

Large-Scale Numerical Investigations into the Dynamics of Nonlinear Classical Systems

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Outline

Introduction

Numerical simulations

Conclusions

Acknowledgements

- I would like to thank A.I. Nicolin, and V. Băran for helping and motivating me.
- The author has been supported by PN-III-P4-ID-PCE-2016-0792.
- All numerical simulations were performed on the computing cluster of Department of Computational Physics and Information Technologies, “Horia Hulubei” National Institute for Physics and Nuclear Engineering.

Introduction

The model

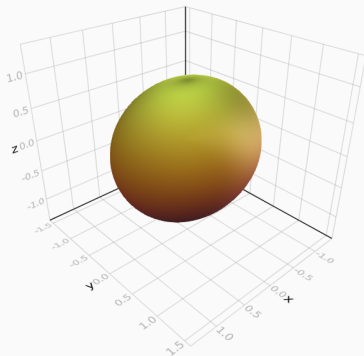
- The physical system that we model is the surface of heavy nuclei.
- The Hamiltonian describes the constrained motion of the vibrational quadrupole degrees of freedom of nuclear surface.

The model

The Hamiltonian of the system

$$H = \frac{A}{2} (p_0^2 + p_2^2) + \frac{A}{2} (q_0^2 + q_2^2) + \frac{B}{\sqrt{2}} q_0 (3q_2^2 - q_0^2) + \frac{D}{4} (q_0^2 + q_2^2)^2$$

- Harmonic oscillator part
- Integrable part
- Non-integrable term

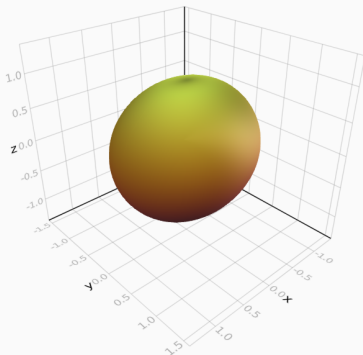


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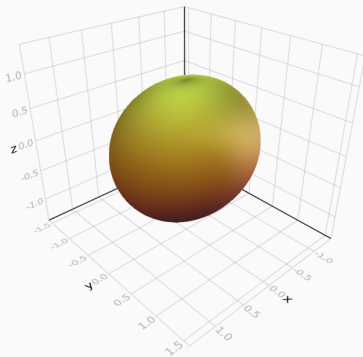


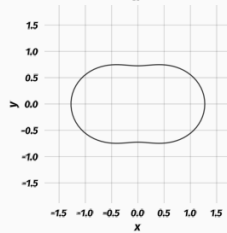
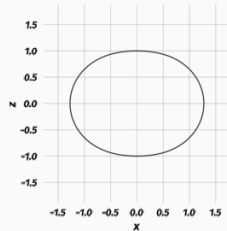
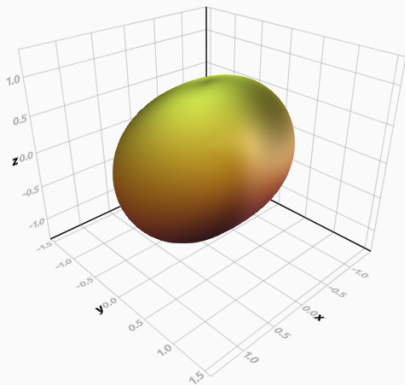
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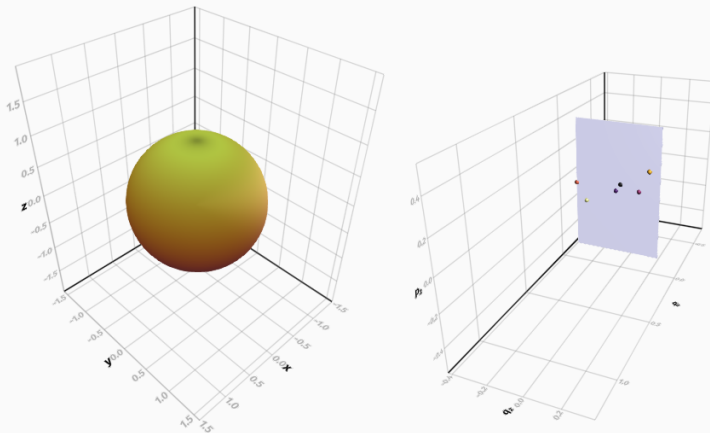


