LASER WAKEFIELD ACCELERATION: Studies using Particle in Cell Method

Master Thesis

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Bucharest, 10th September 2019

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Outline

Introduction

- The aim of this thesis is to investigate the interaction of high power laser pulses with solid and gaseous targets.
- The fundamentals of the Particle in Cell method are detailed.
- A method of accelerating electrons with very intense lasers is presented.

Classical Electrodynamics

Maxwell's Equations

- The dynamcis of the electromagnetic fields are given by Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

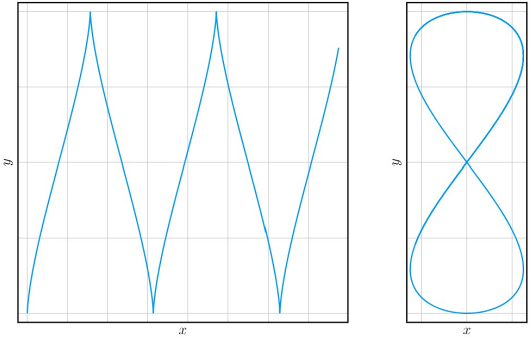
- The dynamcis of the charged particles are given by

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) .$$

Electron in a plane wave

- In a plane wave a relativistic electron has a non-trivial oscillation.
- In a particular frame of reference, its motion looks like an 8.



The Ponderomotive force

- In a spatially uniform electric field an electron would oscillate around its equilibriunm position (in a figure 8 pattern).
- If there is a spatial gradient of the field, a ponderomotrive force will appear

$$F_p = -\frac{e^2}{4m\omega^2}\nabla \mathrm{E}^2$$

Laser Wakefield

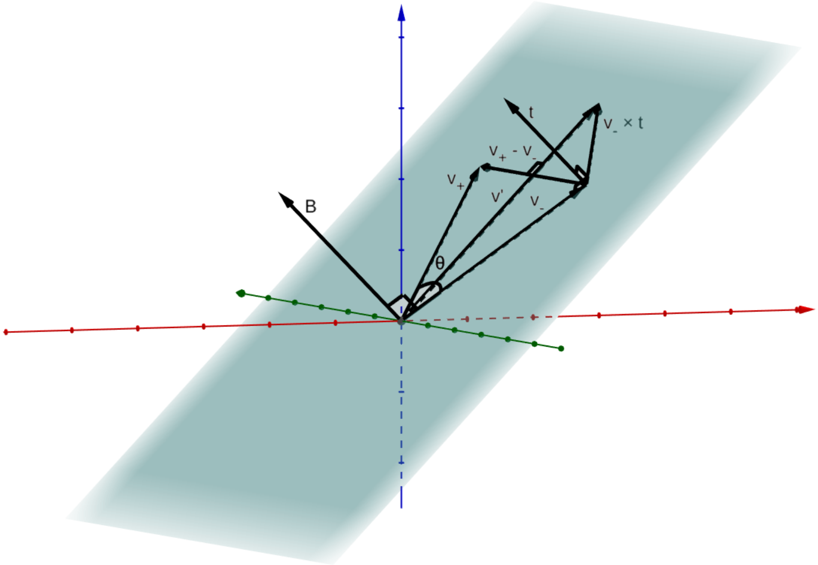
- When a high intensity laser propagates in plasma, the highly intense electric field pushes the electrons far away from their equilibriunm position (via the ponderomotrive force). This is known as the “blowout phase”.
- This creates a high intensity field between the electrons and the nuclei left behind. Any charged particles trapped in this region will be accelerated.
- This low density bubble propagates through the plasma as the laser advances.

The Particle in Cell Method

- Solves the equations of motion for the electron

$$\frac{\mathbf{x}_{n+1} - \mathbf{x}_n}{\Delta t} = \mathbf{v}_{n+1}$$
$$\frac{\mathbf{v}_{n+1/2} - \mathbf{v}_{n-1/2}}{\Delta t} = \frac{q}{m}\left(\mathbf{E}(\mathbf{x}_n) + \frac{\mathbf{v}_{n+1/2} + \mathbf{v}_{n-1/2}}{2} \times \mathbf{B}(\mathbf{x}_n)\right)$$

- The Boris push algorithm: the motion is decomposed in the contribution from the electric and magnetic field.


