Large-Scale Numerical Investigations into the Dynamics of Nonlinear Classical Systems

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May 17, 2019

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Outline

Introduction

Numerical simulations

- Choosing an integrator
- The maximal Lyapunov coefficient

Conclusions

Acknowledgements

- I would like to thank A.I. Nicolin, and V. Băran for helping and motivating me.
- The author has been supported by the research project PN-III-P4-ID-PCE-2016-0792 funded by the Romanian Minisrty of Research and Inovation.
- All numerical simulations were performed on the computing cluster of Department of Computational Physics and Information Technologies, “Horia Hulubei” National Institute for Physics and Nuclear Engineering.

Introduction

The model

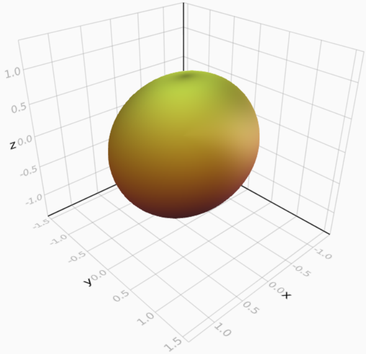
- The physical system that we model is the surface of heavy nuclei.
- The Hamiltonian describes the constrained motion of the vibrational quadrupole degrees of freedom of nuclear surface.

The model

The Hamiltonian of the system

$$H = \frac{A}{2}\left(p_0^2 + p_2^2\right) + \frac{A}{2}\left(q_0^2 + q_2^2\right) + \frac{B}{\sqrt{2}} q_0 \left(3q_2^2 - q_0^2\right) + \frac{D}{4}\left(q_0^2 + q_2^2\right)^2$$

- Harmonic oscillator part
- Integrable part
- Non-integrable term



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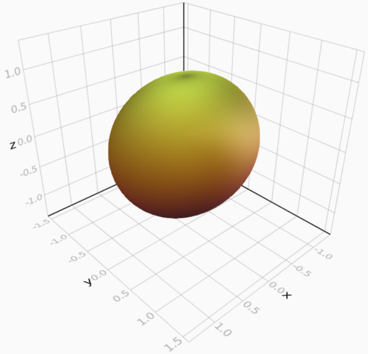

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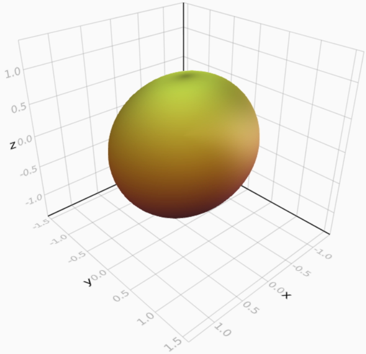

