Escuela Politécnica Nacional

Nombres: Moisés Pineda y Sebastián Morales

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Repositorio:

https://github.com/SebastianMoralesEpn/Github1.0/tree/0bf724d0529e52b252de022dc205debd6085ebed/Talleres/Taller7

[Taller 2b] Métodos iterativos

Grafique la trayectoria de los siguientes sistemas de ecuaciones:

$$x_1 + x_2 = 7$$
$$-2x_1 + 5x_2 = 0$$

- $X_0 = (0, 0)$
- $X_0 = (5, 2)$

$$x_1 + x_2 = 6$$
$$-2x_1 + x_2 = 0$$

Prueba 3 posiciones iniciales, encuentre una en la que diverge el sistema.

In [28]:
```
%load_ext autoreload
```

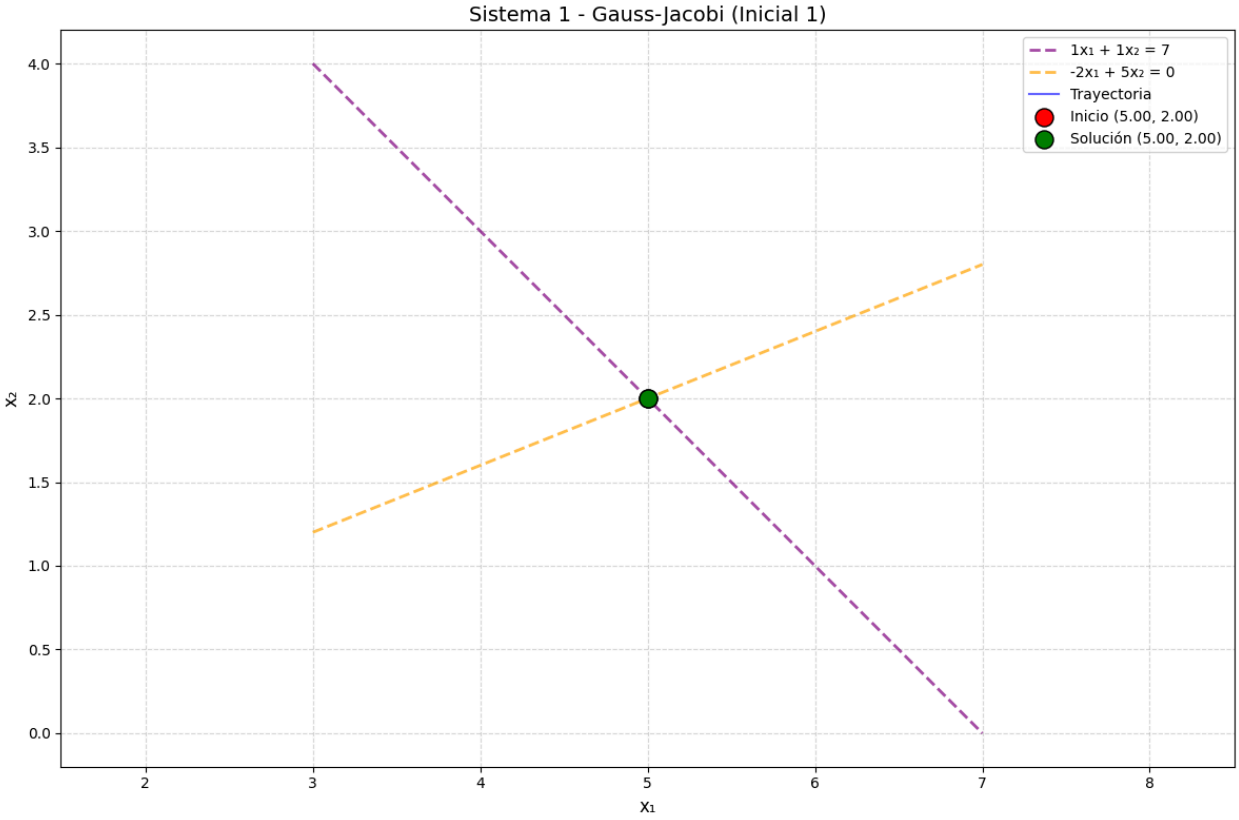
```
The autoreload extension is already loaded. To reload it, use:
 %reload_ext autoreload
```

In [29]:
```
# %% Configuración inicial
%autoreload 2
from src import gauss_jacobi, gauss_seidel
import numpy as np
import matplotlib.pyplot as plt

# Función mejorada para graficar sistemas y trayectorias
def plot_system_with_trajectory(A, b, tray, title):
    tray_array = np.vstack([np.array(x).flatten() for x in tray])
    x_vals = np.linspace(min(tray_array[:, 0])-2, max(tray_array[:, 0])+2, 400

    plt.figure(figsize=(12, 8))

    # Graficar cada ecuación del sistema
    colors = ['purple', 'orange']
```

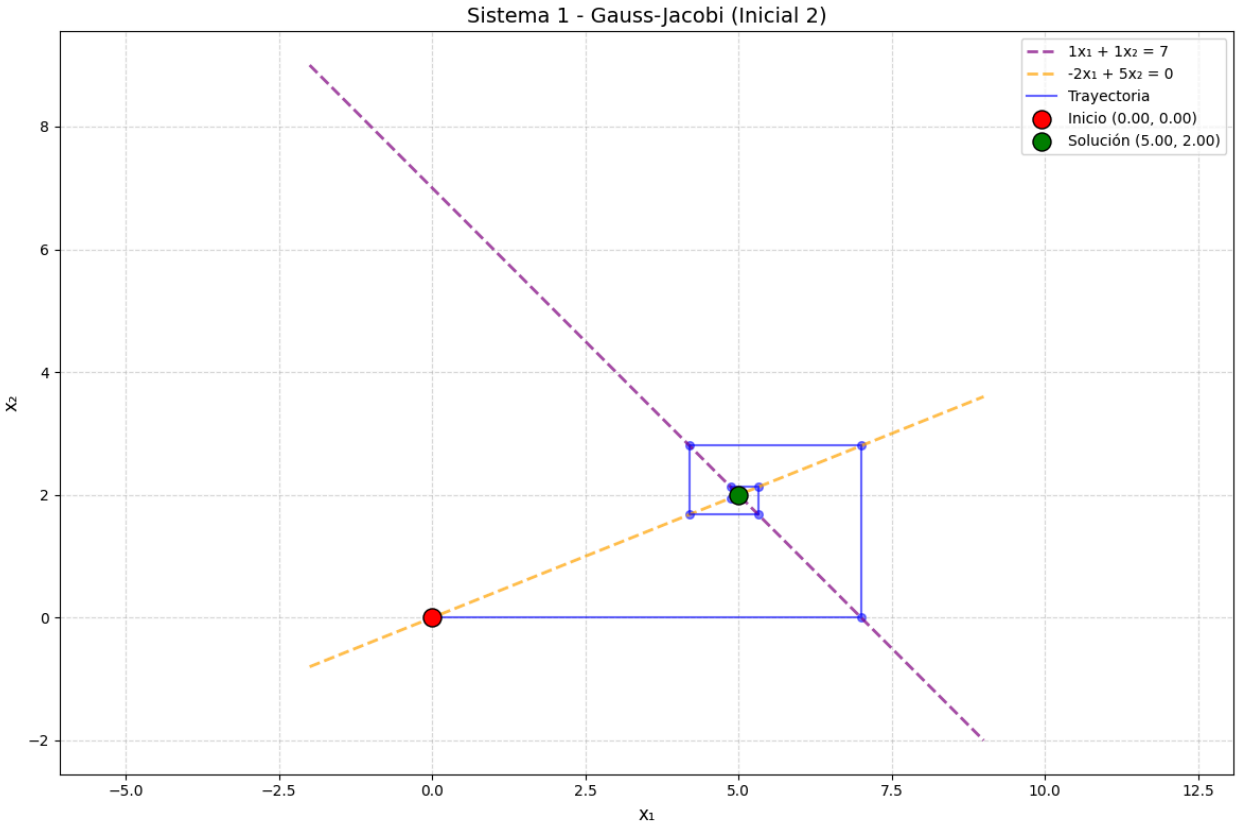
```
    for i in range(A.shape[0]):
        if A[i, 1] != 0:  # Para evitar división por cero
            y_vals = (b[i] - A[i, 0]*x_vals)/A[i, 1]
            plt.plot(x_vals, y_vals, color=colors[i], linestyle='--',
                     linewidth=2, alpha=0.7,
                     label=f'{A[i,0]}x₁ + {A[i,1]}x₂ = {b[i]}')

    # Graficar trayectoria
    plt.plot(tray_array[:, 0], tray_array[:, 1], 'b-', linewidth=1.5, alpha=0.
    plt.scatter(tray_array[:, 0], tray_array[:, 1], color='blue', s=30, alpha=

    # Destacar puntos inicial y final
    plt.scatter(tray_array[0, 0], tray_array[0, 1], color='red', s=150,
                edgecolor='black', zorder=5, label=f'Inicio ({tray_array[0,0]:
    plt.scatter(tray_array[-1, 0], tray_array[-1, 1], color='green', s=150,
                edgecolor='black', zorder=5, label=f'Solución ({tray_array[-1,

    plt.xlabel('x₁', fontsize=12)
    plt.ylabel('x₂', fontsize=12)
    plt.title(title, fontsize=14)
    plt.legend(loc='best', fontsize=10)
    plt.grid(True, linestyle='--', alpha=0.5)
    plt.axis('equal')
    plt.tight_layout()
    plt.show()
```

In [30]:
```
# %% Sistema 1 - GAUSS-JACOBI
A1 = np.array([[1, 1], [-2, 5]])
b1 = np.array([7, 0])
initial_positions = [np.array([5, 2]), np.array([0, 0]), np.array([-2, 4])]

print("=== SISTEMA 1 - MÉTODO DE GAUSS-JACOBI ===")
for i, x0 in enumerate(initial_positions, 1):
    print(f"\nCaso {i}: Posición inicial {x0}")
    try:
        x_j1, tray_j1 = gauss_jacobi(A=A1, b=b1, x0=x0, tol=1e-6, max_iter=50)
        plot_system_with_trajectory(A1, b1, tray_j1, f"Sistema 1 - Gauss-Jacob
        print(f"Solución encontrada: [{x_j1[0,0]:.6f}, {x_j1[1,0]:.6f}]")
    except Exception as e:
        print(f"¡Error! El método divergió: {str(e)}")
```

