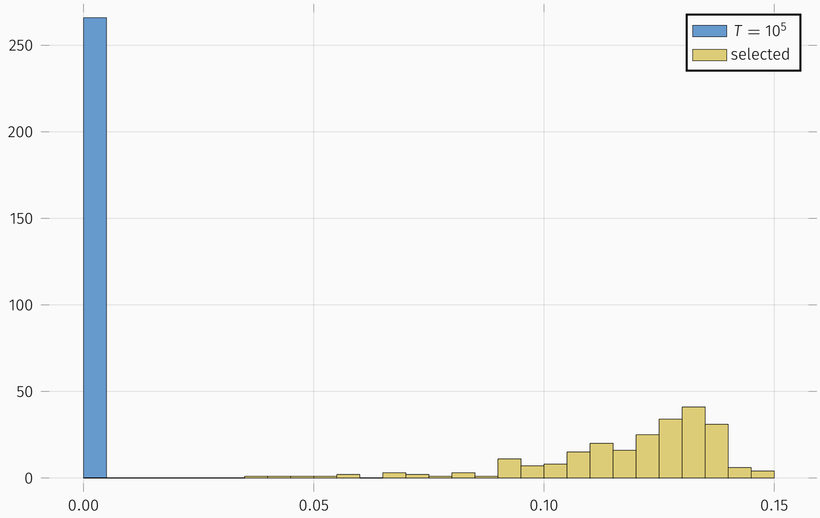


Figure 10: Selecting the chaotic trajectories for $B = 0.55, E = 120$.

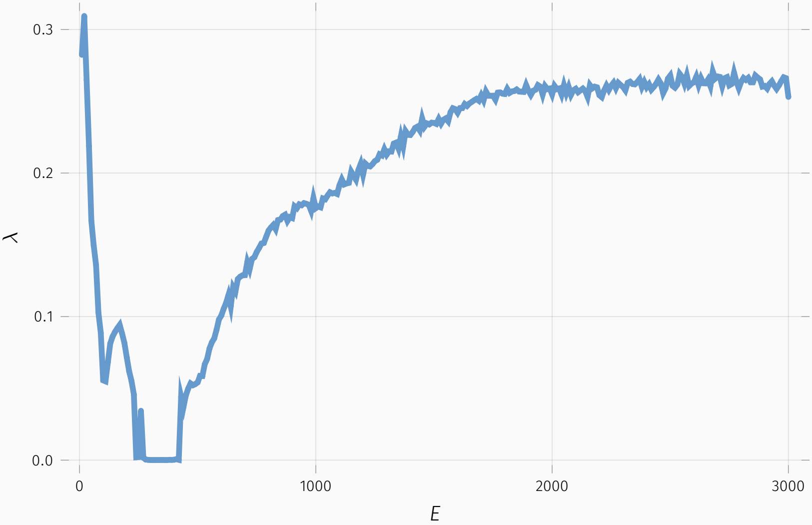


Figure 11: Averaged λ for $B = 0.5, E \in (10, 3000)$.

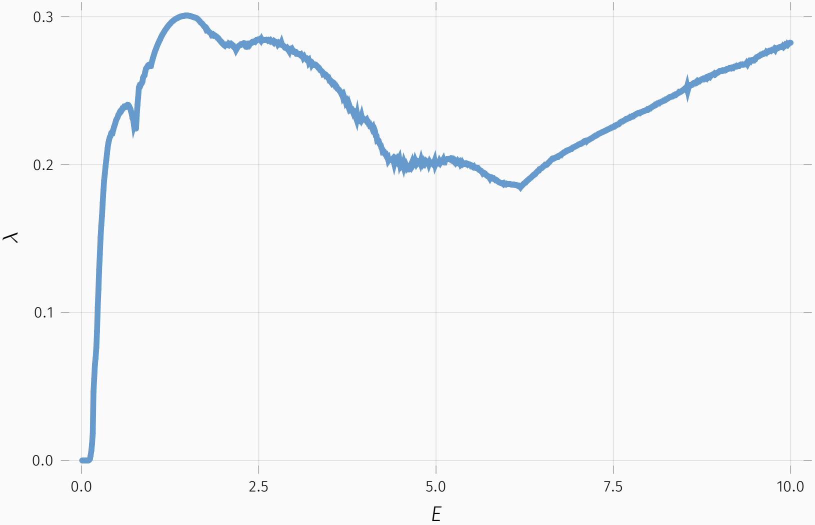


Figure 12: Averaged λ for $B = 0.5, E \in (0.01, 10)$.