Figure 1: The nucleus and the Poincaré section at $B = 0.5, E = 0.3$

Numerical simulations

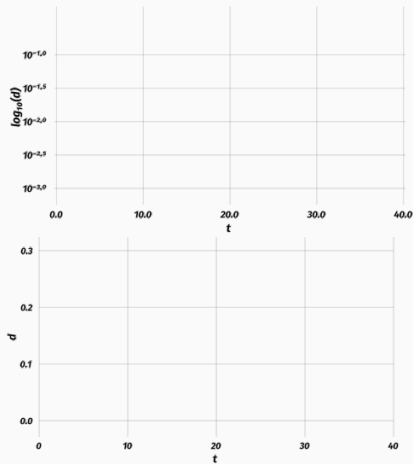
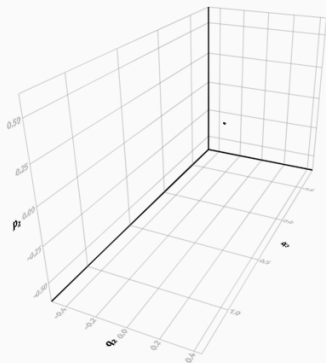


Figure 2: The distance between nearby trajectories for $B = 0.5, E = 0.3$

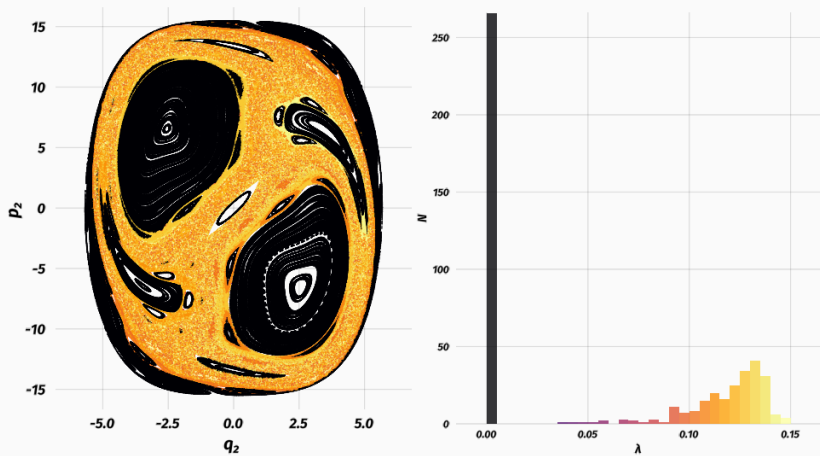


Figure 3: A Poincaré section at $B = 0.55, E = 120$

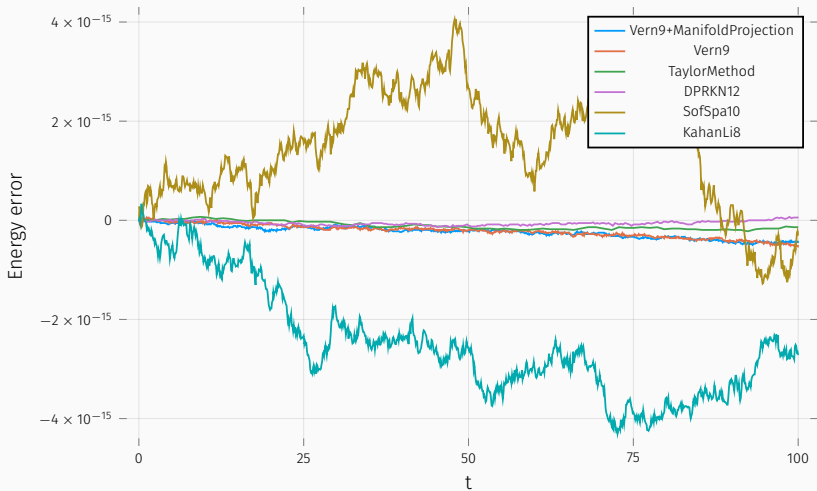


