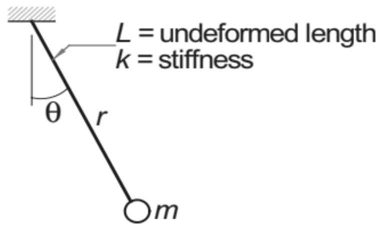
PROBLEMA 1

```
clearvars
clear all
clc
```

15. ■



The mass m is suspended from an elastic cord with an extensional stiffness k and undeformed length L. If the mass is released from rest at $\theta = 60^\circ$ with the cord unstretched, find the length r of the cord when the position $\theta = 0$ is reached for the first time. The differential equations describing the motion are

$$\ddot{r} = r\dot{\theta}^2 + g\cos\theta - \frac{k}{m}(r - L)$$

$$\ddot{\theta} = \frac{-2\dot{r}\dot{\theta} - g\sin\theta}{r}$$

Use $g = 9.80665\ \mathrm{m/s^2}$, $k = 40\ \mathrm{N/m}$, $L = 0.5\ \mathrm{m}$, and $m = 0.25\ \mathrm{kg}$.

Introducimos los datos:

```
g = 9.80665; k = 40; L = 0.5; m = 0.25;
```

La condiciones cinemáticas iniciales son:

```
t0 = pi/3; dt0 = 0; r0 = L; dr0 = 0;
```

Luego de hacer el cambio de variable usual, el sistema de ecuaciones es:

```
sist = @(t,z) [z(2);z(1)*z(4)^2+g*cos(z(1))-(k/m)*(z(1)-L);...
    z(4);(-2*z(2)*z(4)-g*sin(z(3)))/z(1)];
```

Sujetamos al vector de valor inicial:

```
z0 = [r0;dr0;t0;dt0];
```

Introducimos el intervalo de integración y el número de puntos que suavizará la curva solución:

```
np = 500; t0 = 0; tf = 5; dt = (tf-t0)/(np-1);
```

Hacemos uso del método RK4

```
[x,z] = RK4(sist,t0,tf,z0,dt,np-1);
nz = 0;
zero = [];
```

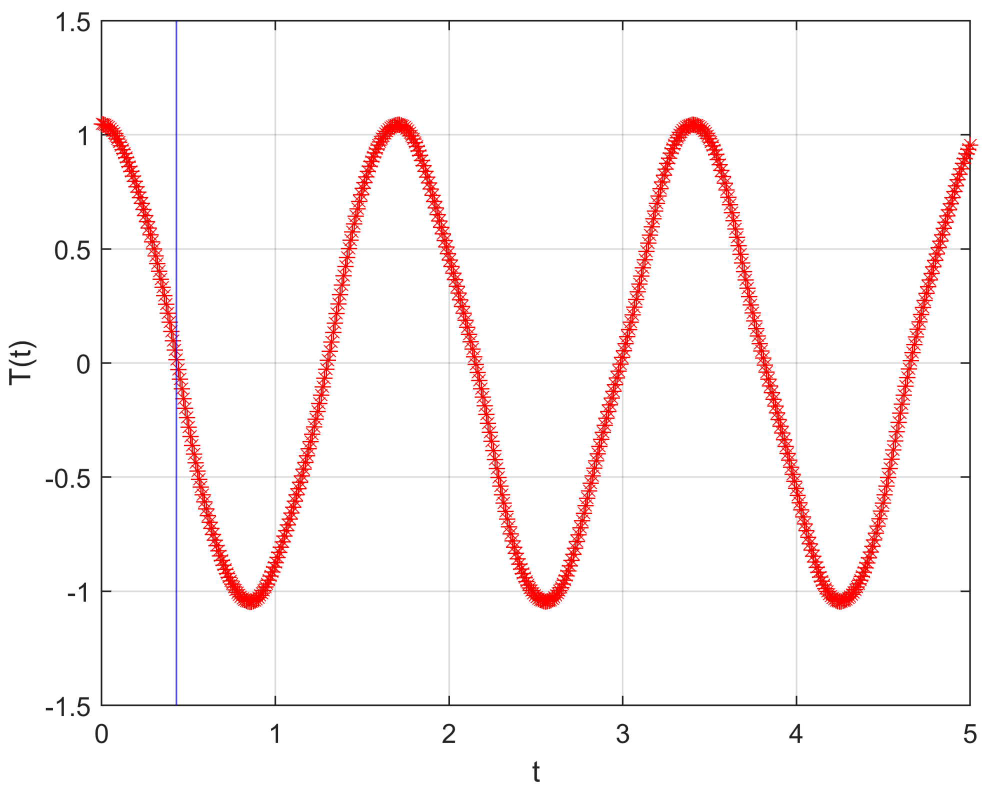
Por el teorema de Bolzano (TVI), determinamos un cero en la gráfica, del problema, ÁNGULO vs TIEMPO:

```
for k = 1:length(x)-1
    if sign(z(3,k)) ~= sign(z(3,k+1))
        nz = nz + 1;
        zero(nz,1) = x(k);
        zero(nz,2) = x(k+1);
        break;
    end
end
```