t
v_ × t
v+ - v-
v+
v'
v-
B
θ

The Boris push

1. $\mathbf{v}^- = \mathbf{v}_{n-1/2} + \frac{q\mathbf{E}}{m}\frac{\Delta t}{2}$
2. rotate $\mathbf{v}^-$ to obtain $\mathbf{v}^+$ using
    2.1 $\mathbf{v}' = \mathbf{v}^- + \mathbf{v}^- \times \mathbf{t}$, where $\mathbf{t} = \frac{q\mathbf{B}}{m}\frac{\Delta t}{2}$
    2.2 $\mathbf{v}^+ = \mathbf{v}^- + \mathbf{v}' \times \mathbf{s}$, where $\mathbf{s} = \frac{2\mathbf{t}}{1+t^2}$
3. $\mathbf{v}_{n+1/2} = \mathbf{v}^+ + \frac{q\mathbf{E}}{m}\frac{\Delta t}{2}$

Conservaion properties

- We want the algorithm to be as close as possible to the original continuous system in terms of symmetries and conserved quantities.
- A linear map $A : \mathbb{R}^{2d} \to \mathbb{R}^{2d}$ is called *symplectic* if

$$A^T J^{-1} A = J^{-1}, \qquad \text{where } J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix},$$

- The Boris push algorithm is not symplectic, but it is volume preserving.

The Field solver

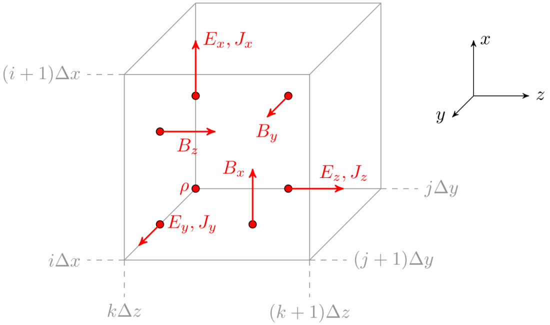
- Solve Maxwell's equations using

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$
$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \frac{1}{\varepsilon_0}\mathbf{j}\,.$$

- Let us consider the 1D case first

$$\frac{B_y^{n+1/2}(k+\frac{1}{2}) - B_y^{n-1/2}(k+\frac{1}{2})}{\Delta t} = -\frac{E_x^n(k+1) - E_x^n(k)}{\Delta z}$$
$$\frac{E_x^{n+1}(k) - E_x^n(k)}{\Delta t} = -c^2 \frac{B_y^{n+1/2}(k+\frac{1}{2}) - B_y^{n+1/2}(k-\frac{1}{2})}{\Delta z} - \frac{1}{\varepsilon_0} j_x^{n+1/2}(k)\,.$$

- Yee's method uses a leapfrog-like algorithm for staggering in both space and time.
- Continuity equations for **E** and **B** are automatically satisfied at cell boundaries.



The 3D case

- The magnetic field is given by

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} + \frac{\partial E_z}{\partial x}$$

- which is discretized as

$$\frac{B_y^{n+1/2}(i+\frac{1}{2}, j, k+\frac{1}{2}) - B_y^{n-1/2}(i+\frac{1}{2}, j, k+\frac{1}{2})}{\Delta t} =$$
$$-\frac{{E_x}^n(i+\frac{1}{2}, j, k+1) - {E_x}^n(i+\frac{1}{2}, j, k)}{\Delta z}$$
$$+\frac{{E_z}^n(i+1, j, k+\frac{1}{2}) - {E_z}^n(i, j, k+\frac{1}{2})}{\Delta x}$$

- or

$$\partial_t B_y|^n_{i+\frac{1}{2},j,k+\frac{1}{2}} = -\partial_z E_x|^n_{i+\frac{1}{2},j,k+\frac{1}{2}} + \partial_x E_z|^n_{i+\frac{1}{2},j,k+\frac{1}{2}}$$

What about the other equations?

