A survey on policy search algorithms
for learning robot controllers in a handful of trials

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Most policy search algorithms require thousands of training episodes to find an effective policy, which is often infeasible with a physical robot. This survey article focuses on the extreme other end of the spectrum: how can a robot adapt with only a handful of trials (a dozen) and a few minutes? By analogy with the word "big-data", we refer to this challenge as "micro-data reinforcement learning". We show that a first strategy is to leverage prior knowledge on the policy structure (*e.g.*, dynamic movement primitives), on the policy parameters (*e.g.*, demonstrations), or on the dynamics (*e.g.*, simulators). A second strategy is to create data-driven surrogate models of the expected reward (*e.g.*, Bayesian optimization) or the dynamical model (*e.g.*, model-based policy search), so that the policy optimizer queries the model instead of the real system. Overall, all successful micro-data algorithms combine these two strategies by varying the kind of model and prior knowledge. The current scientific challenges essentially revolve around scaling up to complex robots (*e.g.*, humanoids), designing generic priors, and optimizing the computing time.

1 Introduction

Reinforcement learning (RL) [148] is a generic framework that allows robots to learn and adapt by trial-and-error. There is currently a renewed interest in RL owing to recent advances in deep learning [90]. For example, RL-based agents can now learn to play many of the Atari 2600 games directly from pixels [108, 109], that is, without explicit feature engineering, and beat the world's best players at Go and chess with minimal human knowledge [138]. Unfortunately, these impressive successes are difficult to transfer to robotics because the algorithms behind them are highly data-intensive: 4.8 million games were required to learn to play Go from scratch [138], 38 days of play (real time) for Atari 2600 games [108], and, for example, about 100 hours of simulation time (much more for real time) for a 9-DOF mannequin that learns to walk [61].

By contrast, robots have to face the real world, which cannot be accelerated by GPUs nor parallelized on large clusters. And the real world will not become faster in a few years, contrary to computers so far (Moore's law). In concrete terms, this means that most of the experiments that are successful in simulation cannot be replicated in the real world because they would take too much time to be technically feasible. What is more, online adaptation is much more useful when it is fast than when it requires hours — or worse, days — of trial-and-error. For instance, if a robot is stranded in a nuclear plant and has to discover a new way to use its arm to open a door; or if a walking robot encounters a new kind of terrain for which it is required to alter its gait; or if a humanoid robot falls, damages its knee, and needs to learn how to limp: in most cases, adaptation has to occur in a few minutes or within a dozen trials to be of any use.

By analogy with the word "big-data", we refer to the challenge of learning by trial-and-error in a handful of trials as "micro-data reinforcement learning" [112]. This concept is close to "data-efficient reinforcement learning" [41], but we think it captures a slightly different meaning. The main difference is that efficiency is a ratio between a cost and benefit, that is, data-efficiency is a ratio between a quantity of data and, for instance, the complexity of the task. In addition, efficiency is a relative term: a process is more efficient than another; it is not simply "efficient"[1]. In that sense, many deep learning algorithms are data-efficient because they require fewer trials than the previous generation, regardless of the fact that they might need millions of time-steps. By contrast, we propose the terminology "micro-data learning" to represent an absolute value, not a relative one: how can a robot learn in a few minutes of interaction? or how can a robot learn in less than 20 trials[2]? Importantly, a micro-data algorithm might reduce the number of trials by incorporating appropriate prior knowledge; this does not necessarily make it more "data-efficient" than another algorithm that would use more trials but less prior knowledge: it simply makes them different because the two algorithms solve a different challenge.

Among the different approaches for RL, most of the recent work in robotics focuses on *Policy Search* (PS), that is, on viewing the RL problem as the optimization of the parameters of a given policy [39]. A few PS algorithms are explicitly focused on requiring very little *interaction time* with the robot, which often implies that they authorize themselves to substantially increase the *computing time* and the amount of *prior knowledge*. The purpose of this paper is to survey such existing micro-data policy search techniques that have been successfully used for robot control[3], and to identify the challenges in this emerging field. In particular, we focus on policy search approaches that have the *explicit goal* of reducing the interaction time between the robot and the environment to a few seconds or minutes.

Most published algorithms for micro-data policy search implement and sometimes combine two main strategies (Fig. 1): leveraging prior knowledge and building surrogate models.

Using prior knowledge requires balancing carefully between what can be realistically known before learning and what is left to be learnt. For instance, some experiments assume that demonstrations can be provided, but that they are imperfect [79, 116]; some others assume that a damaged robot knows its model in its intact form, but not the damaged model [23, 31, 120]. This

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[1]In some rare cases, a process can be "optimally efficient".

[2]It is challenging to put a precise limit for "micro-data learning" as each domain has different experimental constraints, this is why we will refer in this article to "a few minutes" or a "a few trials". The commonly used word "big-data" has a similar "fuzzy" limit that depends on the exact domain.

[3]Planning-based and model-predictive control [52] methods do not search for a full policy, this is why they do not fit into the scope of this paper.
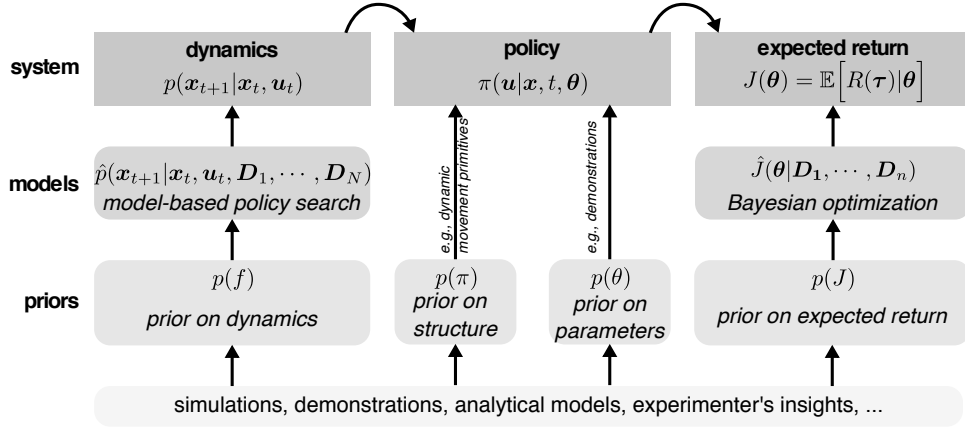
Fig. 1. Overview of possible strategies for Micro-Data Policy Search (MDPS). The first strategy (bottom) is to leverage prior knowledge on the dynamics, on the policy parameters, on the structure of the policy, or on the expected return. A second strategy is to learn surrogate models of the dynamics or of the expected return; these models can also be initialized or guided by some prior knowledge.

knowledge can be introduced at different places, typically in the structure of the policy (*e.g.*, dynamic movement primitives; DMPs) [66], in the reward function (*e.g.*, reward shaping), or in the dynamical model [1, 23].

The second strategy is to create models from the data gathered during learning and utilize them to make better decisions about what to try next on the robot. We can further categorize these methods into (a) algorithms that learn a surrogate model of the expected return (*i.e.*, long-term reward) from a starting state [16, 135]; and (b) algorithms that learn models of the transition dynamics and/or the immediate reward function (*e.g.*, learning a controller for inverted helicopter flight by first learning a model of the helicopter's dynamics [116]). The two strategies — priors and surrogates — are often combined; for example, most works with a surrogate model impose a policy structure and some of them use prior information to shape the initial surrogate function, before acquiring any data.

The rest of the paper is structured as follows. Section 2 presents the problem formulation for micro-data policy search. Section 3 describes the work about priors on the policy structure and parameters. Section 4 provides an overview of the work on learning surrogate models of the expected return, with and without priors, while Section 5 is focused on learning models of the dynamics and the immediate reward. Section 6 lists the few noteworthy approaches for micro-data policy search that do not fit well into the previous sections. Finally, Section 7 sketches the challenges that the field needs to overcome and Section 8 enumerates a few "precepts" to guide future work in this field.

2 Problem formulation

We model the robots as discrete-time dynamical systems that can be described by transition probabilities of the form:

$$p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t) \tag{1}$$

with continuous-valued states $\boldsymbol{x} \in \mathbb{R}^E$ and controls $\boldsymbol{u} \in \mathbb{R}^F$.

If we assume deterministic dynamics and Gaussian system noise, this equation is often written as:

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) + \boldsymbol{w}. \tag{2}$$

Here, $\boldsymbol{w}$ is i.i.d. Gaussian system noise, and f is a function that describes the unknown transition dynamics.

We assume that the system is controlled through a parameterized *policy* $\pi(\boldsymbol{u}|\boldsymbol{x}, t, \boldsymbol{\theta})$ that is followed for T steps. Throughout the paper we adopt the episode-based, fixed time-horizon formulations for clarity and pedagogical reasons, but also because most of the micro-data policy search approaches use this formulation.

In the general case, $\pi(\boldsymbol{u}|\boldsymbol{x}, t, \boldsymbol{\theta})$ outputs a distribution (*e.g.*, a Gaussian) that is sampled in order to get the action to apply; *i.e.*, we have *stochastic policies*. Most algorithms utilize policies that are not time-dependent (*i.e.*, they drop t), but we include it here for completeness. Several algorithms use *deterministic policies*; a deterministic policy means that $\pi(\boldsymbol{u}|\boldsymbol{x}, t, \boldsymbol{\theta}) \Rightarrow \boldsymbol{u} = \pi(\boldsymbol{x}, t|\boldsymbol{\theta})$.

When following a particular policy from an initial state distribution $p(\boldsymbol{x}_0)$, the system's states and actions jointly form *trajectories* $\boldsymbol{\tau} = (\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{x}_1, \boldsymbol{u}_1, \dots, \boldsymbol{x}_T)$, which are often also called *rollouts* or *paths*. We assume that a scalar performance system exists, $R(\boldsymbol{\tau})$, that evaluates the performance of the system given a trajectory $\boldsymbol{\tau}$. This *long-term reward* (or *return*) is defined as the sum of the immediate rewards along the trajectory $\boldsymbol{\tau}$:

$$R(\boldsymbol{\tau}) = \sum_{t=0}^{T-1} r_{t+1} = \sum_{t=0}^{T-1} r(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{x}_{t+1}) \tag{3}$$

where $r_{t+1} = r(\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{x}_{t+1}) \in \mathbb{R}$ is the *immediate reward* of being in state $\boldsymbol{x}_t$ at time t, taking the action $\boldsymbol{u}_t$ and reaching the state $\boldsymbol{x}_{t+1}$ at time $t + 1$.

