Large-Scale Numerical Investigations into the Dynamics of Nonlinear Classical Systems

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Outline

Acknowledgements

- I would like to thank A.I. Nicolin, and V. Băran for helping and motivating me.
- The author has been supported by PN-III-P4-ID-PCE-2016-0792.
- All numerical simulations were performed on the computing cluster of Department of Computational Physics and Information Technologies, “Horia Hulubei” National Institute for Physics and Nuclear Engineering.

Introduction

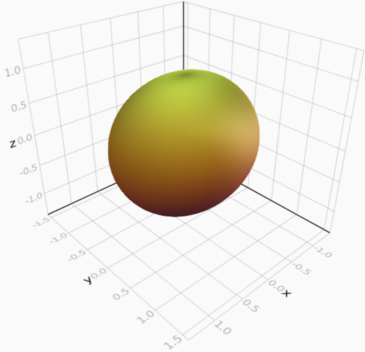
The model

- The physical system that we model is the surface of heavy nuclei.
- The Hamiltonian describes the constrained motion of the vibrational quadrupole degrees of freedom of nuclear surface.

The Hamiltonian of the system

$$H = \frac{A}{2}\left(p_0^2 + p_2^2\right) + \frac{A}{2}\left(q_0^2 + q_2^2\right) + \frac{B}{\sqrt{2}} q_0 \left(3q_2^2 - q_0^2\right) + \frac{D}{4}\left(q_0^2 + q_2^2\right)^2$$

- Harmonic oscillator part
- Integrable part
- Non-integrable term



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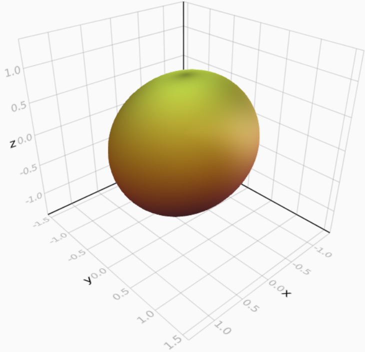

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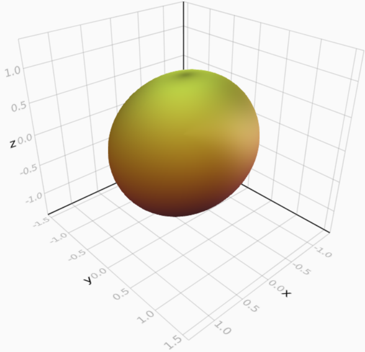

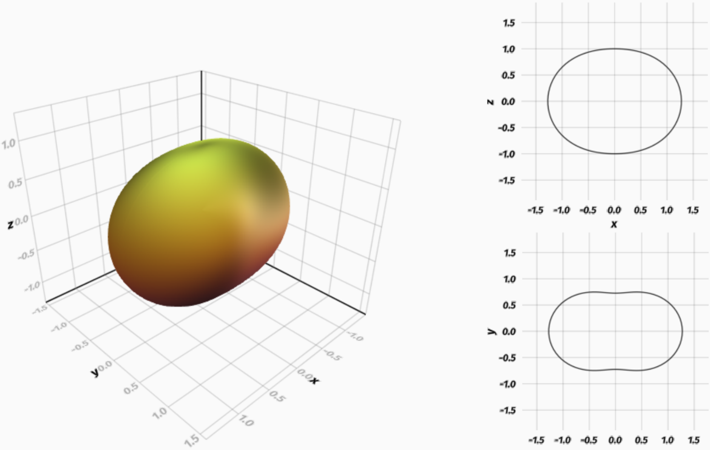
z
y
x
1.0
0.5
0.0
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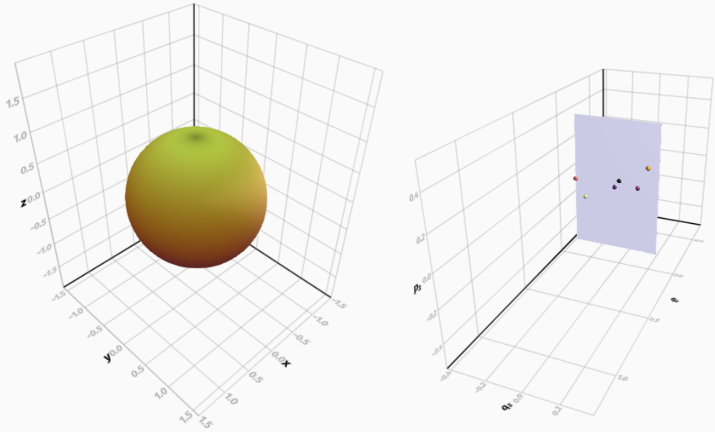