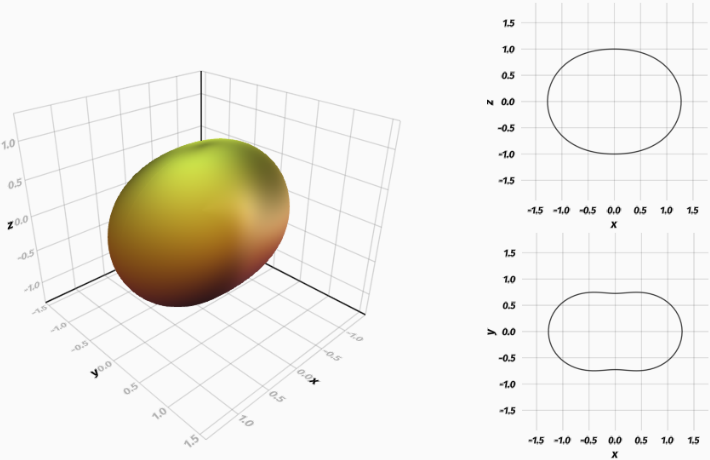

Figure 1: The nuclear surface and its sections

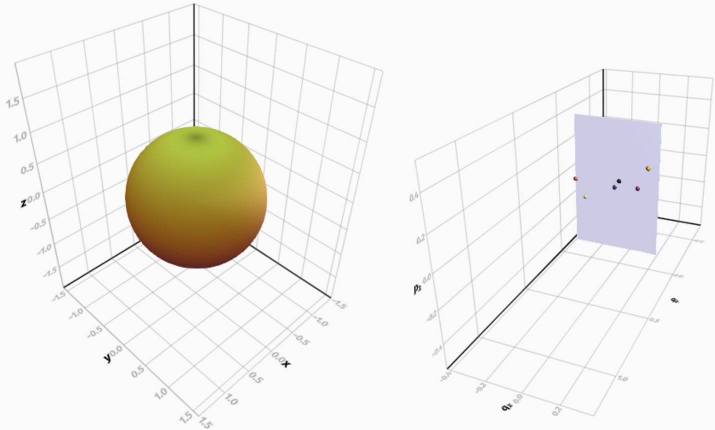

Figure 2: The nucleus and the Poincaré section at $B = 0.5, E = 0.3$

Numerical simulations

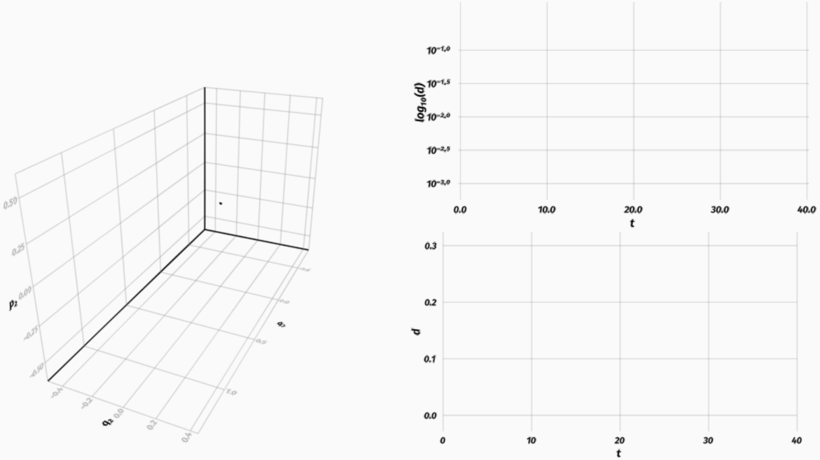

Figure 3: The distance between nearby trajectories for $B = 0.5, E = 0.3$

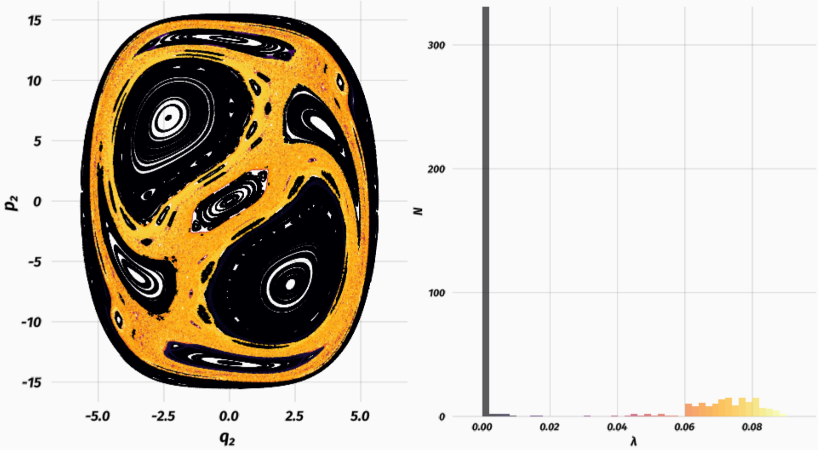