We define the *expected return* $J(\boldsymbol{\theta})$ when following a policy $\pi_{\boldsymbol{\theta}}$ for t time-steps as:

$$J(\boldsymbol{\theta}) = \mathbb{E}\Big[R(\boldsymbol{\tau})|\boldsymbol{\theta}\Big]$$
$$= \int R(\boldsymbol{\tau})P(\boldsymbol{\tau}|\boldsymbol{\theta}) \tag{4}$$

where $P(\boldsymbol{\tau}|\boldsymbol{\theta})$ is the distribution over trajectories $\boldsymbol{\tau}$ for any given policy parameters $\boldsymbol{\theta}$ applied on the actual system:

$$\underbrace{P(\boldsymbol{\tau}|\boldsymbol{\theta})}_{\text{trajectories for } \boldsymbol{\theta}} = \underbrace{p(\boldsymbol{x}_0)}_{\text{initial state}} \prod_t \underbrace{p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)}_{\text{transition dynamics}} \underbrace{\pi(\boldsymbol{u}_t|\boldsymbol{x}_t, t, \boldsymbol{\theta})}_{\text{policy}}. \tag{5}$$

The objective of a *policy search algorithm* is to find the parameters $\boldsymbol{\theta}^*$ that maximize the *expected return* $J(\boldsymbol{\theta})$ when following the policy $\pi_{\boldsymbol{\theta}^*}$:

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\arg\max} \, J(\boldsymbol{\theta}). \tag{6}$$

Algorithm 1 Generic policy search algorithm

1: Apply initialization strategy using INITSTRATEGY
2: Collect data, D_0, with COLLECTSTRATEGY
3: **for** $n = 1 \rightarrow N_{iter}$ **do**
4:     Learn models using LEARNSTRATEGY and D_{n-1}
5:     Calculate θ_{n+1} using UPDATESTRATEGY
6:     Apply policy $\pi_{\theta_{n+1}}$ on the system
7:     Collect data, D_n, with COLLECTSTRATEGY
8: **end for**

Algorithm 2 Gradient-free direct policy search algorithm

1: **procedure** INITSTRATEGY
2:     Select θ_1 randomly
3: **end procedure**
4: **procedure** COLLECTSTRATEGY
5:     Collect samples of the form $(\theta, \frac{\sum_i^N R(\tau)_i}{N}) = (\theta, \tilde{J}_\theta)$ by running policy π_θ N times.
6: **end procedure**

Most policy search algorithms can be described with a generic algorithm (Algo. 1) and they: (1) start with an initialization strategy (INITSTRATEGY), for instance using random actions, and (2) collect data from the robot (COLLECTSTRATEGY), for instance the states at each discrete time-steps or the reward at the end of the episode; they then (3) enter a loop (for N_{iter} iterations) that alternates between learning one or more models (LEARNSTRATEGY) with the data acquired so far, and selecting the next policy π_{θ^*} to try on the robot (UPDATESTRATEGY).

This generic outline allows us to describe direct (*e.g.*, policy gradient algorithms [149]), surrogate-based (*e.g.*, Bayesian optimization [16]) and model-based policy search algorithms, where each algorithm implements in a different way each of INITSTRATEGY, COLLECTSTRATEGY, LEARNSTRATEGY and UPDATESTRATEGY. We will also see that in this outline we can also fit policy search algorithms that utilize priors; coming from simulators, demonstrations or any other source.

To better understand how policy search is performed, let's use a gradient-free optimizer (UPDATESTRATEGY) and learn directly on the system (*i.e.*, LEARNSTRATEGY $= \emptyset$). This type of algorithms falls in the category of *model-free* or *direct* policy search algorithms [82, 148].

INITSTRATEGY can be defined as randomly choosing some policy parameters, θ_1 (Algo. 2), and COLLECTSTRATEGY collects samples of the form $(\theta, \frac{\sum_i^N R(\tau)_i}{N})$ by running N times the policy π_θ. We execute the same policy multiple times because we are interested in approximating the expected return (Eq. (3)). $\tilde{J}_\theta = \frac{\sum_i^N R(\tau)_i}{N}$ is then used as the value for the sample θ in a regular optimization loop that tries to maximize it (*i.e.*, the UPDATESTRATEGY is optimizer-dependent).

This straightforward approach to policy search typically requires a large amount of interaction time with the system to find a high-performing solution [148]. Many approaches have been suggested to improve the sample efficiency of model-free approaches (*e.g.*, [2, 27, 28, 35, 99, 109, 134, 137, 149, 154]). Nevertheless, the objective of the present article is to describe algorithms that require several orders of magnitude less interaction time by leveraging priors and models.

3 Using priors on the policy parameters/representation

When designing the policy $\pi(u|x, t, \theta)$, the key design choices are what the space of θ is, and how it maps states to actions. This design is guided by a trade-off between having a representation that is *expressive*, and one that provides a space that is *efficiently searchable*.

Expressiveness can be defined in terms of the optimal policy π_ζ^*. For a given task ζ, there is theoretically always at least one optimal policy π_ζ^*. Here, we drop θ to express that we do not mean a specific representation parameterized by θ. Rather π_ζ^* emphasizes that there is some policy (with some representation, perhaps unknown to us) that cannot be outperformed by any other policy (whatever its representation). We use $J_\zeta(\pi_\zeta^*)$ to denote this highest possible expected reward.

A parameterized policy π_θ should be expressive enough to *represent* this optimal policy π_ζ^* (or at least come close), *i.e.*,

$$J_\zeta(\pi_\zeta^*) - \max_\theta J_\zeta(\theta) < \delta \tag{7}$$

where δ is some acceptable margin of suboptimality. Note that absolute optimality is rarely required in robotics; in many everyday applications, small tracking errors may be acceptable, and the quadratic command cost needs not be the absolute minimum.

On the other hand, the policy representation should be such that it is easy (or at least feasible) to find θ^*, *i.e.*, it should be *efficiently searchable*[4]. In general, smaller values of $\dim(\theta)$ lead to more efficiently searchable spaces.

In the following subsections, we describe several common policy representations which make different trade-offs between expressiveness and being efficiently searchable.

3.1 Hand-designed policies

One approach to reducing the policy parameter space is to hand-tailor it to the task ζ to be solved. In [48], for instance, a policy for ball acquisition is designed. The resulting policy only has only four parameters, *i.e.*, $\dim(\theta)$ is 4. This low-dimensional policy parameter space is easily searched, and only 672 trials are required to optimize the policy. Thus, prior knowledge is used to find a compact representation, and policy search is used to find the optimal θ^* for this representation.

One disadvantage of limiting $\dim(\theta)$ to a very low dimensionality is that δ may become quite large, and we have no estimate of how much more the reward could have been optimized with a more expressive policy representation. Another disadvantage is that the representation is very specific to the task ζ for which it was designed. Thus, such a policy cannot be reused to learn other tasks. It then greatly limits the transfer learning capabilities of the approaches, since the learned policy can hardly be re-used for any other task.

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[4]Analogously, the universal approximation theorem states that a feedforward network with single hidden layer suffices to *represent* any continuous function, but it does not imply that the function is *learnable* from data.

3.2 Policies as function approximators

Ideally, our policy representation Θ is expressive enough so that we can apply it to many different tasks, *i.e.*,

$$\underset{\Theta}{\arg\min} \sum_{n=1}^{N} J_{\zeta_n}(\pi^*_{\zeta_n}) - \max_{\boldsymbol{\theta}} J_{\zeta_n}(\boldsymbol{\theta}), \text{ with } \boldsymbol{\theta} \in \Theta, \qquad (8)$$

i.e., over a set of tasks, we minimize the sum of differences between the theoretically optimal policy π^* for each task, and the optimal policy *given the representation* $\pi_{\boldsymbol{\theta}}$ for each task[5].

Two examples of such generally applicable policy representations are linear policies (9), radial basis function networks (10), or neural networks, namely

$$\pi_{\boldsymbol{\theta}}(\boldsymbol{x}) = \boldsymbol{\theta}^{\mathsf{T}} \boldsymbol{\psi}(\boldsymbol{x}) \qquad (9)$$
$$\pi_{\boldsymbol{\theta}}(\boldsymbol{x}) = \boldsymbol{w}^{\mathsf{T}} \boldsymbol{\psi}_{\boldsymbol{\theta}}(\boldsymbol{x}). \qquad (10)$$

These more general policies can be used for many tasks [54, 80]. However, prior knowledge is still required to determine the appropriate number of basis functions and their shape. Again, a lower number of basis functions will usually lead to more efficient learning, but less expressive policies and thus potentially higher δ.

One advantage of using a function approximator is that programming by demonstration can often be used to determine an initial policy. The initial parameters $\boldsymbol{\theta}$ are obtained through supervised learning, by providing the demonstration as training data $(\boldsymbol{x}_i, \boldsymbol{u}_i)_{i=1:N}$. This is discussed in more detail in Section 3.6

The function approximator can be used to generate a single estimate (corresponding to a first order moment in statistics), but it can also be extended to higher order moments. Typically, extending it to second order moments allows the system to get information about the variations that we can exploit to fulfill a task, as well as the synergies between the different policy parameters in the form of covariances. This is typically more expensive to learn—or it requires multiple demonstrations [105]—but the learned representation can typically be more expressive, facilitating adaptation and generalization.

3.3 Dynamical Movement Primitives

Dynamical Movement Primitives (DMPs) combine the generality of function approximators with the advantages of dynamical systems, such as robustness towards perturbations and convergence guarantees [64, 65]. DMPs limit the expressiveness of the policy to typical classes of tasks in robotics, such as point-to-point movements ('discrete DMPs') or repetitive movements ('rythmic DMPs').

Discrete DMPs are summarized in Eq. 11. The canonical system represents the movement *phase s*, which starts at 1, and converges to 0 over time. The transformation systems combines a spring-damper system with a function approximator $f_{\boldsymbol{\theta}}$, which, when integrated, generates accelerations $\ddot{y}$. Multi-dimensional DMPs are achieved by coupling multiple transformation systems with one canonical system. The vector $\boldsymbol{y}$ typically represents the end-effector pose or the joint angles.

As the spring-damper system converges to y^g, and s (and thus $s f_{\boldsymbol{\theta}}(s)$) converges to 0, the overall system y is guaranteed to converge to y^g. We have:

$$\omega \ddot{y} = \underbrace{\alpha(\beta(y^g - y) - \dot{y})}_{\text{Spring-damper system}} + \underbrace{s\, f_{\boldsymbol{\theta}}(s)}_{\text{Forcing term}}. \qquad \text{(Transf.)} \quad (11)$$

$$\omega \dot{s} = -\alpha_s s. \qquad \text{(Canonical)} \quad (12)$$

This facilitates learning, because, whatever parameterization $\boldsymbol{\theta}$ of the function approximator we choose, a discrete DMP is guaranteed to converge towards a goal y^g. Similarly, a rhythmic DMP will always generate a repetitive motion, independent of the values in $\boldsymbol{\theta}$. The movement can be made slower or faster by changing the time constant ω.

Another advantage of DMPs is that only one function approximator is learned for each dimension of the DMP, and that the input of each function approximator is the phase variable s, which is always 1D. Thus, whereas the overall DMP closes the loop on the state y, the part of the DMP that is learned ($f_{\boldsymbol{\theta}}(s)$) is an open-loop system. This greatly facilitates learning, and simple black-box optimization algorithms have been shown to outperform state-of-the-art RL algorithms for such policies [143]. Approaches for learning the goal y^g of a discrete movement have also been proposed [144]. Since the goal is constant throughout the movement, few trials are required to learn it.