Con esto, encontramos el instante de tiempo en donde, por primera vez,

se tiene que el ÁNGULO VERTICAL es NULO.

```
plot(x,z(3,:),'r*')
xline(zero(nz,1),'b')
xlabel('t')
ylabel('T(t)')
grid on
```



Ahora bien, aparte, tenemos que la distancia radial toma los siguientes valores conforme transcurre el tiempo:

```
r = z(1,:);
```

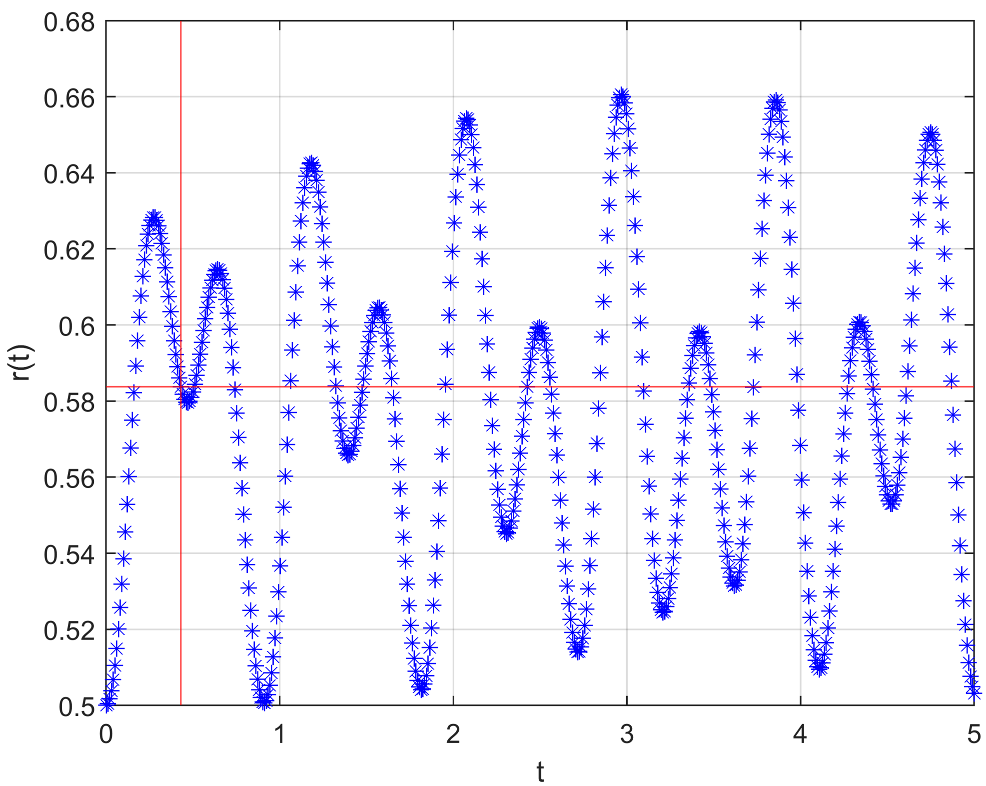
Con esto, encontramos el valor de "r" en el momento en el que el ÁNGULO VERTICAL es NULO.

```
rtzero = r(find(x==zero(nz,1)))
```

```
rtzero = 0.5838
```

Verificamos gráficamente:

```
plot(x,z(1,:),'b*')
xlabel('t')
ylabel('r(t)')
grid on
xline(zero(nz,1),'r')
yline(rtzero,'r')
```



PROBLEMA 2

```
clearvars
clear all
clc
```

18. ■ Integrate the following problems from $x = 0$ to 20 and plot y versus x:

(a) $y'' + 0.5(y^2 - 1) + y = 0 \qquad y(0) = 1 \qquad y'(0) = 0$

(b) $y'' = y \cos 2x \qquad y(0) = 0 \qquad y'(0) = 1$

These differential equations arise in nonlinear vibration analysis.

a)

Condiciones iniciales:

```
y0 = 1; dy0 = 0;
```

Sistema de ecuaciones asociado, luego de hacer el cambio de variable:

```
sist = @(x,z) [z(2);-0.5*(z(1)^2-1)-z(1)];
```

Vector valor inicial:

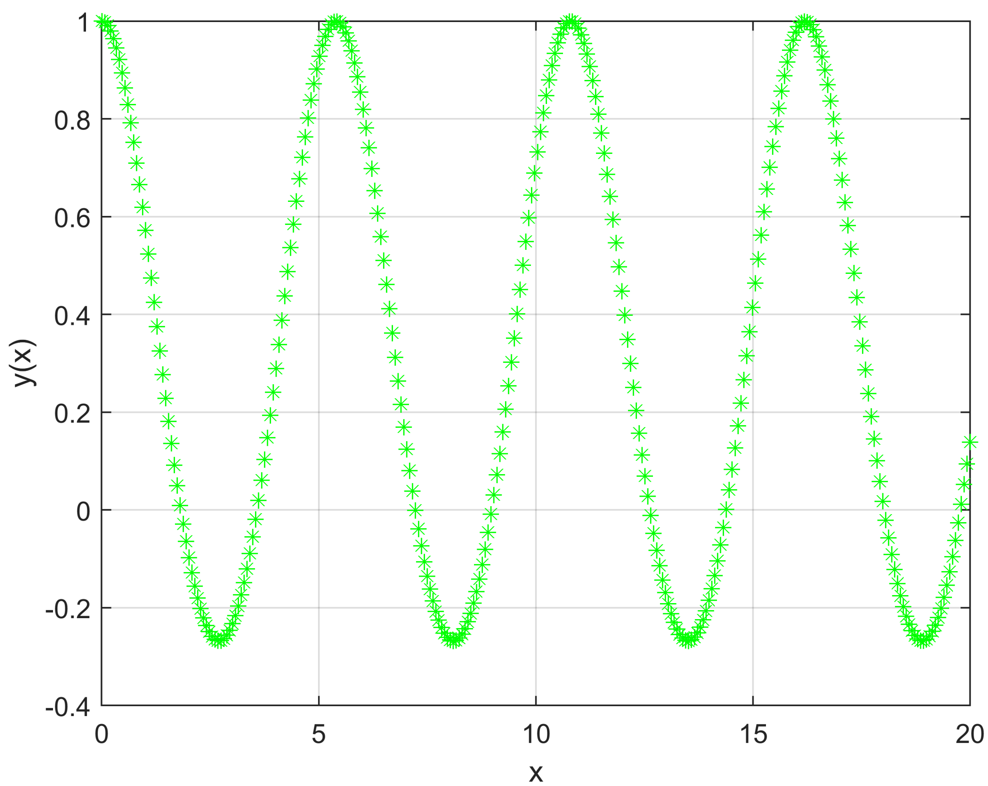
```
z0 = [y0;dy0];
```

Imponemos los parametros de suavidad y el intervalo de solución de la EDO:

```
np = 300; x0 = 0; xf = 20;
dx = (xf-x0)/(np-1);
```

Resolvinedo por RK4, tenemos:

```
[x,z] = RK4(sist,x0,xf,z0,dx,np-1);
plot(x,z(1,:),'g*')
xlabel('x')
ylabel('y(x)')
grid on
```



b)

Condiciones iniciales:

```
y0 = 0; dy0 = 1;
```

Sistema de ecuaciones asociado, luego de hacer el cambio de variable:

```
sist = @(x,z) [z(2);z(1)*cos(2*x)];
```

Vector valor inicial:

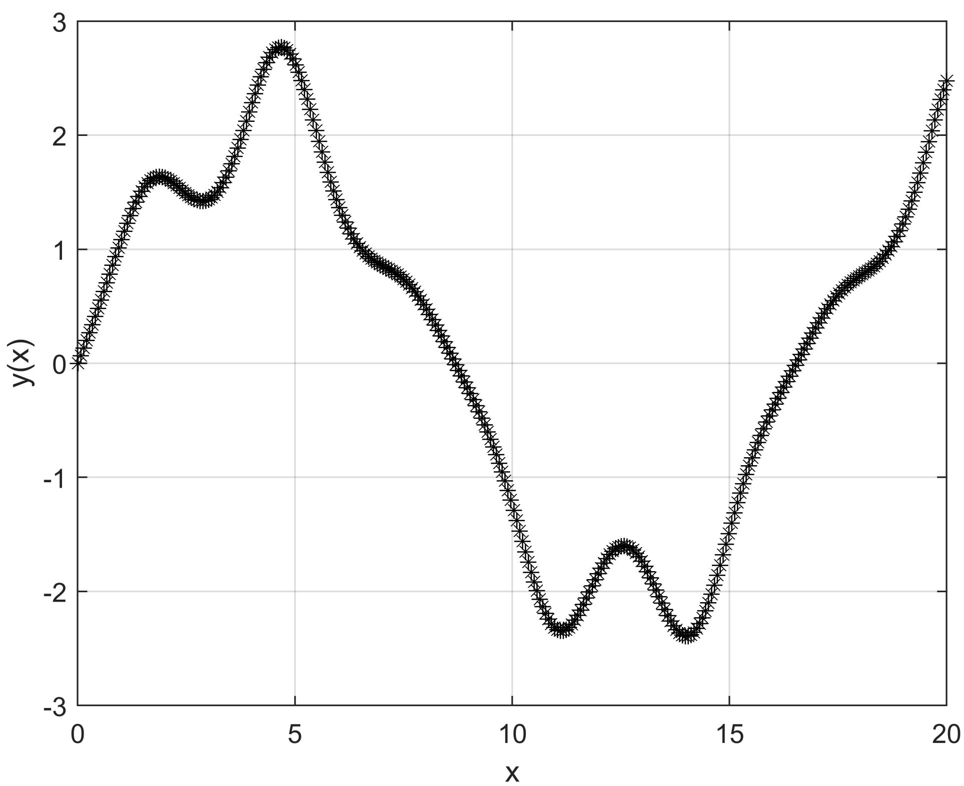
```
z0 = [y0;dy0];
```

Imponemos los parametros de suavidad y el intervalo de solución de la EDO:

```
np = 300; x0 = 0; xf = 20;
dx = (xf-x0)/(np-1);
```

Resolvinedo por RK4, tenemos:

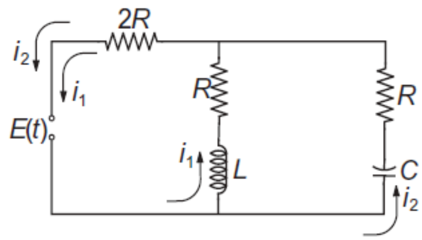
```
[x,z] = RK4(sist,x0,xf,z0,dx,np-1);
plot(x,z(1,:),'k*')
xlabel('x')
ylabel('y(x)')
grid on
```



PROBLEMA 3

```
clearvars
clear all
clc
```

21. ■



Kirchoff's equations for the circuit shown are

$$L\frac{di_1}{dt} + Ri_1 + 2R(i_1 + i_2) = E(t) \qquad \text{(a)}$$

$$\frac{q_2}{C} + Ri_2 + 2R(i_2 + i_1) = E(t) \qquad \text{(b)}$$

Differentiating Eq. (b) and substituting the charge–current relationship $dq_2/dt = i_2$, we get

$$\frac{di_1}{dt} = \frac{-3Ri_1 - 2Ri_2 + E(t)}{L} \qquad \text{(c)}$$

$$\frac{di_2}{dt} = -\frac{2}{3}\frac{di_1}{dt} - \frac{i_2}{3RC} + \frac{1}{3R}\frac{dE}{dt} \qquad \text{(d)}$$

We could substitute di_1/dt from Eq. (c) into Eq. (d), so that the latter would assume the usual form $di_2/dt = f(t, i_1, i_2)$, but it is more convenient to leave the equations as they are. Assuming that the voltage source is turned on at time $t = 0$,

plot the loop currents i_i and i_2 from $t = 0$ to 0.05 s. Use $E(t) = 240\sin(120\pi t)$ V, $R = 1.0\,\Omega$, $L = 0.2 \times 10^{-3}$ H, and $C = 3.5 \times 10^{-3}$ F.