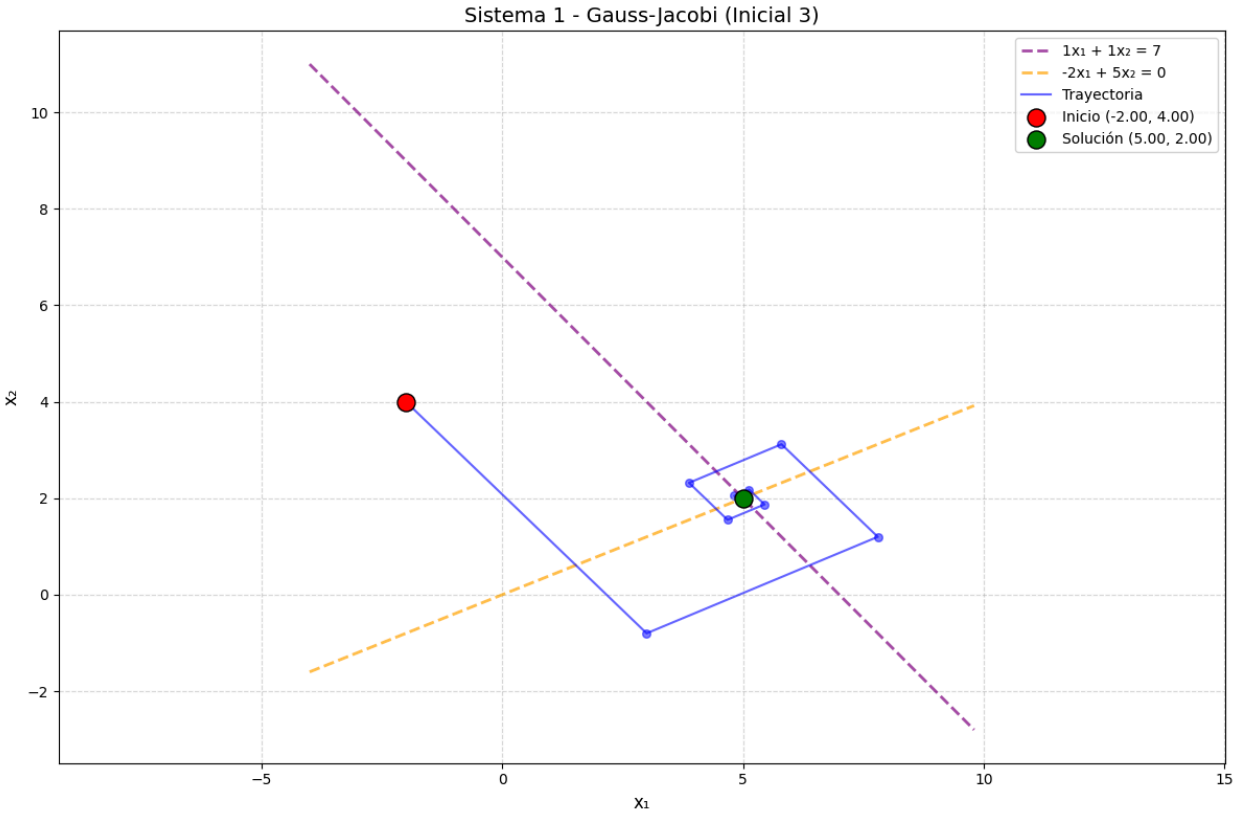
```
=== SISTEMA 1 - MÉTODO DE GAUSS-JACOBI ===

Caso 1: Posición inicial [5 2]
[07-15 20:25:40][INFO] i= 0 x: [5 2]
[07-15 20:25:40][INFO] i= 1 x: [[5. 2.]]
```


Sistema 1 - Gauss-Jacobi (Inicial 1)
1x₁ + 1x₂ = 7
-2x₁ + 5x₂ = 0
Trayectoria
Inicio (5.00, 2.00)
Solución (5.00, 2.00)
x₂
x₁
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
3
4
5
6
7
8

```
Solución encontrada: [5.000000, 2.000000]

Caso 2: Posición inicial [0 0]
[07-15 20:25:41][INFO] i= 0 x: [0 0]
[07-15 20:25:41][INFO] i= 1 x: [[7. 0.]]
[07-15 20:25:41][INFO] i= 2 x: [[7.  2.8]]
[07-15 20:25:41][INFO] i= 3 x: [[4.2 2.8]]
[07-15 20:25:41][INFO] i= 4 x: [[4.2  1.68]]
[07-15 20:25:41][INFO] i= 5 x: [[5.32 1.68]]
[07-15 20:25:41][INFO] i= 6 x: [[5.32  2.128]]
[07-15 20:25:41][INFO] i= 7 x: [[4.872 2.128]]
[07-15 20:25:41][INFO] i= 8 x: [[4.872  1.9488]]
[07-15 20:25:41][INFO] i= 9 x: [[5.0512 1.9488]]
[07-15 20:25:41][INFO] i= 10 x: [[5.0512  2.02048]]
[07-15 20:25:41][INFO] i= 11 x: [[4.97952 2.02048]]
[07-15 20:25:41][INFO] i= 12 x: [[4.97952  1.991808]]
[07-15 20:25:41][INFO] i= 13 x: [[5.008192 1.991808]]
[07-15 20:25:41][INFO] i= 14 x: [[5.008192  2.0032768]]
[07-15 20:25:41][INFO] i= 15 x: [[4.9967232 2.0032768]]
[07-15 20:25:41][INFO] i= 16 x: [[4.9967232  1.99868928]]
[07-15 20:25:41][INFO] i= 17 x: [[5.00131072 1.99868928]]
[07-15 20:25:41][INFO] i= 18 x: [[5.00131072 2.00052429]]
[07-15 20:25:41][INFO] i= 19 x: [[4.99947571 2.00052429]]
[07-15 20:25:41][INFO] i= 20 x: [[4.99947571 1.99979028]]
[07-15 20:25:41][INFO] i= 21 x: [[5.00020972 1.99979028]]
[07-15 20:25:41][INFO] i= 22 x: [[5.00020972 2.00008389]]
[07-15 20:25:41][INFO] i= 23 x: [[4.99991611 2.00008389]]
[07-15 20:25:41][INFO] i= 24 x: [[4.99991611 1.99996645]]
[07-15 20:25:41][INFO] i= 25 x: [[5.00003355 1.99996645]]
[07-15 20:25:41][INFO] i= 26 x: [[5.00003355 2.00001342]]
[07-15 20:25:41][INFO] i= 27 x: [[4.99998658 2.00001342]]
[07-15 20:25:41][INFO] i= 28 x: [[4.99998658 1.99999463]]
[07-15 20:25:41][INFO] i= 29 x: [[5.00000537 1.99999463]]
[07-15 20:25:41][INFO] i= 30 x: [[5.00000537 2.00000215]]
[07-15 20:25:41][INFO] i= 31 x: [[4.99999785 2.00000215]]
[07-15 20:25:41][INFO] i= 32 x: [[4.99999785 1.99999914]]
[07-15 20:25:41][INFO] i= 33 x: [[5.00000086 1.99999914]]
[07-15 20:25:41][INFO] i= 34 x: [[5.00000086 2.00000034]]
[07-15 20:25:41][INFO] i= 35 x: [[4.99999966 2.00000034]]
```


Sistema 1 - Gauss-Jacobi (Inicial 2)
1x₁ + 1x₂ = 7
-2x₁ + 5x₂ = 0
Trayectoria
Inicio (0.00, 0.00)
Solución (5.00, 2.00)
x₂
x₁
8
6
4
2
0
−2
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5