The optimal parameters $\boldsymbol{\theta}^*$ for a certain DMP are specific to one specific task ζ. Task-parameterized (dynamical) motion primitives aim at generalizing them to variations of a task, which are described with the task parameter vector $\boldsymbol{q}$ (*e.g.*, the 3D pose to place an object on a table [145]). Learning a motion primitive that is optimal for all variations of a task (*i.e.*, all $\boldsymbol{q}$ within a range) is much more challenging, because the curse of dimensionality applies to the task parameter vector $\boldsymbol{q}$ just as it does for the state vector $\boldsymbol{x}$ in reinforcement learning. Task-parameterized representations based on the use of multiple coordinate systems have been developed to cope with this curse of dimensionality [20], but these models have only been applied to learning from demonstration applications so far.

Another approach to avoid the curse of dimensionality is to consider a hierarchical organization of the policy. In [33], Daniel *et al.* propose the use of a hierarchical policy composed of a gating network and multiple sub-policies, and introducing an entropy-based constraint ensuring that the agent finds distinct solutions with different sub-policies. These sub-policies are treated as latent variables in an expectation-maximization procedure, allowing the distribution of the update information between the sub-policies. In Queisser and Steil [126], an upper-level policy is used to interpolate between policy parameterizations for different task variations. This substantially speeds up learning when many variations of the same task must be learned.

3.4 Learning the controller

If the policy generates a reference trajectory, a controller is required to map this trajectory (and the current state) to robot control commands (typically torques or joint angle velocity commands). This can be done for instance with a *proportional-integral-derivative* (PID) controller [18], or a *linear quadratic tracking* (LQT) controller [22]. The parameters of this controller can also be included in $\boldsymbol{\theta}$, so that both the reference trajectory and controller parameters are learned at the same time. By doing so, appropriate gains [18, 21] or forces [74] for the task can be learned together with the movement required to reproduce the task. Typically, such representation provides a way to coordinate

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[5]Note that this optimization is never actually performed. It is a mathematical description of what the policy representation designer is implicitly aiming for.

motor commands to react to perturbations, by rejecting perturbations only in the directions that would affect task performance.

3.5 Learning the policy representation

So far we have described how the policy representation is determined with prior knowledge, and the $\boldsymbol{\theta}$ of this policy is then optimized through policy search. Another approach is to learn the policy representation and its parameters at the same time, as in NeuroEvolution of Augmenting Topologies (NEAT) [142]. It is even possible, in simulation, to co-evolve an appropriate body morphology and policy [13, 139]. As in natural evolution itself, these approaches require massive amounts of rollouts, and do not focus on learning in a handful of trials.

3.6 Initialization with demonstrations / imitation learning

An advantage of using expressive policies is that they are able to learn (close to) optimal policies for many different tasks. A downside is that such policies are also able to represent many suboptimal policies for a particular task, *i.e.*, there will be many local minima. To ensure convergence, it is important that the initial policy parameters are close to the global optimum. In robotics, this is possible through imitation, *i.e.* the initialization of $\boldsymbol{\theta}$ from a demonstrated trajectory. This is possible if we know the general movement a robot should make to solve the task, and are able to demonstrate it by recording our movement, or physically guiding the robot through kinesthetic teaching. Starting with a $\boldsymbol{\theta}$ that is close $\boldsymbol{\theta}^*$ greatly reduces the number of samples to find $\boldsymbol{\theta}^*$, and the interplay between imitation and policy search is therefore an important component in micro-data learning.

4 Learning models of the expected return

With the appropriate policy representation (and/or initial policy parameters) chosen, the policy search in Algorithm 1 is then executed. The most important step is determining the next parameter vector $\boldsymbol{\theta}_{n+1}$ to test on the physical robot.

In order to choose the next parameter vector $\boldsymbol{\theta}_{n+1}$ to test on the physical robot, a strategy is to learn a model $\hat{J}(\boldsymbol{\theta})$ of the expected return $J(\boldsymbol{\theta})$ (Eq. (4)) using the values collected during the previous episodes, and then choose the optimal $\boldsymbol{\theta}_{n+1}$ according to this model. Put differently, the main concept is to optimize $J(\boldsymbol{\theta})$ by leveraging $\hat{J}(\boldsymbol{\theta}|R(\boldsymbol{\tau}|\boldsymbol{\theta}_1), \cdots, R(\boldsymbol{\tau}|\boldsymbol{\theta}_N))$.

4.1 Bayesian optimization

Algorithm 3 Policy search with Bayesian optimization

1: **procedure** COLLECTSTRATEGY
2:     Collect samples of the form $(\boldsymbol{\theta}, R(\boldsymbol{\tau}))$
3: **end procedure**
4: **procedure** LEARNSTRATEGY
5:     Learn model $\hat{J} : \boldsymbol{\theta} \rightarrow J(\boldsymbol{\theta})$
6: **end procedure**
7: **procedure** UPDATESTRATEGY
8:     $\boldsymbol{\theta}_{n+1} = \operatorname{argmax}_{\boldsymbol{\theta}} \text{ACQUISITIONFUNCTION}(\boldsymbol{\theta})$
9: **end procedure**

The most representative class of algorithms that falls in this category is Bayesian optimization (BO) [16]. Bayesian optimization consists of two main components: a model of the *expected return*, and an *acquisition function*, which uses the model to define the utility of each point in the search space.

Bayesian optimization, for policy search, follows the generic policy search algorithm (Algo. 1) and implements COLLECTSTRATEGY, LEARNSTRATEGY and UPDATESTRATEGY (Algo. 3). More specifically, a surrogate model, $\hat{J}(\boldsymbol{\theta})$, of the expected return is learned from the data, then the next policy to test is selected by optimizing the ACQUISITIONFUNCTION. The ACQUISITIONFUNCTION tries to intelligently exploit the model and its uncertainties in order to trade-off exploration and exploitation.

The main axes of variation are: (a) the way INITSTRATEGY is defined (the most usual approaches are random policy parameters or random actions), (b) the type of model used to learn J, (c) which ACQUISITIONFUNCTION is used, and (d) the optimizer used to optimize the ACQUISITIONFUNCTION.

Gaussian Processes Gaussian Process (GP) regression [128] is the most popular choice for the model. A GP is an extension of multivariate Gaussian distribution to an infinite-dimension stochastic process for which any finite combination of dimensions will be a Gaussian distribution [128]. More precisely, it is a distribution over functions, completely specified by its mean function, $\mu(\cdot)$ and covariance function, $k(\cdot, \cdot)$ and it is computed as follows:

$$\hat{J}(\boldsymbol{\theta}) \sim \mathcal{GP}(\mu(\boldsymbol{\theta}), k(\boldsymbol{\theta}, \boldsymbol{\theta}')). \tag{13}$$

Assuming $D_{1:t} = \{R(\boldsymbol{\tau}|\boldsymbol{\theta}_1), ..., R(\boldsymbol{\tau}|\boldsymbol{\theta}_t)\}$ is a set of observations, we can query the GP at a new input point $\boldsymbol{\theta}_*$ as follows:

$$p(\hat{J}(\boldsymbol{\theta}_*)|D_{1:t}, \boldsymbol{\theta}_*) = \mathcal{N}(\mu(\boldsymbol{\theta}_*), \sigma^2(\boldsymbol{\theta}_*)). \tag{14}$$

The mean and variance predictions of the GP are computed using a kernel vector $\boldsymbol{k} = k(D_{1:t}, \boldsymbol{\theta}_*)$, and a kernel matrix K, with entries $K_{ij} = k(\boldsymbol{\theta}_i, \boldsymbol{\theta}_j)$:

$$\mu(\boldsymbol{\theta}_*) = \boldsymbol{k}^T K^{-1} D_{1:t},$$
$$\sigma^2(\boldsymbol{\theta}_*) = k(\boldsymbol{\theta}_*, \boldsymbol{\theta}_*) - \boldsymbol{k}^T K^{-1} \boldsymbol{k}. \tag{15}$$

For the acquisition function, most algorithms use the Expected Improvement, the Upper Confidence Bound or the Probability of Improvement [16, 62].

Probability of Improvement. One of the first acquisition functions is the Probability of Improvement [89] (PI). PI defines the probability that a new test point $\hat{J}(\boldsymbol{\theta})$ will be better than the best observation so far $\boldsymbol{\theta}^+$; since we cannot directly get this information from $D_{1:t}$, in practice we query the approximated model $\hat{J}$ on $D_{1:t}$ and get the best parameters. When using GPs as the surrogate model, this can be analytically computed:

$$PI(\boldsymbol{\theta}) = p(\hat{J}(\boldsymbol{\theta}) > \hat{J}(\boldsymbol{\theta}^+))$$
$$= \Phi\left(\frac{\mu(\boldsymbol{\theta}) - \hat{J}(\boldsymbol{\theta}^+)}{\sigma(\boldsymbol{\theta})}\right) \tag{16}$$

where $\Phi(\cdot)$ denotes the CDF of the standard normal distribution. The main drawback of PI is that it basically performs pure exploitation; in practice, a slightly modified version of PI is used where a trade-off parameter ξ is added [16].

Expected Improvement. The Expected Improvement [16] (EI) acquisition function is an extension of PI, where the expected improvement (deviation) from the current maximum is calculated. Again, when using GPs as the surrogate model, EI can be analytically computed:

$$
\begin{aligned}
I(\boldsymbol{\theta}) &= \max\{0, \hat{J}(\boldsymbol{\theta}) - \hat{J}(\boldsymbol{\theta}^+)\} \\
EI(\boldsymbol{\theta}) &= \mathbb{E}(I(\boldsymbol{\theta})) \\
&= \begin{cases} (\mu(\boldsymbol{\theta}) - \hat{J}(\boldsymbol{\theta}^+))\Phi(Z) + \sigma(\boldsymbol{\theta})\phi(Z), & \text{if } \sigma(\boldsymbol{\theta}) > 0. \\ 0, & \text{otherwise.} \end{cases} \\
Z &= \frac{\mu(\boldsymbol{\theta}) - \hat{J}(\boldsymbol{\theta}^+)}{\sigma(\boldsymbol{\theta})}
\end{aligned}
\tag{17}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the PDF and CDF of the standard normal distribution respectively.

Upper Confidence Bound. The Upper Confidence Bound (UCB) acquisition function is the easiest to grasp and works very well in practice [62]. When using GPs as the surrogate model, it is defined as follows:

$$
UCB(\boldsymbol{\theta}) = \mu(\boldsymbol{\theta}) + \alpha\sigma(\boldsymbol{\theta})
\tag{18}
$$

where α is a user specified parameter.