Introducimos los datos:

```
R = 1; L = 0.2*10^(-3); C = 3.5*10^(-3);
```

Condiciones iniciales:

```
i10 = 0; i20 = 0;
```

La fuente electromotriz, y su derivada es (dato):

```
E = @(t) 240*sin(120*pi*t);
dEdt = @(t) 240*120*pi*cos(120*pi*t);
```

La (a) del problema es:

```
di1dt = @(x,z) (-3*R*z(1)-2*R*z(2)+E(x))/L;
```

Haciendo los cambios de variables adecuados, se construye el sistema de ecuaciones:

```
sist = @(x,z) [di1dt(x,z);(-2/3)*di1dt(x,z)-(1/(3*R*C))*z(2)+(1/(3*R))*dEdt(x)];
```

Sujeto al valor inicial:

```
z0 = [i10;i20];
```

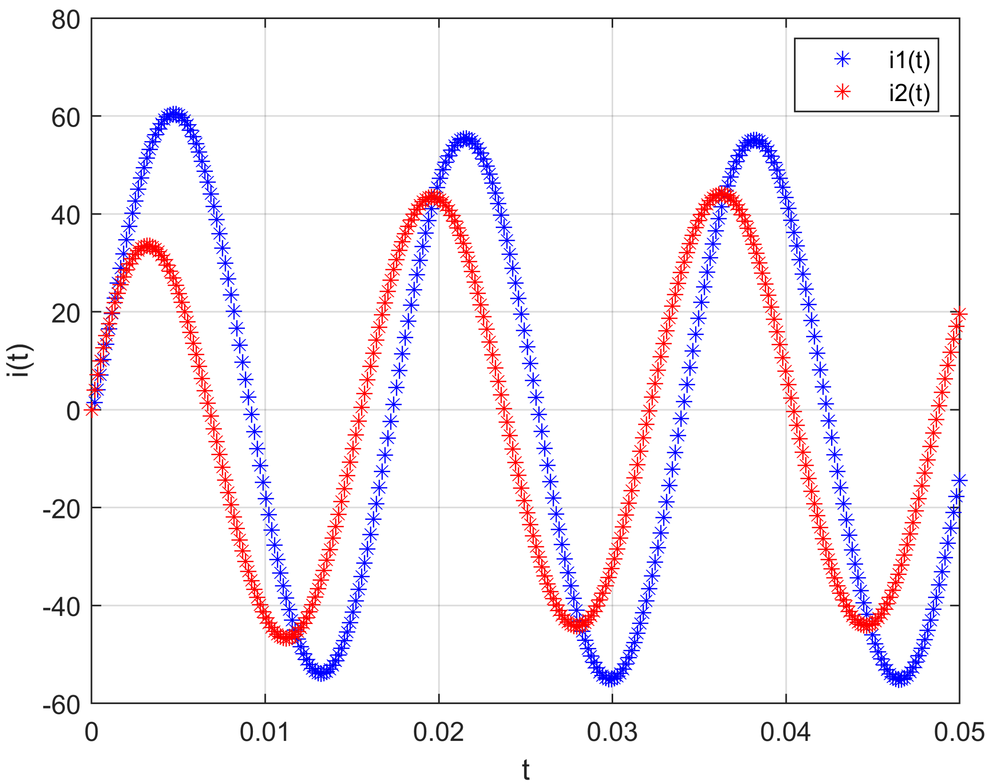
Imponemos el intervalo de solución de la EDO y el número de puntos que suavizará la curva:

```
np = 300; t0 = 0; tf = 0.05;
```

```
dt = (tf-t0)/(np-1);
```

Hacemos uso de RK4:

```
[x,z] = RK4(sist,t0,tf,z0,dt,np-1);
plot(x,z(1,:),'b*',x,z(2,:),'r*')
xlabel('t')
ylabel('i(t)')
legend('i1(t)','i2(t)')
grid on
```



PROBLEMA 4

```
clearvars
clear all
clc
```

EXAMPLE 7.4

Solve

$$y'' = -0.1y' - x \qquad y(0) = 0 \qquad y'(0) = 1$$

from $x = 0$ to 2 in increments of $h = 0.25$ with the fourth-order Runge–Kutta method.

23. Write a function for second-order Runge–Kutta method of integration. You may use `runKut4` as a model. Use the function to solve the problem in Example 7.4. Compare your results with those in Example 7.4.