Figure 1: The nucleus and the Poincaré section at $B = 0.5, E = 0.3$

Numerical simulations

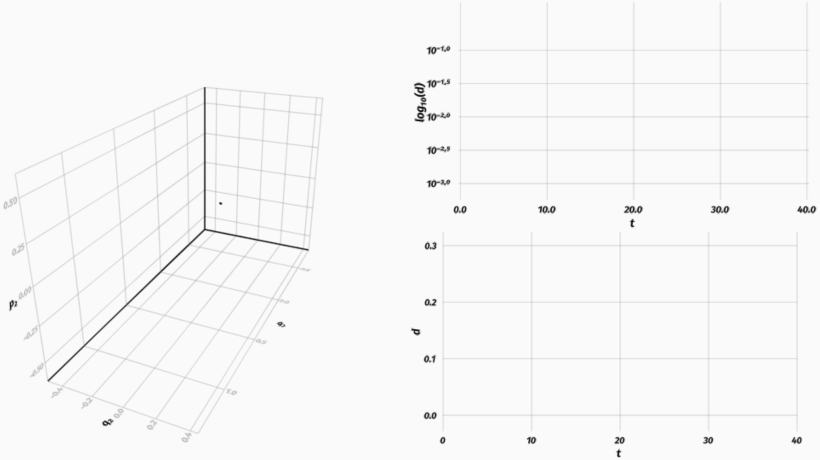

Figure 2: The distance between nearby trajectories for $B = 0.5, E = 0.3$

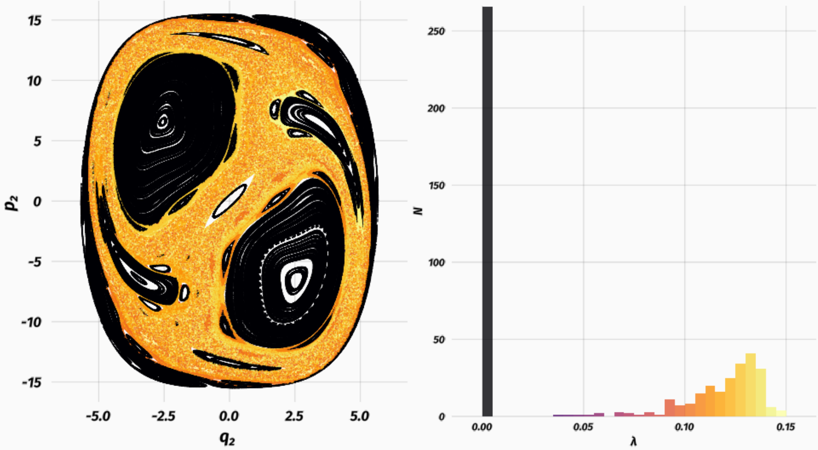

Figure 3: A Poincaré section at $B = 0.55$, $E = 120$

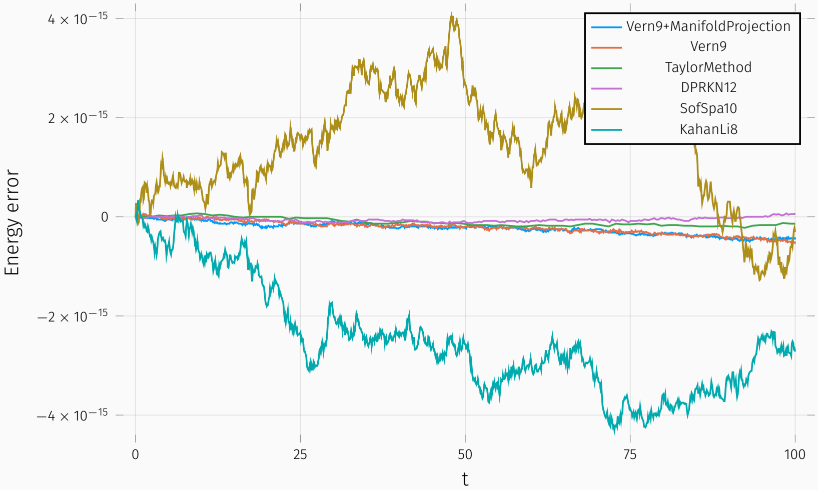

Figure 4: Energy error benchmark for short integration time