Henning et al. [62] performed a thorough experimental analysis and concluded that EI can perform better on artificial objective functions than PI and UCB, but more recent experiments on gait learning on a physical robot suggested that UCB can outperform EI in real situations [19].

Since BO does not depend on the dimensionality of the state space, it can be effective for learning policies for robots with complex (*e.g.*, locomotion tasks, because of the non-linearity created by the contacts) or high-dimensional dynamics. For instance, Bayesian optimization was successfully used to learn policies for a quadruped robot [100] (around 100 trials with a well-chosen 15D policy space), a small biped "compass robot" [19] (around 100 trials with a finite state automata policy), and a pocket-sized, vibrating soft tensegrity robot [129] (around 30 trials with directly controlling the motors). In all of these cases, BO was at least an order of magnitude more data-efficient than competing methods.

Unfortunately, BO scales badly with respect to the dimensionality of the policy space because modeling the objective function (*i.e.*, the expected return) becomes exponentially more difficult when the dimension increases [9]. This is why all the aforementioned studies employed low-dimensional policy spaces and very well chosen policy structures (*i.e.*, they all use a strong prior on the policy structure). Scaling up BO is, however, an active field of research in optimization and some ideas might be applied to robotics in the future; according to the optimization literature, random embeddings [157] and additive models [76, 130] are among the most promising ideas.

4.2 Bayesian optimization with priors

One of the most interesting features of BO is that it can leverage priors (*e.g.*, from simulation or from previous tasks) to accelerate learning on the actual task. Perhaps the most representative algorithm in this area is the "Intelligent Trial & Error" (IT&E) algorithm [31]. IT&E first uses MAP-Elites [31], an evolutionary

illumination [113, 155] (also known as quality-diversity [125]) algorithm, to create a repertoire of about 15000 high-performing policies and stores them in a low-dimensional map (*e.g.*, 6-dimensional whereas the policy space is 36-dimensional). When the robot needs to adapt, a BO algorithm searches for the best policy in the low-dimensional map and uses the reward stored in the map as the mean function of a GP. This algorithm allowed a 6-legged walking robot to adapt to several damage conditions (*e.g.*, a missing or a shortened leg) in less than 2 minutes (less than a dozen of trials), whereas it used a simulator of the intact robot to generate the prior.

Gaussian processes with priors Assuming $D_{1:t} = \{R(\boldsymbol{\tau}|\boldsymbol{\theta}_1), ..., R(\boldsymbol{\tau}|\boldsymbol{\theta}_t)\}$ is a set of observations and $R_m(\boldsymbol{\theta})$ being the reward in the map, we can query the GP at a new input point $\boldsymbol{\theta}_*$ as follows:

$$
p(\hat{J}(\boldsymbol{\theta}_*)|D_{1:t}, \boldsymbol{\theta}_*) = \mathcal{N}(\mu(\boldsymbol{\theta}_*), \sigma^2(\boldsymbol{\theta}_*)).
\tag{19}
$$

The mean and variance predictions of this GP are computed using a kernel vector $\boldsymbol{k} = k(D_{1:t}, \boldsymbol{\theta}_*)$, and a kernel matrix K, with entries $K^{ij} = k(\boldsymbol{\theta}_i, \boldsymbol{\theta}_j)$ and where $k(\cdot, \cdot)$ is the kernel of the GP:

$$
\begin{aligned}
\mu(\boldsymbol{\theta}_*) &= R_m(\boldsymbol{\theta}_*) + \boldsymbol{k}^T K^{-1}(D_{1:t} - R_m(\boldsymbol{\theta}_{1:t})), \\
\sigma^2(\boldsymbol{\theta}_*) &= k(\boldsymbol{\theta}_*, \boldsymbol{\theta}_*) - \boldsymbol{k}^T K^{-1} \boldsymbol{k}.
\end{aligned}
\tag{20}
$$

The formulation above allows us to combine observations from the prior and the real-world smoothly. In areas where real-world data is available, the prior's prediction will be corrected to match the real-world ones. On the contrary, in areas far from real-world data, the predictions resort to the prior function [25, 31, 91].

Following a similar line of thought but implemented differently, a few recent works [5, 6] use a simulator to learn the kernel function of a GP, instead of utilizing it to create a mean function like in IT&E [31]. In particular, Antonova et al. [5] used domain knowledge for bipedal robots (*i.e.*, *Determinants of Gait* (DoG) [68]) to produce a kernel that encodes the differences in walking gaits rather than the Euclidean distance of the policy parameters. In short, for each controller parameter $\boldsymbol{\theta}$ a score $\text{sc}(\boldsymbol{\theta})$ is computed by summing the 5 DoG and the kernel $k(\cdot, \cdot)$ is defined as $k(\boldsymbol{\theta}_i, \boldsymbol{\theta}_j) = k(\text{sc}(\boldsymbol{\theta}_i), \text{sc}(\boldsymbol{\theta}_j))$. This proved to be beneficial and their approach outperformed both traditional BO and state-of-the-art black-box optimizers (CMA-ES). Moreover, in their follow-up work [6], the same authors use neural networks to model this kernel instead of hand-specifying it. Their evaluation shows that the learned kernels perform almost as good as hand-tuned ones and outperform traditional BO. Lastly, in this work they were able to make a physical humanoid robot (ATRIAS) to walk in a handful of trials.

A similar but more general idea (*i.e.*, no real assumption about the underlying system) was introduced by [159]. The authors propose a Behavior-Based Kernel (BBK) that utilizes trajectory data to compare policies, instead of using the distance in parameters (as is usually done). More specifically, they define the behavior of a policy to be the associated trajectory density $P(\boldsymbol{\tau}|\boldsymbol{\theta})$ and the kernel $k(\cdot, \cdot)$ is defined as $k(\boldsymbol{\theta}_i, \boldsymbol{\theta}_j) = \alpha \exp D(\boldsymbol{\theta}_i, \boldsymbol{\theta}_j)$, where $D(\boldsymbol{\theta}_i, \boldsymbol{\theta}_j)$ is defined as a sum of KL-divergences between the trajectory densities of different policies. Their approach was able to efficiently learn on several benchmarks; *e.g.*, it required on average less than 20 episodes on the mountain car, acrobot and cartpole swing-up tasks. One could argue that this approach does not utilize any prior information, but rather creates it on the fly; nevertheless, the evaluation was only performed with low-dimensional and well-chosen policy spaces.

Wilson et al. [159] proposed to learn models of the dynamics and the immediate reward to compute an approximate mean function of the GP, which is then used in a traditional BO procedure. They also combine this idea with the BBK kernel and follow a regular BO procedure where at each iteration they re-compute the mean function of the GP with the newly learned models. Although, their approach successfully learned several tasks in less than 10 episodes (*e.g.*, mountain car, cartpole swing-up), there is an issue that might not be visible at first sight: the authors combine model learning, which scales badly with the state/action space dimensionality (see Section 5), with Bayesian optimization, which scales badly with the dimensionality of the policy space. As such, their approach can only work with relatively small state/action spaces and small policy spaces. Using priors in the dynamics (see Section 5.2) and recent improvements on BO (see Section 4.1) could make their approach more practical.

Instead of using the simulator to precompute priors, Alonso et al. [104] propose an approach that has the ability to automatically decide whether it will gain crucial information from a real sample or it can use the simulator that is cheaper. More specifically, they present a BO algorithm for multiple information sources. Their approach relies on Entropy Search (ES) [62], which selects parameters in order to maximally reduce the uncertainty about the location of the maximum of $J(\boldsymbol{\theta})$ in each step. It quantifies this uncertainty through the entropy of the distribution over the location of the maximum, $p_{\max}(\boldsymbol{\theta}) = \mathbb{P}(\boldsymbol{\theta} \in \operatorname{argmin}_{\boldsymbol{\theta}} J(\boldsymbol{\theta}))$. ES basically defines a different ACQUISITIONFUNCTION for BO as follows:

$$ES(\boldsymbol{\theta}) = \operatorname*{argmax}_{\boldsymbol{\theta}} \mathbb{E}[\Delta H(\boldsymbol{\theta})] \qquad (21)$$

where $\Delta H(\boldsymbol{\theta})$ is the change in entropy of $p_{\max}$ caused by retrieving a new cost value at location $\boldsymbol{\theta}$. They use entropy to measure the information content of simulations and real experiments. Since this is an appropriate unit of measure for the utility of both sources, the algorithm is able to compare physically meaningful quantities in the same units, and trade off accuracy for cost. As a result, the algorithm can automatically decide whether to evaluate cheap, but inaccurate simulations or perform expensive and precise real experiments. They applied the method to fine-tune the policy of a cart-pole system and showed that their approach can speed up the optimization process significantly compared to standard BO.

Pautrat et al. [120] also recently proposed to combine BO with multiple information sources (or *priors*). They define a new ACQUISITIONFUNCTION function for BO, which they call *Most Likely Expected Improvement* (MLEI). MLEI attempts to have the right balance between the likelihood of the priors and the potential for high-performing solutions. In other words, a good expected improvement according to an unlikely model should be ignored; conversely, a likely model with a low expected improvement might be too pessimistic ("nothing works") and not helpful. A model that is "likely enough" and lets us expect some good improvement might be the most helpful to find the maximum of the objective function. The MLEI acquisition function is defined as follows:

$$EIP(\boldsymbol{\theta}, \mathcal{P}) = EI(\boldsymbol{\theta}) \times p(\hat{J}(\boldsymbol{\theta}_{1..t}) \mid \boldsymbol{\theta}_{1..t}, \mathcal{P}(\boldsymbol{\theta}_{1..t}))$$
$$MLEI(\boldsymbol{\theta}, \mathcal{P}_1, \cdots, \mathcal{P}_m) = \max_{\boldsymbol{p} \in \mathcal{P}_1, \cdots, \mathcal{P}_m} EIP(\boldsymbol{\theta}, \boldsymbol{p}) \qquad (22)$$

where $\mathcal{P}_i, i = 1 \ldots m$ is the set of available priors (where each $\mathcal{P}_i$ is defined similarly to R_m in Eq.(20)). They evaluated their

approach in a transfer learning scenario with a simulated arm and in a damage recovery one with both a simulated and a physical hexapod robot. Their approach demonstrates improved performance relative to random trials or a hand-chosen prior (when that prior does not correspond to the new task). Interestingly, this method also is able to outperform the real prior in some circumstances.

Lober et al. [101] use a BO procedure that selects parameterizations of a QP-based whole body controller [132, 140] in order to control a humanoid robot. In particular, they formulate a policy that includes the QP-based controller (that contains an approximate model of the system and an optimizer) and is parameterized by way-points (and/or switching times). Their approach was able to allow an iCub robot to move a heavy object while maintaining body balance and avoid collisions [101, 102].