Figure 4: A Poincaré section at $B = 0.5$, $E = 120$

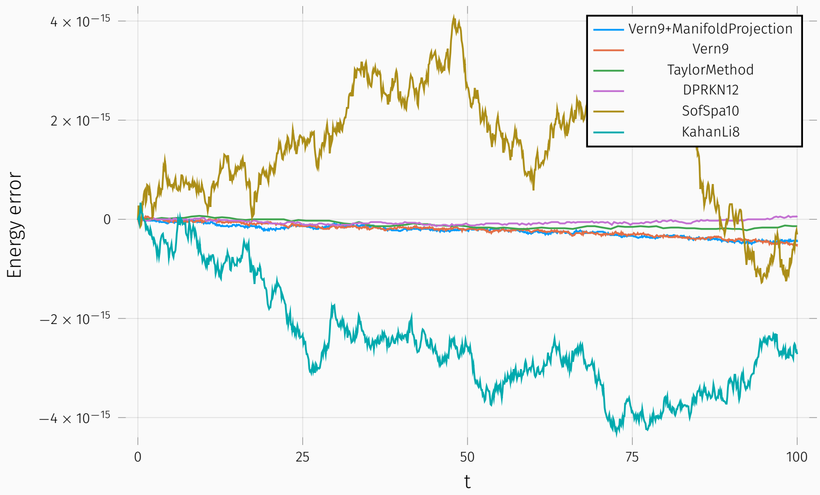

Figure 5: Energy error benchmark for short integration time

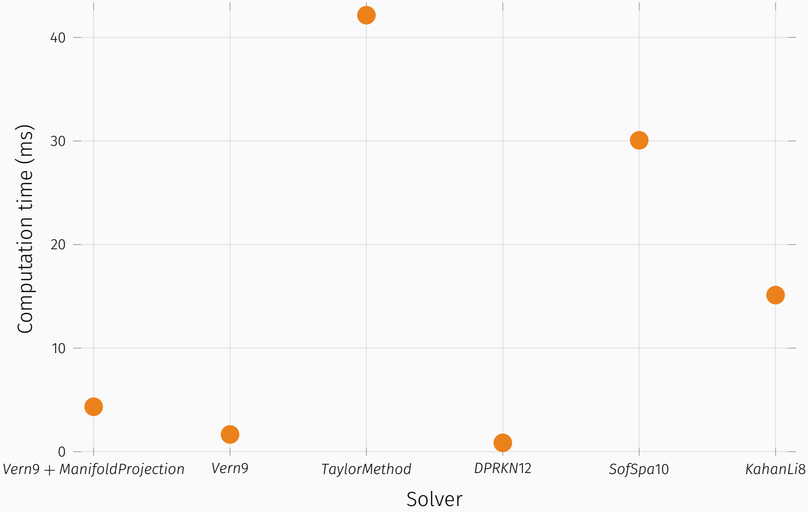

Figure 6: Computational time benchmark for short integration time

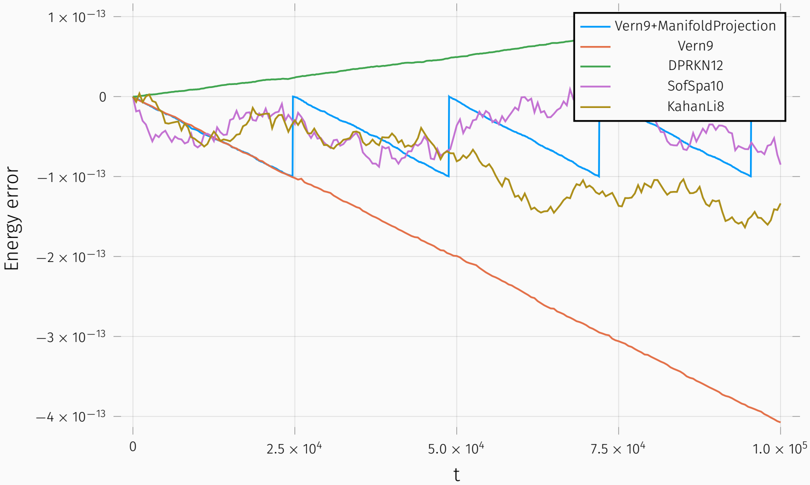

Figure 7: Energy error benchmark for long integration time

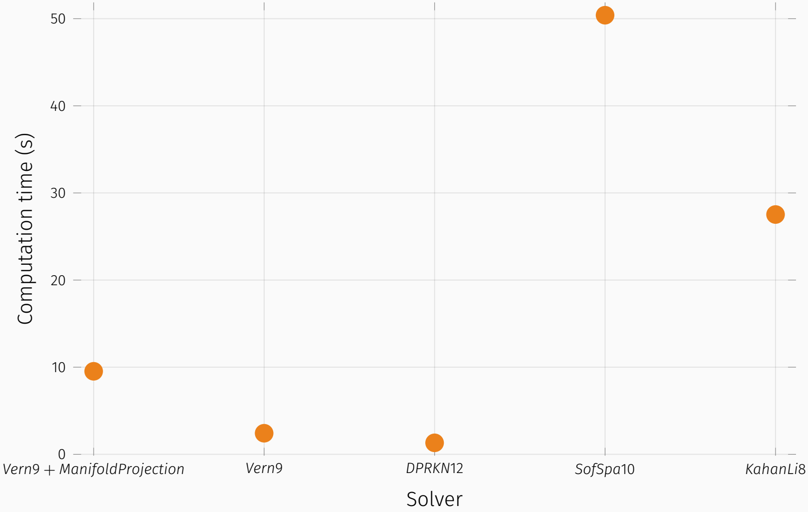