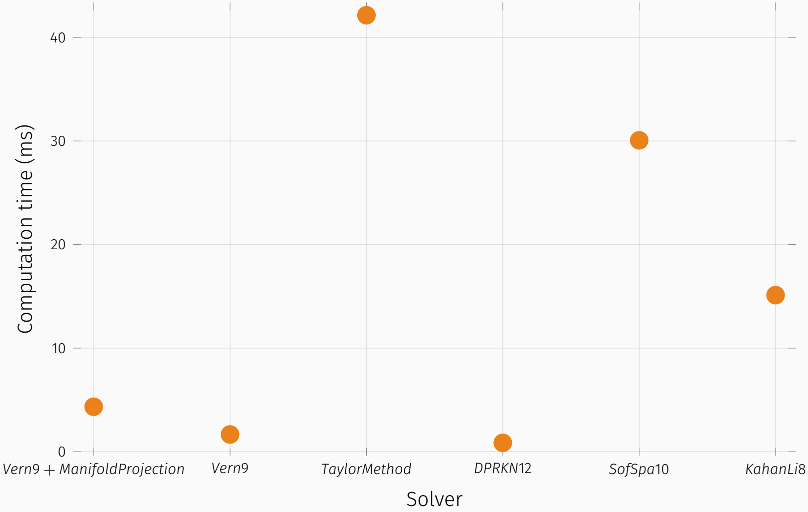

Figure 5: Computational time benchmark for short integration time

Figure 6: Energy error benchmark for long integration time

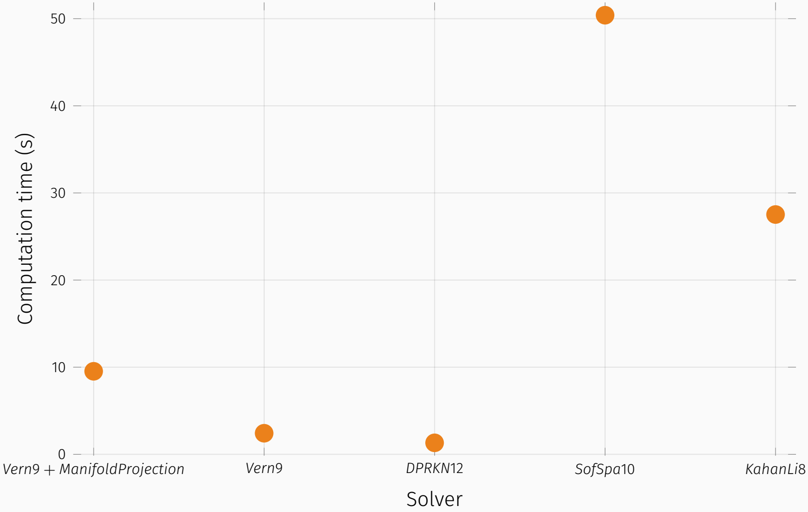

Figure 7: Computational time benchmark for long integration time

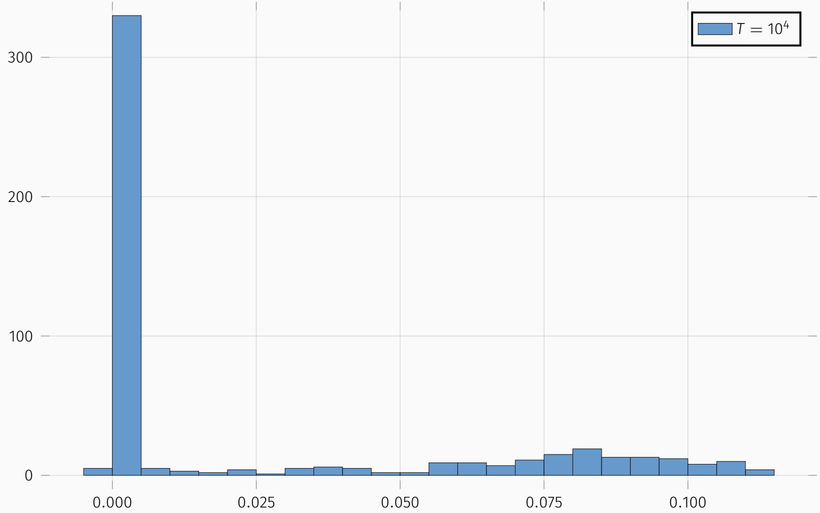

Figure 8: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

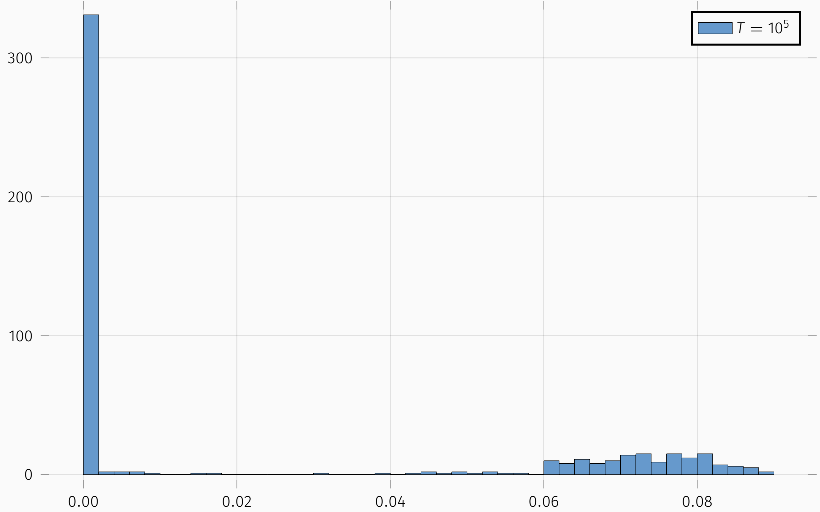