Safety-Aware Approaches Another interesting direction of research is using variants of BO for safety-aware learning; that is learning that actively tries to avoid regions that might cause harm to the robot. In [118] the authors proposed an extension of IT&E that safely trades-off between exploration and exploitation in a damage recovery scenario. To achieve this, (1) they generate, through MAP-Elites, a diverse archive of estimations concerning performance and safety criteria and (2) they use this as prior knowledge in an online constrained BO [53] procedure that guides the search towards a compensatory behavior and with respect to the safety beliefs. Their algorithm, sI&TE, allowed a simulated damaged iCub to crawl again safely.

Similarly, in [10] Berkenkamp et al. introduced SafeOpt, a BO procedure to automatically tune controller parameters by trading-off between exploration and exploitation only within a safe zone of the search space. Their approach requires minimal knowledge, such as an initial, not optimal, safe controller to bootstrap the search. Using this approach a quadrotor vehicle was able to safely improve its performance over the initial sub-optimal policy.

5 Learning models of the dynamics

Instead of learning a model of the expected long-term reward (section 4.1), one can also learn a model of the dynamics of the robot. By repeatedly querying this surrogate model, it is then possible to make a prediction of the expected return. This idea leads to *model-based policy search algorithms* [39, 124], in which the trajectory data are used to learn the dynamics model, then policy search is performed on the model [73, 147].

Put differently, the algorithms leverage the trajectories $\boldsymbol{\tau}_1, \cdots \boldsymbol{\tau}_N$ observed so far to learn a function $\hat{f}(\boldsymbol{x}, \boldsymbol{u})$ so that:

$$\hat{\boldsymbol{x}}_{t+1} = \hat{f}(\boldsymbol{x}_t, \boldsymbol{u}_t). \qquad (23)$$

This function, $\hat{f}(\boldsymbol{x}_t, \boldsymbol{u}_t)$, is then used to compute an estimation of the expected return, $\hat{J}(\boldsymbol{\theta} | \boldsymbol{\tau}_1, \cdots \boldsymbol{\tau}_N)$.

5.1 Model-based Policy Search

Let us consider that the actual dynamics f (and consequently the transition probabilities) are approximated by a model $\hat{f}$ and the immediate reward function r is approximated by a model $\hat{r}$. As such, in model-based policy search we are alternating between learning the models ($\hat{f}$ and $\hat{r}$) and maximizing the expected long-

term reward on the model:

$$\hat{J}(\boldsymbol{\theta}) = \mathbb{E}[\hat{R}(\boldsymbol{\tau})|\boldsymbol{\theta}] = \int \hat{R}(\boldsymbol{\tau})\hat{P}(\boldsymbol{\tau}|\boldsymbol{\theta}) \qquad (24)$$

where

$$\hat{P}(\boldsymbol{\tau}|\boldsymbol{\theta}) = p(\boldsymbol{x}_0)\prod_t \hat{p}(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)\pi_{\boldsymbol{\theta}}(\boldsymbol{u}_t|\boldsymbol{x}_t, t). \qquad (25)$$

This iterative scheme can be seen as follows:

$$\boldsymbol{\tau}_n \sim P(\boldsymbol{\tau}|\boldsymbol{\theta}_n) \qquad (26)$$
$$\boldsymbol{D}_n = \boldsymbol{D}_{n-1} \cup \{\boldsymbol{\tau}_n, R(\boldsymbol{\tau}_n)\} \qquad (27)$$
$$\boldsymbol{\theta}_{n+1} = \operatorname*{argmax}_{\boldsymbol{\theta}} \hat{J}(\boldsymbol{\theta}|\boldsymbol{D}_n) \qquad (28)$$

where $\boldsymbol{\theta}_0$ is randomly determined or initialized to some value, $\boldsymbol{D}_0 = \emptyset$ and $\hat{J}(\boldsymbol{\theta}|\boldsymbol{D})$ means calculating $\hat{J}(\boldsymbol{\theta})$ once the models $\hat{f}$ and $\hat{r}$ are learned using the dataset of trajectories and rewards $\boldsymbol{D}$. After some stopping criteria is met (usually number of episodes or convergence of policy parameters), the policy that approximately maximizes Eq. (6) can be straightforwardly retrieved:

$$\pi_{\boldsymbol{\theta}^*} = \boldsymbol{\theta}_{\text{last}}. \qquad (29)$$

If the system (and the policy) is not stochastic, one could retrieve the best policy differently:

$$\pi_{\boldsymbol{\theta}^*} = \operatorname*{argmax}_{\boldsymbol{\theta}} R(\boldsymbol{\tau}_n). \qquad (30)$$

Algorithm 4 Model-based policy search

1: **procedure** COLLECTSTRATEGY
2:     Collect samples of the form $(\boldsymbol{x}_t, \boldsymbol{u}_t, r_{t+1})$
3: **end procedure**
4: **procedure** LEARNSTRATEGY
5:     Learn model $\hat{f} : (\boldsymbol{x}_t, \boldsymbol{u}_t) \to \boldsymbol{x}_{t+1}$
6:     Learn model $\hat{r} : (\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{x}_t) \to r_{t+1}$
7: **end procedure**
8: **procedure** UPDATESTRATEGY
9:     $\boldsymbol{\theta}_{n+1} = \operatorname{argmax}_{\boldsymbol{\theta}} \hat{J}(\boldsymbol{\theta}|\boldsymbol{D}_n)$
10: **end procedure**

Model-based policy search follows the generic policy search algorithm (Algo. 1) and implements COLLECTSTRATEGY, LEARNSTRATEGY and UPDATESTRATEGY (Algo. 4). The main axes of variation are: (a) the way INITSTRATEGY is defined (the most usual approaches are random policy parameters or random actions), (b) the type of models used to learn $\hat{f}$ and $\hat{r}$, (c) the optimizer used to optimize $\hat{J}(\boldsymbol{\theta}|\boldsymbol{D}_n)$, and (d) how are the long-term predictions, given the models, performed (i.e., how Eq. (24) is calculated or approximated).

Model-based policy search algorithms are usually more data-efficient than both direct and surrogate-based policy search methods as they do not depend much on the dimensionality of the policy space. On the other hand, since they are modelling the transition dynamics, practical algorithms are available only for relative small state-action spaces [24, 39, 41, 124].

5.1.1 Model Learning

There exist many approaches to learn the models $\hat{f}$ and $\hat{r}$ (for model-based policy search) in the literature [41, 96, 150]. We can categorize the learned models in deterministic (e.g., neural networks or linear regression) and probabilistic ones (e.g., Gaussian Processes).

Probabilistic models usually rely on Bayesian methods and are typically non-parametric (and thus exhibit potentially infinite capacity), whereas deterministic models are typically parametric (and thus do not have infinite capacity). Probabilistic models are usually more effective than deterministic models in model-based policy search [39] because they provide uncertainty information that can be incorporated into the long-term predictions, thus giving the capability to the optimizer to find more robust controllers (and not over-exploit the model biases). Black-DROPS [24] and PILCO [36] both utilize Gaussian processes (GPs) to greatly reduce the interaction time to solve several tasks, although Black-DROPS is not tied to them and any other deterministic or probabilistic model can be used.

Recently, the model-based Policy Gradients with Parameter-based Exploration (M-PGPE) algorithm [150] suggested instead of learning the model $\hat{f}$, to directly try to estimate the transition probabilities $p(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)$ using least-squares conditional density estimation [146]. Using this formulation they were able to bypass some drawbacks of GPs such as computation speed and smoothness assumption (although choosing appropriate kernels in the GPs can produce non-smooth predictions).

Another way of learning models of the dynamics is to use local linear models [86, 96, 98]; i.e., models that are trained on and are only correct in the regions where one controller/policy can drive the system. Guided policy search with unknown dynamics utilizes this scheme and is able to learn efficiently even in high-dimensional states and discontinuous dynamics, like 2D walking and peg-in-the-hole tasks [96, 98] and even dexterous manipulation tasks [86].

There has, also, recently been some work on using Bayesian Neural Networks (BNNs) [50] to improve the scaling of model-based policy search algorithms [51, 63]. Compared to GPs, BNNs scale much better with the number of samples. Nevertheless, BNNs require more tedious hyper-parameter optimization and there is no established, intuitive way to include prior knowledge (apart from the structure). A combination of ensembles and probabilistic neural networks has been recently proposed [26] for learning probabilistic dynamics models of higher dimensional systems; for example, state-of-the-art performance was obtained in controlling the half-cheetah benchmark [158] by combining these models with model-predictive control. Recent works showcase that using BNNs with stochastic inputs (and the appropriate policy search procedure) is beneficial when learning in scenarios with multi-modality and heteroskedasticity [42]; traditional model learning approaches (e.g., Gaussian processes) fail to properly model these scenarios. Moreover, decomposing aleatoric (i.e., inherent uncertainty of the underlying system) and epistemic (i.e., uncertainty due to limited data) uncertainties in BNNs (with latent input variables) can provide useful information on which points to sample next [43].

Lastly, when performing model-based policy search under partial observability, different model learning techniques should be used. One interesting idea is to optimize the model with the explicit goal of explaining the already observed trajectories instead of focusing on the step-by-step predictions. Doerr et al. [44] recently proposed a principled approach to incorporate these ideas

into GP modeling and were able to outperform other robust models in long-term predictions and showcase improved performance for model-based policy search on a real robot with noise and latencies.

5.1.2 Long-term predictions

Traditionally, we would categorize the model-based policy search algorithms in those that perform *stochastic long-term predictions* by means of samplings and those that perform *deterministic long-term predictions* by deterministic inference techniques [39]. Recently, an alternative way of computing the expected long-term reward was introduced by [24] (*Policy Evaluation as a Noisy Observation*), where the trajectory generation is combined with the optimization process in order to achieve high-quality predictions with fewer monte-carlo rollouts.

Stochastic Long-Term Predictions The actual dynamics of the system are approximated by the model $\hat{f}$, and the immediate reward function by the model $\hat{r}$. The model $\hat{f}$ provides the transition probabilities $\hat{p}(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)$. Similarly, the model $\hat{r}$ provides the immediate reward distribution $\hat{p}(\hat{r}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{x}_{t+1})$. When applying a policy (with some parameters $\boldsymbol{\theta}$) on the model, we get a *rollout* or *trajectory*:

$$\boldsymbol{\tau} = (\boldsymbol{x}_0, \boldsymbol{u}_0, \boldsymbol{x}_1, \boldsymbol{u}_1, \ldots, \boldsymbol{x}_T) \tag{31}$$

$$\boldsymbol{r} = (\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_T) \tag{32}$$

$$\hat{R}(\boldsymbol{\tau}) = \sum_{t=0}^{T-1} \hat{r}_{t+1} \tag{33}$$

where

$$\boldsymbol{x}_0 \sim p(\boldsymbol{x}_0) \tag{34}$$

$$\hat{r}_{t+1} \sim \hat{p}(\hat{r}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{x}_{t+1}) \tag{35}$$

$$\boldsymbol{u}_t \sim \pi_{\boldsymbol{\theta}}(\boldsymbol{u}_t|\boldsymbol{x}_t, t) \tag{36}$$

$$\boldsymbol{x}_{t+1} \sim \hat{p}(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t). \tag{37}$$

This is basically sampling the distribution over trajectories, $\hat{P}(\boldsymbol{\tau}|\boldsymbol{\theta})$, which is feasible since the sampling is performed with the models. When applying the same policy (*i.e.*, a policy with the same parameters $\boldsymbol{\theta}$), the trajectories $\boldsymbol{\tau}$ (and consequently $\boldsymbol{r}$) can be different (*i.e.*, stochastic) because (of at least one of the following):

- The policy is stochastic. If the policy is deterministic, then $\boldsymbol{u}_t = \pi_{\boldsymbol{\theta}}(\boldsymbol{x}_t, t)$;

- The models ($\hat{f}$ and/or $\hat{r}$) are probabilistic;

- Of the initial state distribution, $p(\boldsymbol{x}_0)$.

