Valuation of
Natural Gas Contracts and Storages

Based on Stochastic Optimization (Scenario Tree)
Under Consideration of Asset Backed Trading

February 2014

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Overview

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Decision Trees

- Founded by Dr. Georg Ostermaier in 2006
- Spin Off from the Institute for Operations Research und Computational Finance at the university of St.Gallen (Switzerland)
- Cooperation with the IOR/CF
- Cooperation with Imperial College (London, Dr. Daniel Kuhn)
- Employees:
  - Dr. Georg Ostermaier (Mathematical Modeling, Support, Software Development)
  - Karsten Hentsch (Support, Software Development)
  - Ines Weber (Mathematical Modeling, Software Development)
  - Jan Hofmann (Mathematical Modeling, Software Development)
  - Ömer Kuzugüden (Mathematical Modeling, Software Development)
  - Klaus Rossmann (Mathematical Modeling, Software Development)
  - Student workers

- Software and Consulting for the introduction of advanced Stochastic Optimization into the Energy Industry

- Added value of Stochastic versus Deterministic Optimization

Solid customer base

decision trees
stochastic optimization

Trianel
Salzburg AG
mark E
e&t
Verbund
OMV
Statkraft
powel
CEL
Süd West Strom
PÖYRY
DONG energy
ExxonMobil

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DT.Energy Suite

- DT.Analytics
  - Stochastic Processes for Power, Gas and CO_2, demands
  - Parameter Estimation for Stochastic processes
  - Monte Carlo Scenario Generation
  - Scenario Tree Generation

- DT.PFC
  - Calculation of Daily Price Forward Curves for gas markets
  - Calculation of Hourly Price Forward Curves for power markets

- DT.Plant
  - Stochastic and deterministic valuation and operation planning of thermal power plants
  - Stochastic and deterministic asset portfolio optimization (power plants, gas storages, steam generators, gas supply contracts etc.)

- DT.Storage
  - Stochastic and deterministic valuation and operation planning for
    - Gas storages, gas contracts
    - Gas procurement portfolios, gas trading portfolios

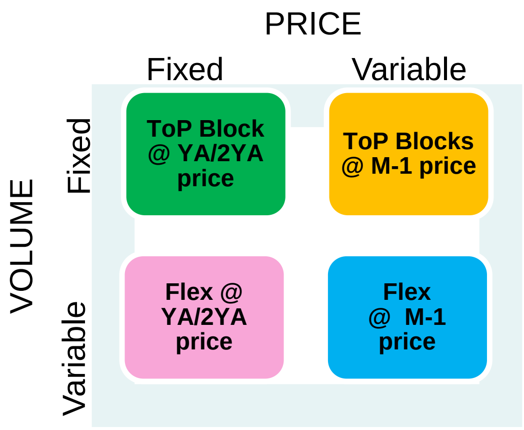
- DT.Hydro
  - Stochastic and deterministic valuation and operation planning of hydro power systems (pumped storage systems)
  - Cross Market Hydro Power optimization (ancillary services, spot, intra day)

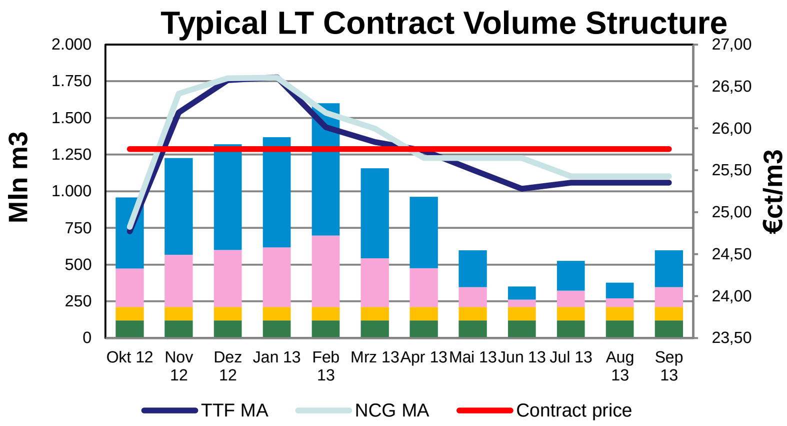
Part I : Introduction

Business Needs

- Legacy Long Term Contracts historically offered flexibility to accommodate physical demand swing
- Flexibility was valued at price levels based on storage tariffs (S/W spreads)
- With EU ‘long’ on storage capacity, S/W spreads have fallen, putting legacy capacity charges under pressure
- Price renegotiations leading to substantial levels of hub indexation (reducing the contract price/commodity price) increase the opportunity to optimize the contracts against the trading market
- Contract holders more and more maximise the value of swing contracts value beyond the traditional S/W optimization by trading in the market
- Increasingly, flexibility sellers adjust pricing to reflect changing market conditions and maximize value obtained, capturing part of buyer’s contract flex optimization value
- Sophisticated optimization software is widely used amongst sellers / buyers

Contract Flexibility





Flex Value		Optimization Potential Examples
Daily/Weekly		*e.g Any indexed priced contract will provide the opportunity to optimize in the DA market*
Monthly/Quarterly		*Optimization between Months and Quarters for a season/year priced contract*
Seasonal		*Optimization between Seasons*
Across Years		*e.g Make-up/Carry Forward rights with attached penalties*
Overall (ACQ vs ToP)		

Contract Flexibility

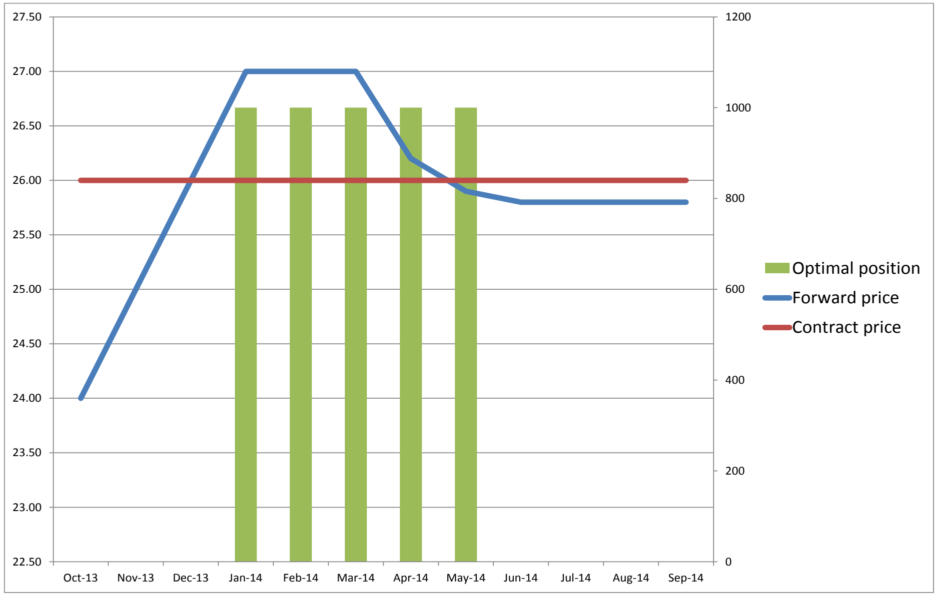
Issues

- Mixed bag of indices and volume takes increases the complexity of flex valuation of these contracts
- Total Flex Value of contract is not necessarily the addition of all of the above; overlapping constraints make these not mutually exclusive
- Legacy analysis relies on:
  - Limited Dataset :
    - i.e S/W spread of last x years
    - DA vs M-1 of last x years
  - Perfect Hindsight
  - Single value planning basis price (Represents average long-term value, Provides baseline value)

Intrinsic / Deterministic Valuation

Minimum amount of money locked in by maximising spread between market price and contract price