Figure 4: Energy error benchmark for short integration time

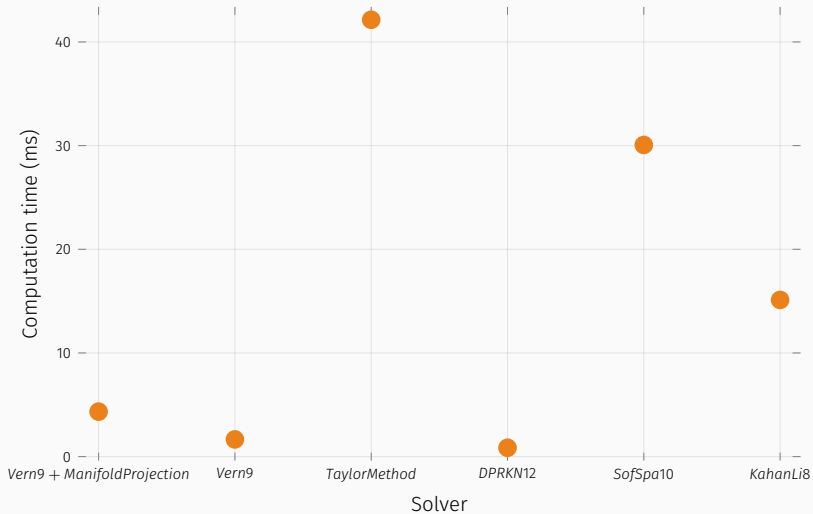


Figure 5: Computational time benchmark for short integration time



Figure 6: Energy error benchmark for long integration time

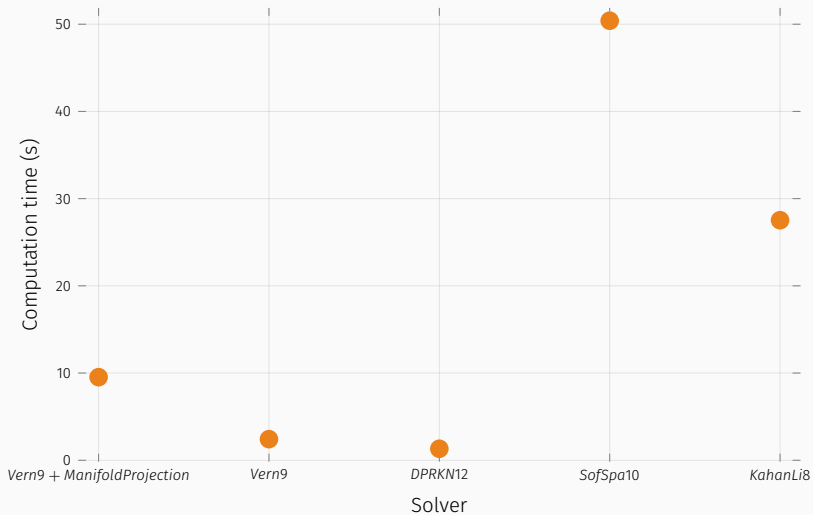


Figure 7: Computational time benchmark for long integration time

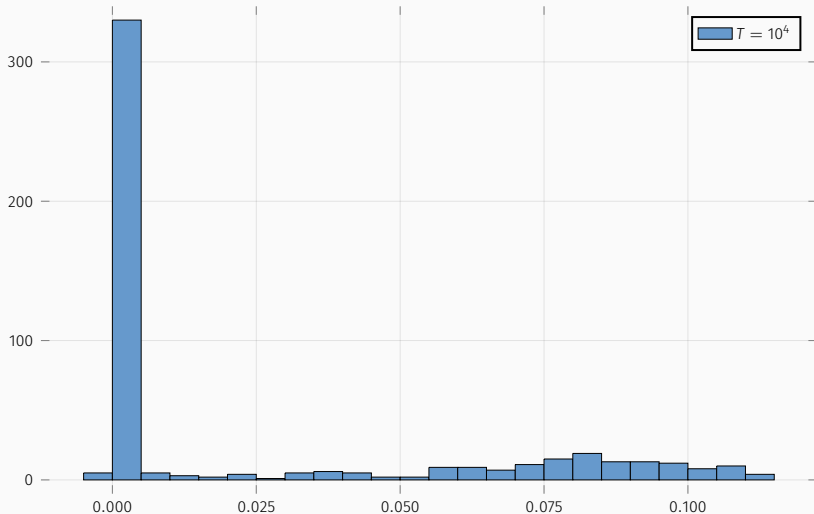