Condiciones iniciales:

```
y0 = 0; dy0 = 1;
```

El sistema de ecuaciones asociado es:

```
sist = @(x,z) [z(2);-0.1*z(2)-x];
```

Sujeto al valor inicial:

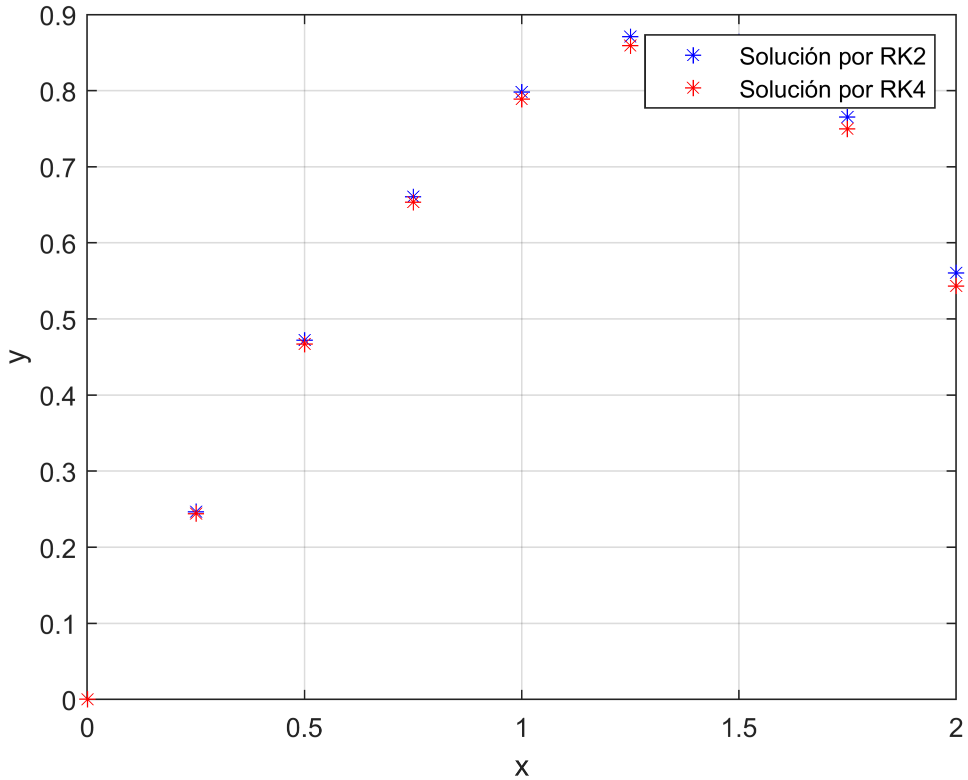
```
z0 = [y0;dy0];
```

Imponemos el intervalo de solución de la EDO y el número de puntos que suavizará la curva:

```
x0 = 0; xf = 2; h = 0.25; np = ((xf-x0)/h)+1;
```

Hacemos uso de RK2 y RK4 y comparamos los resultados en una gráfica:

```
[xRK2,zRK2] = RK2(sist,x0,xf,z0,h,np-1);
[xRK4,zRK4] = RK4(sist,x0,xf,z0,h,np-1);
plot(xRK2,zRK2(1,:),'b*',xRK4,zRK4(1,:),'r*')
xlabel('x')
ylabel('y')
legend
legend('Solución por RK2','Solución por RK4')
grid on
```



El código de la función usada fue:

```
function [x,z] = RK2(df,t0,tf,z0,h,n)

x = [t0:h:tf];

z(:,1) = z0;

for i = 1:n

k1 = df(x(i),z(:,i));

k2 = df(x(i)+(h/2),z(:,i)+(h/2)*k1);

z(:,i+1) = z(:,i)+h*k2;

end

end
```


```
function [x,z] = RK2(df,t0,tf,z0,h,n)
    x = [t0:h:tf];
    z(:,1) = z0;
    for i = 1:n
        k1 = df(x(i),z(:,i));
        k2 = df(x(i)+(h/2),z(:,i)+(h/2)*k1);
        z(:,i+1) = z(:,i)+h*k2;
    end
```

```
end
function [x,z] = RK4(df,t0,tf,z0,h,n)
    x = [t0:h:tf];
    z(:,1) = z0;
    for i = 1:n
        k1 = df(x(i),z(:,i));
        k2 = df(x(i)+(h/2),z(:,i)+(h/2)*k1);
        k3 = df(x(i)+(h/2),z(:,i)+(h/2)*k2);
        k4 = df(x(i)+h,z(:,i)+h*k3);
        z(:,i+1) = z(:,i)+(h/6)*(k1+2*k2+2*k3+k4);
    end
end
```