- Market Price is obtained directly from traded products in the gas market **at one point in time**
- Optimal strategy is based on today's forward curve
- Output is a set of optimal forward contracts covering valuation period
- Intrinsic value can be based on quoted products, a monthly price forward curve (MPFC) or a daily price forward curve (DPFC)

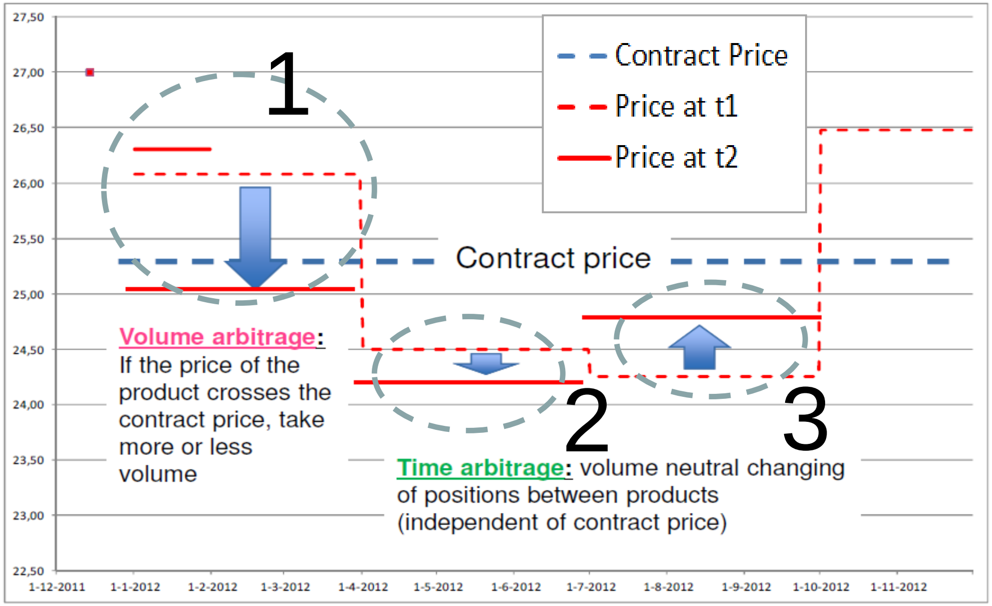


Extrinsic Valuation

Additional value created by changing off take pattern when **market prices change**

- Volume Arbitrage: take more/less over contract period
- Time Arbitrage: volume neutral changing of off take

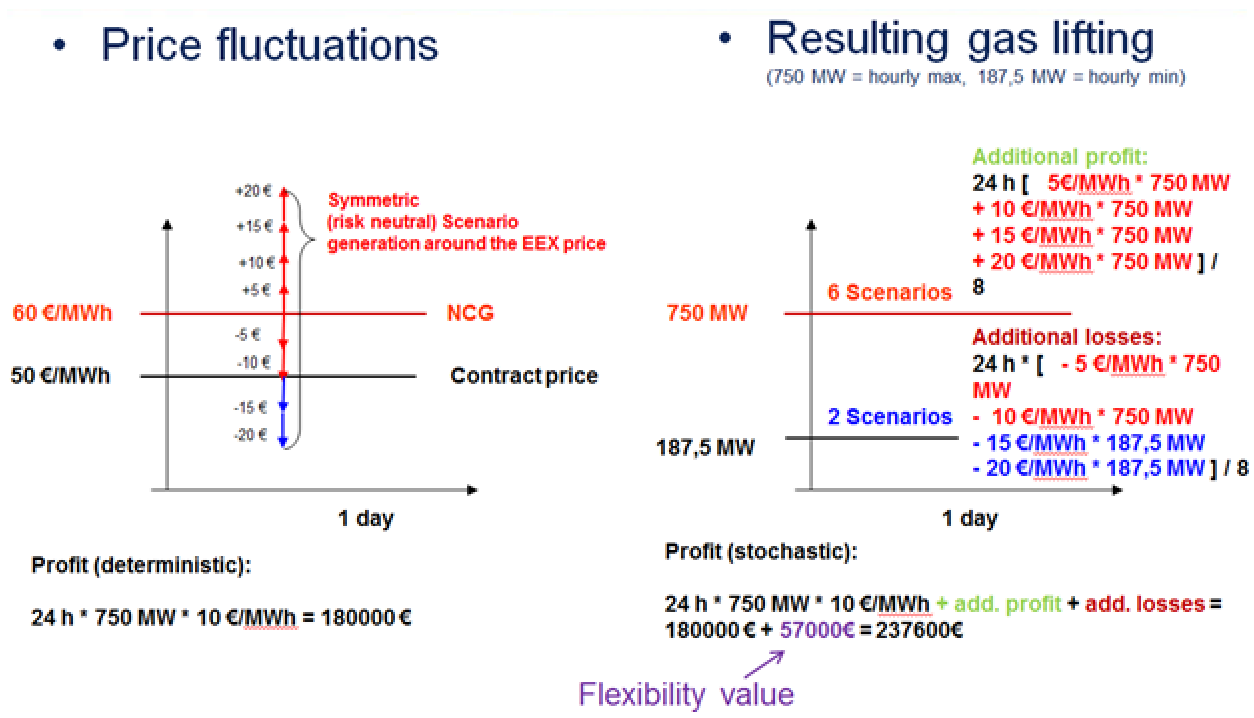
Common trading practise for flexible contract holders



The generation of multiple scenarios of price movements allows for the valuation of extrinsic value

Calculation of Intrinsic/Extrinsic Value

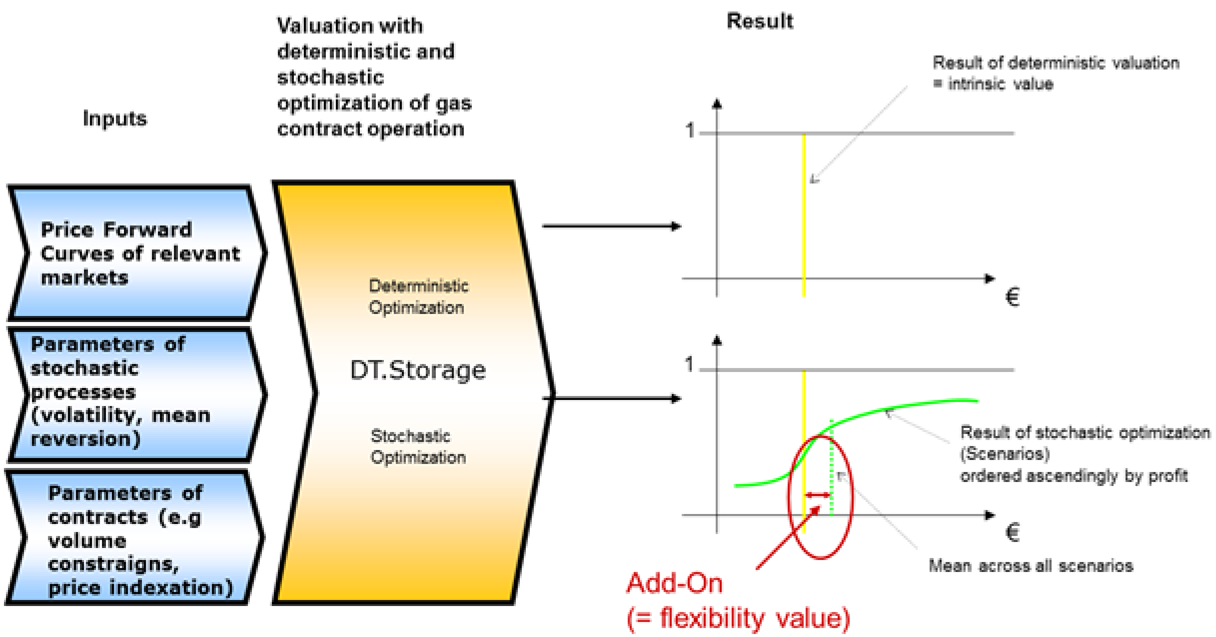
- From intrinsic valuation point of view, profit achieved is 'just' spread between quoting market price (60 €/MWh) and contract price (50 €/MWh) times the volume sold, i.e 24 h x 750 MW x 10€/MWh
- However, when market price fluctuates around expected market price, contract is fully withdrawn in just six out of the eight scenarios, because in two of the scenarios the spread is negative. The average profit across all eight scenarios is higher than the intrinsic value



decision trees
stochastic optimization

Tool Workflow & Functionality

- Intrinsic valuation results in one contract value (yellow vertical line). Extrinsic valuation results in a whole set of contract values, each of which belonging to an individual scenario of market and contract price evolutions resulting in a green line, which is also called “profit distribution curve”.
- Difference between intrinsic and extrinsic value is denoted as the flexibility value or time value. As flexibility adds value to a gas supply contract, the flexibility value is always positive, or – in other words – the extrinsic value is always higher than the intrinsic value.