- For $\nabla \cdot \mathsf{B} = 0$

$$\frac{\partial \nabla \cdot \mathsf{B}}{\partial t} = \nabla \cdot \frac{\partial \mathsf{B}}{\partial t} = \nabla \cdot (-\nabla \times \mathsf{E}) = 0\,.$$

- For $\nabla \cdot \mathsf{E} = \frac{\rho}{\varepsilon_0}$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\nabla \cdot \mathsf{E} - \frac{\rho}{\varepsilon_0}\right) &= -\frac{1}{\varepsilon_0}\left[\frac{\partial \rho}{\partial t} - \nabla \cdot \left(\frac{1}{\mu_0}\nabla \times \mathsf{B} - \mathsf{j}\right)\right] \\ &= -\frac{1}{\varepsilon_0}\left(\frac{\partial \rho}{\partial t} + \nabla \cdot \mathsf{j}\right)\end{aligned}$$

Numerical Stability

- In order to accurately represent the physics, the temporal and spatial discretizations must be chosen appropriately.
- For the spatial discretizations, the dimension of a cell must be significantly smaller than the laser wavelength.

$$D_{cell} \ll \lambda_L$$

- The timestep is then computed from the spatial discretizations automatically via the Courant condition.

$$c\Delta t \leq \frac{1}{\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}}$$

- PIC codes require significant computational resources (HPC).
- The implementation must take advantage of all the available computing resources (MIP, SIMD, CUDA).
- Scalability is crucial for employing large parallel simulations.
- Amdahl's law

$$S_A = \frac{1}{(1-p) + \frac{p}{n}} \qquad \text{and} \qquad \lim_{n\to\infty} S_A = \frac{1}{1-p}$$

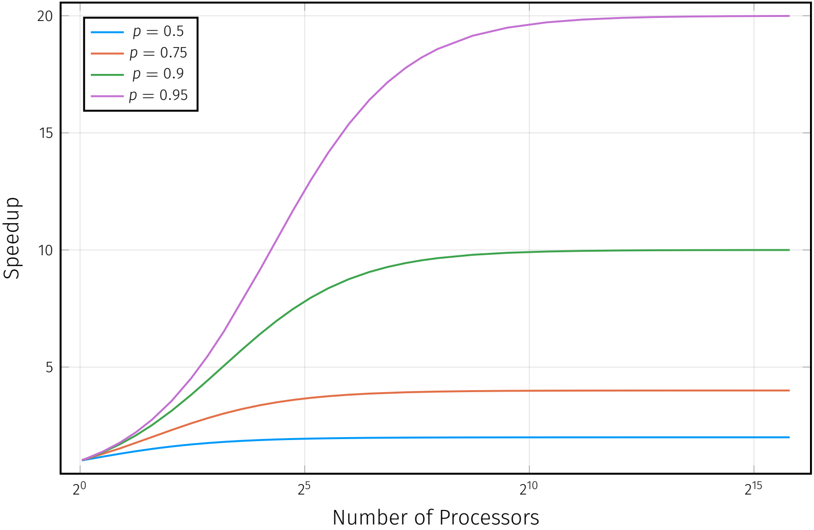


Figure 1: The speedup computed with Amdahl's law

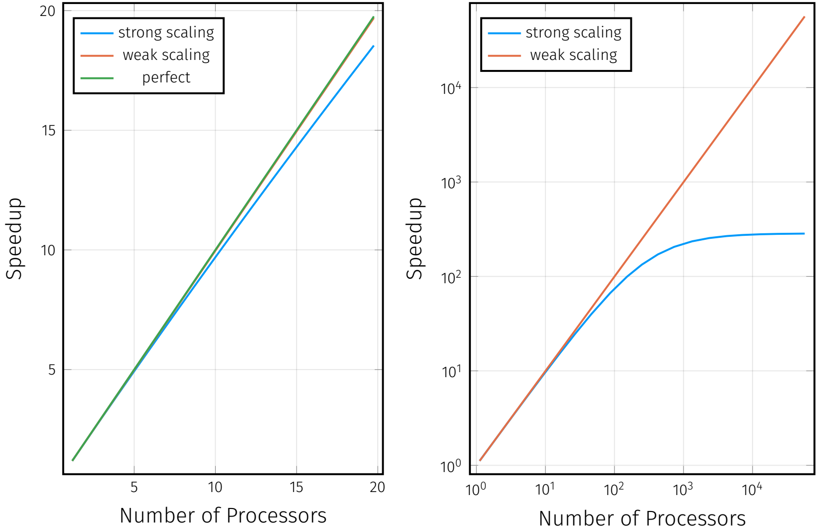
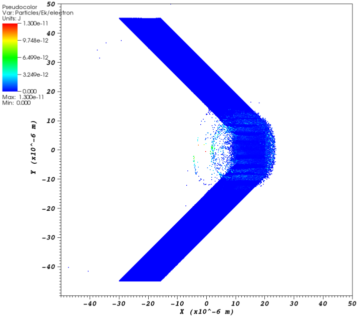