Figure 8: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

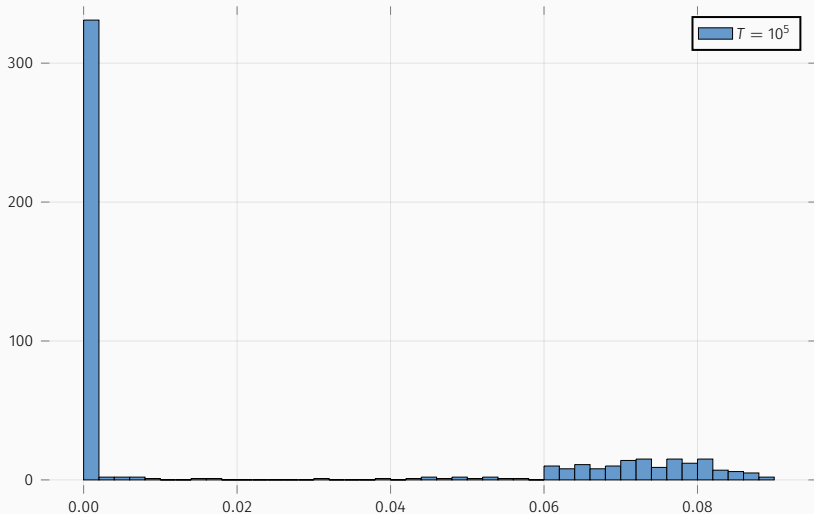


Figure 9: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

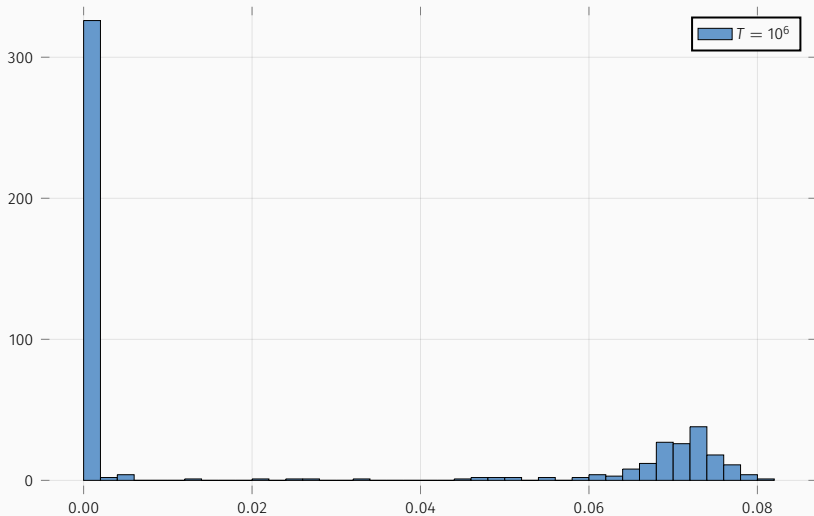


Figure 10: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

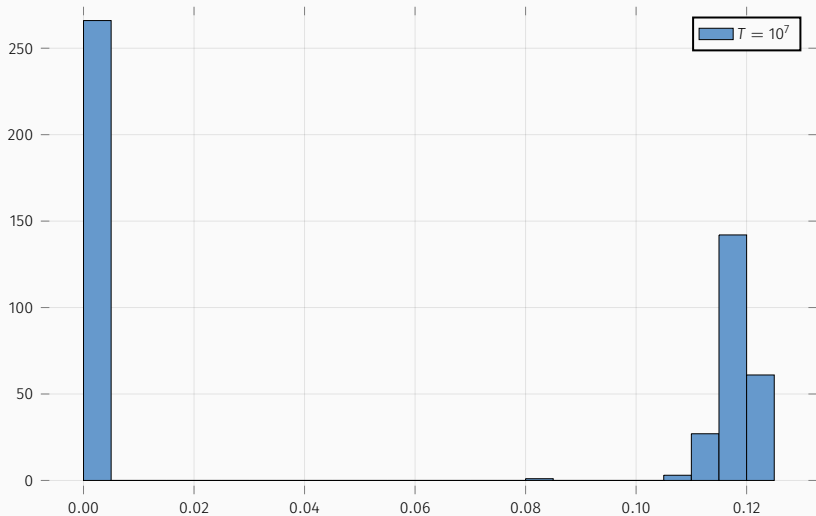


Figure 11: Maximal Lyapunov coefficient histogram for $B = 0.5, E = 120$.