Types of Trading

- **Asset-Backed Trading**
  - Selling contract gas to forward market and potentially re-buy and re-sell it is a form of “Asset backed trading”
    - Different from speculative trading, which involves opening positions in market and speculating on favourable price evolutions to close position later on with a profit
  - All asset-backed trading within the DT.Energy contract valuation needs to respect the constraint that no more gas is sold to the market than can be delivered from the contract
  - The trading strategy can be set to “back-to-back”, i.e. no gas can be re-purchased without selling the same amount to the market at the same time
  - The trading strategy can be set to “open positions”, which allows for not buying and selling at the same time

- **Spot Trading**
  - A substantial share of the flexibility value of a contract comes from spot trading

Types of Trading

- **Forward Trading**
  - Selling contract gas to forward market upfront is limiting risk of making losses from unfavourable spot price evolutions in future
  - Following limitations can be set up:
    - Allowed trading period for each individual product
    - Maximum quantity per trade per day for each product for sales and purchases
    - Maximum long and short position for each product (cumulative quantity over all trades)
    - Maximum daily quantity across overlapping forward products
    - Maximum overall forward sales quantity is limited to the ACQ of the contract.

- **Unwinding**
  - An “Unwinding” algorithm is implemented in the DT.Energy model to capture value of selling and re-buying forward positions – potentially multiple times – before delivery period of relevant forward products

Main Tool Functional Features

- Contract Duration
- Minimum and Maximum Daily Volume Constraints
- Multiple overlapping Take-or-Pay and Maximum Volume Constraints
- Make-up and Carry-Forward with fixed and indexed penalization
- Contract / Commodity Price Setup
- Multiple markets Valuation – Delivery Point Flexibility
- Roll-out of non-trading products
- Scenario Analysis
- Daily Shaping Curve Regression analysis
- Parameter Estimation of gas and oil markets from Historic prices

Part II : THEORY OF MATHEMATICAL APPROACHES

Stochastic Optimisation

- Methodology to value flexibility of gas is referred to as **'Stochastic Optimization'**
- Stochastic processes are set-up based on random walks, which are iteratively and randomly sampled series of numbers out of a distribution
  - Refers to process of generating multiple price scenarios (both for market and contract prices)
  - Parameters (volatility, mean reversion etc) that enter stochastic processes define dynamics to what extent scenarios fluctuate
- Aims to **maximize average value of flexibility** obtained from a swing contract based on knowledge of historical and today's prices
  - Does not give **one** optimum value but a distribution of possible profits

Step 1. Price Pathing

- ✓ Simulation of multiple price scenarios
- ✓ Prices can take different paths and still have the same long-term average
- ✓ The path taken impacts value
- ✓ Simulating price paths increases understanding of potential outcomes

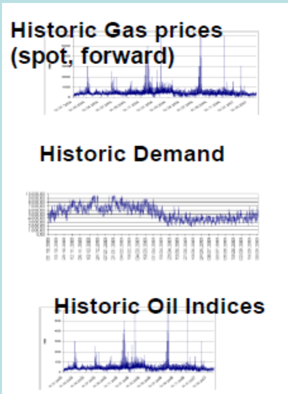
IN CONJUCTION WITH

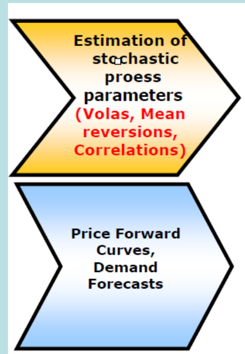
Step 2. Volume Allocation

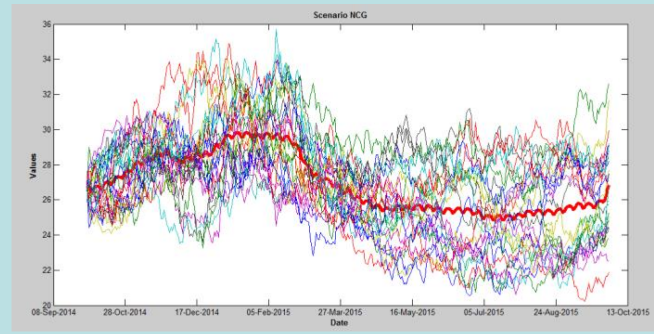
- ✓ Optimum Volume allocation/'takes' throughout contract duration

Tool Building Blocks

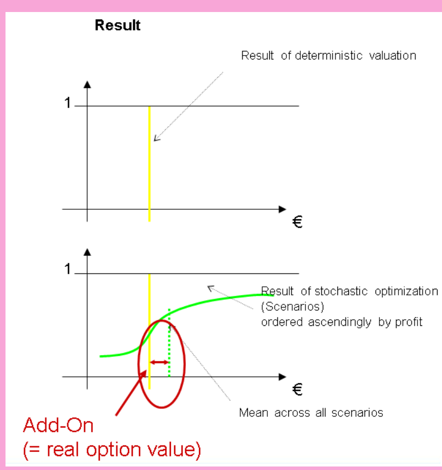
Step 1 : Price Pathing - PARAMETER based price scenario development

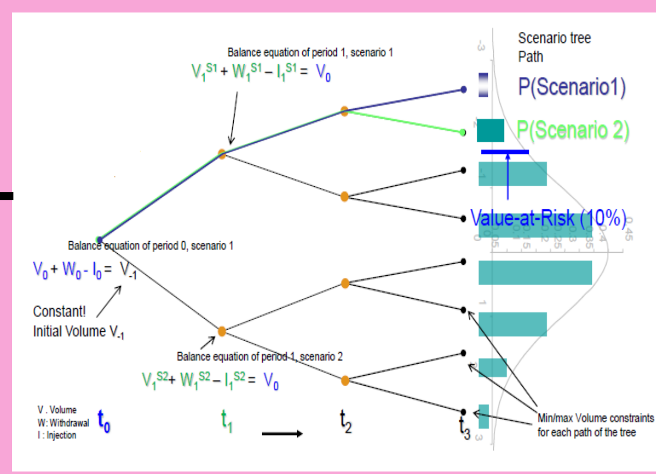






Step 2 : Volume allocation - based on average optimal decisions



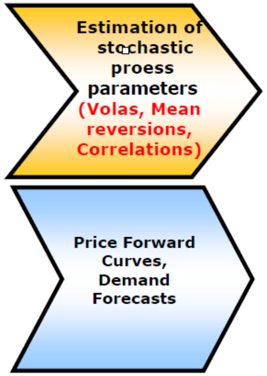


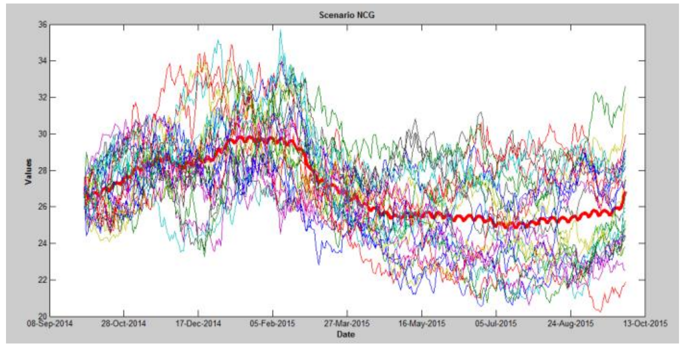
Contract Constraints

Solver (CPLEX™)

decision trees
stochastic optimization

Price Pathing





The current forward quotations are the baseline for the scenario generation. The scenarios are generated in such a way that for every day the average over all scenarios is equal the forward curve, thus satisfying the 'arbitrage free' condition