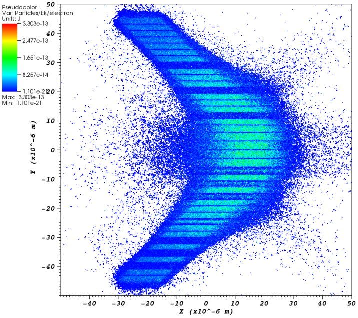
Figure 2: The theoretical speedup compared with linear axis and small number of processors on the left and log-log scale with a large number of processors on the right

Results

Solid targets



(a) The kinetic energy of the electrons right after the pulse hits the cone



(b) The kinetic energy of the electrons a long time after the pulse hits the cone

Figure 3: The kinetic energy of the electrons for the $I = 10^{21}\,\mathrm{W\,cm^{-2}}$ laser pulse

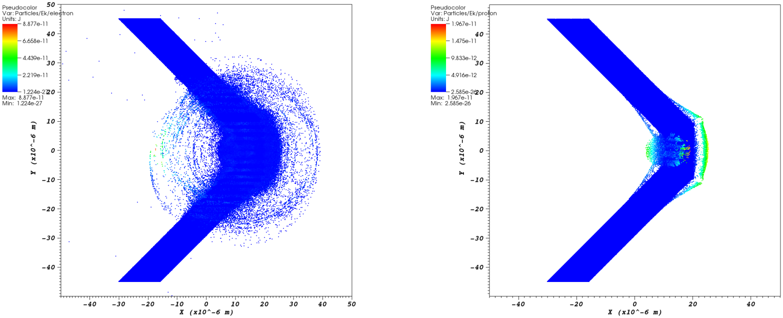
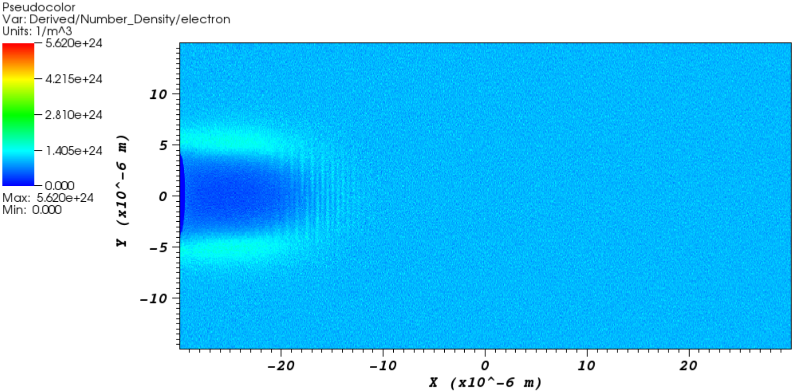
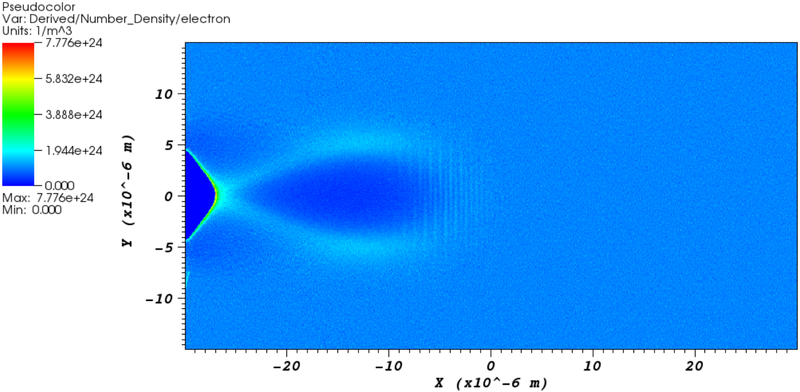
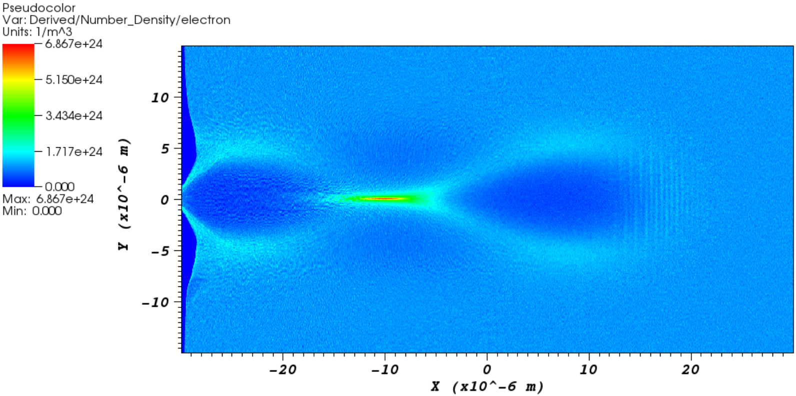