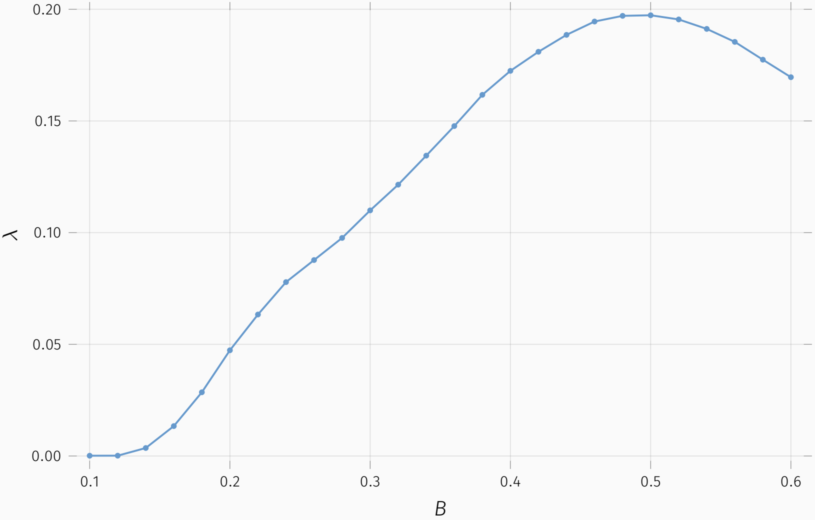


Figure 13: Averaged λ for $B = 0.5, E \in (10, 3000)$ and $B \in (0.1, 0.6)$.

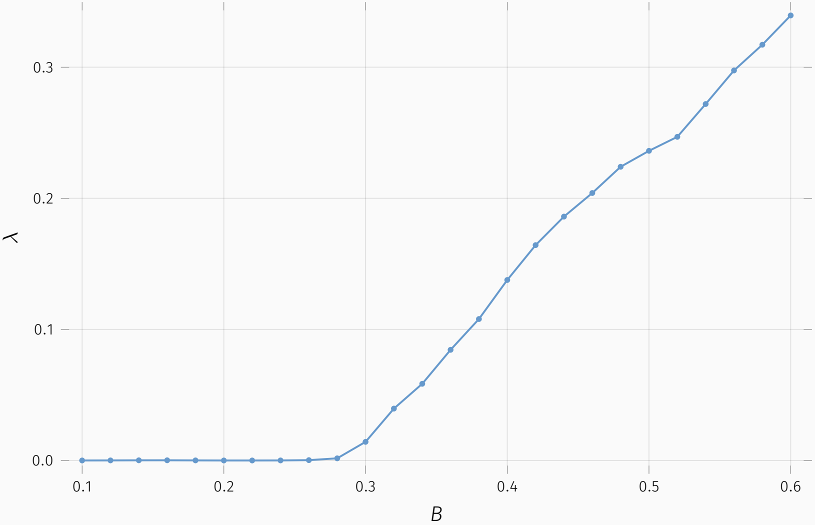


Figure 14: Averaged λ for $B = 0.5$, $E \in (0.01, 10)$ and $B \in (0.1, 0.6)$.

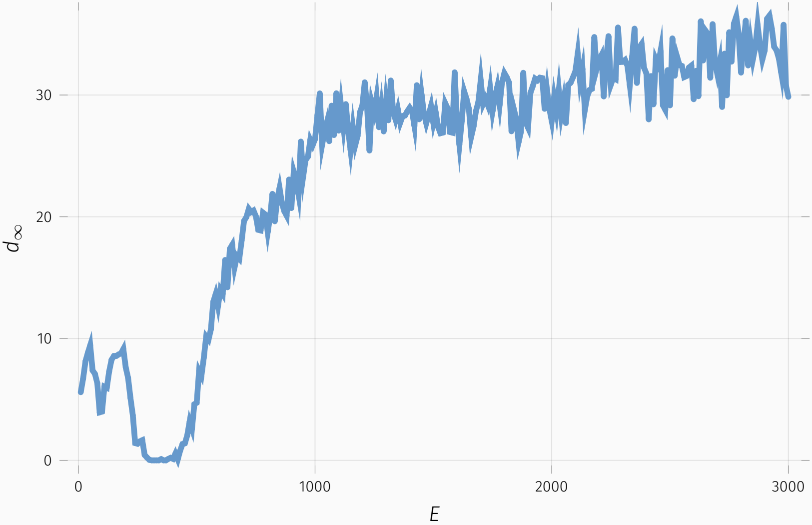


Figure 15: Averaged $d\infty$ for $B = 0.5, E \in (10, 3000)$.

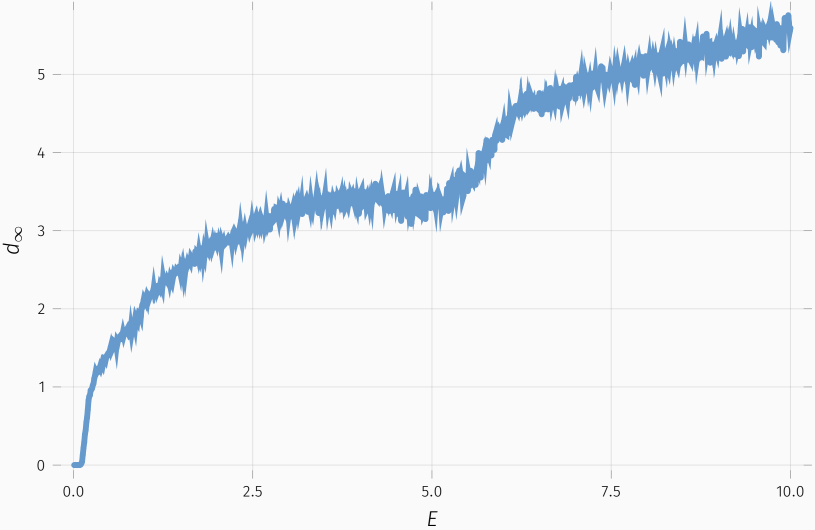


Figure 16: Averaged $d\infty$ for $B = 0.5, E \in (0.01, 10)$.

