Question 2

1 Q_x in reprezentarea pozitiilor

$$\overline{\Delta Q_x^2} = (\overline{Q_x^2}) - (\overline{Q_x})^2$$

$$\overline{Q_x} = \langle\Psi|Q_x|\Psi\rangle = \int\limits_{-\infty}^{\infty} \langle\Psi|\, Q_x\, |x\rangle \langle x|\Psi\rangle \, dx = \int\limits_{-\infty}^{\infty} \langle\Psi|\, x\, |x\rangle \langle x|\Psi\rangle \, dx = \int\limits_{-\infty}^{\infty} x|\langle x|\Psi\rangle|^2 dx$$

$$= \int\limits_{-\infty}^{\infty} x|\Psi(x)|^2 dx$$

$$\overline{Q_x^2} = \langle\Psi|Q_x^2|\Psi\rangle = \int\limits_{-\infty}^{\infty} x^2|\Psi(x)|^2 dx$$

$$|\Psi(x)|^2 = \frac{|A|^2}{\left[1+\left(\frac{\hbar t}{md^2}\right)^2\right]^2} \exp\left[-\frac{\left(x^2 - \frac{\hbar k_0}{m}t\right)^2}{d^2\left[1+\left(\frac{\hbar t}{md^2}\right)^2\right]}\right]$$

Not. $\alpha = \frac{|A|^2}{\left[1+\left(\frac{\hbar t}{md^2}\right)^2\right]^2}, \beta = \frac{\hbar k_0}{m}t, \gamma = d^2\left[1+\left(\frac{\hbar t}{md^2}\right)^2\right]$

$$|\Psi(x)|^2 = \alpha \exp\left[-\frac{\left(x^2-\beta\right)^2}{\gamma}\right]$$

$$\overline{Q_x} = \alpha \int\limits_{-\infty}^{\infty} x \exp\left[-\frac{\left(x^2-\beta\right)^2}{\gamma}\right] dx = 0 \qquad \text{(functie impara)}$$

$$\overline{Q_x^2} = \alpha \int\limits_{-\infty}^{\infty} x^2 \exp\left[-\frac{\left(x^2-\beta\right)^2}{\gamma}\right] dx$$

2 Q_x in reprezentarea impulsurilor

$$\overline{Q_x} = \langle\Psi|Q_x|\Psi\rangle = \int\limits_{-\infty}^{\infty} \langle\Psi|\, Q_x \,|p\rangle \,\langle p|\Psi\rangle \, dp = \int\limits_{-\infty}^{\infty} \Phi^*(p) i\hbar \frac{\partial}{\partial p}\Phi(p)dp = \left(\overline{Q_x}\right)_0 + \frac{t}{m}\overline{P_x}$$

3 $\overline{Q_x^2}$

$$\overline{Q_x^2} = \langle\Psi|Q_x^2|\Psi\rangle = -\hbar^2 \int\limits_{-\infty}^{\infty} \Phi^*(p)\frac{\partial^2}{\partial p^2}\Phi(p)dp \tag{1}$$

$$\begin{aligned}
\Phi(p) &= \varphi(p)\exp\left(-\frac{ip^2}{2\hbar m}t\right)\\
\Phi^*(p) &= \varphi^*(p)\exp\left(\frac{ip^2}{2\hbar m}t\right)\\
\frac{\partial}{\partial p}\Phi(p) &= \left(\frac{\partial\varphi}{\partial p} - \frac{ip}{\hbar m}t\varphi\right)\exp\left(-\frac{ip^2}{2\hbar m}t\right)\\
\frac{\partial^2}{\partial p^2}\Phi(p) &= \left(\frac{\partial^2\varphi}{\partial p^2} - \frac{i}{m\hbar}t\varphi - 2\frac{ip}{m\hbar}t\frac{\partial\varphi}{\partial p} - \frac{p^2}{\hbar^2 m^2}t^2\varphi\right)\exp\left(-\frac{ip^2}{2\hbar m}t\right)\\
\overline{Q_x^2} &= -\hbar^2\int\limits_{-\infty}^{\infty} \varphi^*\frac{\partial^2\varphi}{\partial p^2} - \frac{i}{m\hbar}t|\varphi|^2 - 2\frac{ip}{m\hbar}t\varphi^*\frac{\partial\varphi}{\partial p} - \frac{p^2}{\hbar^2 m^2}t^2|\varphi|^2 dp\\
&= \left(\overline{Q_x^2}\right)_0 + \frac{i\hbar}{m}t\int\limits_{-\infty}^{\infty}\left(|\varphi|^2 + 2p\varphi * \frac{\partial\varphi}{\partial p}\right)dp + \frac{t^2}{m^2}\overline{P_x^2}
\end{aligned}$$

Integrand prin parti (1) obtinem:

$$\begin{aligned}
\overline{Q_x^2} &= \hbar^2\int\limits_{-\infty}^{\infty}\frac{\partial\Phi^*}{\partial p}\frac{\partial\Phi}{\partial p}dp\\
&= \left(\overline{Q_x^2}\right)_0 + \frac{i\hbar}{m}t\int\limits_{-\infty}^{\infty} p\left(\varphi^*\frac{\partial\varphi}{\partial p} - \frac{\partial\varphi^*}{\partial p}\varphi\right)dp + \frac{t^2}{m^2}\overline{P_x^2}
\end{aligned}$$