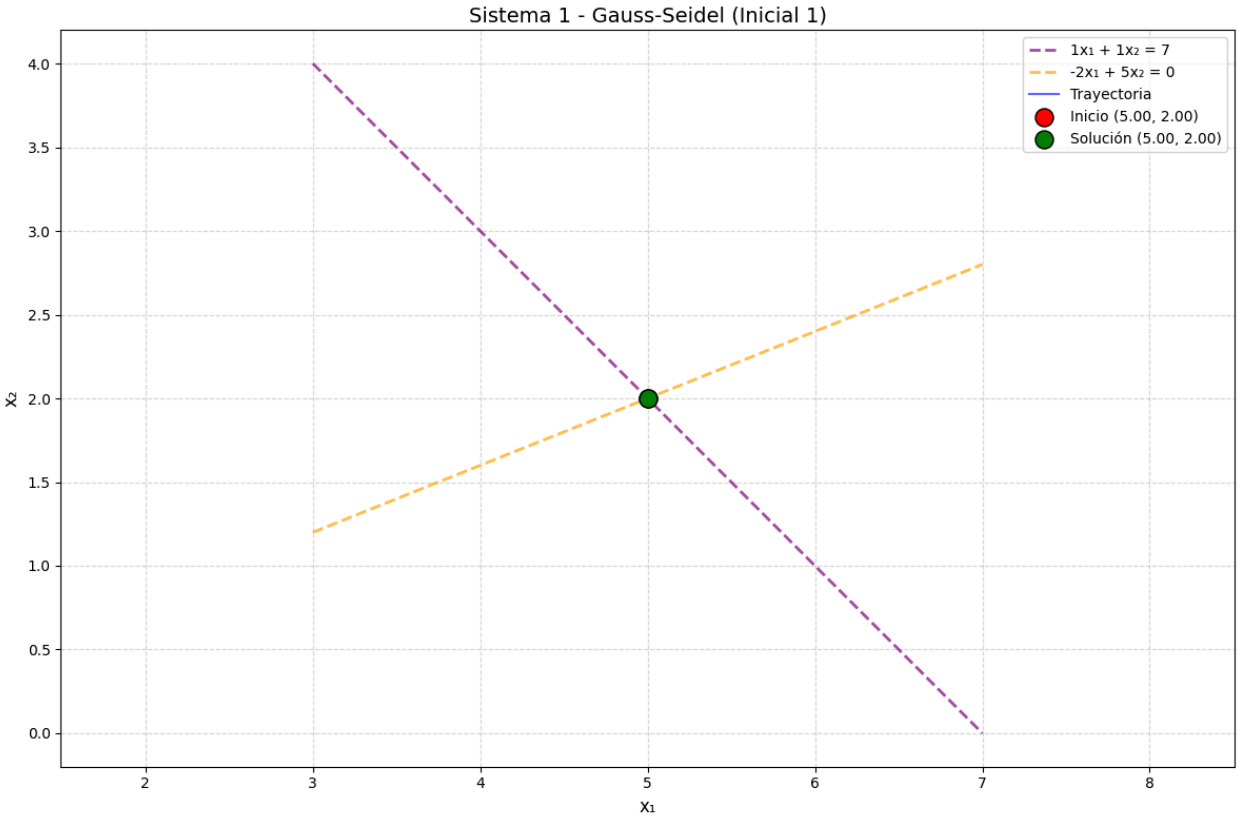
```
Solución encontrada: [5.000000, 2.000000]

Caso 3: Posición inicial [-2  4]
[07-15 20:25:41][INFO] i= 0 x: [-2  4]
[07-15 20:25:41][INFO] i= 1 x: [[ 3.  -0.8]]
[07-15 20:25:41][INFO] i= 2 x: [[7.8 1.2]]
[07-15 20:25:41][INFO] i= 3 x: [[5.8  3.12]]
[07-15 20:25:41][INFO] i= 4 x: [[3.88 2.32]]
[07-15 20:25:41][INFO] i= 5 x: [[4.68  1.552]]
[07-15 20:25:41][INFO] i= 6 x: [[5.448 1.872]]
[07-15 20:25:41][INFO] i= 7 x: [[5.128  2.1792]]
[07-15 20:25:41][INFO] i= 8 x: [[4.8208 2.0512]]
[07-15 20:25:41][INFO] i= 9 x: [[4.9488  1.92832]]
[07-15 20:25:41][INFO] i= 10 x: [[5.07168 1.97952]]
[07-15 20:25:41][INFO] i= 11 x: [[5.02048  2.028672]]
[07-15 20:25:41][INFO] i= 12 x: [[4.971328 2.008192]]
[07-15 20:25:41][INFO] i= 13 x: [[4.991808  1.9885312]]
[07-15 20:25:41][INFO] i= 14 x: [[5.0114688 1.9967232]]
[07-15 20:25:41][INFO] i= 15 x: [[5.0032768  2.00458752]]
[07-15 20:25:41][INFO] i= 16 x: [[4.99541248 2.00131072]]
[07-15 20:25:41][INFO] i= 17 x: [[4.99868928 1.99816499]]
[07-15 20:25:41][INFO] i= 18 x: [[5.00183501 1.99947571]]
[07-15 20:25:41][INFO] i= 19 x: [[5.00052429 2.000734  ]]
[07-15 20:25:41][INFO] i= 20 x: [[4.999266   2.00020972]]
[07-15 20:25:41][INFO] i= 21 x: [[4.99979028 1.9997064 ]]
[07-15 20:25:41][INFO] i= 22 x: [[5.0002936  1.99991611]]
[07-15 20:25:41][INFO] i= 23 x: [[5.00008389 2.00011744]]
[07-15 20:25:41][INFO] i= 24 x: [[4.99988256 2.00003355]]
[07-15 20:25:41][INFO] i= 25 x: [[4.99996645 1.99995302]]
[07-15 20:25:41][INFO] i= 26 x: [[5.00004698 1.99998658]]
[07-15 20:25:41][INFO] i= 27 x: [[5.00001342 2.00001879]]
[07-15 20:25:41][INFO] i= 28 x: [[4.99998121 2.00000537]]
[07-15 20:25:41][INFO] i= 29 x: [[4.99999463 1.99999248]]
[07-15 20:25:41][INFO] i= 30 x: [[5.00000752 1.99999785]]
[07-15 20:25:41][INFO] i= 31 x: [[5.00000215 2.00000301]]
[07-15 20:25:41][INFO] i= 32 x: [[4.99999699 2.00000086]]
[07-15 20:25:41][INFO] i= 33 x: [[4.99999914 1.9999988 ]]
[07-15 20:25:41][INFO] i= 34 x: [[5.0000012  1.99999966]]
[07-15 20:25:41][INFO] i= 35 x: [[5.00000034 2.00000048]]
```



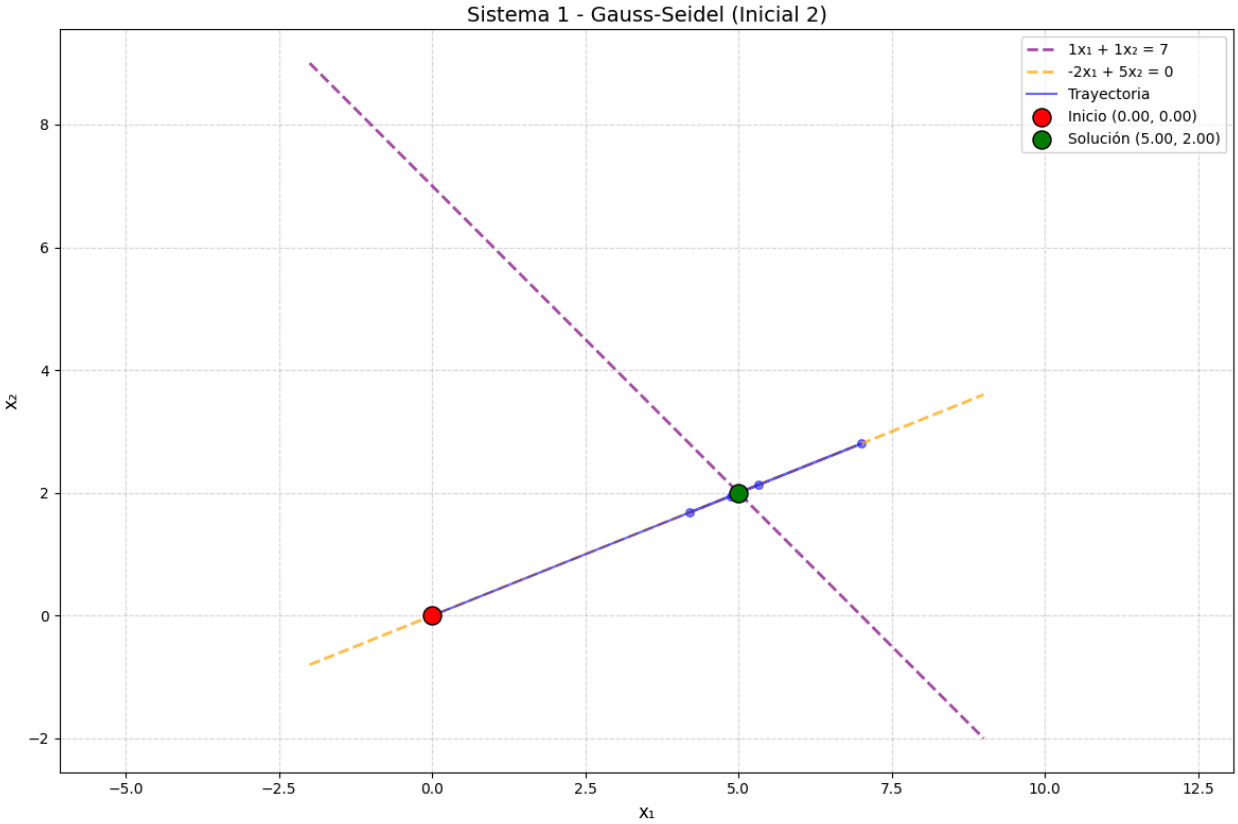

```
Solución encontrada: [5.000000, 2.000000]
```

In [31]:
```python
# %% Sistema 1 - GAUSS-SEIDEL
print("\n=== SISTEMA 1 - MÉTODO DE GAUSS-SEIDEL ===")
for i, x0 in enumerate(initial_positions, 1):
    print(f"\nCaso {i}: Posición inicial {x0}")
    try:
        x_gs1, tray_gs1 = gauss_seidel(A=A1, b=b1, x0=x0, tol=1e-6, max_iter=5
        plot_system_with_trajectory(A1, b1, tray_gs1, f"Sistema 1 - Gauss-Seic
        print(f"Solución encontrada: [{x_gs1[0,0]:.6f}, {x_gs1[1,0]:.6f}]")
    except Exception as e:
        print(f"¡Error! El método divergió: {str(e)}")
```