Figure 9: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

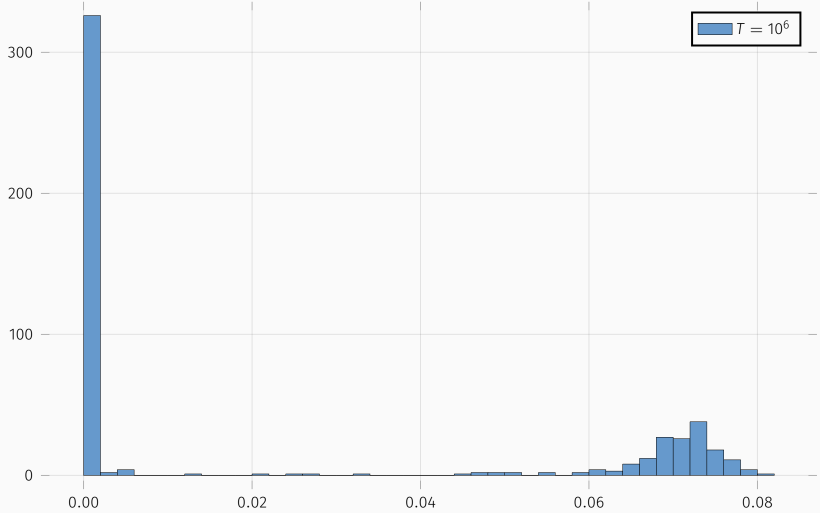

Figure 10: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

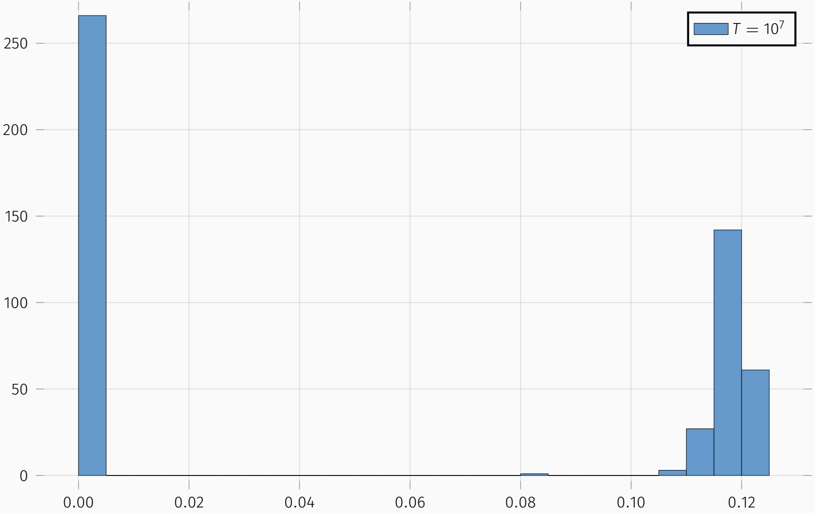

Figure 11: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

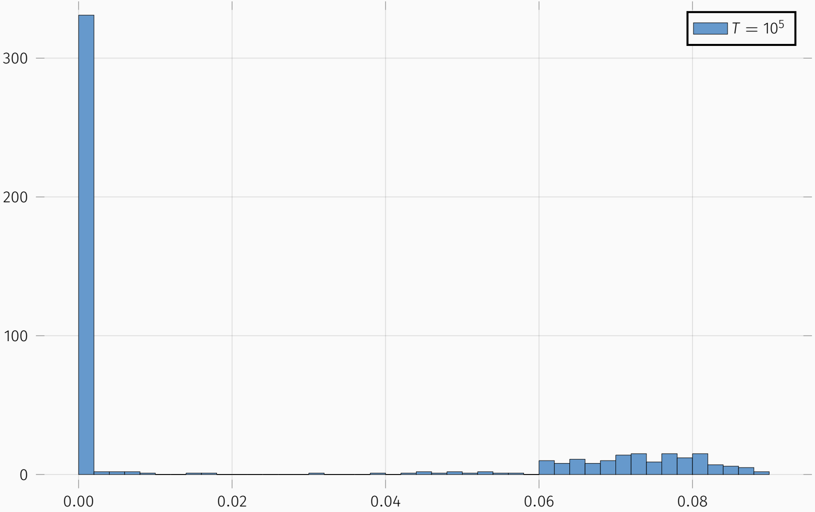

Figure 12: Selecting the chaotic trajectories for $B = 0.5, E = 120$.