Figure 8: Computational time benchmark for long integration time

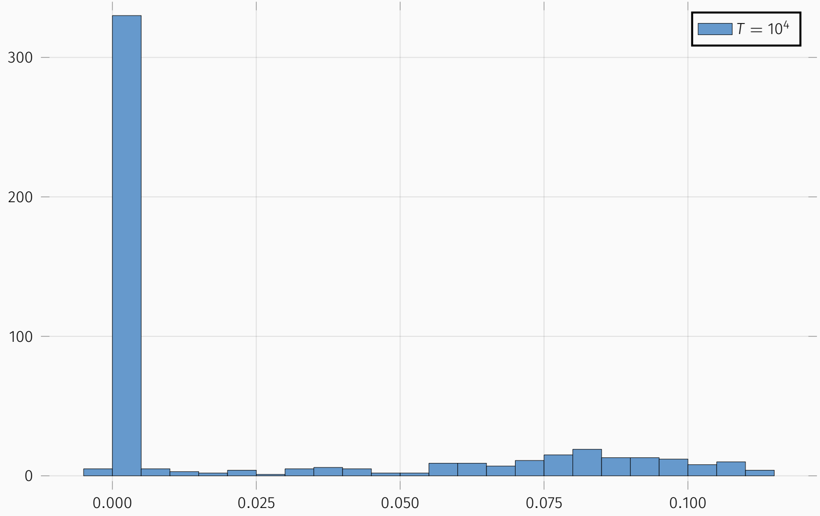

Figure 9: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

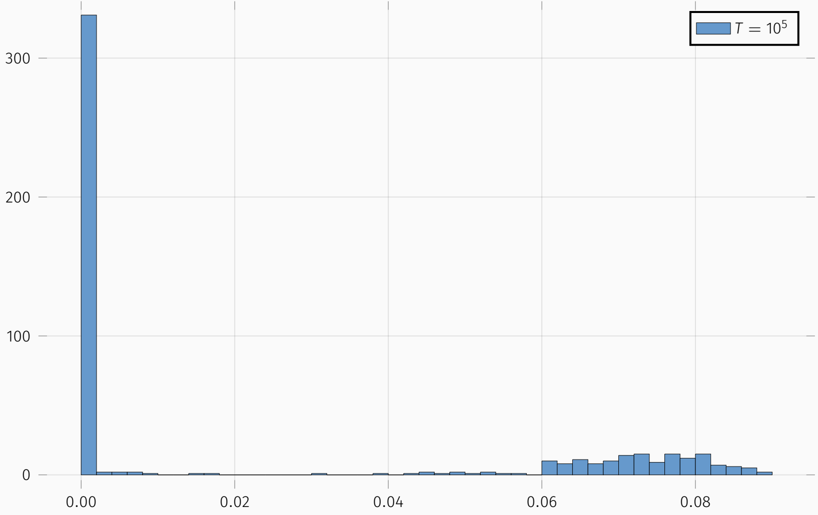

Figure 10: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

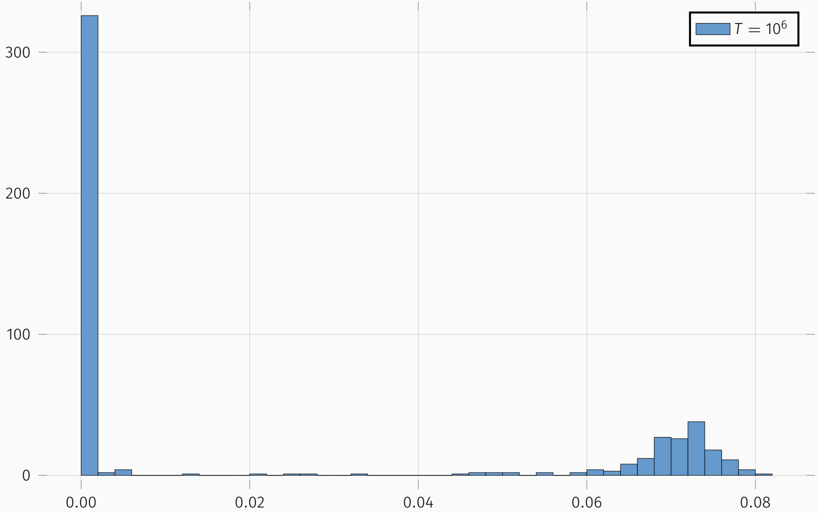

Figure 11: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

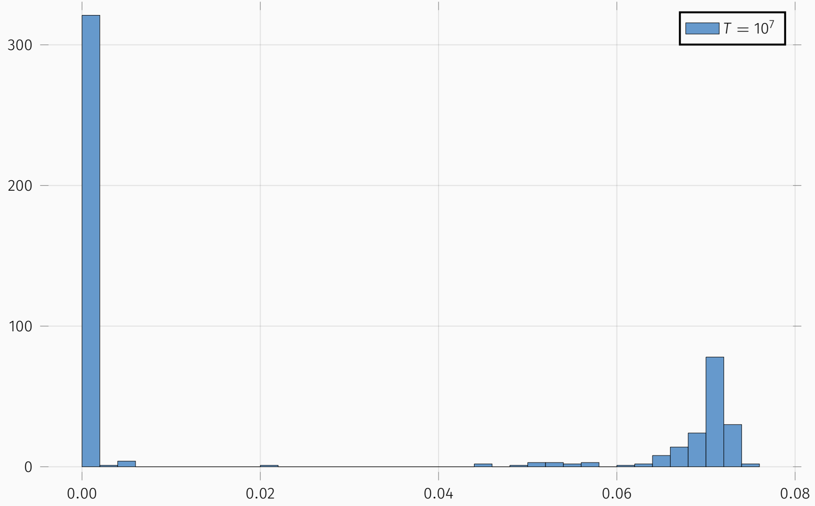