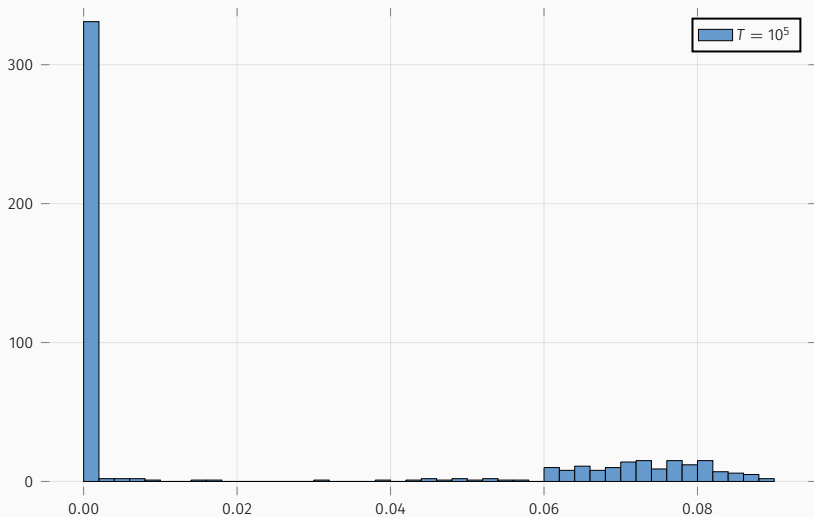


Figure 12: Selecting the chaotic trajectories for $B = 0.5, E = 120$.

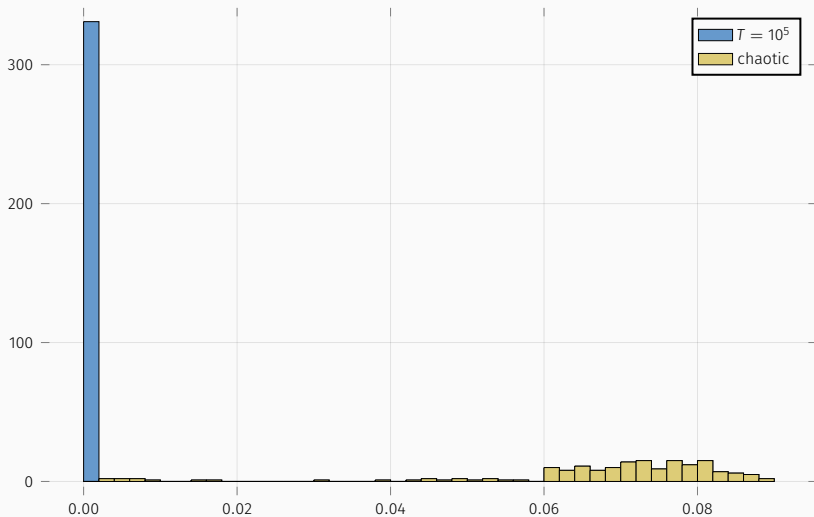


Figure 13: Selecting the chaotic trajectories for $B = 0.5, E = 120$.

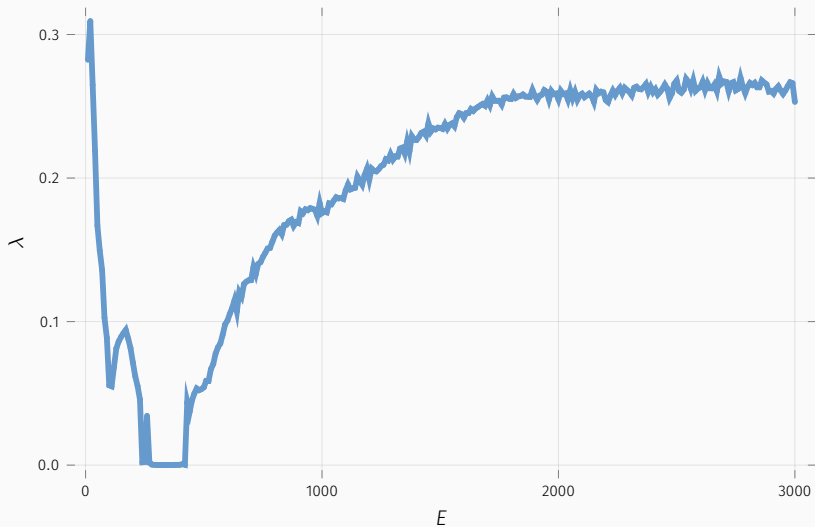


Figure 14: Averaged λ for $B = 0.5, E \in (10, 3000)$.