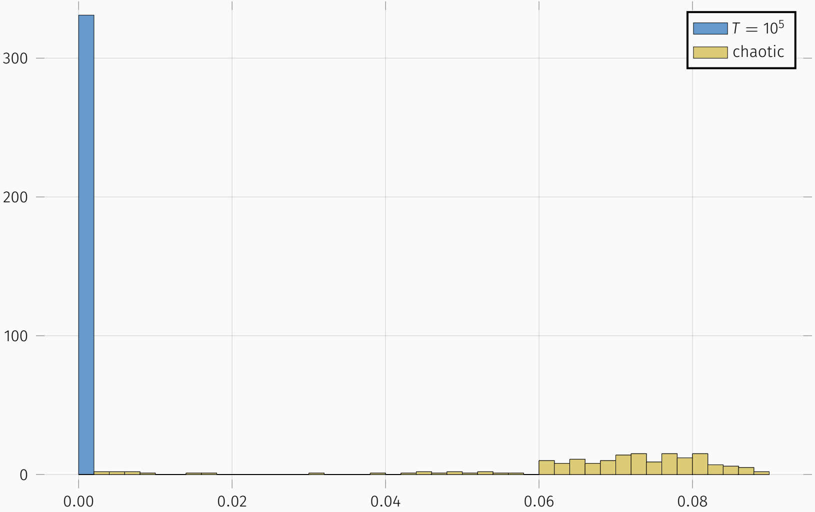

Figure 13: Selecting the chaotic trajectories for $B = 0.5, E = 120$.

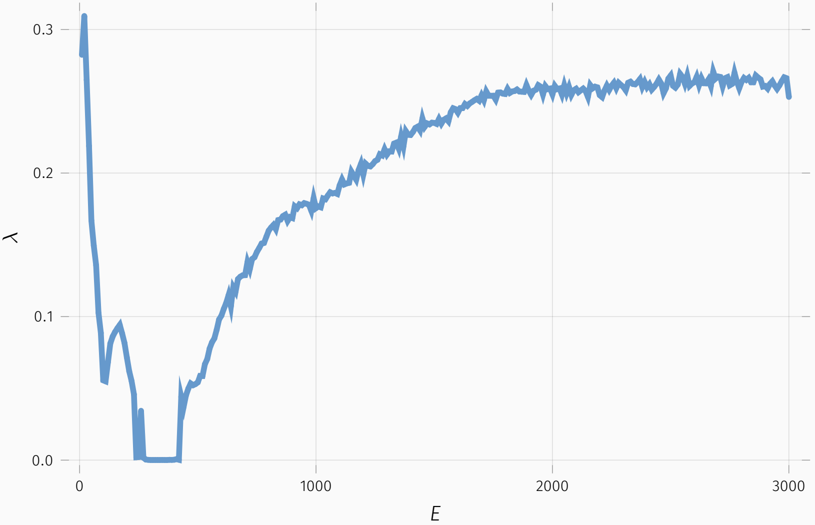

Figure 14: Averaged λ for $B = 0.5, E \in (10, 3000)$.

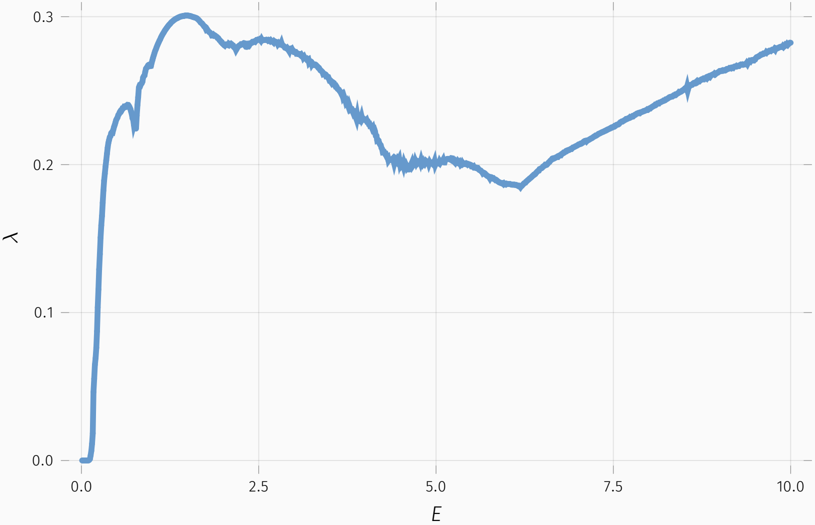

Figure 15: Averaged λ for $B = 0.5, E \in (0.01, 10)$.