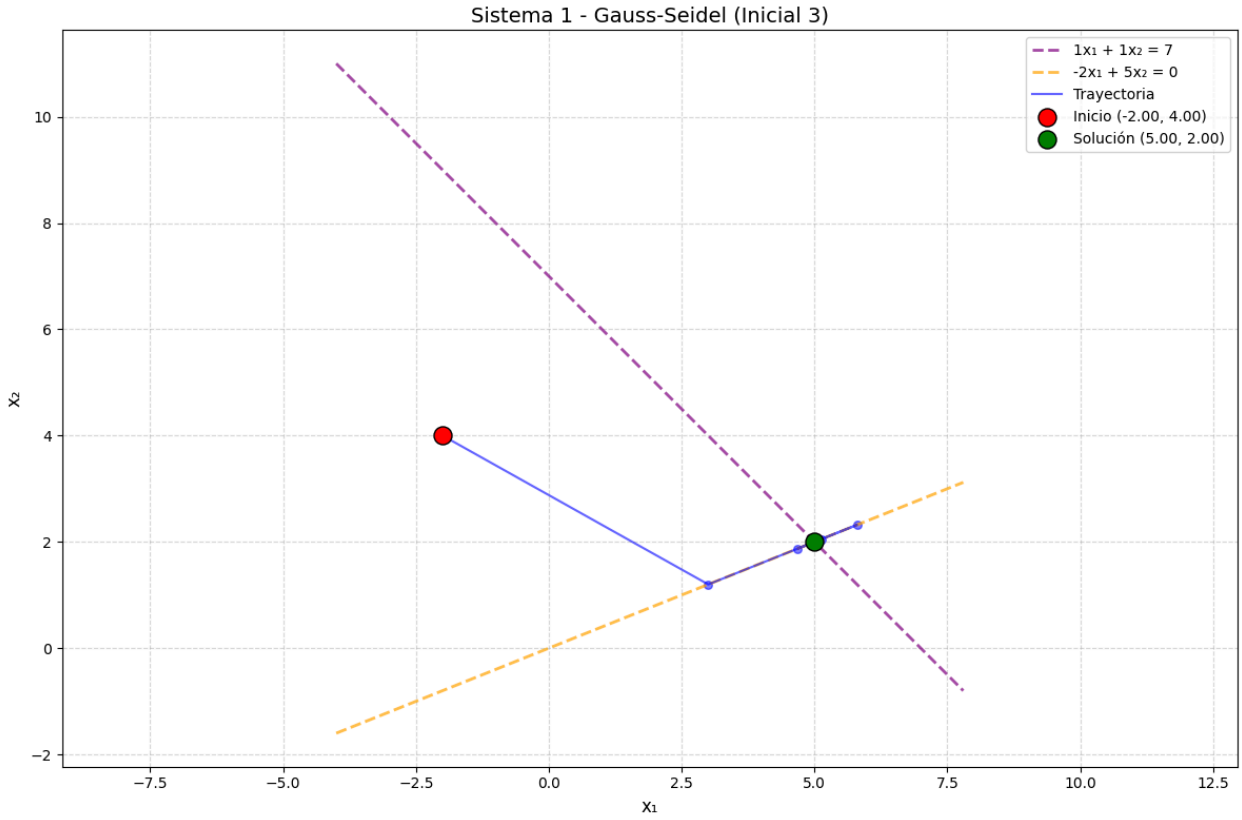
```
=== SISTEMA 1 - MÉTODO DE GAUSS-SEIDEL ===

Caso 1: Posición inicial [5 2]
[07-15 20:26:23][INFO] i= 0 x: [5 2]
[07-15 20:26:23][INFO] i= 1 x: [[5. 2.]]
```




```
Solución encontrada: [5.000000, 2.000000]

Caso 2: Posición inicial [0 0]
[07-15 20:26:23][INFO] i= 0 x: [0 0]
[07-15 20:26:23][INFO] i= 1 x: [[7.  2.8]]
[07-15 20:26:23][INFO] i= 2 x: [[4.2  1.68]]
[07-15 20:26:23][INFO] i= 3 x: [[5.32  2.128]]
[07-15 20:26:23][INFO] i= 4 x: [[4.872  1.9488]]
[07-15 20:26:23][INFO] i= 5 x: [[5.0512  2.02048]]
[07-15 20:26:23][INFO] i= 6 x: [[4.97952  1.991808]]
[07-15 20:26:23][INFO] i= 7 x: [[5.008192  2.0032768]]
[07-15 20:26:23][INFO] i= 8 x: [[4.9967232  1.99868928]]
[07-15 20:26:23][INFO] i= 9 x: [[5.00131072 2.00052429]]
[07-15 20:26:23][INFO] i= 10 x: [[4.99947571 1.99979028]]
[07-15 20:26:23][INFO] i= 11 x: [[5.00020972 2.00008389]]
[07-15 20:26:23][INFO] i= 12 x: [[4.99991611 1.99996645]]
[07-15 20:26:23][INFO] i= 13 x: [[5.00003355 2.00001342]]
[07-15 20:26:23][INFO] i= 14 x: [[4.99998658 1.99999463]]
[07-15 20:26:23][INFO] i= 15 x: [[5.00000537 2.00000215]]
[07-15 20:26:23][INFO] i= 16 x: [[4.99999785 1.99999914]]
[07-15 20:26:23][INFO] i= 17 x: [[5.00000086 2.00000034]]
[07-15 20:26:23][INFO] i= 18 x: [[4.99999966 1.99999986]]
```




```
Solución encontrada: [5.000000, 2.000000]

Caso 3: Posición inicial [-2  4]
[07-15 20:26:24][INFO] i= 0 x: [-2  4]
[07-15 20:26:24][INFO] i= 1 x: [[3.  1.2]]
[07-15 20:26:24][INFO] i= 2 x: [[5.8  2.32]]
[07-15 20:26:24][INFO] i= 3 x: [[4.68  1.872]]
[07-15 20:26:24][INFO] i= 4 x: [[5.128  2.0512]]
[07-15 20:26:24][INFO] i= 5 x: [[4.9488  1.97952]]
[07-15 20:26:24][INFO] i= 6 x: [[5.02048  2.008192]]
[07-15 20:26:24][INFO] i= 7 x: [[4.991808  1.9967232]]
[07-15 20:26:24][INFO] i= 8 x: [[5.0032768  2.00131072]]
[07-15 20:26:24][INFO] i= 9 x: [[4.99868928 1.99947571]]
[07-15 20:26:24][INFO] i= 10 x: [[5.00052429 2.00020972]]
[07-15 20:26:24][INFO] i= 11 x: [[4.99979028 1.99991611]]
[07-15 20:26:24][INFO] i= 12 x: [[5.00008389 2.00003355]]
[07-15 20:26:24][INFO] i= 13 x: [[4.99996645 1.99998658]]
[07-15 20:26:24][INFO] i= 14 x: [[5.00001342 2.00000537]]
[07-15 20:26:24][INFO] i= 15 x: [[4.99999463 1.99999785]]
[07-15 20:26:24][INFO] i= 16 x: [[5.00000215 2.00000086]]
[07-15 20:26:24][INFO] i= 17 x: [[4.99999914 1.99999966]]
[07-15 20:26:24][INFO] i= 18 x: [[5.00000034 2.00000014]]
```

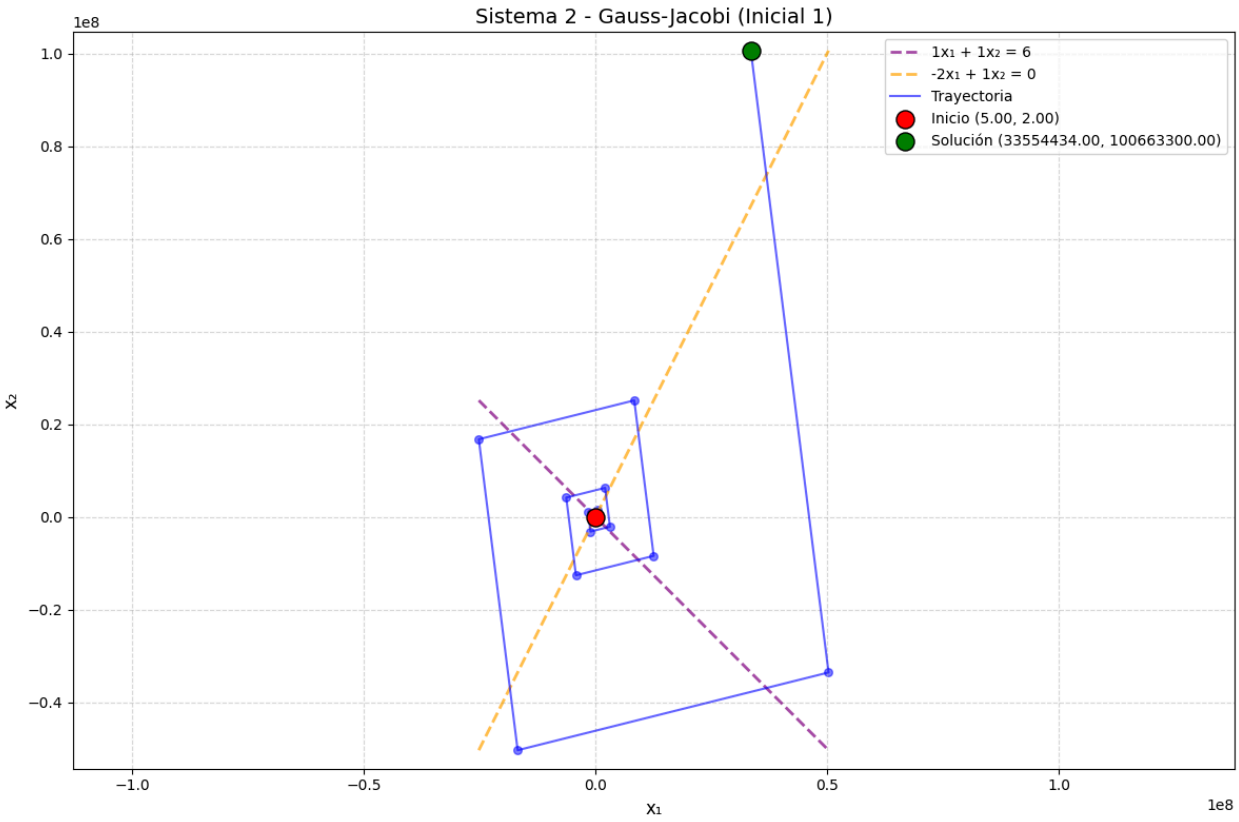



```
Solución encontrada: [5.000000, 2.000000]
```