Figure 12: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

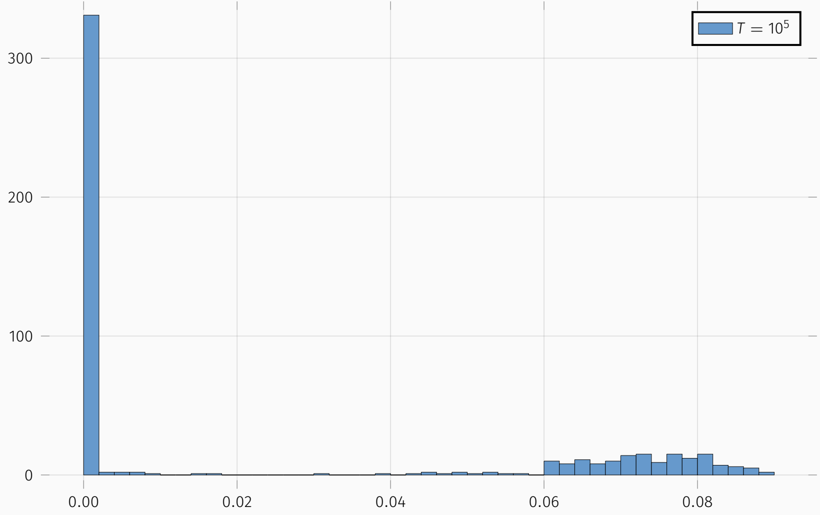

Figure 13: Selecting the chaotic trajectories for $B = 0.5, E = 120$.

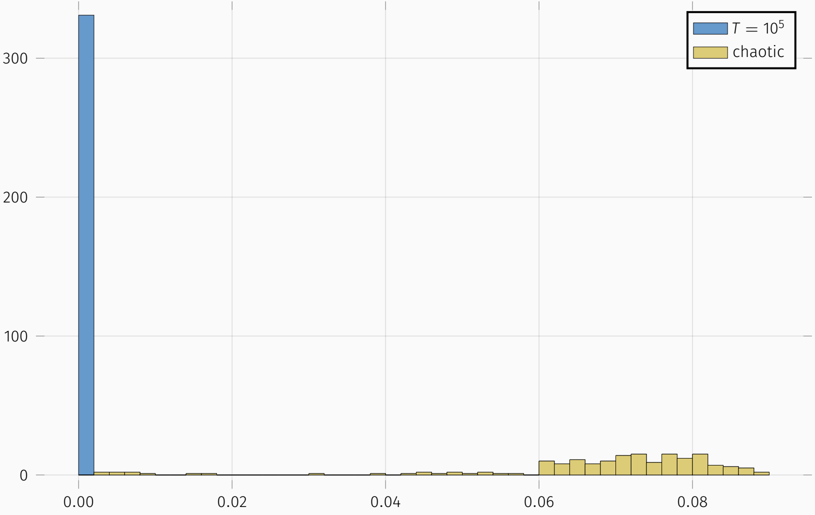

Figure 14: Selecting the chaotic trajectories for $B = 0.5$, $E = 120$.

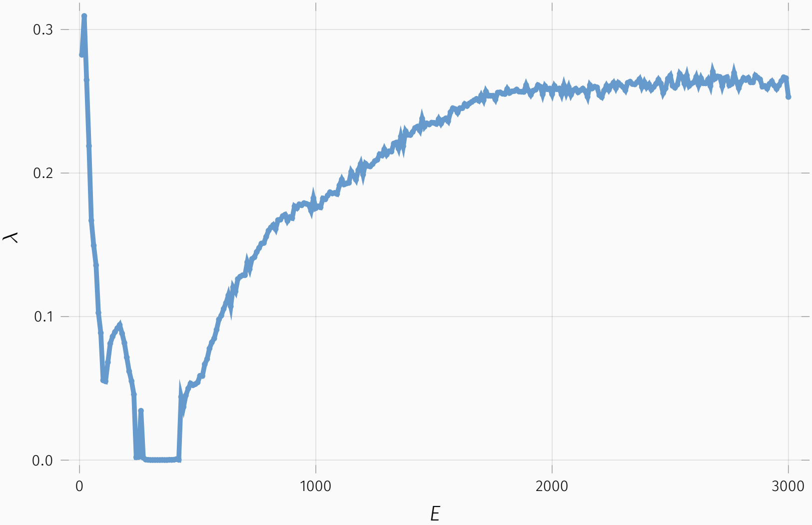

Figure 15: Averaged λ for $B = 0.5, E \in (10, 3000)$.