Monte-Carlo & PEGASUS Policy Evaluation: Once we know how to generate trajectories given some policy parameters, we need to define the way to evaluate the performance of these policy parameters. Perhaps the most straightforward way of computing the expected log-term reward of some policy parameters is to generate m trajectories with the same policy along with their long-term costs and then compute the average (*i.e.*, perform Monte-Carlo sampling):

$$\tilde{\tilde{J}}(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \hat{R}_i(\boldsymbol{\tau}^i). \tag{38}$$

One more efficient way of computing the expected long-term reward with stochastic trajectories is the PEGASUS sampling procedure [115]. In the PEGASUS sampling procedure the random seeds for each time step are fixed. As a result, repeating the same experiment (*i.e.*, the same sequence of control inputs and the same initial state) would result into exactly the same trajectories. This significantly reduces the sampling variance compared to pure Monte-Carlo sampling and can be shown that optimizing this *semi-stochastic* version of the model is equivalent to optimizing the actual model.

The advantages of the sampling-based policy evaluations schemes are that each *rollout* can be performed in parallel and that they require much less implementation effort than the deterministic long-term predictions (see Section 5.1.2). Nevertheless, these sampling-based procedures can experience big variances in the predictions that can negatively affect the optimization process.

Model-based contextual REPS [88] heavily uses sampling-based policy evaluations and showed that when using enough sample trajectories, you can get better approximations than deterministic long-term predictions (see Section 5.1.2); another recent work also strongly justifies the usage of sampling-based policy/action evaluations over deterministic inference methods [26] (especially in higher dimensional systems). They were also able to greatly reduce the computation time by exploiting the parallelization capabilities of modern GPUs. In their paper, model-based contextual REPS is able to learn policies for controlling robots that play table tennis and hockey, where different goal positions are handled as different contexts.

Deterministic Long-Term Predictions Instead of sampling trajectories $\boldsymbol{\tau}$, the probability distribution $\hat{P}(\boldsymbol{\tau}|\boldsymbol{\theta})$ can be computed with deterministic approximations, such as linearization [3], sigma-point methods [72] or moment matching [41]. All these inference methods attempt to approximate the original distribution with a Gaussian.

Assuming a joint probability distribution $\hat{p}(\boldsymbol{x}_t, \boldsymbol{u}_t) = \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$, the distribution $\hat{P}(\boldsymbol{\tau}|\boldsymbol{\theta})$ can be computed by successively computing the distribution of $\hat{p}(\boldsymbol{x}_{t+1})$ given $\hat{p}(\boldsymbol{x}_t, \boldsymbol{u}_t)$. Computing $\hat{p}(\boldsymbol{x}_{t+1})$ corresponds to solving the integral:

$$\hat{p}(\boldsymbol{x}_{t+1}) = \iiint \hat{p}(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t, \boldsymbol{u}_t)\hat{p}(\boldsymbol{x}_t, \boldsymbol{u}_t)d\boldsymbol{x}_t d\boldsymbol{u}_t d\boldsymbol{w}. \tag{39}$$

This integral can be computed analytically only if the transition dynamics $\hat{f}$ are linear (in that case $\hat{p}(\boldsymbol{x}_{t+1})$ is Gaussian). This is rarely the case and as such, approximate inference techniques are used. Usually, we approximate $\hat{p}(\boldsymbol{x}_{t+1})$ as a Gaussian; this can be done either by linearization [3], sigma-point methods [72] or moment matching [41]. The PILCO algorithm [36] uses moment matching, which is the best unimodal approximation of the predictive distribution in the sense that it minimizes the Kullback-Leibler divergence between the true predictive distribution and the unimodal approximation [39].

One big advantage of using deterministic inference techniques for long-term predictions is the low-variance they exhibit in the predictions. In addition, using these inference techniques allows for analytic gradient computation and as such we can exploit efficient gradient-based optimization. However, each of these inference techniques has its own disadvantages; for example, exact moments (for moment matching) can be computed only in special cases since the required integrals might be intractable, which

limits the overall approach (*e.g.*, PILCO requires that the reward function is known and differentiable).

The PILCO algorithm [41] uses this type of long-term predictions and it was the first algorithm that showed remarkable data-efficiency on several benchmark tasks (*e.g.*, less than 20 seconds of interaction time to solve the cart-pole swing-up task) [36]. It was also able to learn on a physical low-cost manipulator [37] and simulated walking tasks [38] among the many successful applications of the algorithm [41].

Policy Evaluation as a Noisy Observation One alternative long-term prediction scheme was introduced by Chatzilygeroudis et al. [24], where the authors combine the trajectory generation with the optimization process in an attempt to reduce the computation time of the policy search on the model.

In their approach, they exploit population, rank-based optimizers, like CMA-ES [57], in order to combine the policy evaluation with the optimization procedure. In particular, they exploit the *implicit averaging* property that they possess [70, 107] in order to efficiently perform sampling based evaluation of the trajectories. Their key idea is that when using this type of optimizers, there's no need to (fully) compute the expected long-term reward but thanks to the implicit averaging the problem can be transformed into a noisy optimization one (and the optimizer implicitly computes the expectation). They introduced Black-DROPS, one of the first purely black-box, flexible and data-efficient model-based policy search methods.

In more detail, instead of performing deterministic long-term predictions, like PILCO, or Monte-Carlo evaluation, like PEGASUS, Black-DROPS stochastically generates trajectories, but considers that each of these trajectories (or rollouts) is a measurement of a function $G(\boldsymbol{\theta})$ that is the actual function $\hat{J}(\boldsymbol{\theta})$ perturbed by a noise $N(\boldsymbol{\theta})$:

$$G(\boldsymbol{\theta}) = \hat{J}(\boldsymbol{\theta}) + N(\boldsymbol{\theta}). \tag{40}$$

It is easy to verify that maximizing $\mathbb{E}\left[G(\boldsymbol{\theta})\right]$ is equivalent to maximizing $\hat{J}(\boldsymbol{\theta})$, when $\mathbb{E}\left[N(\boldsymbol{\theta})\right] =$ constant.

Implicit Averaging and Noisy Functions: Seeing the maximization of $\hat{J}(\boldsymbol{\theta})$ as the optimization of a noisy function allows to maximize it without computing or estimating it explicitly: we only use the noisy measurements in the optimization algorithm. To do so, they use the Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) [59], which is one of the most successful evolutionary algorithms for optimizing noisy and black-box functions [58, 60, 70]. CMA-ES performs four steps at each generation k:

(1) sample λ new candidates according to a multi-variate Gaussian distribution of mean m_k and covariance $\sigma_k^2 C_k$, that is, $\boldsymbol{\theta}_i \sim \mathcal{N}(m_k, \sigma_k^2 C_k)$ for $i \in 1, \cdots, \lambda$;

(2) rank the λ sampled candidates based on their (noisy) performance $G(\boldsymbol{\theta}_i)$;

(3) compute m_{k+1} using a weighted average of the μ best candidates: $m_{k+1} = \sum_{i=1}^{\mu} w_i \boldsymbol{\theta}_i$, where $\sum_{i=1}^{\mu} w_i = 1$;

(4) update the covariance matrix to reflect the distribution of the successful search steps.

Overall, these steps are only marginally impacted by noise in the performance function, as confirmed by empirical experiments with noisy functions [70]. More precisely, the only decision that

matters is whether a solution belongs to the μ best ones (step 2), that is, a precise ranking is not needed and errors can only happen at the boundaries between the low-performing and high-performing solutions. In addition, if a candidate is misclassified because of the noise, the impact of this error will be smoothed out by the average when computing m_{k+1} (step 3). One can also observe that because CMA-ES samples several solutions around a mean m_k, it performs many evaluations of similar parameters, which are then averaged: this *implicit averaging* [70, 107] has many similarities with re-evaluating noisy solutions to estimate their expectation.

In the extreme case where the population is infinite, it can be shown that the selection part of evolutionary algorithms is not affected at all by noise [107]. Moreover, it is shown that in genetic algorithms with ranking-based selection, adding random perturbations to the design variables at each generation is equivalent to optimizing the expected fitness function [153]. The Black-DROPS algorithm ulitizes a recent variant of CMA-ES that uses a similar scheme and combines random pertubations with re-evaluation for uncertainty handling [60] along with restart strategies for better exploration [8].

While Black-DROPS has the same data-efficiency as PILCO, it has the added benefit of being able to exploit multi-core architectures, thus, greatly reducing the computation time [24]. In addition, it is one of the first purely black-box model-based policy search algorithms; *i.e.*, one can swap the model types, reward functions and/or initialization procedure with minimal effort and be able to exploit different properties that they possess. This is an important feature as it allows us to more easily exploit good sources of prior information [23]. Black-DROPS was able to learn in less than 20 seconds of interaction time to solve the cart-pole swing-up task as well as to control a physical 4-DOF physical manipulator in less than 5-6 episodes.