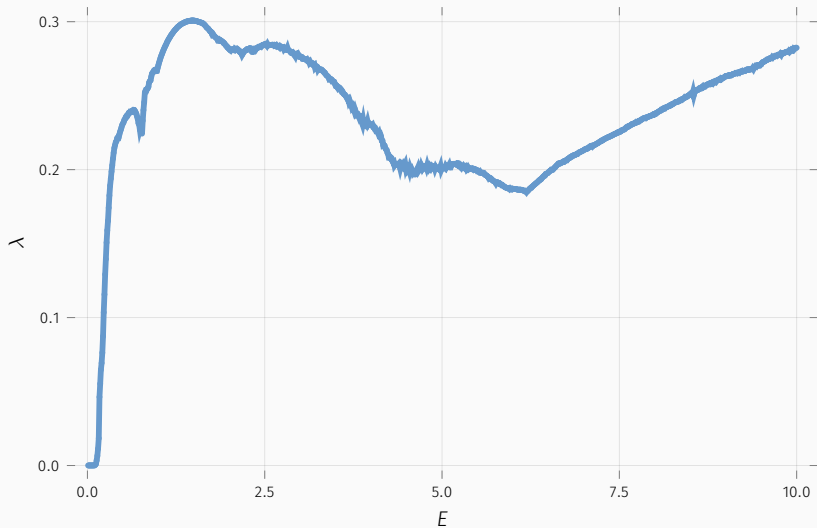


Figure 15: Averaged λ for $B = 0.5$, $E \in (0.01, 10)$.

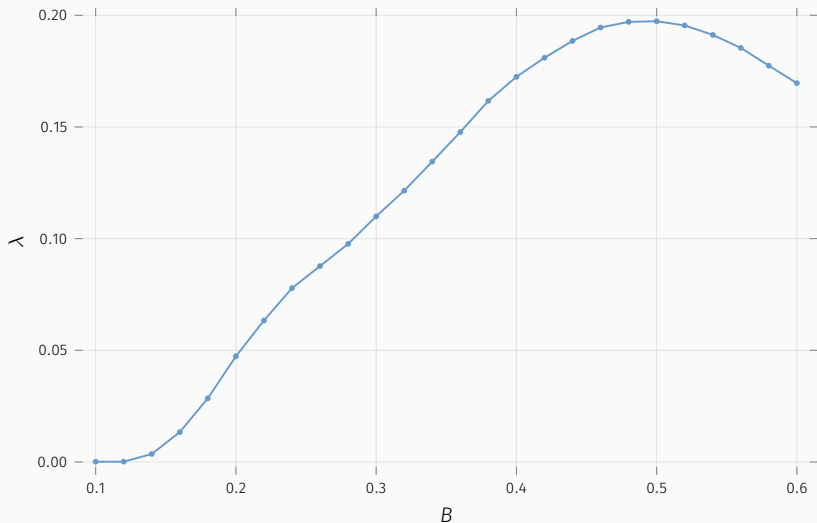


Figure 16: Averaged λ for $B = 0.5, E \in (10, 3000)$ and $B \in (0.1, 0.6)$.