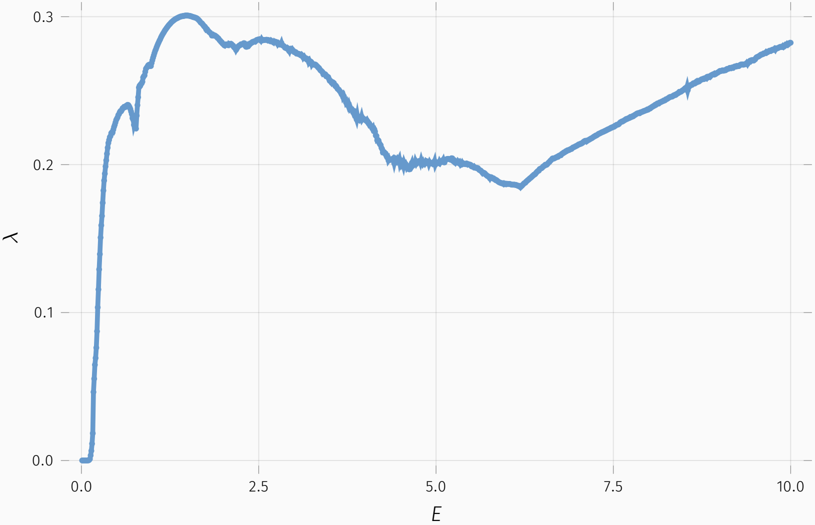

Figure 16: Averaged λ for $B = 0.5, E \in (0.01, 10)$.

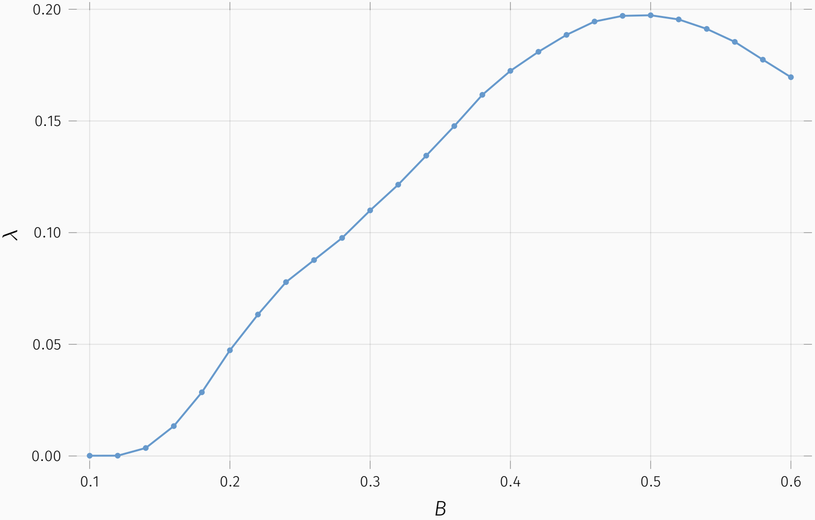

Figure 17: Averaged λ for $B = 0.5, E \in (10, 3000)$ and $B \in (0.1, 0.6)$.

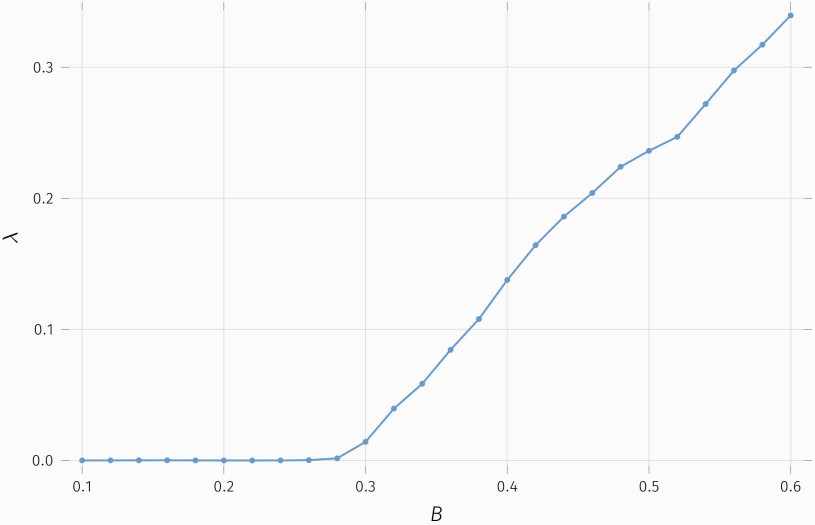

Figure 18: Averaged λ for $B = 0.5, E \in (0.01, 10)$ and $B \in (0.1, 0.6)$.