5.2 Using priors on the dynamics

Reducing the interaction time in model-based policy search can be achieved by using priors on the models [11, 23, 32, 40, 91, 133, 160]; *i.e.*, starting with an initial guess of the dynamics (and/or the reward function) and then learning the residual model. This type of algorithms follow the general model-based policy search framework (Algo. 4) and usually implement different types of INITSTRATEGY. Notably, most of the approaches (and the most successful ones) rely on Gaussian processes to model the dynamics, as priors can be very elegantly incorporated (the formulation is identical to Eq. (19), but re-stated with the proper the notation for clarity):

Gaussian Processes with priors for dynamical models Assuming $D_{1:t} = \{f(\tilde{\boldsymbol{x}}_1), ..., f(\tilde{\boldsymbol{x}}_t)\}$ is a set of observations, $\tilde{\boldsymbol{x}}_t = (\boldsymbol{x}_t, \boldsymbol{u}_t) \in \mathbb{R}^{E+F}$ and $M(\tilde{\boldsymbol{x}})$ being the simulator function (*i.e.*, the initial guess of the dynamics), we can query the GP at a new input point $\tilde{\boldsymbol{x}}_*$ as follows (of course we have E independent GPs; one for each output dimension [24, 36]):

$$p(\hat{f}(\tilde{\boldsymbol{x}}_*)|D_{1:t}, \tilde{\boldsymbol{x}}_*) = \mathcal{N}(\mu(\tilde{\boldsymbol{x}}_*), \sigma^2(\tilde{\boldsymbol{x}}_*)). \tag{41}$$

The mean and variance predictions of this GP are computed using a kernel vector $\boldsymbol{k} = k(D_{1:t}, \tilde{\boldsymbol{x}}_*)$, and a kernel matrix K, with entries $K^{ij} = k(\tilde{\boldsymbol{x}}_i, \tilde{\boldsymbol{x}}_j)$ and where $k(\cdot, \cdot)$ is the kernel of the GP:

$$\mu(\tilde{\boldsymbol{x}}_*) = M(\tilde{\boldsymbol{x}}_*) + \boldsymbol{k}^T K^{-1}(D_{1:t} - M(\tilde{\boldsymbol{x}}_{1:t})),$$
$$\sigma^2(\tilde{\boldsymbol{x}}_*) = k(\tilde{\boldsymbol{x}}_*, \tilde{\boldsymbol{x}}_*) - \boldsymbol{k}^T K^{-1} \boldsymbol{k}. \tag{42}$$

A few approaches [11, 78] use simple analytic and fast simulators prior to create a Gaussian process prior of the dynamics (and assume the reward function to be known). PILCO with priors [32] uses simulated data (from running PILCO in the simulator) to create a GP prior for the dynamics and then performs policy search with PILCO. PILCO with priors was able to increase the data-efficiency of PILCO in a real inverted pendulum using a very simple model as a prior. A similar approach, PI-REM [133], utilizes analytic equations for the dynamics prior and tries to actively bring the real trials as close as possible to the simulated ones (*i.e.*, reference trajectory) using a slightly modified PILCO policy search procedure. PI-REM was also able to increase the data-efficiency of PILCO in a real inverted pendulum (with variable stiffness actuators) using a simple model as a prior.

An approach that splits the self-modeling process from the policy search is presented in [12]. The authors were among the first ones to combine a self-modelling procedure (close to model identification [136]) with policy search. The self-modelling part of their approach consists of 3 steps: (a) action executing and data-collection, (b) synthesization of 15 candidate self-models that explain the sensory data and (c) active selection of the action that will elicit the most information from the robot/system. After a few cycles of these steps (*i.e.*, around 15), the most accurate model is selected and policy search is performed to produce a desired behavior. Their approach was able to control efficiently (*i.e.*, less than 20 episodes) a four-legged robot and it was also able to adapt to damages in a few trials (by re-running the self-modeling procedure).

Black-DROPS with priors [23] proposes a new GP learning scheme that combines model identification and non-parametric model learning (called GP-MI) and then performs policy search with Black-DROPS. The main idea of GP-MI is to use simulators with tunable parameters, *i.e.*, mean functions of the form $M(\tilde{\boldsymbol{x}}, \boldsymbol{\phi}_M)$ where each vector $\boldsymbol{\phi}_M \in \mathbb{R}^{n_M}$ corresponds to a different prior model of the system (*e.g.*, different lengths of links). Searching for the $\boldsymbol{\phi}_M$ that best matches the observations can be seen as a model identification procedure, which could be solved via minimizing the mean squared error; nevertheless, the authors formulate it in a way so that they can exploit the GP framework to jointly optimize for the kernel hyper-parameters and the mean parameters, which allows the modeling procedure to balance between non-parametric and parametric modeling.

Black-DROPS with GP-MI was able to robustly learn controllers for a pendubot swing-up task [141] even when the priors were misleading. More precisely, it was able to outperform Black-DROPS, PILCO, PILCO with priors, Black-DROPS with fixed priors (*i.e.*, this should be similar to PI-REM) and IT&E. Moreover, Black-DROPS with GP-MI was able to find high-performing walking policies for a physical damaged hexapod robot (48D state and 18D action space) in less than 1 minute of interaction time and outperformed IT&E that excels in this setting [23, 31].

Following a similar rationale, VGMI [163], uses a Bayesian optimization procedure to find the simulator's mechanical parameters so as to match the real-world trajectories (*i.e.*, it performs model identification) and then performs policy search on the updated simulator. In particular, VGMI was able to learn policies for a physical dual-arm collaborative task and out-performed PILCO.

6 Other approaches

6.1 Guided policy search

Guided policy search (GPS) with unknown dynamics [96, 98] is a somewhat hybrid approach that combines local trajectory optimization (that happens directly on the real system), learning local models of the dynamics (see Section 5.1.1) and indirect policy search where it attempts to approximate the local controllers with one big neural network policy (using supervised learning). In more detail, GPS consists of two loops: an outer loop that executes the local linear-Gaussian policies on the real system, records data and fits the dynamics models and an inner loop where it alternates between optimizing the local linear-Gaussian policies (using trajectory optimization and the fitted dynamics models) and optimizing the global policy to match all the local policies (via supervised learning and without utilizing the learned models) [98].

The results of GPS show that it is less data-efficient than model-based policy search approaches, but more data-efficient than traditional direct policy search. Moreover, GPS is able to handle bigger state-action spaces (*i.e.*, it has also been used with image observations [98]) than traditional model-based policy search approaches as it reduces the final policy optimization step in a supervised one that can be efficiently tackled with all the recent deep learning methods [90]. GPS was able to learn in less than 100 episodes even in high-dimensional states and discontinuous dynamics like 2D walking, peg-in-the-hole task and controlling an octopus robot [96, 98] among the many successful applications of the algorithm [97, 110].

6.2 Transferability approaches

The main hypothesis of the transferability approach [84, 85] is that physics simulators are accurate for some policies, *e.g.*, static gaits, and innacurate for some others, *e.g.*, highly dynamic gaits. As a consequence, it is possible to learn in simulation if the search is constrained to policies that are simulated accurately.

As no simulator currently comes with an estimate of its accuracy, the key idea of the transferability approach is to learn a model of a *transferability function*, which predicts the accuracy of a simulator given policy parameters or a trajectory in simulation. This function is often easier to learn than the expected return (Section 4.1) because it is essentially a classification problem (instead of a regression problem). In addition, small errors in the model have often little consequences, because the search is mainly driven by the expected return in simulation (and not by the transferability optimization).

The resulting learning process requires only a handful trials on the physical robot (in most of the experiments, less than 25); however, the main drawback is that it can only find policies that perform similarly in simulation and in reality (*e.g.*, static gaits versus highly dynamic gaits). These type of algorithms were able to efficiently learn policies for mobile robots that have to navigate in mazes [85] (15 trials on the robot), for a walking quadruped robot [83, 85] (about 10 trials) and for a 6-legged robot that had to learn how to walk in spite of a damaged leg without updating the simulator [84] (25 trials). Similar ideas were recently developed for humanoid robots with QP-based controllers [140].

7 Challenges and Frontiers

7.1 Scalability

Most of the works we described so far have been demonstrated with simple robots and simple tasks, such as the cartpole swing-up task (4D state space, 1D action space) [36] or simple manipulators (4D state space, 4D action space) [24]. By contrast, humanoid robots have orders of magnitude larger state-action spaces; for example, the 53-DOF iCub robot [152] has a state space of more than 100 dimensions (not counting tactile and visual sensors [103]). Most of the current micro-data approaches are unable to learn with such complex robots.

On the one hand, model-based policy search algorithms (Section 5.1) generalize well to new tasks (since the model does not depend on the task) and learn high-dimensional policies with little interaction time (since the policy search happens within the model and not in interaction with the robot); but they do not scale well with the size of the state space: in the general case, the quantity of data to learn a good approximation of the forward model scales exponentially with the dimensionality of the state-space (this is the curse of dimensionality, see [9]). A factored state representation may provide the means to tackle such complexity, for example, by using dynamic Bayesian networks [34] to represent the forward model [14], but we are not aware of any recent work in this direction.

On the other hand, direct policy search algorithms (Sections 3.6 and 4), scale well with the dimensionality of the state space because the complexity of the learning problem mostly depends on the number of parameters of the policy, and not on the dimensionality of the state-space; however, they do not generalize well to new tasks (when there is a model, it is specific to the reward) and they require a low-dimensional policy. Such a low-dimensional policy is an important, task-specific prior that constrains what can be learnt. For example, central pattern generators can be used for rhythmic tasks such as locomotion [67], but they are unlikely to work well for a manipulation task; similarly, quadratic programming-based controllers (and in general model-based controllers) can facilitate learning whole body controllers for humanoid robots [87, 140], but they impose the control strategy and the model.

In summary, model-based policy search algorithms scale well with the dimensionality of the policy, but they do not scale with the dimensionality of the state space; and direct policy search algorithms scale well with the dimensionality of the state-space, but not with the dimensionality of the policy. None of these two approaches will perform well on every task: future work should focus on either scaling model-based policy search algorithms so that they can learn in high-dimensional state spaces, or scaling direct policy search algorithms so that they can use higher-dimensional policies.

The dimensionality of the sensory observations is also an important challenge for micro-data learning: to our knowledge, no micro-data approach can perform "end-to-end learning", that is, learning with a raw data stream like a camera. Deep RL has recently made possible to learn policies from raw pixel input [108], largely because of the prior provided by convolutional networks. However, deep RL algorithms typically require a very large interaction time with the environment (*e.g.*, 38 days of play for Atari 2600 games [108]), which is not compatible with most robotics experiments and applications. To address this challenge, a potential starting point is to use unsupervised learning to learn low-dimensional features, which can then be used as inputs for policies. Interestingly, it is possible to leverage priors to learn such state representations from raw observations in a reasonable interaction time [71, 95]. It is also possible to create forward models in image space, that is, predicting the next image knowing the current one and the actions, which would allow to design model-based policy search algorithms that work with an image stream [7, 46, 55, 117].

7.2 Priors

Evolution has endowed animals and humans with substantial prior knowledge. For instance, hatchling turtles are prewired to run towards the sea [114]; or marine iguanas are able to run and jump within moments of their birth in order to avoid being eaten by snakes[6]. These species cannot rely on online learning mechanisms for mastering these behaviors: without such priors they would simply cease to exist.

Similarly to priors obtained from nature, artificial agents or robots can learn very quickly when provided with the right priors, as we presented in Sections 3, 4.2, and 5.2. In other words, priors play a catalytic role in reducing the interaction time of policy search methods. Thus, the following questions naturally arise: what should be innate and what should be learned? and how should the innate part be designed?

Most of the existing methodologies use task-specific priors (*e.g.*, demonstrations). Such priors can greatly accelerate policy search, but have the disadvantage of requiring an expert to provide them for all the different tasks the robots might face. More generic or task-agnostic priors (*e.g.*, properties of the physical world) could relax these assumptions while still providing a learning speedup. Some steps have been made into identifying such task-agnostic priors for robotics, and using them for state representation [71, 94]. We believe this is an important direction that requires more investigation. Meta-learning [29, 47, 49, 131], that is, "learning to learn", is a related line of work that can provide a principled and potentially automatic way of designing priors.