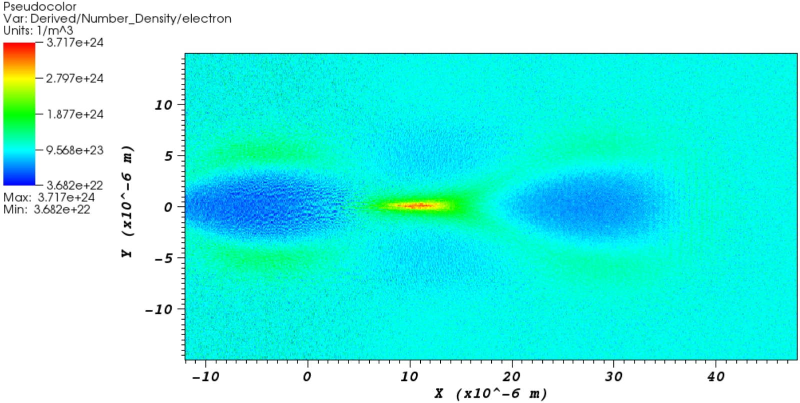
Figure 4: The kinetic energy of the electrons and protons for the $I = 10^{22}\,\mathrm{W\,cm^{-2}}$ laser pulse

Gaseous targets




Pseudocolor
Var: Derived/Number_Density/electron
Units: 1/m^3
7.776e+24
5.832e+24
3.888e+24
1.944e+24
0.000
Max: 7.776e+24
Min: 0.000
10
5
0
-5
-10
Y (x10^-6 m)
-20
-10
0
10
20
X (x10^-6 m)


Pseudocolor
Var: Derived/Number_Density/electron
Units: 1/m^3
6.867e+24
5.150e+24
3.434e+24
1.717e+24
0.000
Max: 6.867e+24
Min: 0.000
10
5
0
-5
-10
Y (x10^-6 m)
-20
-10
0
10
20
X (x10^-6 m)


Pseudocolor
Var: Derived/Number_Density/electron
Units: 1/m^3
3.717e+24
2.797e+24
1.877e+24
9.568e+23
3.682e+22
Max: 3.717e+24
Min: 3.682e+22
10
5
0
-5
-10
Y (x10^-6 m)
-10
0
10
20
30
40
X (x10^-6 m)

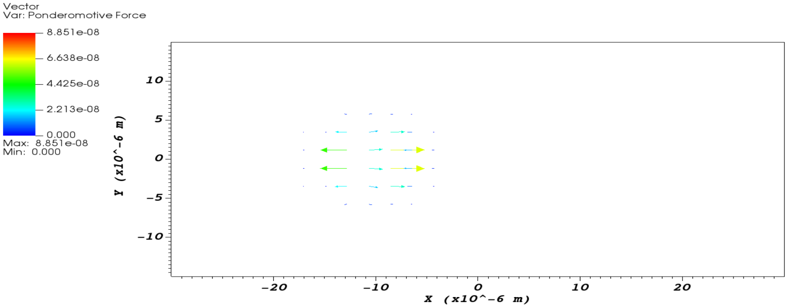


Figure 5: The ponderomotive force for the $I = 10^{18}\,\mathrm{W\,cm^{-2}}$ laser pulse

Conclusions

Conclusions

- This thesis addresses the large field of laser plasma interactions in a computational framework.
- This thesis is devoted to a class of self-consistent methods of solving the dynamics of electrically charged particles in strong electromagnetic fields.
- The most prominent effect observed in my numerical simulations is the laser wakefield acceleration visible in the case of gaseous targets subjected to highly intense laser pulses.

Thank you!