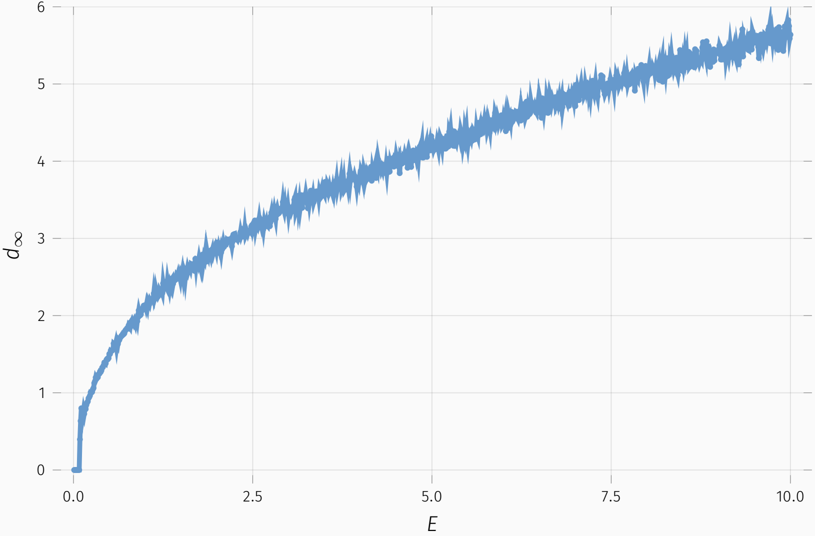

Figure 19: Averaged d_∞ for $B = 0.5, E \in (0.01, 10)$.

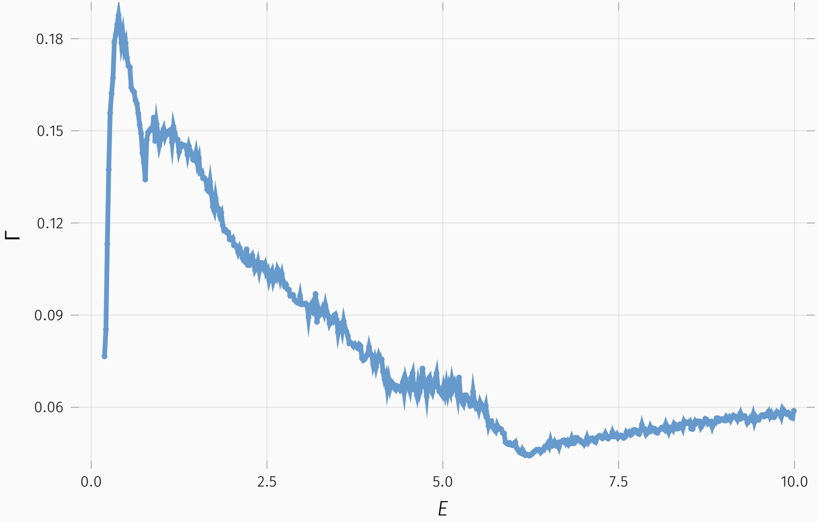

Figure 20: Averaged Γ for $B = 0.5, E \in (0.01, 10)$.

Conclusions

Thank you!