Pilipovic Model

Monte Carlo Scenario Generation

- **Pilipovic** price process aims to model (scenario) deviation from forward curve using following logic
  - For a given day, forward settlement prices are known and fixed, and thus the mean value to which the spot price scenario is supposed to revert to is also fixed
  - Tomorrow however the expected value of the forward curve will have changed and thus the spot prices will be affected by the uncertainty of this mean-value as well
- Purpose of **Monte Carlo scenario generation** is to produce sufficiently many scenarios of joint (correlated) evolutions of forward products quotes in future

Stochastic Pilipovic Process

Mean Reversion

Short Term Volatility

$$dG_t^{(s/w)_t} = \boxed{\alpha^{(s/w)}(Y_t^G - G_t^{(s/w)})dt} + \boxed{\sigma_G^{(s/w)} dW_t^{(s/w)}}$$

$$\boxed{dY_t^{(G)} = \nu^{(G)} dW_T^{G_2}}$$

Long Term Evolution

$$\longrightarrow G(t) = G(0)\exp(G_t(t))$$

G = Simulated Deviation of Spot Price

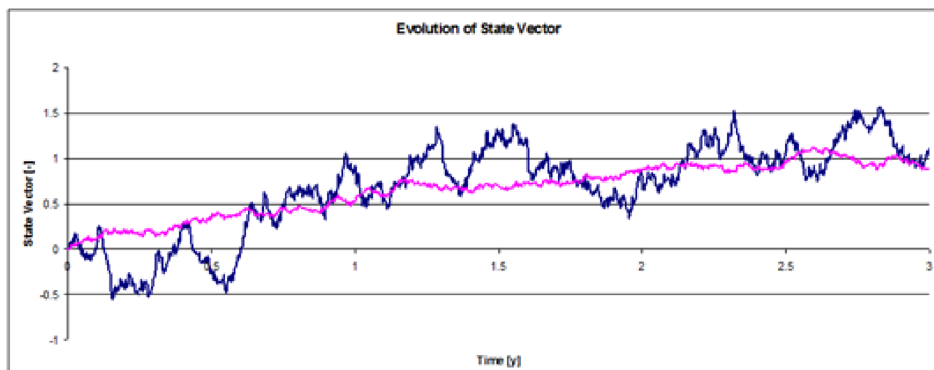
G = Simulated Deviation of Spot Price

$\alpha^{(S/W)}$	mean-reversion of gas spotprices in summer- and winter-band, resp.
$\sigma^{(S/W)}$	volatility of gas spotprices in summer- and winter-band, resp
$\nu^{(G)}$	volatility of long-term gas price evolution
$dW_t^{(s/w)}$	increment of uncorrelated standard brownian motions in summer- and winter-band for gas prices
$dW_t^{(G_2)}$	increment of a standard brownian motion for the long-term gasprice process

- Equation is referred to as a 2-risk Factor Stochastic Pilipovic Process because the simulated deviation of the Spot Price depends on a mean reversion and a short term volatility

$dY_t^{(G)}$ Deviation form Long Term Price

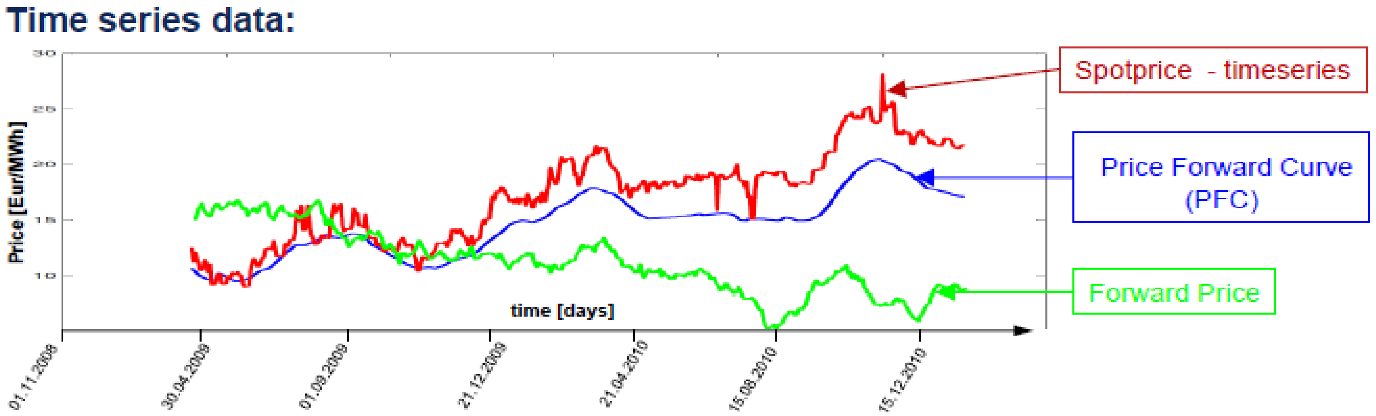
$dG_t^{(s/w)_t}$ Simulated Deviation of Spot Price



decision trees stochastic optimization

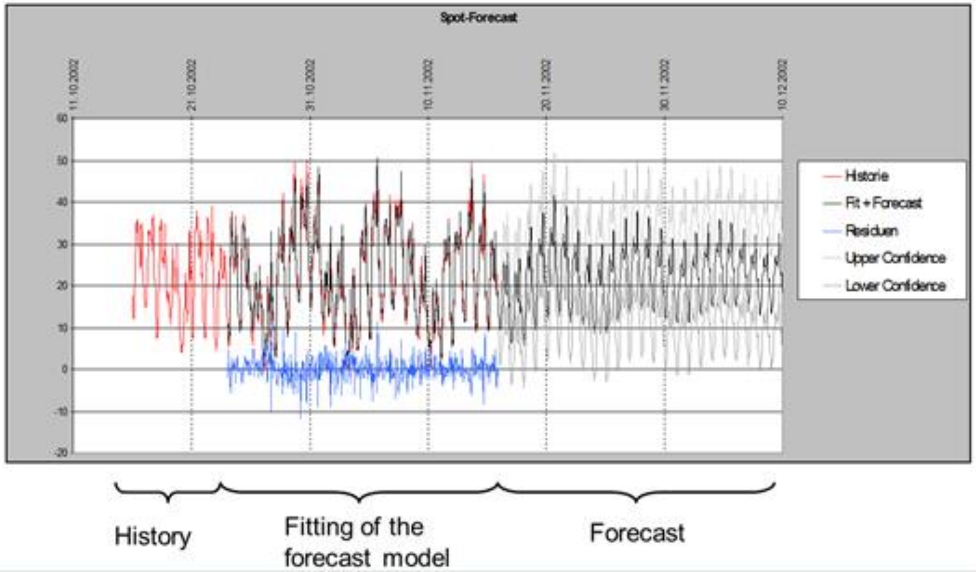
Parameter Estimation

- Aims to provide parameters that enter into stochastic differential equations (Pilipovic equation) describing natural gas prices on different hubs, oil and other commodity prices
  - Assuming that future is not much unlike the past, parameters can be extracted from historical prices
- Parameters to be estimated
  - Short Term Mean Reversion $\alpha^{(s/w)}$
  - Spot Volatility $\sigma_G^{(s/w)}$
  - Long Term Mean Reversion $\nu^{(G)}$
- Input curves required for Parameter Estimation
  - Historic spot price curve (red line)
  - Historic Price Forward Curve (blue line)
  - Historic quote of a selected forward product (green line)