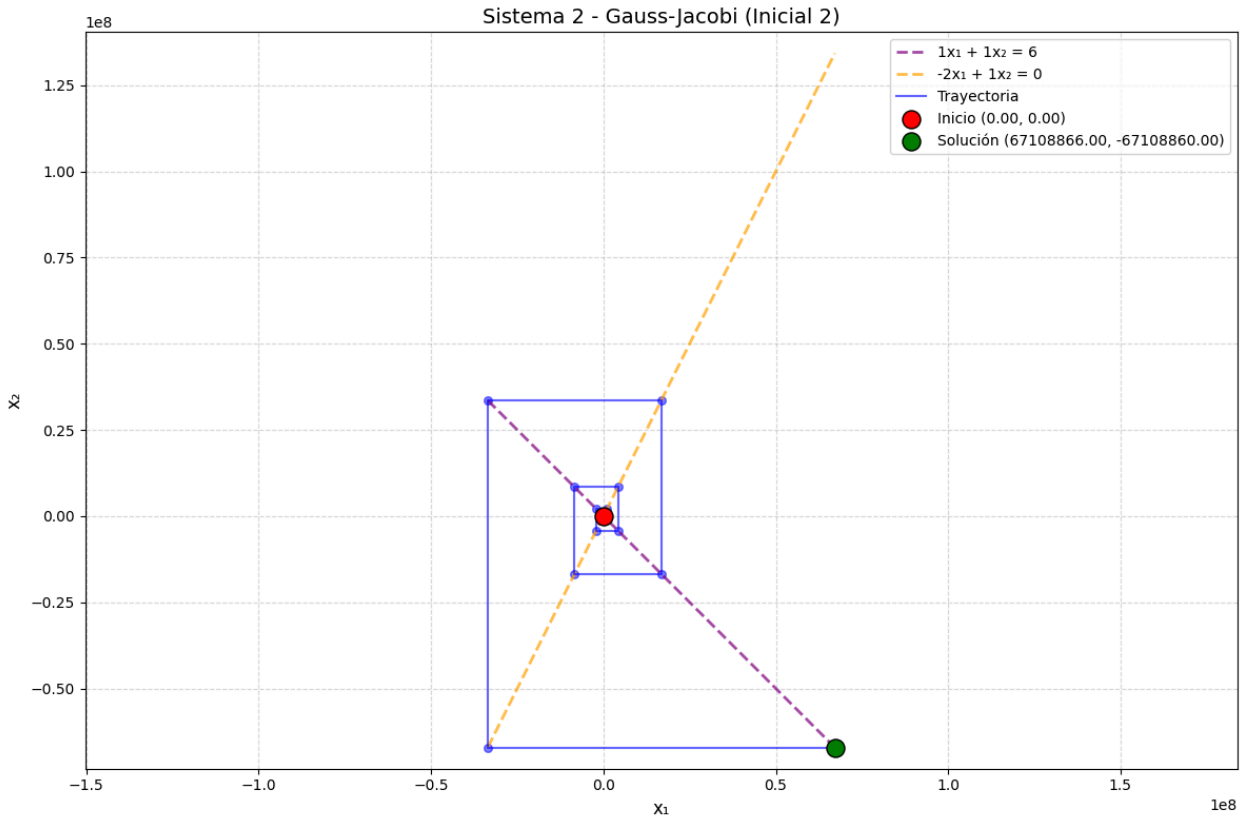
In [32]:

```python
# %% Sistema 2 - GAUSS-JACOBI
A2 = np.array([[1, 1], [-2, 1]])
b2 = np.array([6, 0])
initial_positions = [np.array([5, 2]), np.array([0, 0]), np.array([-2, 4])]

print("\n=== SISTEMA 2 - MÉTODO DE GAUSS-JACOBI ===")
for i, x0 in enumerate(initial_positions, 1):
    print(f"\nCaso {i}: Posición inicial {x0}")
    try:
        x_j2, tray_j2 = gauss_jacobi(A=A2, b=b2, x0=x0, tol=1e-6, max_iter=50)
        plot_system_with_trajectory(A2, b2, tray_j2, f"Sistema 2 - Gauss-Jacob
        print(f"Solución encontrada: [{x_j2[0,0]:.6f}, {x_j2[1,0]:.6f}]")
    except Exception as e:
        print(f"¡Error! El método divergió: {str(e)}")
```

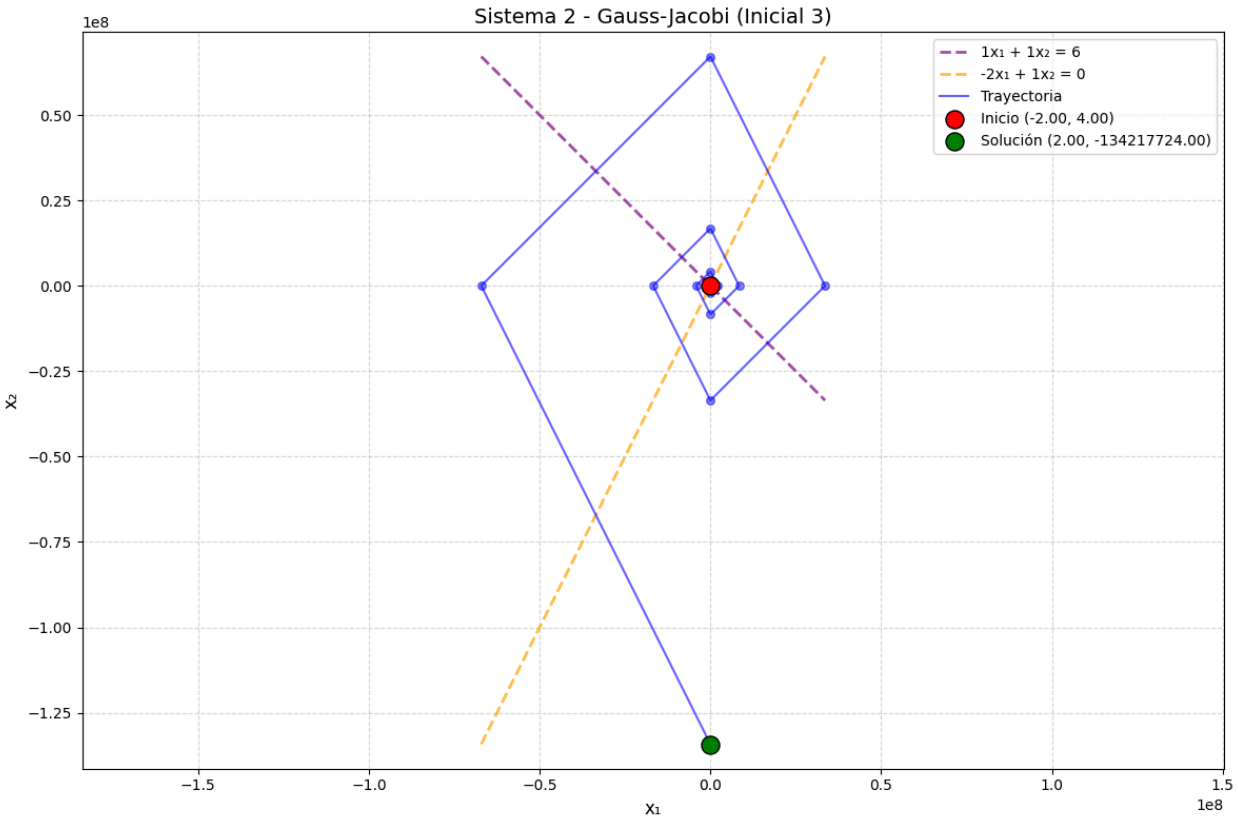
```
=== SISTEMA 2 - MÉTODO DE GAUSS-JACOBI ===

Caso 1: Posición inicial [5 2]
[07-15 20:26:57][INFO] i= 0 x: [5 2]
[07-15 20:26:57][INFO] i= 1 x: [[ 4. 10.]]
[07-15 20:26:57][INFO] i= 2 x: [[-4.  8.]]
[07-15 20:26:57][INFO] i= 3 x: [[-2. -8.]]
[07-15 20:26:57][INFO] i= 4 x: [[14. -4.]]
[07-15 20:26:57][INFO] i= 5 x: [[10. 28.]]
[07-15 20:26:57][INFO] i= 6 x: [[-22.  20.]]
[07-15 20:26:57][INFO] i= 7 x: [[-14. -44.]]
[07-15 20:26:57][INFO] i= 8 x: [[ 50. -28.]]
[07-15 20:26:57][INFO] i= 9 x: [[ 34. 100.]]
[07-15 20:26:57][INFO] i= 10 x: [[-94.  68.]]
[07-15 20:26:57][INFO] i= 11 x: [[ -62. -188.]]
[07-15 20:26:57][INFO] i= 12 x: [[ 194. -124.]]
[07-15 20:26:57][INFO] i= 13 x: [[130. 388.]]
[07-15 20:26:57][INFO] i= 14 x: [[-382.  260.]]
[07-15 20:26:57][INFO] i= 15 x: [[-254. -764.]]
[07-15 20:26:57][INFO] i= 16 x: [[ 770. -508.]]
[07-15 20:26:57][INFO] i= 17 x: [[ 514. 1540.]]
[07-15 20:26:57][INFO] i= 18 x: [[-1534.  1028.]]
[07-15 20:26:57][INFO] i= 19 x: [[-1022. -3068.]]
[07-15 20:26:57][INFO] i= 20 x: [[ 3074. -2044.]]
[07-15 20:26:57][INFO] i= 21 x: [[2050. 6148.]]
[07-15 20:26:57][INFO] i= 22 x: [[-6142.  4100.]]
[07-15 20:26:57][INFO] i= 23 x: [[ -4094. -12284.]]
[07-15 20:26:57][INFO] i= 24 x: [[12290. -8188.]]
[07-15 20:26:57][INFO] i= 25 x: [[ 8194. 24580.]]
[07-15 20:26:57][INFO] i= 26 x: [[-24574.  16388.]]
[07-15 20:26:57][INFO] i= 27 x: [[-16382. -49148.]]
[07-15 20:26:57][INFO] i= 28 x: [[ 49154. -32764.]]
[07-15 20:26:57][INFO] i= 29 x: [[32770. 98308.]]
[07-15 20:26:57][INFO] i= 30 x: [[-98302.  65540.]]
[07-15 20:26:57][INFO] i= 31 x: [[ -65534. -196604.]]
[07-15 20:26:57][INFO] i= 32 x: [[ 196610. -131068.]]
[07-15 20:26:57][INFO] i= 33 x: [[131074. 393220.]]
[07-15 20:26:57][INFO] i= 34 x: [[-393214.  262148.]]
[07-15 20:26:57][INFO] i= 35 x: [[-262142. -786428.]]
[07-15 20:26:57][INFO] i= 36 x: [[ 786434. -524284.]]
[07-15 20:26:57][INFO] i= 37 x: [[ 524290. 1572868.]]
[07-15 20:26:57][INFO] i= 38 x: [[-1572862.  1048580.]]
[07-15 20:26:57][INFO] i= 39 x: [[-1048574. -3145724.]]
[07-15 20:26:57][INFO] i= 40 x: [[ 3145730. -2097148.]]
[07-15 20:26:57][INFO] i= 41 x: [[2097154. 6291460.]]
[07-15 20:26:57][INFO] i= 42 x: [[-6291454.  4194308.]]
[07-15 20:26:57][INFO] i= 43 x: [[ -4194302. -12582908.]]
[07-15 20:26:57][INFO] i= 44 x: [[12582914. -8388604.]]
[07-15 20:26:57][INFO] i= 45 x: [[ 8388610. 25165828.]]
[07-15 20:26:57][INFO] i= 46 x: [[-25165822.  16777220.]]
[07-15 20:26:57][INFO] i= 47 x: [[-16777214. -50331644.]]
[07-15 20:26:57][INFO] i= 48 x: [[ 50331650. -33554428.]]
[07-15 20:26:57][INFO] i= 49 x: [[3.3554434e+07 1.0066330e+08]]
```