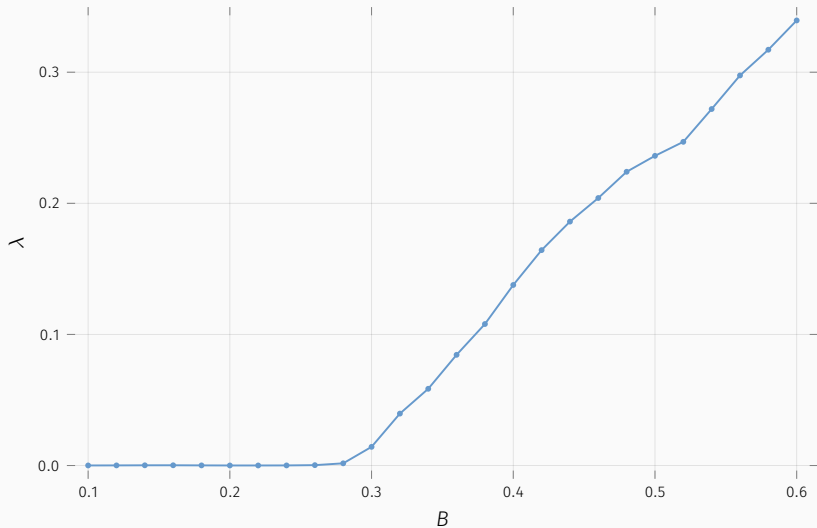


Figure 17: Averaged λ for $B = 0.5, E \in (0.01, 10)$ and $B \in (0.1, 0.6)$.

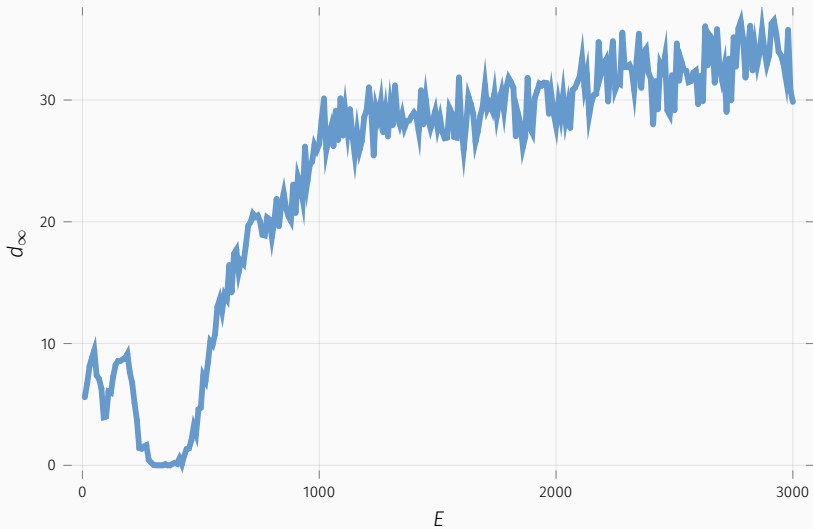


Figure 18: Averaged d_∞ for $B = 0.5, E \in (10, 3000)$.

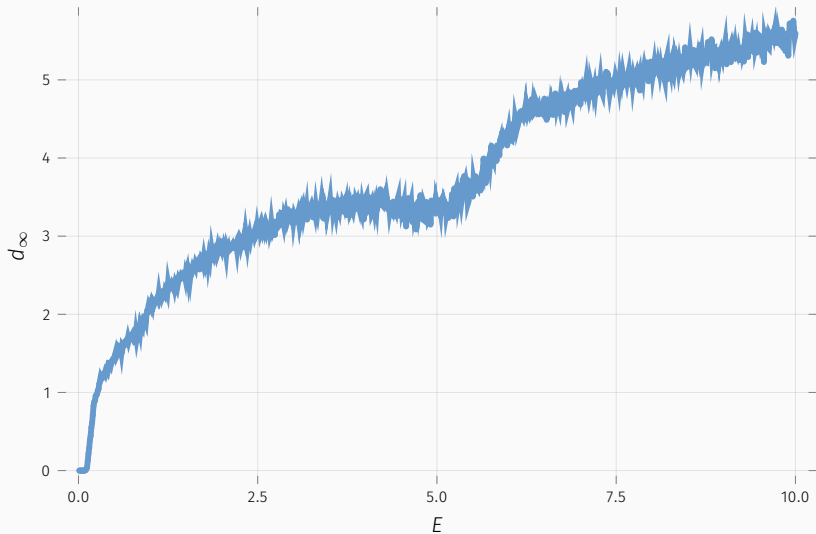


Figure 19: Averaged d_∞ for $B = 0.5, E \in (0.01, 10)$.

Conclusions

Thank you!



