

Question 2

1 Q_x in reprezentarea pozitiiilor

$$\overline{\Delta Q_x^2} = (\overline{Q_x^2}) - (\overline{Q_x})^2$$

$$\begin{aligned}\overline{Q_x} &= \langle \Psi | Q_x | \Psi \rangle = \int_{-\infty}^{\infty} \langle \Psi | Q_x | x \rangle \langle x | \Psi \rangle dx = \int_{-\infty}^{\infty} \langle \Psi | x | x \rangle \langle x | \Psi \rangle dx = \int_{-\infty}^{\infty} x |\langle x | \Psi \rangle|^2 dx \\ &= \int_{-\infty}^{\infty} x |\Psi(x)|^2 dx\end{aligned}$$

$$\overline{Q_x^2} = \langle \Psi | Q_x^2 | \Psi \rangle = \int_{-\infty}^{\infty} x^2 |\Psi(x)|^2 dx$$

$$|\Psi(x)|^2 = \frac{|A|^2}{\sqrt{1 + \left(\frac{\hbar t}{md^2}\right)^2}} \exp \left[-\frac{\left(x - \frac{\hbar k_0}{m} t\right)^2}{d^2 \left[1 + \left(\frac{\hbar t}{md^2}\right)^2\right]} \right]$$

$$\text{Not. } \alpha = \frac{|A|^2}{\sqrt{1 + \left(\frac{\hbar t}{md^2}\right)^2}}, \beta = \frac{\hbar k_0}{m} t, \gamma = d^2 \left[1 + \left(\frac{\hbar t}{md^2}\right)^2\right]$$

$$|\Psi(x)|^2 = \alpha \exp \left[-\frac{(x - \beta)^2}{\gamma} \right]$$

$$\overline{Q_x} = \alpha \int_{-\infty}^{\infty} x \exp \left[-\frac{(x - \beta)^2}{\gamma} \right] dx = \alpha \beta \sqrt{\gamma \pi} = \frac{\hbar k_0}{m} t$$

$$\overline{Q_x^2} = \alpha \int_{-\infty}^{\infty} x^2 \exp \left[-\frac{(x - \beta)^2}{\gamma} \right] dx = \alpha \sqrt{\gamma \pi} \left(\frac{\gamma}{2} + \beta^2 \right) = \frac{d^2}{2} \left[1 + \left(\frac{\hbar t}{md^2} \right)^2 \right] + \frac{\hbar^2 k_0^2}{m^2} t^2$$

$$\overline{\Delta Q_x^2} = \frac{d^2}{2} \left[1 + \left(\frac{\hbar t}{md^2} \right)^2 \right]$$

2 Q_x in reprezentarea impulsurilor

$$\overline{Q_x} = \langle \Psi | Q_x | \Psi \rangle = \int_{-\infty}^{\infty} \langle \Psi | Q_x | p \rangle \langle p | \Psi \rangle dp = \int_{-\infty}^{\infty} \Phi^*(p) i\hbar \frac{\partial}{\partial p} \Phi(p) dp = (\overline{Q_x})_0 + \frac{t}{m} \overline{P_x}$$

3 $\overline{Q_x^2}$

$$\overline{Q_x^2} = \langle \Psi | Q_x^2 | \Psi \rangle = -\hbar^2 \int_{-\infty}^{\infty} \Phi^*(p) \frac{\partial^2}{\partial p^2} \Phi(p) dp \quad (1)$$

$$\Phi(p) = \varphi(p) \exp\left(-\frac{ip^2}{2\hbar m} t\right)$$

$$\Phi^*(p) = \varphi^*(p) \exp\left(\frac{ip^2}{2\hbar m} t\right)$$

$$\frac{\partial}{\partial p} \Phi(p) = \left(\frac{\partial \varphi}{\partial p} - \frac{ip}{\hbar m} t \varphi\right) \exp\left(-\frac{ip^2}{2\hbar m} t\right)$$

$$\frac{\partial^2}{\partial p^2} \Phi(p) = \left(\frac{\partial^2 \varphi}{\partial p^2} - \frac{i}{\hbar m} t \varphi - 2 \frac{ip}{\hbar m} t \frac{\partial \varphi}{\partial p} - \frac{p^2}{\hbar^2 m^2} t^2 \varphi\right) \exp\left(-\frac{ip^2}{2\hbar m} t\right)$$

$$\overline{Q_x^2} = -\hbar^2 \int_{-\infty}^{\infty} \varphi^* \frac{\partial^2 \varphi}{\partial p^2} - \frac{i}{\hbar m} t |\varphi|^2 - 2 \frac{ip}{\hbar m} t \varphi^* \frac{\partial \varphi}{\partial p} - \frac{p^2}{\hbar^2 m^2} t^2 |\varphi|^2 dp$$

$$= (\overline{Q_x^2})_0 + \frac{i\hbar}{m} t \int_{-\infty}^{\infty} \left(|\varphi|^2 + 2p\varphi^* \frac{\partial \varphi}{\partial p} \right) dp + \frac{t^2}{m^2} \overline{P_x^2}$$

Integrand prin parti (1) obținem:

$$\begin{aligned} \overline{Q_x^2} &= \hbar^2 \int_{-\infty}^{\infty} \frac{\partial \Phi^*}{\partial p} \frac{\partial \Phi}{\partial p} dp \\ &= (\overline{Q_x^2})_0 + \frac{i\hbar}{m} t \int_{-\infty}^{\infty} p \left(\varphi^* \frac{\partial \varphi}{\partial p} - \frac{\partial \varphi^*}{\partial p} \varphi \right) dp + \frac{t^2}{m^2} \overline{P_x^2} \end{aligned}$$