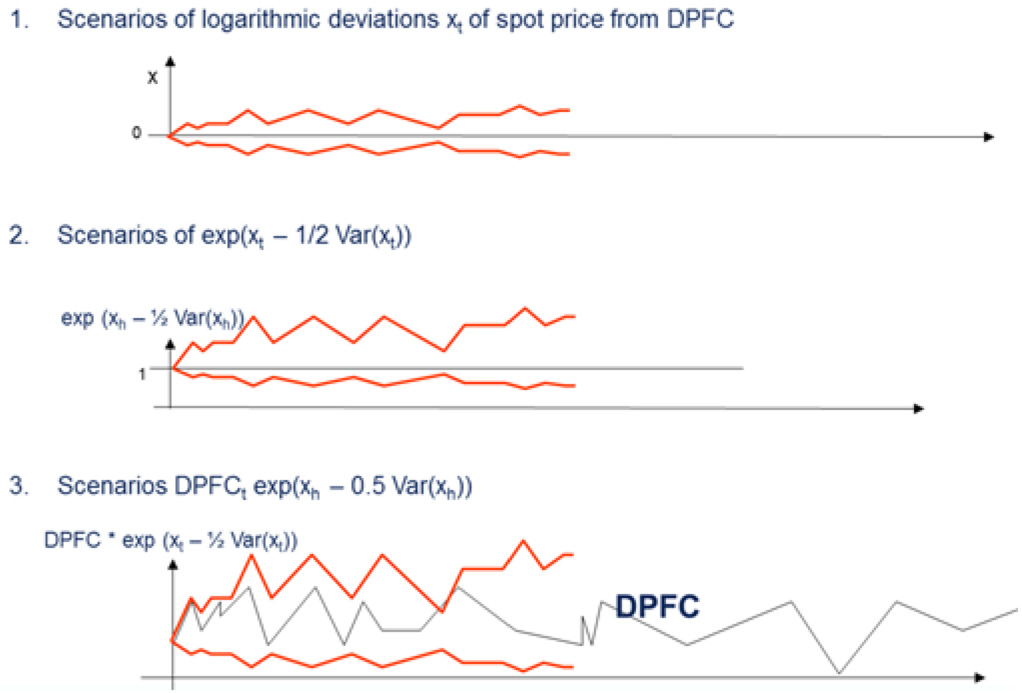
Daily Price Forward Curve (DPFC) creation

- Starting point for creation of a daily price forward curve (DPFC) is a daily spot price shaping curve
  - As daily shaping of future spot prices is hard to predict, the basic approach here is again to assume that daily shaping in the future is as it was in the past
- Regression model to derive the daily price shaping curve is a multivariate regression model, using historic gas price a historic temperature curve as exogenous inputs
- After shaping curve is formed it needs to be scaled to the Price Forward market quotes which will result in a daily granularity scaled forward curve.



Monte Carlo Scenario Generation

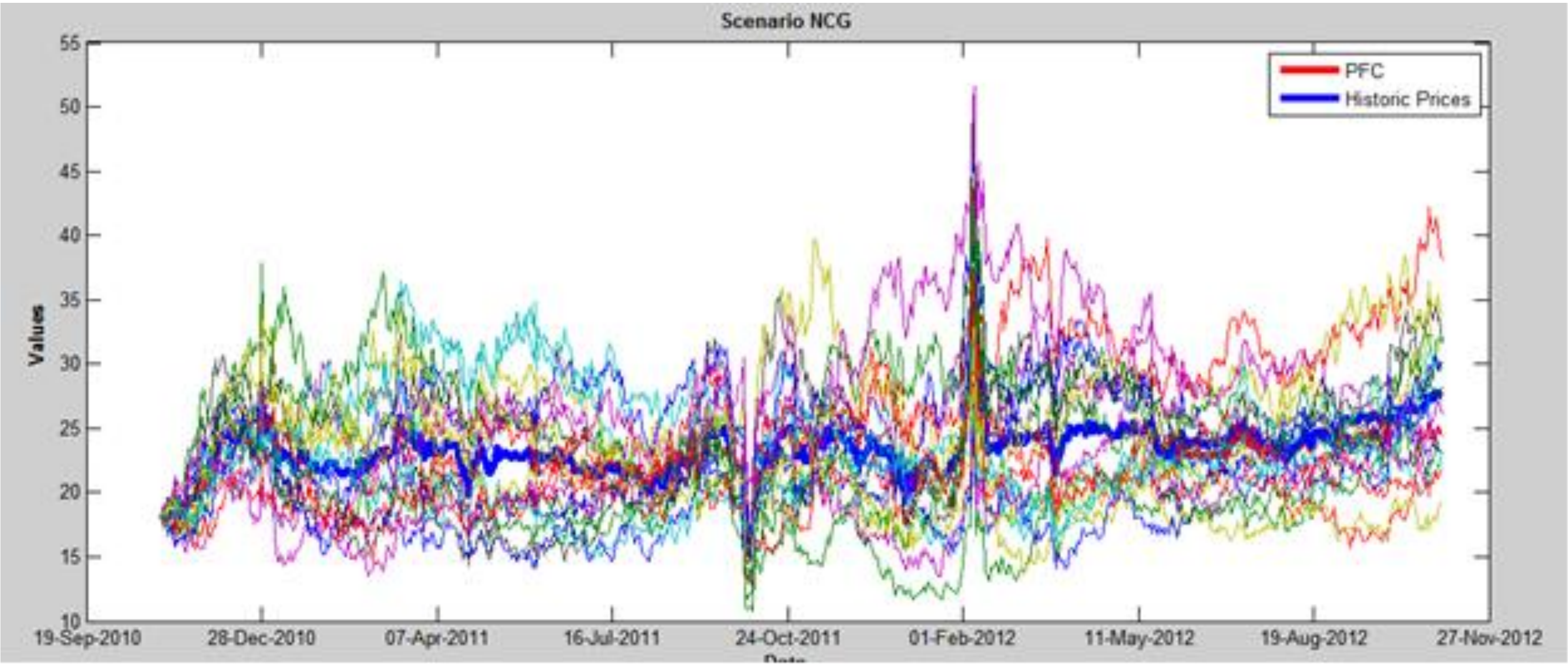
- Purpose of Monte Carlo scenario generation is to produce sufficiently many scenarios of joint (correlated) evolutions of forward products quotes in future
  - Note: if correlations between product prices other than 1, it can occasionally occur that two product prices cross each other, known as flipping prices
- Monte Carlo scenario generation principally runs in three steps, as shown in



- **Step 1:** scenarios of logarithmic deviations of spot price from DPFC are generated (Pilipovic)
- **Step 2:** deviations are taken into exponential function, by which factors of deviation from DPFC are generated.
  - Factor 0.5 $Var(x_t)$ is needed to make sure that average of all deviation factor scenarios is still 1, which means in average all scenarios meet DPFC
- **Step 3:** DPFC is multiplied with deviation factor scenarios, which results in scenarios of gas spot prices