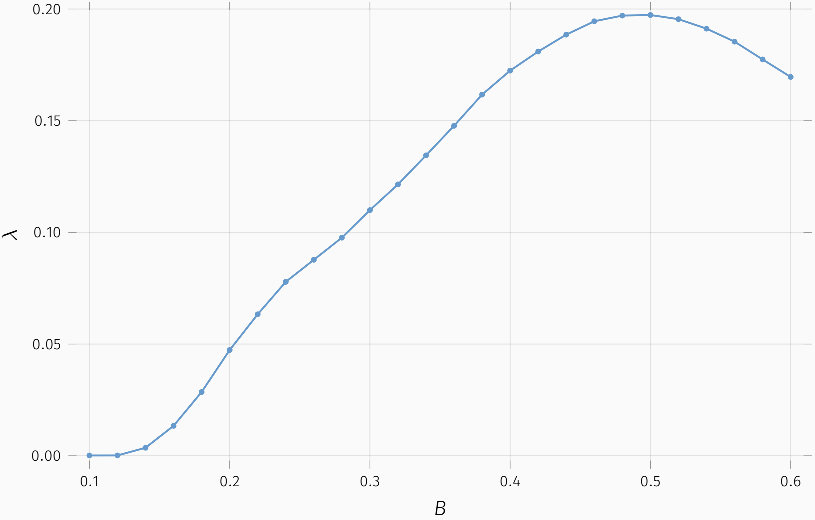

Figure 16: Averaged λ for $B = 0.5, E \in (10, 3000)$ and $B \in (0.1, 0.6)$.

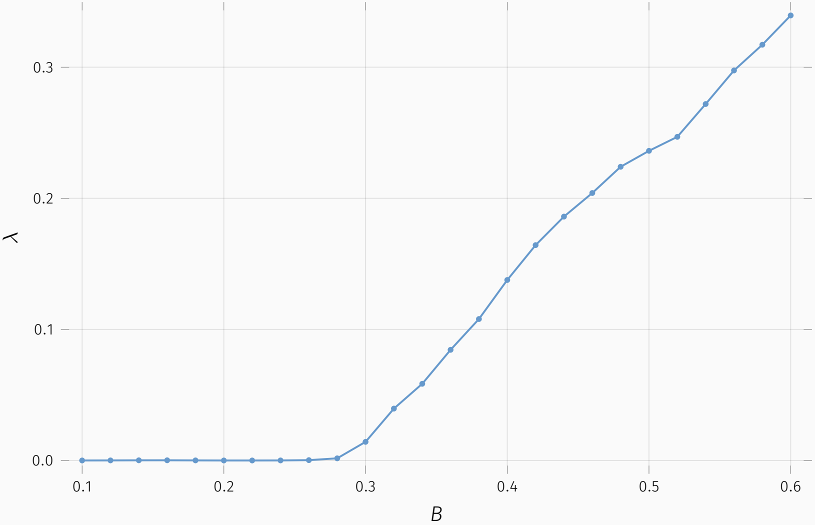

Figure 17: Averaged λ for $B = 0.5, E \in (0.01, 10)$ and $B \in (0.1, 0.6)$.

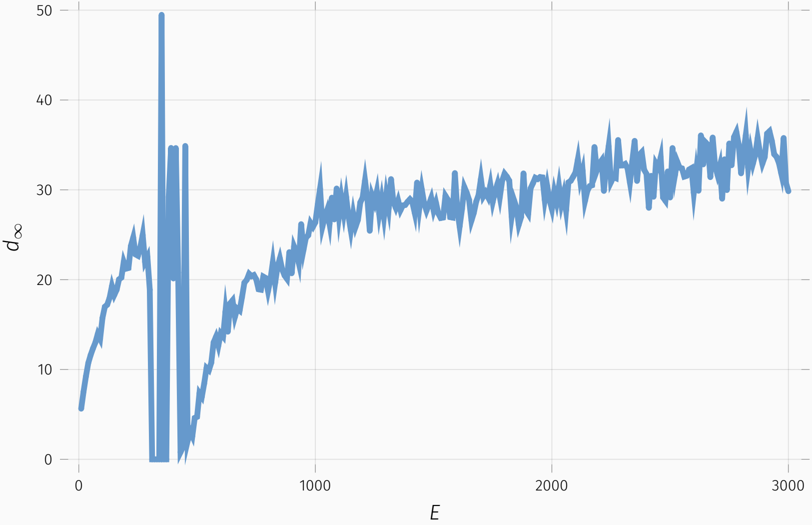

Figure 18: Averaged d_∞ for $B = 0.5, E \in (10, 3000)$.