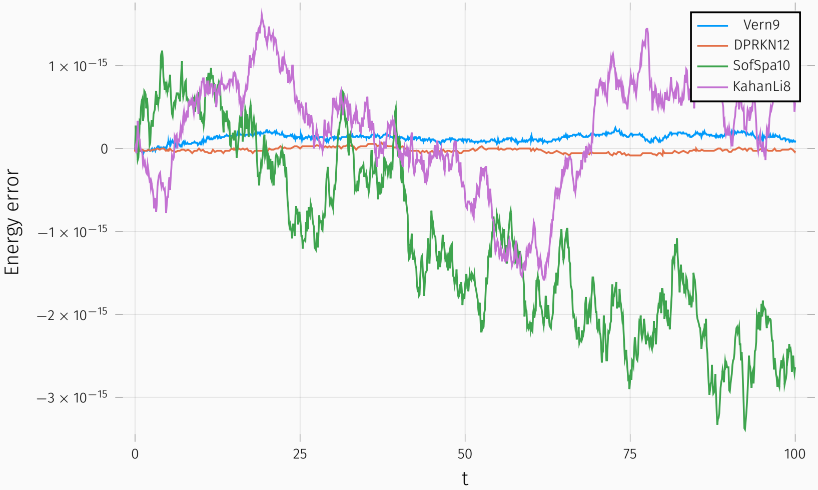

Figure 21: Energy error benchmark for short integration time with rescaling

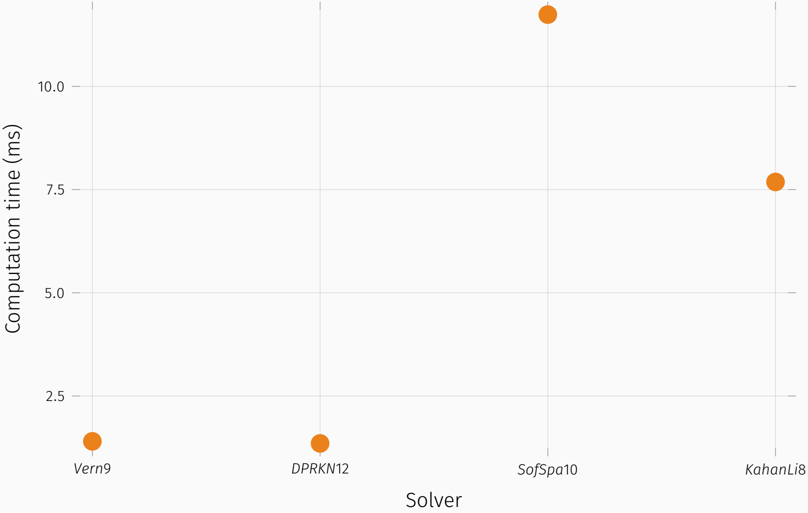

Figure 22: Computational time benchmark for short integration time with rescaling

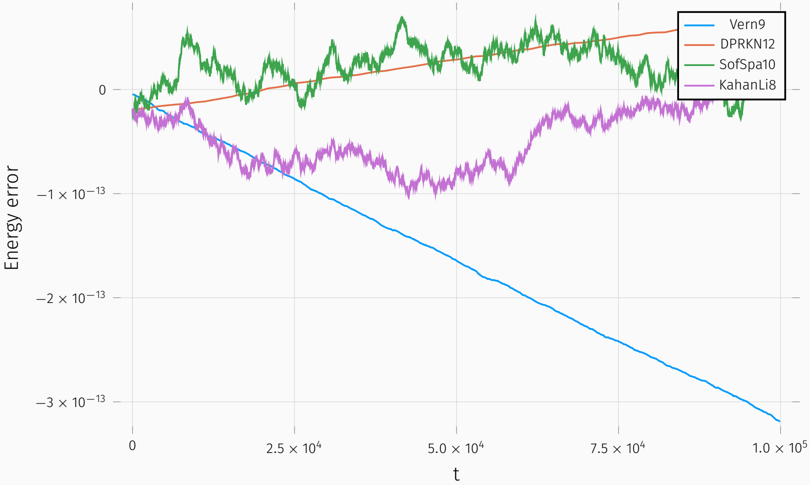

Figure 23: Energy error benchmark for long integration time with rescaling

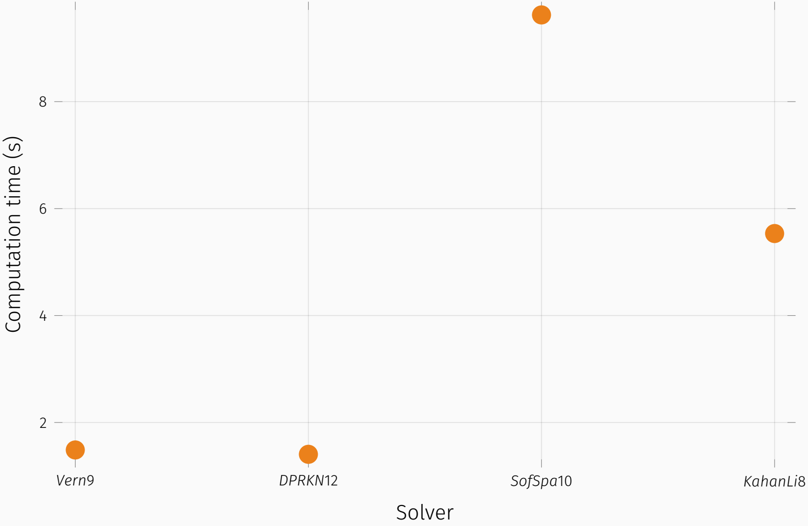

Figure 24: Computational time benchmark for long integration time with rescaling

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

References i