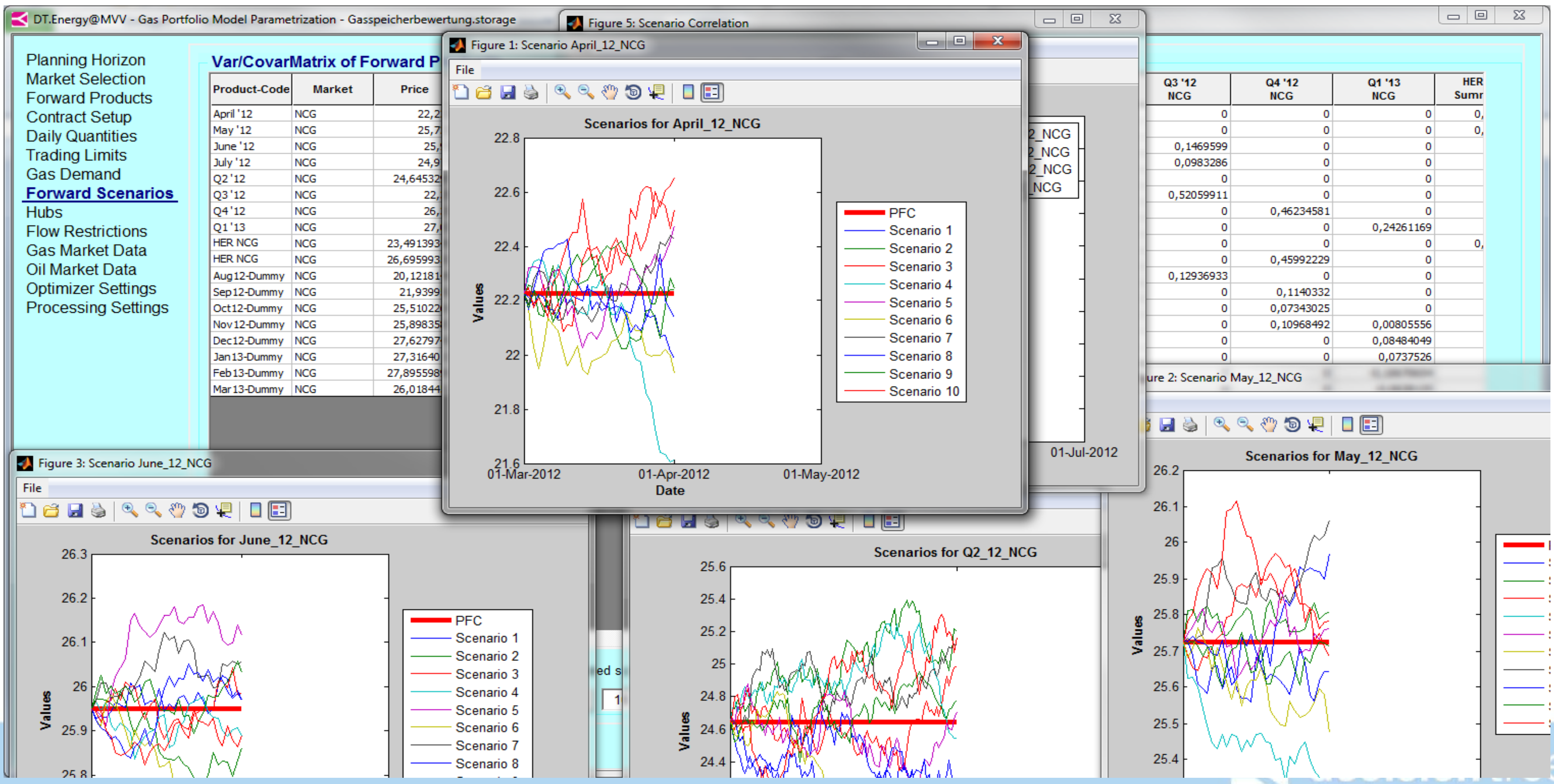
MC Scenario Generation Spot - example

- Exemplary result of a MC generation of 20 single path scenarios of German NCG gas spot price
- Seasonal behaviour of the HPFC (Historic Price Forward Curve) is maintained in the scenarios



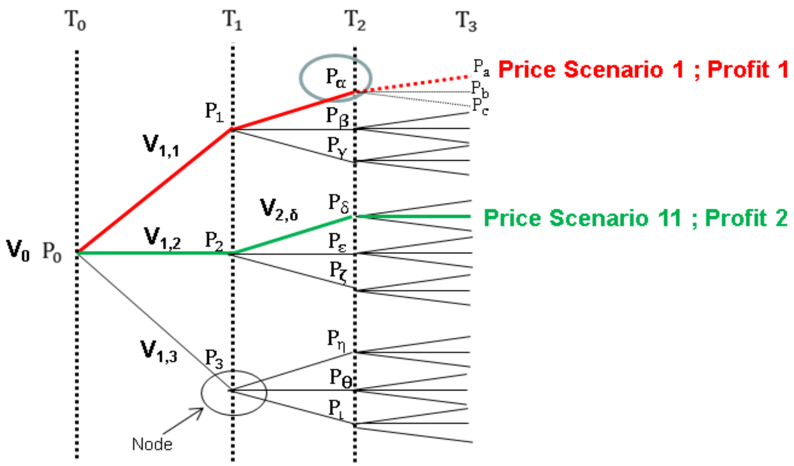
MC Scenario Generation Forward - example

- Exemplary result of a MC generation of 10 single path scenarios of German NCG gas forward prices
- Individual scenario generation for each tradable product considering all correlations
- Based on multidimensional stochastic process, VaR/Covar-Matrix estimated from past price quotations



Tree Approach

- Create a discrete tree that matches price behaviour as observed in all generated price scenarios
- Contract duration is split into relevant time periods which form ‘nodes’ of the tree (i.e. tree branches)
- Each node is associated with a price, derived from the price scenarios
- At each node a decision needs to be made on quantity to be lifted under contract and each node branches to 3 equal probability price scenarios every time
- Contract constraints (like max/min DCQ, ACQ, ToP) are fed into optimization model and need to be met
- Solving a decision tree is a linear program example whereby volumes are allocated such that average of all profits at the end of the tree is maximized
- Profit results are equal in probability, and form a cumulative probability distribution function
- Max Expected Profit is calculated at every node for every scenario path on the tree and are summed to give the maximum expected profit for each scenario (Σprofit 1,1 = Price 1,1 x Volume 1,1)



At time T_2 and price P_α, there are 3 possible price outcomes forecast at time T_3, all of which can occur with equal probability:
$E(P_{a,T_3}|P_{\alpha,T_2}) = E(P_{b,T_3}|P_{\alpha,T_2}) = E(P_{cT_3}|P_{\alpha,T_2}) = \frac{1}{3}$

Variable	Symbol
Time period	T
Price	P
Volume	V

Numerical Example of Tree Optimization

- Differences between Monte Carlo simulation, tree based stochastic optimization and deterministic optimization of a (very simple) swing option
  - Note: Perfect foresight in each of four MC Scenarios leads to an over estimation of the value of swing option

Comparison of the valuation of a (very simple) Swing Option by MC-Simulation, Stochastic and Deterministic Optimization

Number of exercise periods: 3
Max Power: 20 MW
Max Energy: 30 MWh
Red: Prices
Green: optimized decision on exercise (generation)

MC-Simulation:

Period 1		*Period 2*		*Period 3*		*cumul. Energye*	*cumul. Profit*
50	0	55	10	90	20	30	2350
50	0	55	20	52.5	10	30	1625
50	20	48	0	47.5	10	30	1475
50	20	48	10	41	0	30	1480

Over estimation of Value due to perfect foresight in each scenario

Average Profit across all paths: **1732.5**

(Wait and See)

Expected Value of Perfect Information (EVPI)

Stochastic Optimization (Scenario Tree with path dependent exercise):

				90	20	30	2300
		55	0				
				52.5	20	30	1550
50	10						
				47.5	0	30	1460
		48	20				
				41	0	30	1460

Average Profit across all paths: **1692.5**

(Here and Now)

Vaulue of the Stochastic Solution (VSS)

Intrinsic/Deterministic Optimization

50	0	51.5	10	57.75	20	30

Average Profit across all paths: **1670**

Intrinsic Value

(Expected Value)