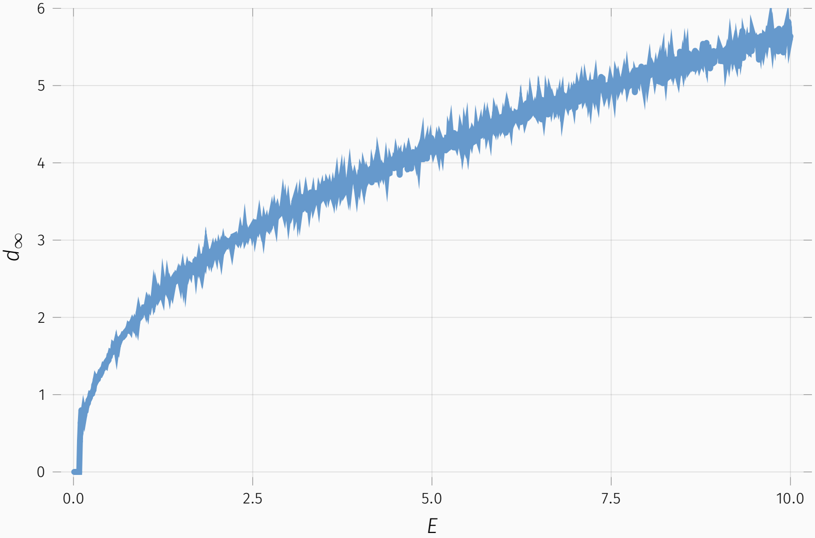

Figure 19: Averaged d_∞ for $B = 0.5, E \in (0.01, 10)$.

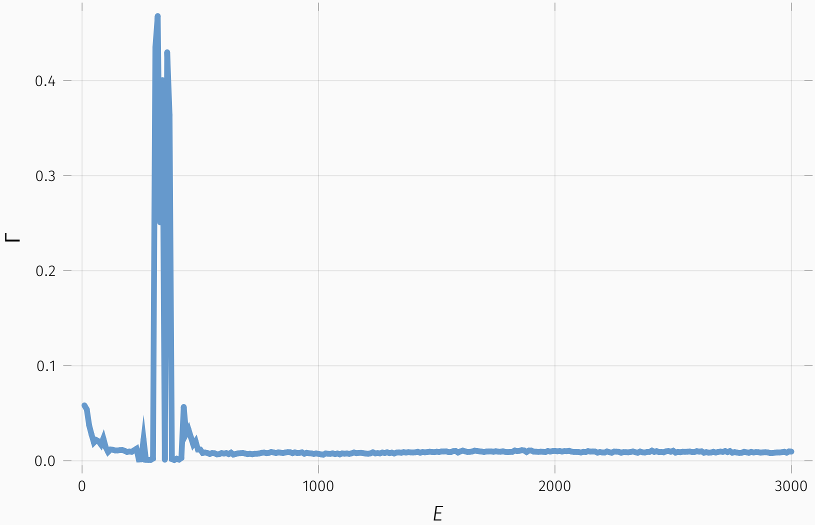

Figure 20: Averaged Γ for $B = 0.5, E \in (10, 3000)$.

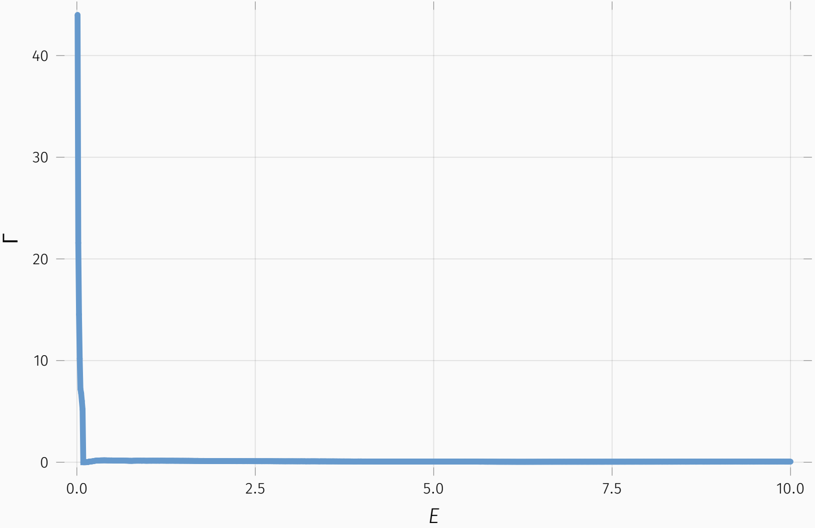

Figure 21: Averaged Γ for $B = 0.5, E \in (0.01, 10)$.

Conclusions

Thank you!