Figure 20: Energy error benchmark for short integration time with rescaling

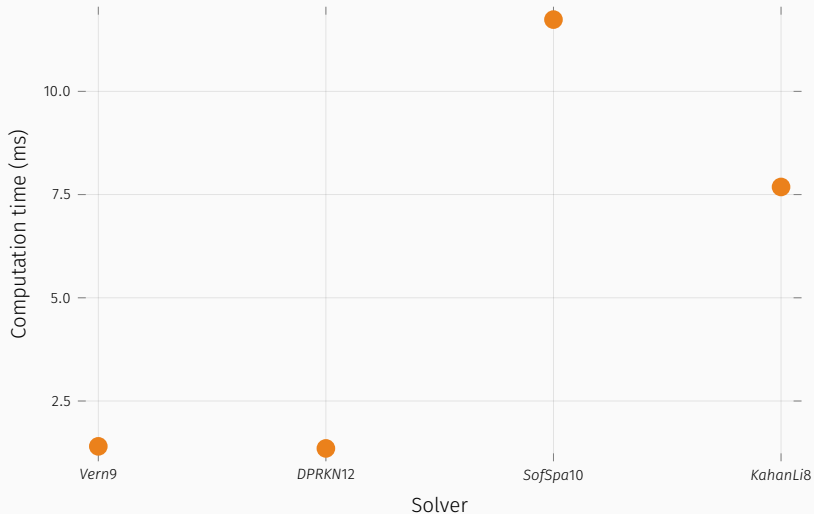


Figure 21: Computational time benchmark for short integration time with rescaling

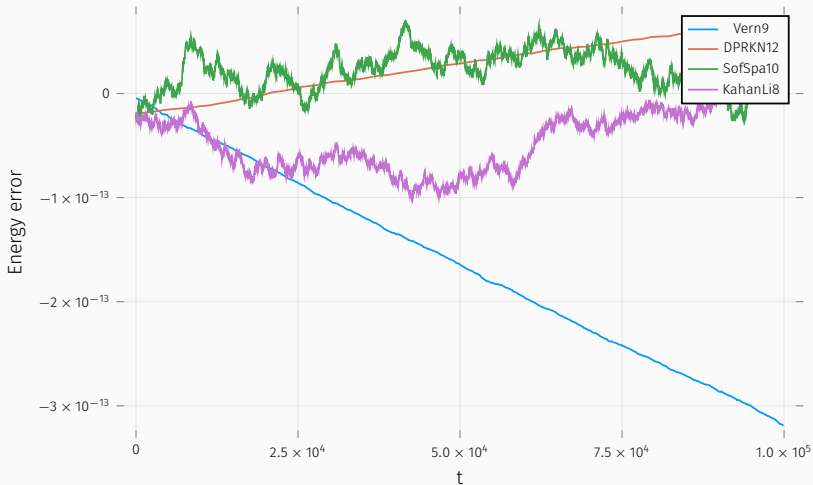


Figure 22: Energy error benchmark for long integration time with rescaling

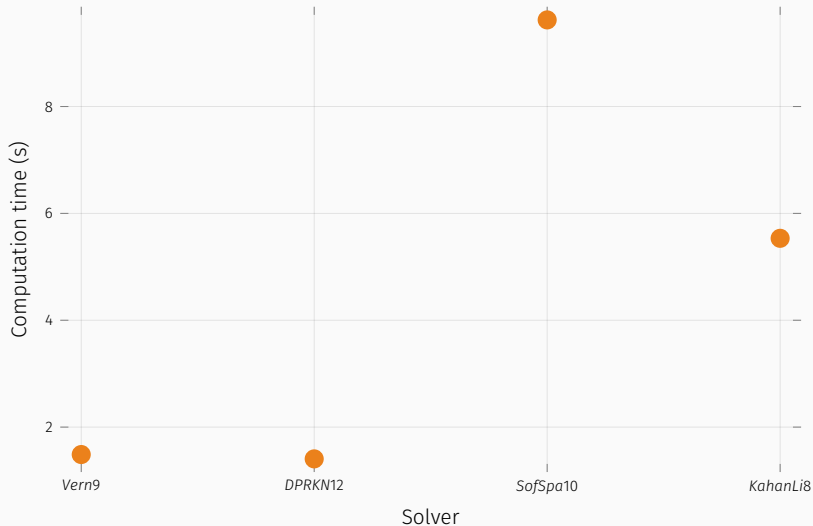


Figure 23: Computational time benchmark for long integration time with rescaling

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