Under estimation due to neglection of volatility

Contract volume constraints, Trading constraints

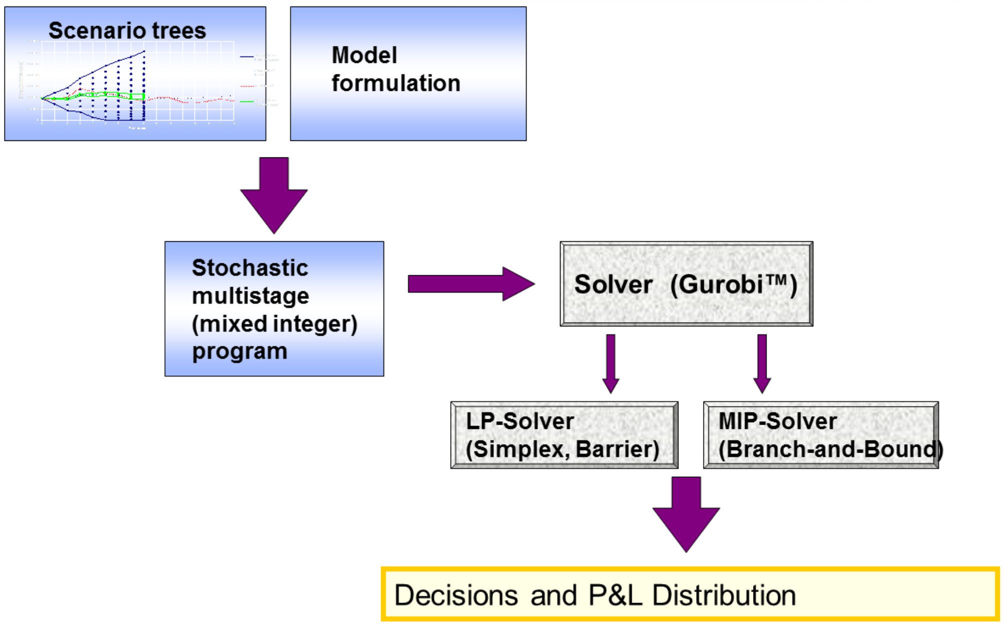
- Daily minimum and daily maximum
- Time integral volume constraints
  - ACC and Take-or-Pay
  - Quarterly, monthly, seasonal volume constraints
  - Arbitrary time integral volume constraints
  - Multiple years volume constraints
- Make up and carry forward
  - Penalization with fixed price
  - Penalization with indexed price (will be finished in March 2014)

- Trading constraints related to the contract
  - Overall Forward short position volume (MWh) cannot be greater than the time integral volume constraints allow for (ACQ)
  - Overall Forward short position (MW) cannot be greater than the maximum DCQ (overlapping products such as quarters and months

- Trading constraints
  - Limitation of daily trade volume (buys and sells) per product
  - Limitation of maximum cumulated long and short position
  - Back-to-Back Trading or Trading strategy with open positions (buy back but not sell at the same time)
  - Limitation of trading period for each product

Technical Implementation (solving the tree)

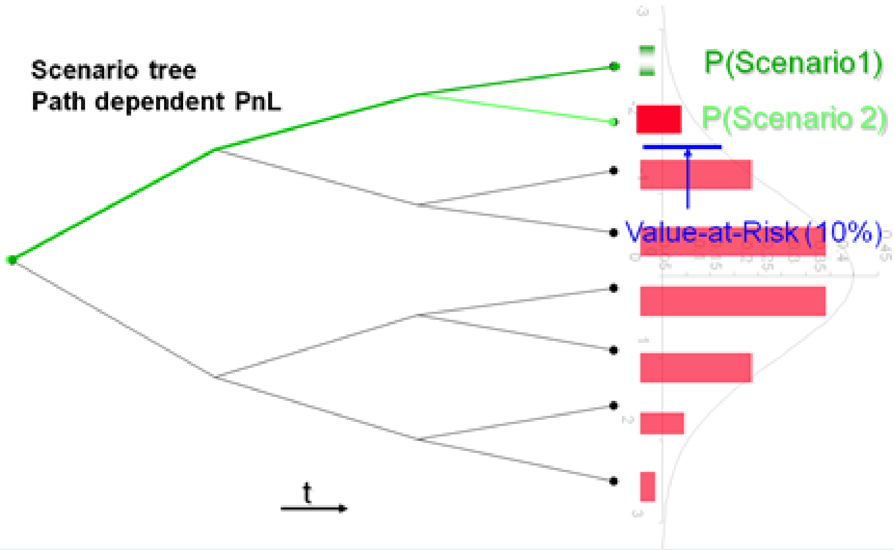
- Technical implementation of the stochastic optimization is based on a C++ optimization kernel
- The principal components of the C++ stochastic optimization kernel are the scenario tree generator, the analytical model formulation, the set-up of the multistage stochastic optimization problem, its solution by Gurobi and finally the provision of results