Sistema 2 - Gauss-Jacobi (Inicial 1)
1e8
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
X₂
−1.0
−0.5
0.0
0.5
1.0
X₁
1e8
1x₁ + 1x₂ = 6
-2x₁ + 1x₂ = 0
Trayectoria
Inicio (5.00, 2.00)
Solución (33554434.00, 100663300.00)

```
Solución encontrada: [33554434.000000, 100663300.000000]

Caso 2: Posición inicial [0 0]
[07-15 20:26:58][INFO] i= 0 x: [0 0]
[07-15 20:26:58][INFO] i= 1 x: [[6. 0.]]
[07-15 20:26:58][INFO] i= 2 x: [[ 6. 12.]]
[07-15 20:26:58][INFO] i= 3 x: [[-6. 12.]]
[07-15 20:26:58][INFO] i= 4 x: [[ -6. -12.]]
[07-15 20:26:58][INFO] i= 5 x: [[ 18. -12.]]
[07-15 20:26:58][INFO] i= 6 x: [[18. 36.]]
[07-15 20:26:58][INFO] i= 7 x: [[-30.  36.]]
[07-15 20:26:58][INFO] i= 8 x: [[-30. -60.]]
[07-15 20:26:58][INFO] i= 9 x: [[ 66. -60.]]
[07-15 20:26:58][INFO] i= 10 x: [[ 66. 132.]]
[07-15 20:26:58][INFO] i= 11 x: [[-126.  132.]]
[07-15 20:26:58][INFO] i= 12 x: [[-126. -252.]]
[07-15 20:26:58][INFO] i= 13 x: [[ 258. -252.]]
[07-15 20:26:58][INFO] i= 14 x: [[258. 516.]]
[07-15 20:26:58][INFO] i= 15 x: [[-510.  516.]]
[07-15 20:26:58][INFO] i= 16 x: [[ -510. -1020.]]
[07-15 20:26:58][INFO] i= 17 x: [[ 1026. -1020.]]
[07-15 20:26:58][INFO] i= 18 x: [[1026. 2052.]]
[07-15 20:26:58][INFO] i= 19 x: [[-2046.  2052.]]
[07-15 20:26:58][INFO] i= 20 x: [[-2046. -4092.]]
[07-15 20:26:58][INFO] i= 21 x: [[ 4098. -4092.]]
[07-15 20:26:58][INFO] i= 22 x: [[4098. 8196.]]
[07-15 20:26:58][INFO] i= 23 x: [[-8190.  8196.]]
[07-15 20:26:58][INFO] i= 24 x: [[ -8190. -16380.]]
[07-15 20:26:58][INFO] i= 25 x: [[ 16386. -16380.]]
[07-15 20:26:58][INFO] i= 26 x: [[16386. 32772.]]
[07-15 20:26:58][INFO] i= 27 x: [[-32766.  32772.]]
[07-15 20:26:58][INFO] i= 28 x: [[-32766. -65532.]]
[07-15 20:26:58][INFO] i= 29 x: [[ 65538. -65532.]]
[07-15 20:26:58][INFO] i= 30 x: [[ 65538. 131076.]]
[07-15 20:26:58][INFO] i= 31 x: [[-131070.  131076.]]
[07-15 20:26:58][INFO] i= 32 x: [[-131070. -262140.]]
[07-15 20:26:58][INFO] i= 33 x: [[ 262146. -262140.]]
[07-15 20:26:58][INFO] i= 34 x: [[262146. 524292.]]
[07-15 20:26:58][INFO] i= 35 x: [[-524286.  524292.]]
[07-15 20:26:58][INFO] i= 36 x: [[ -524286. -1048572.]]
[07-15 20:26:58][INFO] i= 37 x: [[ 1048578. -1048572.]]
[07-15 20:26:58][INFO] i= 38 x: [[1048578. 2097156.]]
[07-15 20:26:58][INFO] i= 39 x: [[-2097150.  2097156.]]
[07-15 20:26:58][INFO] i= 40 x: [[-2097150. -4194300.]]
[07-15 20:26:58][INFO] i= 41 x: [[ 4194306. -4194300.]]
[07-15 20:26:58][INFO] i= 42 x: [[4194306. 8388612.]]
[07-15 20:26:58][INFO] i= 43 x: [[-8388606.  8388612.]]
[07-15 20:26:58][INFO] i= 44 x: [[ -8388606. -16777212.]]
[07-15 20:26:58][INFO] i= 45 x: [[ 16777218. -16777212.]]
[07-15 20:26:58][INFO] i= 46 x: [[16777218. 33554436.]]
[07-15 20:26:58][INFO] i= 47 x: [[-33554430.  33554436.]]
[07-15 20:26:58][INFO] i= 48 x: [[-33554430. -67108860.]]
[07-15 20:26:58][INFO] i= 49 x: [[ 67108866. -67108860.]]
```


Sistema 2 - Gauss-Jacobi (Inicial 2)
1e8
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
x₂
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
1e8
x₁
1x₁ + 1x₂ = 6
-2x₁ + 1x₂ = 0
Trayectoria
Inicio (0.00, 0.00)
Solución (67108866.00, -67108860.00)

```
Solución encontrada: [67108866.000000, -67108860.000000]

Caso 3: Posición inicial [-2  4]
[07-15 20:26:59][INFO] i= 0 x: [-2  4]
[07-15 20:26:59][INFO] i= 1 x: [[ 2. -4.]]
[07-15 20:26:59][INFO] i= 2 x: [[10.  4.]]
[07-15 20:26:59][INFO] i= 3 x: [[ 2. 20.]]
[07-15 20:26:59][INFO] i= 4 x: [[-14.   4.]]
[07-15 20:26:59][INFO] i= 5 x: [[  2. -28.]]
[07-15 20:26:59][INFO] i= 6 x: [[34.  4.]]
[07-15 20:26:59][INFO] i= 7 x: [[ 2. 68.]]
[07-15 20:26:59][INFO] i= 8 x: [[-62.   4.]]
[07-15 20:26:59][INFO] i= 9 x: [[   2. -124.]]
[07-15 20:26:59][INFO] i= 10 x: [[130.   4.]]
[07-15 20:26:59][INFO] i= 11 x: [[  2. 260.]]
[07-15 20:26:59][INFO] i= 12 x: [[-254.    4.]]
[07-15 20:26:59][INFO] i= 13 x: [[   2. -508.]]
[07-15 20:26:59][INFO] i= 14 x: [[514.   4.]]
[07-15 20:26:59][INFO] i= 15 x: [[   2. 1028.]]
[07-15 20:26:59][INFO] i= 16 x: [[-1022.     4.]]
[07-15 20:26:59][INFO] i= 17 x: [[ 2.000e+00 -2.044e+03]]
[07-15 20:26:59][INFO] i= 18 x: [[2050.    4.]]
[07-15 20:26:59][INFO] i= 19 x: [[2.0e+00 4.1e+03]]
[07-15 20:26:59][INFO] i= 20 x: [[-4.094e+03  4.000e+00]]
[07-15 20:26:59][INFO] i= 21 x: [[ 2.000e+00 -8.188e+03]]
[07-15 20:26:59][INFO] i= 22 x: [[8.194e+03 4.000e+00]]
[07-15 20:26:59][INFO] i= 23 x: [[2.0000e+00 1.6388e+04]]
[07-15 20:26:59][INFO] i= 24 x: [[-1.6382e+04  4.0000e+00]]
[07-15 20:26:59][INFO] i= 25 x: [[ 2.0000e+00 -3.2764e+04]]
[07-15 20:26:59][INFO] i= 26 x: [[3.277e+04 4.000e+00]]
[07-15 20:26:59][INFO] i= 27 x: [[2.000e+00 6.554e+04]]
[07-15 20:26:59][INFO] i= 28 x: [[-6.5534e+04  4.0000e+00]]
[07-15 20:26:59][INFO] i= 29 x: [[ 2.00000e+00 -1.31068e+05]]
[07-15 20:26:59][INFO] i= 30 x: [[1.31074e+05 4.00000e+00]]
[07-15 20:26:59][INFO] i= 31 x: [[2.00000e+00 2.62148e+05]]
[07-15 20:26:59][INFO] i= 32 x: [[-2.62142e+05  4.00000e+00]]
[07-15 20:26:59][INFO] i= 33 x: [[ 2.00000e+00 -5.24284e+05]]
[07-15 20:26:59][INFO] i= 34 x: [[5.2429e+05 4.0000e+00]]
[07-15 20:26:59][INFO] i= 35 x: [[2.00000e+00 1.04858e+06]]
[07-15 20:26:59][INFO] i= 36 x: [[-1.048574e+06  4.000000e+00]]
[07-15 20:26:59][INFO] i= 37 x: [[ 2.000000e+00 -2.097148e+06]]
[07-15 20:26:59][INFO] i= 38 x: [[2.097154e+06 4.000000e+00]]
[07-15 20:26:59][INFO] i= 39 x: [[2.000000e+00 4.194308e+06]]
[07-15 20:26:59][INFO] i= 40 x: [[-4.194302e+06  4.000000e+00]]
[07-15 20:26:59][INFO] i= 41 x: [[ 2.000000e+00 -8.388604e+06]]
[07-15 20:26:59][INFO] i= 42 x: [[8.38861e+06 4.00000e+00]]
[07-15 20:26:59][INFO] i= 43 x: [[2.000000e+00 1.677722e+07]]
[07-15 20:26:59][INFO] i= 44 x: [[-1.6777214e+07  4.0000000e+00]]
[07-15 20:26:59][INFO] i= 45 x: [[ 2.0000000e+00 -3.3554428e+07]]
[07-15 20:26:59][INFO] i= 46 x: [[3.3554434e+07 4.0000000e+00]]
[07-15 20:26:59][INFO] i= 47 x: [[2.0000000e+00 6.7108868e+07]]
[07-15 20:26:59][INFO] i= 48 x: [[-6.7108862e+07  4.0000000e+00]]
[07-15 20:26:59][INFO] i= 49 x: [[ 2.00000000e+00 -1.34217724e+08]]
```