Conclusions

Thank you!

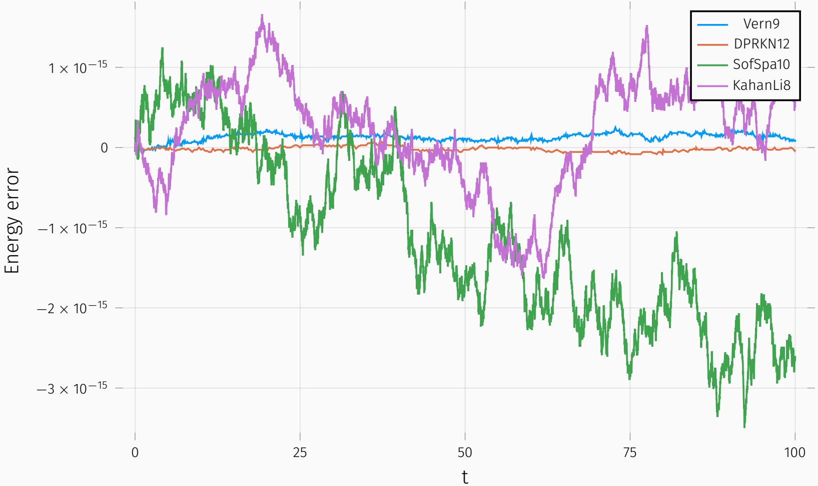


Figure 17: Energy error benchmark for short integration time with rescaling

Figure 18: Computational time benchmark for short integration time with rescaling

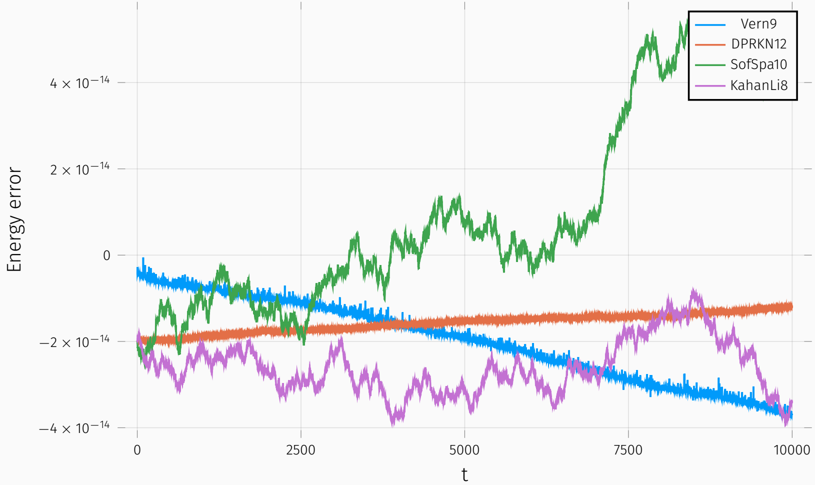


Figure 19: Energy error benchmark for long integration time with rescaling

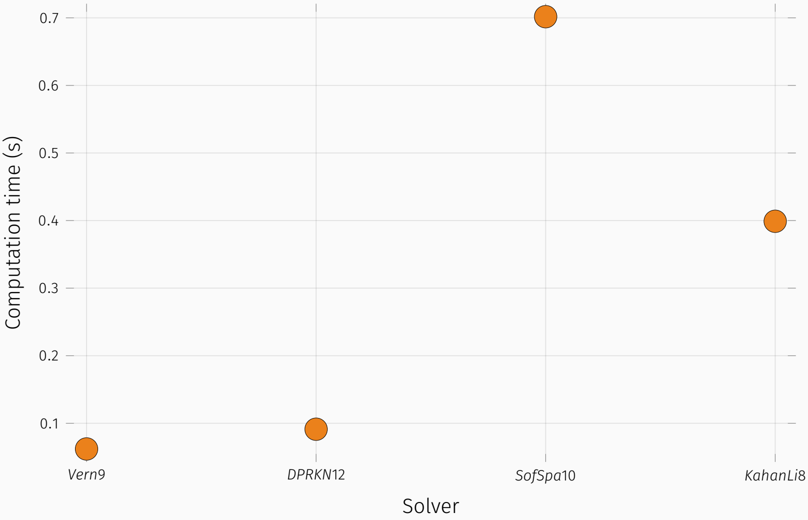


Figure 20: Computational time benchmark for long integration time with rescaling

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

References i