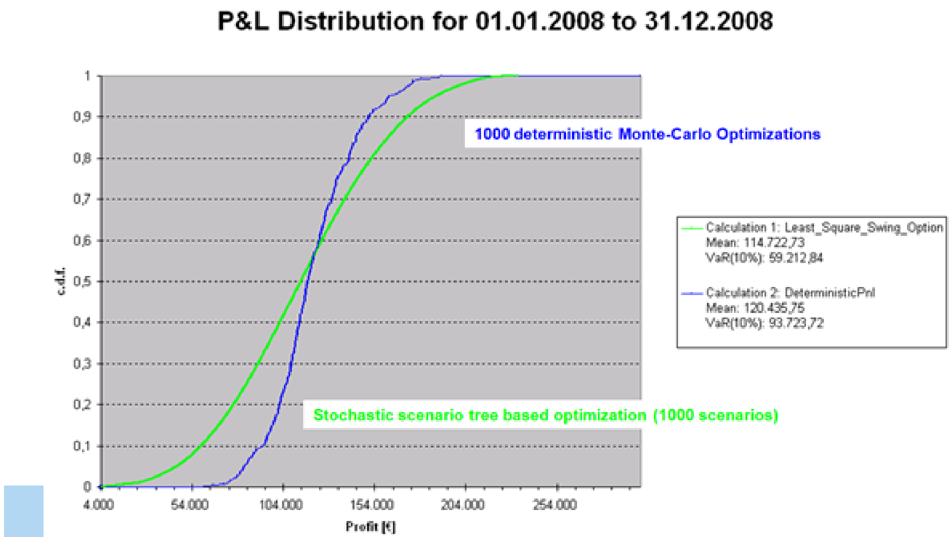


decision trees
stochastic optimization

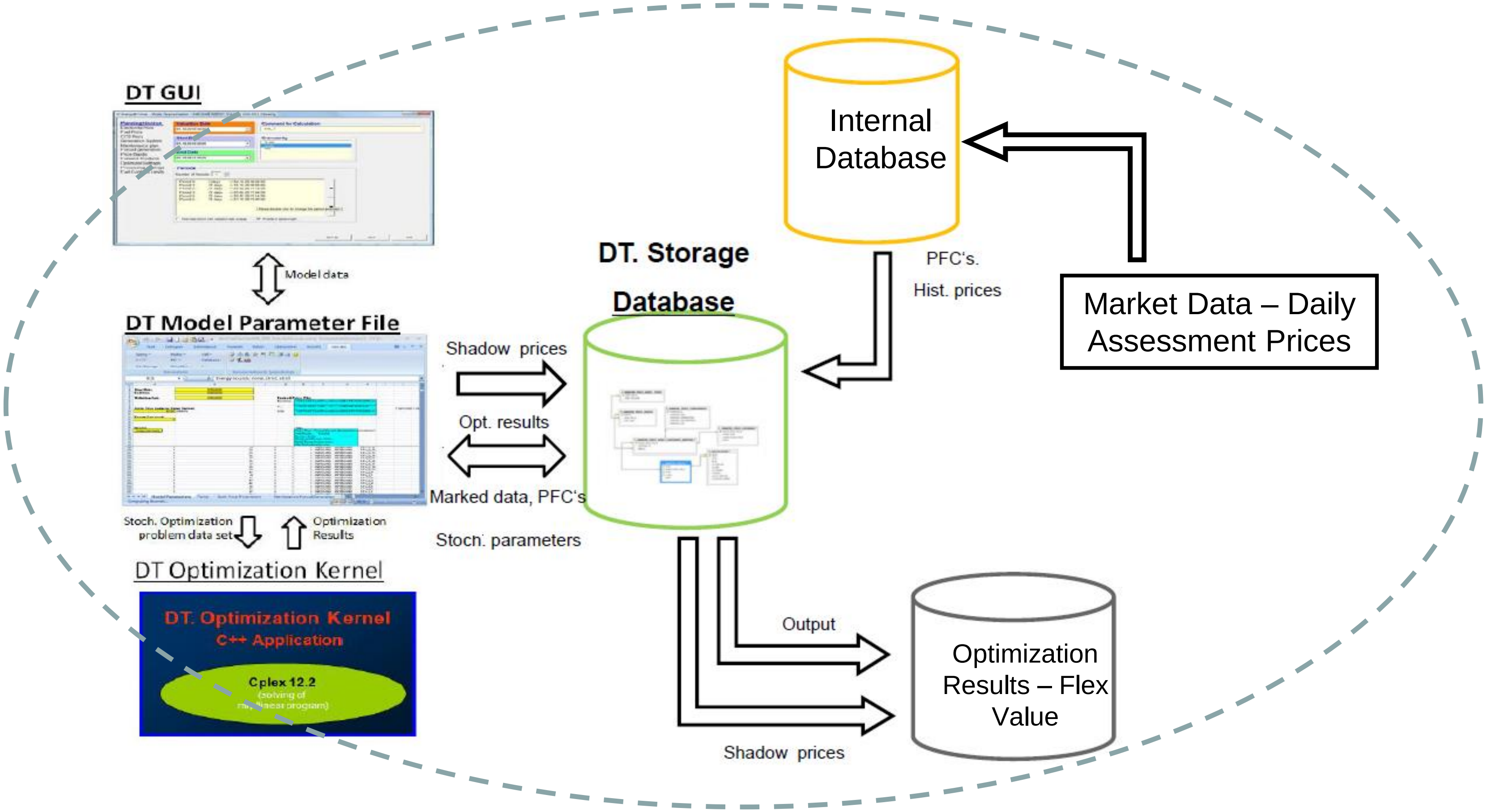
Profit distribution function

- From every scenario tree based stochastic optimization run a profit and loss distribution function can be derived
- Profit and loss distribution functions provide significantly enhanced information on the financial risks and chances that are related to an asset (portfolio) in the defined planning horizon
- Measures used to analyse the output of the risk analysis:
  - Minimum and Maximum profit/loss
  - VaR with confidence level of 97% and 90%
  - Tail-VaR with confidence level of 97%, 90% and 50%
  - Mean and Median
  - Absolute deviance, Standard deviance and Variance
  - Skewness and Kurtosis





IT Architecture



decision trees
stochastic optimization

Example Contract Valuation

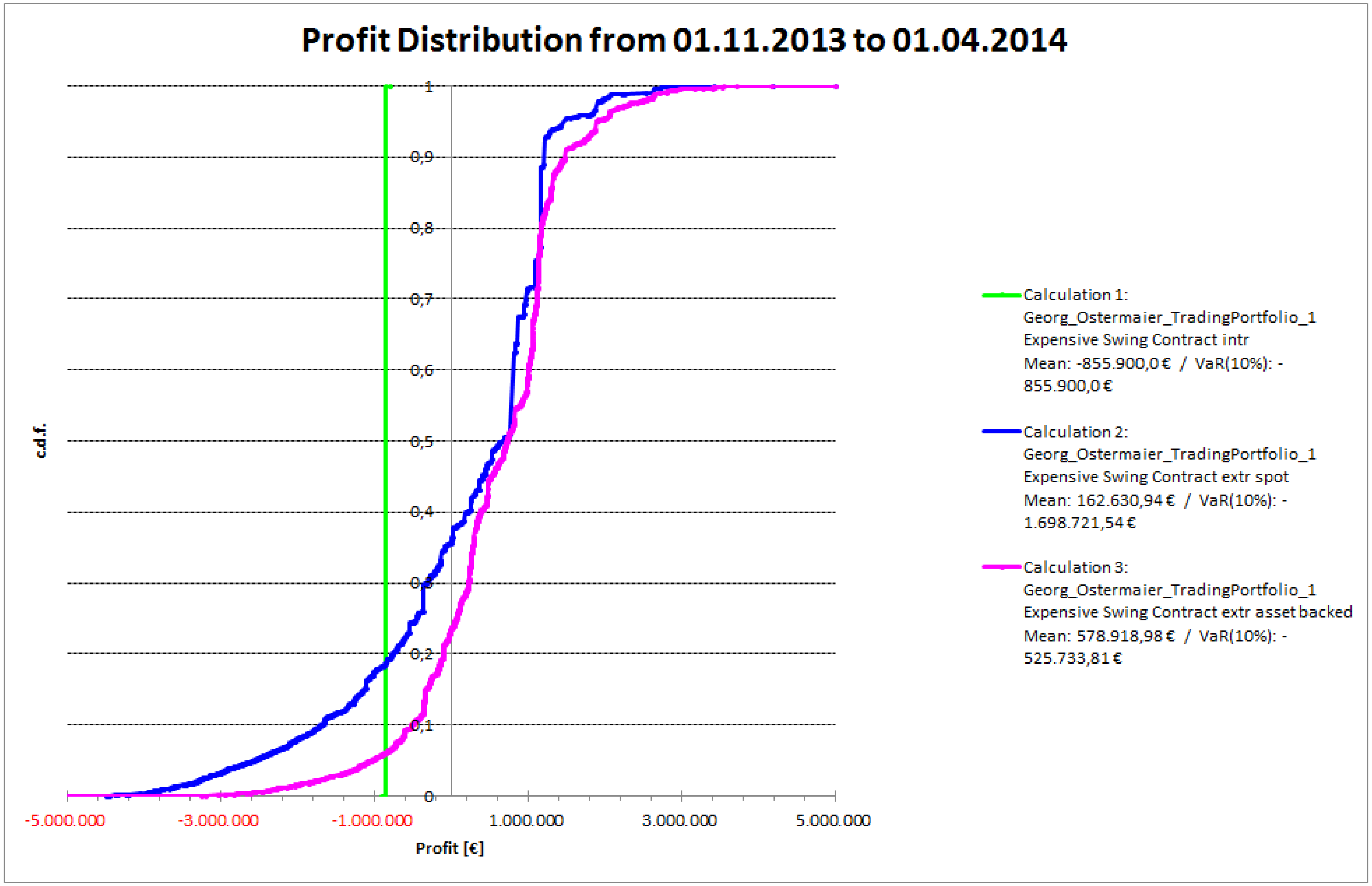
Delivery Point: Austrian virtual gas trading point (VTP)

Contract Period: 01.11.2013 to 01.04.2014

Contract Price: 30,75 EUR/MWh

ACQ: 270,000 MWh

Take or Pay = ACQ



Decision Trees Expertise

- **Valuation of assets and asset portfolios:**
  - thermal power plants, gas storage, hydroelectric power systems, combined heat and power systems, district heating networks
- **Risk management**
- **Market price analysis and modeling, price forward curves, price forecasts and scenario generation**
- **Scheduling and portfolio optimization**
  - Cross-Market-Optimization of complex hydro power systems
  - Cross-Market-Optimization of combined heat and power generation systems considering the district heating supply
  - Integrated optimization of thermal power plants and of the upstream fuel supply chain
  - Optimal management of gas/LNG