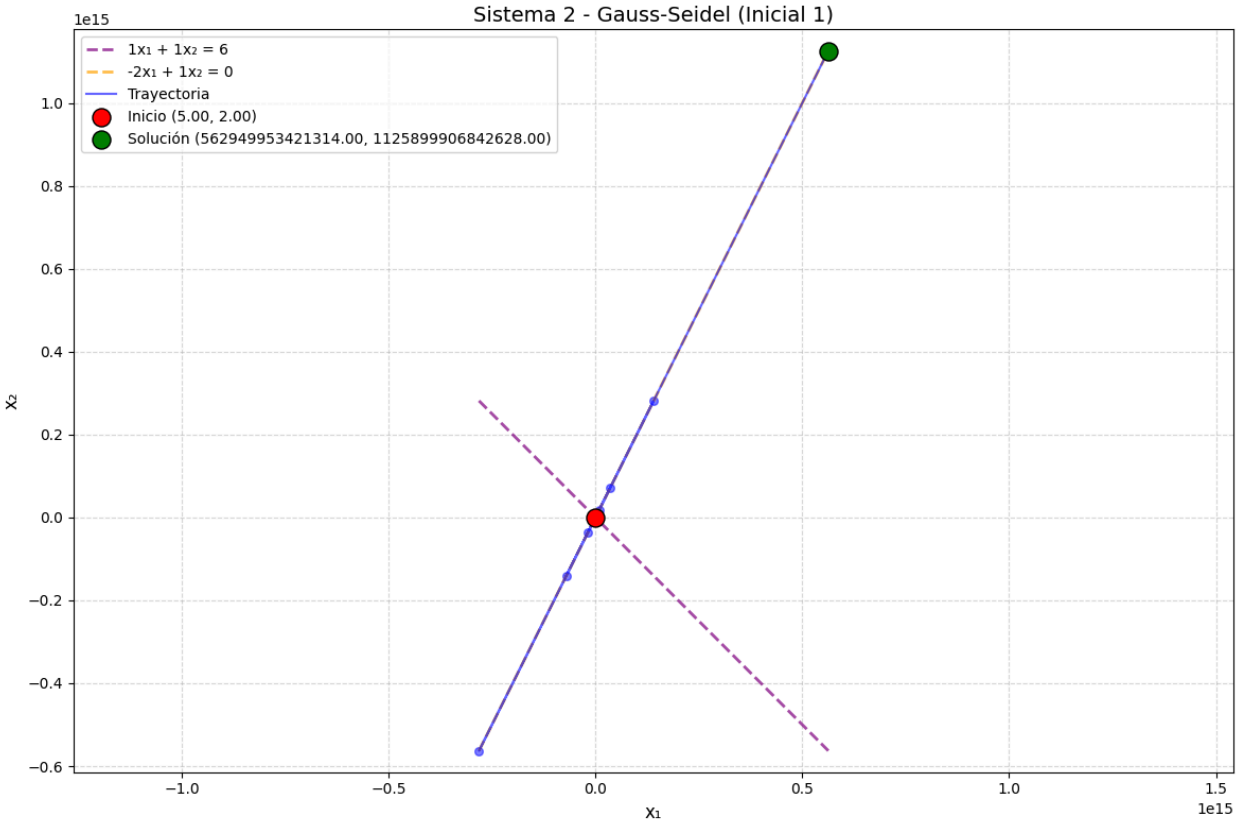



```
Solución encontrada: [2.000000, -134217724.000000]
```

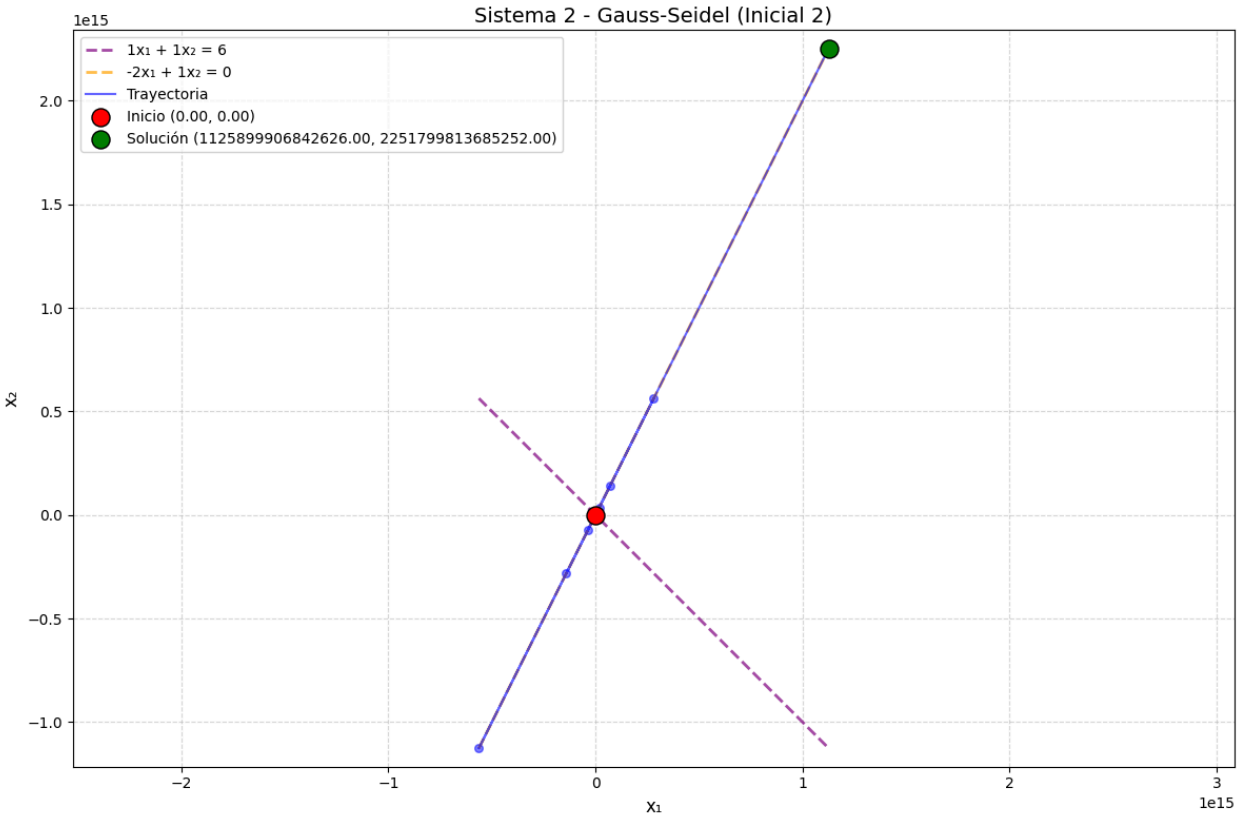
In [33]:

```python
# %% Sistema 2 - GAUSS-SEIDEL
print("\n=== SISTEMA 2 - MÉTODO DE GAUSS-SEIDEL ===")
for i, x0 in enumerate(initial_positions, 1):
    print(f"\nCaso {i}: Posición inicial {x0}")
    try:
        x_gs2, tray_gs2 = gauss_seidel(A=A2, b=b2, x0=x0, tol=1e-6, max_iter=5
        plot_system_with_trajectory(A2, b2, tray_gs2, f"Sistema 2 - Gauss-Seid
        print(f"Solución encontrada: [{x_gs2[0,0]:.6f}, {x_gs2[1,0]:.6f}]")
    except Exception as e:
        print(f"¡Error! El método divergió: {str(e)}")
```

```
=== SISTEMA 2 - MÉTODO DE GAUSS-SEIDEL ===

Caso 1: Posición inicial [5 2]
[07-15 20:27:23][INFO] i= 0 x: [5 2]
[07-15 20:27:23][INFO] i= 1 x: [[4. 8.]]
[07-15 20:27:23][INFO] i= 2 x: [[-2. -4.]]
[07-15 20:27:23][INFO] i= 3 x: [[10. 20.]]
[07-15 20:27:23][INFO] i= 4 x: [[-14. -28.]]
[07-15 20:27:23][INFO] i= 5 x: [[34. 68.]]
[07-15 20:27:23][INFO] i= 6 x: [[ -62. -124.]]
[07-15 20:27:23][INFO] i= 7 x: [[130. 260.]]
[07-15 20:27:23][INFO] i= 8 x: [[-254. -508.]]
[07-15 20:27:23][INFO] i= 9 x: [[ 514. 1028.]]
[07-15 20:27:23][INFO] i= 10 x: [[-1022. -2044.]]
[07-15 20:27:23][INFO] i= 11 x: [[2050. 4100.]]
[07-15 20:27:23][INFO] i= 12 x: [[-4094. -8188.]]
[07-15 20:27:23][INFO] i= 13 x: [[ 8194. 16388.]]
[07-15 20:27:23][INFO] i= 14 x: [[-16382. -32764.]]
[07-15 20:27:23][INFO] i= 15 x: [[32770. 65540.]]
[07-15 20:27:23][INFO] i= 16 x: [[ -65534. -131068.]]
[07-15 20:27:23][INFO] i= 17 x: [[131074. 262148.]]
[07-15 20:27:23][INFO] i= 18 x: [[-262142. -524284.]]
[07-15 20:27:23][INFO] i= 19 x: [[ 524290. 1048580.]]
[07-15 20:27:23][INFO] i= 20 x: [[-1048574. -2097148.]]
[07-15 20:27:23][INFO] i= 21 x: [[2097154. 4194308.]]
[07-15 20:27:23][INFO] i= 22 x: [[-4194302. -8388604.]]
[07-15 20:27:23][INFO] i= 23 x: [[ 8388610. 16777220.]]
[07-15 20:27:23][INFO] i= 24 x: [[-16777214. -33554428.]]
[07-15 20:27:23][INFO] i= 25 x: [[33554434. 67108868.]]
[07-15 20:27:23][INFO] i= 26 x: [[-6.71088620e+07 -1.34217724e+08]]
[07-15 20:27:23][INFO] i= 27 x: [[1.3421773e+08 2.6843546e+08]]
[07-15 20:27:23][INFO] i= 28 x: [[-2.68435454e+08 -5.36870908e+08]]
[07-15 20:27:23][INFO] i= 29 x: [[5.36870914e+08 1.07374183e+09]]
[07-15 20:27:23][INFO] i= 30 x: [[-1.07374182e+09 -2.14748364e+09]]
[07-15 20:27:23][INFO] i= 31 x: [[2.14748365e+09 4.29496730e+09]]
[07-15 20:27:23][INFO] i= 32 x: [[-4.29496729e+09 -8.58993459e+09]]
[07-15 20:27:23][INFO] i= 33 x: [[8.58993459e+09 1.71798692e+10]]
[07-15 20:27:23][INFO] i= 34 x: [[-1.71798692e+10 -3.43597384e+10]]
[07-15 20:27:23][INFO] i= 35 x: [[3.43597384e+10 6.87194767e+10]]
[07-15 20:27:23][INFO] i= 36 x: [[-6.87194767e+10 -1.37438953e+11]]
[07-15 20:27:23][INFO] i= 37 x: [[1.37438953e+11 2.74877907e+11]]
[07-15 20:27:23][INFO] i= 38 x: [[-2.74877907e+11 -5.49755814e+11]]
[07-15 20:27:23][INFO] i= 39 x: [[5.49755814e+11 1.09951163e+12]]
[07-15 20:27:23][INFO] i= 40 x: [[-1.09951163e+12 -2.19902326e+12]]
[07-15 20:27:23][INFO] i= 41 x: [[2.19902326e+12 4.39804651e+12]]
[07-15 20:27:23][INFO] i= 42 x: [[-4.39804651e+12 -8.79609302e+12]]
[07-15 20:27:23][INFO] i= 43 x: [[8.79609302e+12 1.75921860e+13]]
[07-15 20:27:23][INFO] i= 44 x: [[-1.75921860e+13 -3.51843721e+13]]
[07-15 20:27:23][INFO] i= 45 x: [[3.51843721e+13 7.03687442e+13]]
[07-15 20:27:23][INFO] i= 46 x: [[-7.03687442e+13 -1.40737488e+14]]
[07-15 20:27:23][INFO] i= 47 x: [[1.40737488e+14 2.81474977e+14]]
[07-15 20:27:23][INFO] i= 48 x: [[-2.81474977e+14 -5.62949953e+14]]
[07-15 20:27:23][INFO] i= 49 x: [[5.62949953e+14 1.12589991e+15]]
```