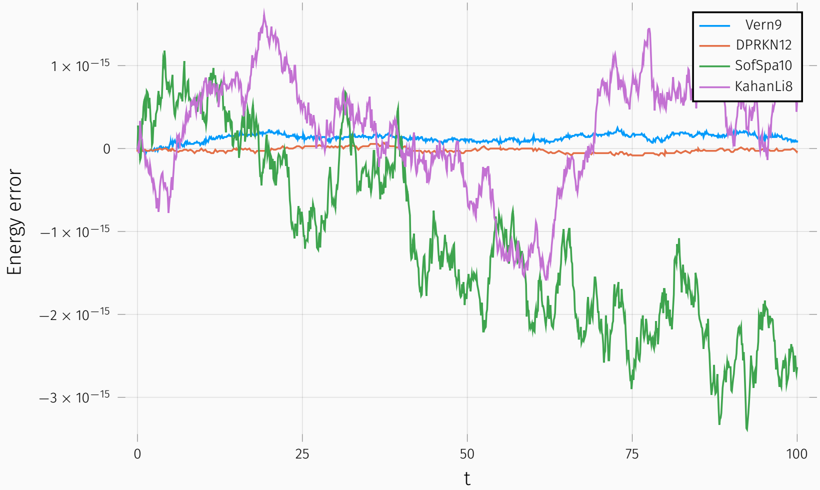

Figure 22: Energy error benchmark for short integration time with rescaling

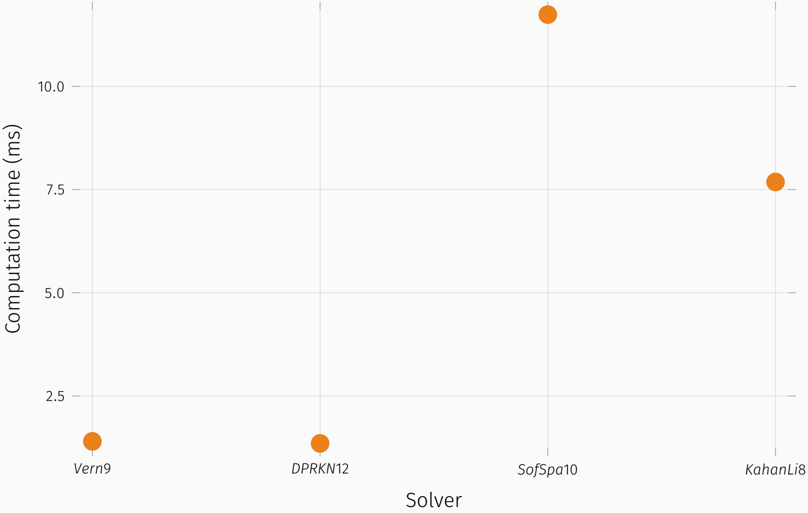

Figure 23: Computational time benchmark for short integration time with rescaling

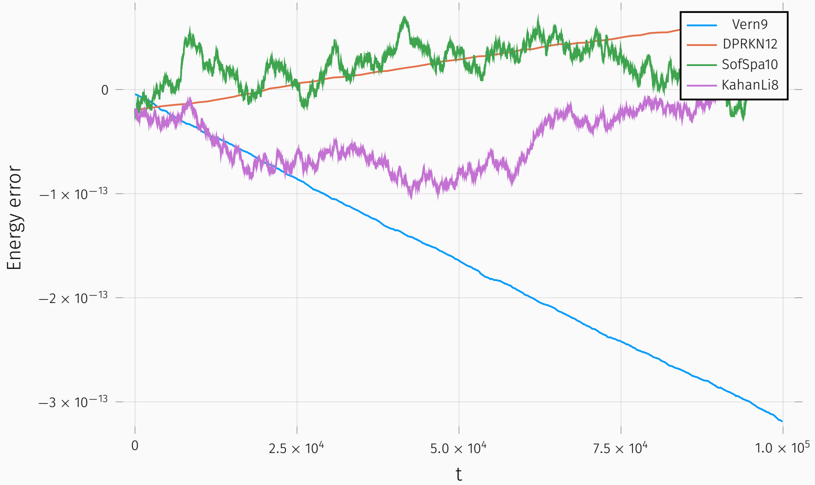

Figure 24: Energy error benchmark for long integration time with rescaling

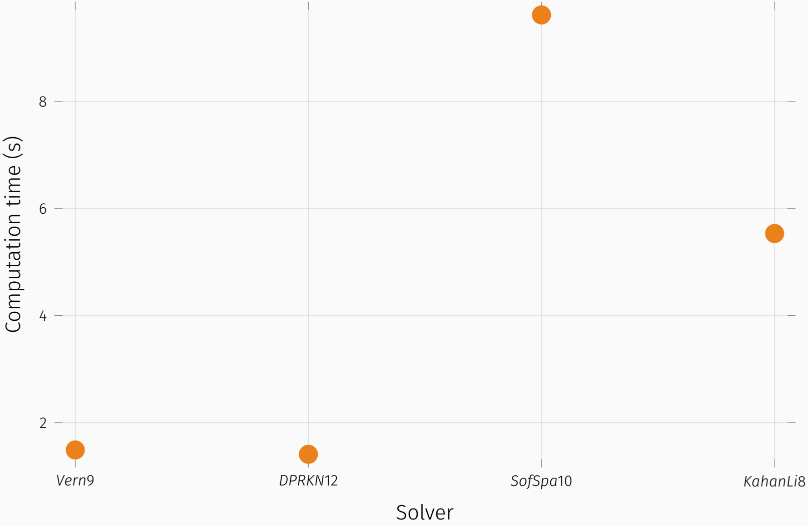

Figure 25: Computational time benchmark for long integration time with rescaling

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

References i