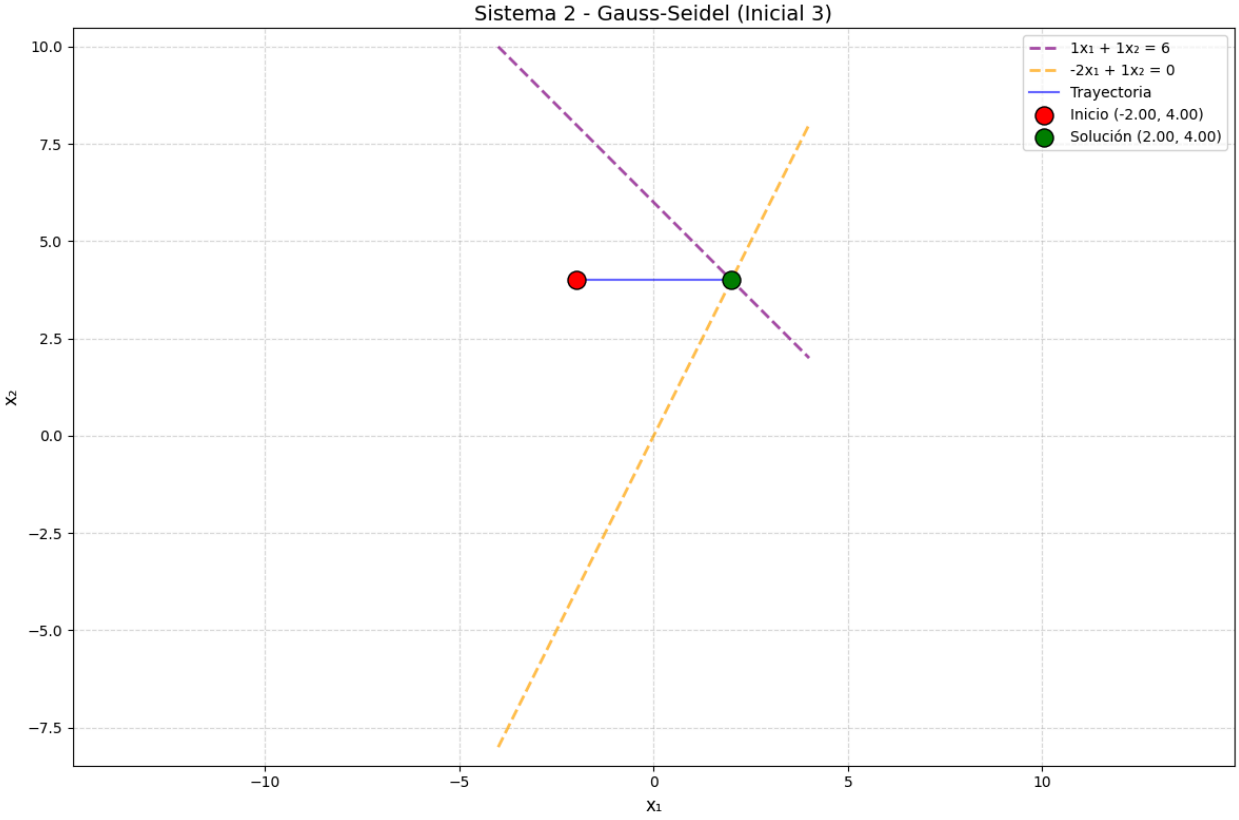

Sistema 2 - Gauss-Seidel (Inicial 1)
1e15
1x₁ + 1x₂ = 6
-2x₁ + 1x₂ = 0
Trayectoria
Inicio (5.00, 2.00)
Solución (562949953421314.00, 1125899906842628.00)
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
x₂
−1.0
−0.5
0.0
0.5
1.0
1.5
1e15
x₁

```
Solución encontrada: [562949953421314.000000, 1125899906842628.000000]

Caso 2: Posición inicial [0 0]
[07-15 20:27:24][INFO] i= 0 x: [0 0]
[07-15 20:27:24][INFO] i= 1 x: [[ 6. 12.]]
[07-15 20:27:24][INFO] i= 2 x: [[ -6. -12.]]
[07-15 20:27:24][INFO] i= 3 x: [[18. 36.]]
[07-15 20:27:24][INFO] i= 4 x: [[-30. -60.]]
[07-15 20:27:24][INFO] i= 5 x: [[ 66. 132.]]
[07-15 20:27:24][INFO] i= 6 x: [[-126. -252.]]
[07-15 20:27:24][INFO] i= 7 x: [[258. 516.]]
[07-15 20:27:24][INFO] i= 8 x: [[ -510. -1020.]]
[07-15 20:27:24][INFO] i= 9 x: [[1026. 2052.]]
[07-15 20:27:24][INFO] i= 10 x: [[-2046. -4092.]]
[07-15 20:27:24][INFO] i= 11 x: [[4098. 8196.]]
[07-15 20:27:24][INFO] i= 12 x: [[ -8190. -16380.]]
[07-15 20:27:24][INFO] i= 13 x: [[16386. 32772.]]
[07-15 20:27:24][INFO] i= 14 x: [[-32766. -65532.]]
[07-15 20:27:24][INFO] i= 15 x: [[ 65538. 131076.]]
[07-15 20:27:24][INFO] i= 16 x: [[-131070. -262140.]]
[07-15 20:27:24][INFO] i= 17 x: [[262146. 524292.]]
[07-15 20:27:24][INFO] i= 18 x: [[ -524286. -1048572.]]
[07-15 20:27:24][INFO] i= 19 x: [[1048578. 2097156.]]
[07-15 20:27:24][INFO] i= 20 x: [[-2097150. -4194300.]]
[07-15 20:27:24][INFO] i= 21 x: [[4194306. 8388612.]]
[07-15 20:27:24][INFO] i= 22 x: [[ -8388606. -16777212.]]
[07-15 20:27:24][INFO] i= 23 x: [[16777218. 33554436.]]
[07-15 20:27:24][INFO] i= 24 x: [[-33554430. -67108860.]]
[07-15 20:27:24][INFO] i= 25 x: [[6.71088660e+07 1.34217732e+08]]
[07-15 20:27:24][INFO] i= 26 x: [[-1.34217726e+08 -2.68435452e+08]]
[07-15 20:27:24][INFO] i= 27 x: [[2.68435458e+08 5.36870916e+08]]
[07-15 20:27:24][INFO] i= 28 x: [[-5.36870910e+08 -1.07374182e+09]]
[07-15 20:27:24][INFO] i= 29 x: [[1.07374183e+09 2.14748365e+09]]
[07-15 20:27:24][INFO] i= 30 x: [[-2.14748365e+09 -4.29496729e+09]]
[07-15 20:27:24][INFO] i= 31 x: [[4.2949673e+09 8.5899346e+09]]
[07-15 20:27:24][INFO] i= 32 x: [[-8.58993459e+09 -1.71798692e+10]]
[07-15 20:27:24][INFO] i= 33 x: [[1.71798692e+10 3.43597384e+10]]
[07-15 20:27:24][INFO] i= 34 x: [[-3.43597384e+10 -6.87194767e+10]]
[07-15 20:27:24][INFO] i= 35 x: [[6.87194767e+10 1.37438953e+11]]
[07-15 20:27:24][INFO] i= 36 x: [[-1.37438953e+11 -2.74877907e+11]]
[07-15 20:27:24][INFO] i= 37 x: [[2.74877907e+11 5.49755814e+11]]
[07-15 20:27:24][INFO] i= 38 x: [[-5.49755814e+11 -1.09951163e+12]]
[07-15 20:27:24][INFO] i= 39 x: [[1.09951163e+12 2.19902326e+12]]
[07-15 20:27:24][INFO] i= 40 x: [[-2.19902326e+12 -4.39804651e+12]]
[07-15 20:27:24][INFO] i= 41 x: [[4.39804651e+12 8.79609302e+12]]
[07-15 20:27:24][INFO] i= 42 x: [[-8.79609302e+12 -1.75921860e+13]]
[07-15 20:27:24][INFO] i= 43 x: [[1.75921860e+13 3.51843721e+13]]
[07-15 20:27:24][INFO] i= 44 x: [[-3.51843721e+13 -7.03687442e+13]]
[07-15 20:27:24][INFO] i= 45 x: [[7.03687442e+13 1.40737488e+14]]
[07-15 20:27:24][INFO] i= 46 x: [[-1.40737488e+14 -2.81474977e+14]]
[07-15 20:27:24][INFO] i= 47 x: [[2.81474977e+14 5.62949953e+14]]
[07-15 20:27:24][INFO] i= 48 x: [[-5.62949953e+14 -1.12589991e+15]]
[07-15 20:27:24][INFO] i= 49 x: [[1.12589991e+15 2.25179981e+15]]
```



Solución encontrada: [1125899906842626.000000, 2251799813685252.000000]

Caso 3: Posición inicial [-2 4]
[07-15 20:27:25][INFO] i= 0 x: [-2 4]
[07-15 20:27:25][INFO] i= 1 x: [[2. 4.]]



```
Solución encontrada: [2.000000, 4.000000]
```

Animaciones:

Las animaciones se encuentran en el repositorio en la carpeta animations..