Physical simulations can also be used to automatically generate priors while being a very generic tool [5, 6, 31, 120]. By essence, physical simulations can run in parallel and take advantage of faster computing hardware (from clusters of CPUs to GPUs): learning priors in simulation could be an analog of the billions of years of evolution that shaped the learning systems of all the current lifeforms.

While priors can bootstrap policy search, they can also be misleading when a new task is encountered. Thus, an important research avenue is to design policy search algorithms that can not only incorporate well-chosen priors, but also ignore those that are irrelevant for the current task [23]. Following this line of thought, a promising idea is to design algorithms that actively select among a variety of priors [120].

7.3 Generalization and robustness

The majority of aforementioned articles are not much concerned with the generalization abilities and the robustness of the learned policy: they are designed to solve a single task, in a single context, with often very little evaluation of the abilities to reject perturbations. For example, IT&E [31] focused on a repertoire of forward walking gaits for a hexapod robot on flat ground, rather than on various surfaces (*e.g.*, incline surface or stairs) or various

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[6]As portrayed in the recent documentary "Planet Earth 2" from BBC.

directions [25]; PILCO was applied for stacking a tower of foam blocks with a robotic manipulator [41], but the task remained fixed over the course of learning (*i.e.*, the tower position on the table did not vary, nor the size of the cubes) and there were no external perturbations (*e.g.*, a wind gust). Put differently, in most of the reported experiments, the algorithms are very likely to have "overfitted" the robot and the task.

This situation could appear surprising because generalization and robustness are two of the most important questions in machine learning and control theory [136]. Its source is, however, straightforward: assessing the robustness or the generalization abilities of a policy typically requires a significant additional interaction time. For example, a typical approach to measure the robustness of a control policy is to evaluate it with many different starting conditions and perturbations; a similar technique is often used to test the generalization abilities. Nevertheless, such an approach multiplies the interaction time by the number of tested conditions, which is likely to make the algorithm very quickly intractable on a real robot. In addition, this problem is usually amplified when the dimension of the state space increases, since there exist many more ways of perturbing a high-dimensional system than a low-dimensional one.

A potential remedy is to use policies that are intrinsically robust to some perturbations, that is, designing the policy space such that a change in the parameter space keeps the policy robust. For instance, the learning algorithm could search for a trajectory and a controller could be designed to follow it in a robust way: this corresponds to traditional trajectory optimization (or planning) in robotics[7] [136]. This is one of the ideas behind dynamic movement primitives (see Section 3), which act like "attractors" towards a trajectory of a fixed point. Similarly, it is possible to learn waypoints [101] or "repulsors" [140] to mix learning with advanced, closed-loop "whole-body" controllers. It is, also, possible to incorporate optimization layers (*e.g.*, a QP program [122]) in a neural network in order to take advantage of the structure they provide. Lastly, one can learn distinct *soft* policies for simpler tasks and then compose them in order to achieve a more complicated task [56].

It is also conceivable to learn models of the generalization abilities [123], although it has, to our knowledge, never been tested with real robots. In that case, a model is trained to distinguish between behaviors (or trajectories) that are likely to overfit from those that are likely to be robust. This model can then be used in a policy search algorithm (*e.g.*, in a constrained Bayesian optimization scheme).

7.4 Interplay between planning, model-predictive control and policy search

The data-efficiency of policy search algorithms like PILCO or Black-DROPS rises from the fact that they learn and use dynamical models (Section 5.1). However, if we assume that the dynamical model is known or can be learnt, there is a large literature on control methods that can be used. So, is policy search the right approach in such a case?

A fundamental controller from control theory is the linear-quadratic regulator (LQR) [75], which is optimal when the the dynamics are linear and the cost function is quadratic. Systems with nonlinear dynamics can be tackled with LQR by linearizing them around the current state and action, however, other approaches can be used such as differential dynamic programming [69, 106] and its simpler variant, the iterative linear-quadratic Gaussian algorithm [151]. Generally, these methods can be used for optimal control with a large horizon lookahead, however, doing so can be computationally costly. For this reason, they are mostly employed to calculate trajectories offline.

A way to permit online trajectory optimization is by reducing the horizon lookahead, thus, gaining in computational efficiency. This is known as model-predictive control (MPC) [52]. Using shorter horizons, MPC is no longer optimal with respect to the overall, high-level task. This means that MPC can be used for short-term tasks, such as tracking a trajectory, which can be produced offline. The advantage of MPC is that it can get feedback from the real system and replan at every step. Such a control scheme can be very effective and has, for example, recently allowed real-time whole-body control of humanoid robots [81].

Although MPC can replan at every step, it still has the disadvantage of relying on models. Models can be inaccurate or wrong (especially in the first episodes of learning when not enough data have been collected), therefore, there needs to be a mechanism that corrects the mismatch. A potential solution could be to combine iterative learning control [15, 111] with MPC (*e.g.*, see [7, 92, 93, 156]). MPC additionally has the disadvantage of requiring full knowledge of the system state. A way to mitigate this problem is to combine MPC with a policy search algorithm, such as guided policy search. This can be realized by using MPC with full state information in some training phase, to learn neural network (NN) policies that take the raw observations as input, thus, not requiring full state information at test time [162]. The execution of the policy can be parallelized and can thus run faster than MPC online.

Should we then learn a big NN policy for complex high-level tasks, such as a humanoid robot helping with the house chores? Firstly, we need to consider that such complex tasks require long planning horizons. Secondly, as the task becomes more complex, so could potentially the policy space. Even if we do not consider the memory requirements for such policies, learning such tasks from scratch would be intractable, even in simulation. One way of addressing such complexity is by decomposing the high-level task into a hierarchy of subtasks. Sampling-based planners [17, 77] could operate at the high to mid levels of the hierarchy, whereas MPC could operate at the mid to low levels. Furthermore, policy search (or other algorithms for optimal control) can be used to discover primitives which themselves are used as components of a higher-level policy (*e.g.*, see [45]) or a planning algorithm (*e.g.*, see [25, 30]).

7.5 Computation time

Micro-data learning focuses on the desirable property of reducing the interaction time. However, most articles purposefully neglect computation time because they assume that it will be tackled automatically with faster hardware in the future. Although this is possible, it is worth investigating how different algorithms can potentially be sped up for near real-time execution with today's hardware.

For illustration, PILCO (see Section 5.1) is a very successful and data-efficient algorithm, but can be very computationally expensive when the state-action or policy space dimensionality increases [24, 159] (*e.g.*, Wilson et al. [159] report that PILCO required 3 weeks of computation time for 20 episodes on a 3-link planar arm task) and cannot take advantage of multi-core ar-

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[7]Please note that most of the planning methods require to know an accurate model of the robot — which is not assumed by many learning methods.

chitectures. Black-DROPS and Black-DROPS with GP-MI (see Section 5.2) can greatly reduce the interaction time and take advantage of multi-core architectures, but they still require a considerable amount of computation time (*e.g.*, Black-DROPS with GP-MI required 24 hours on a modern 16-core computer for 26 episodes of the pendubot task [23]). Both approaches use GP models which have a complexity that is quadratic to the number of points when queried; this is clearly inefficient when millions of such GP queries (*e.g.*, Black-DROPS performs around 64M [24]) are performed in each episode.

On the other hand, IT&E [31] and "robust policies" (*e.g.*, see [121], [161], [119], [127]) can practically run in real-time because the prior is pre-computed offline. This "recipe" is shared by recent meta-learning methodologies, such as [49], that aim to learn an expressive policy (*i.e.*, a deep neural network) that can be optimized online using a single gradient update.

This does not mean that the offline precomputation time should not be optimized. Algorithms such as IT&E or the work in [121] use a form of directed exploration to create such a prior[8]. If, for example, random search were used, it would probably need orders of magnitude more computation time to create a prior of the same quality.

8 Conclusion: five precepts for micro-data learning

Thanks to recent advances in priors, policy representations, reward modeling, and dynamical models, it is now possible to learn policies on robots in a few minutes of interaction time. These micro-data learning algorithms considerably expand the usefulness of learning on robots: with these algorithms, we can envision robots that adapt "in front of our eyes". These algorithms, nonetheless, face critical challenges, most notably to scale-up simultaneously to high-dimensional state spaces and high-dimensional policy spaces.

As guidelines for future work in the field, we propose 5 precepts that summarize the "generic rules" that govern most of the work published so far about micro-data learning for robot control:

1. Use as much data as possible from each trial (*e.g.*, trajectory data, not only reward value): when data is scarce, every bit matters.

2. Every estimate (or model) should come with a measure of its uncertainty: when very little data is available, models will never have enough data to be "right" for the whole search space; algorithms must take this fact into account and reason with this uncertainty.

3. Take the time to choose what to test next (active learning): computers are likely to become faster in the future, but physics will not accelerate; it is therefore a sensible strategy to trade data resources for computational resources. It is still desirable, but less critical on the long term, to design algorithms that are fast enough to run on embedded systems.

4. If needed, use expensive algorithms before the mission: since we mostly care about online adaptation, we can have

access to time and resources before the mission (access to computing clusters, GPUs, etc.)

5. Leveraging prior knowledge is a key for micro-data learning: it should not be feared. However, the prior knowledge used should be as explicit and as generic as possible.

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References

[1] Pieter Abbeel, Morgan Quigley, and Andrew Y Ng. Using inaccurate models in reinforcement learning. In *Proceedings of the 23rd international conference on Machine learning*, pages 1–8. ACM, 2006.

[2] Abbas Abdolmaleki, Rudolf Lioutikov, Jan R Peters, Nuno Lau, Luis Pualo Reis, and Gerhard Neumann. Model-based relative entropy stochastic search. In *NIPS*, 2015.

[3] Brian DO Anderson and John B Moore. Optimal filtering. *Englewood Cliffs*, 21:22–95, 1979.

[4] Marcin Andrychowicz, Filip Wolski, Alex Ray, Jonas Schneider, Rachel Fong, Peter Welinder, Bob McGrew, Josh Tobin, OpenAI Pieter Abbeel, and Wojciech Zaremba. Hindsight experience replay. In *Advances in Neural Information Processing Systems*, pages 5048–5058, 2017.

[5] Rika Antonova, Akshara Rai, and Christopher G Atkeson. Sample efficient optimization for learning controllers for bipedal locomotion. In *International Conference on Humanoid Robots (Humanoids)*, pages 22–28. IEEE, 2016.

[6] Rika Antonova, Akshara Rai, and Christopher G Atkeson. Deep Kernels for Optimizing Locomotion Controllers. In *1st Conference on Robot Learning (CoRL)*, 2017.

[7] J.-A. M Assael, N. Wahlström, T. B. Schön, and M. P. Deisenroth. Data-efficient learning of feedback policies from image pixels using deep dynamical models. *NIPS Deep Reinforcement Learning Workshop*, 2015.

[8] Anne Auger and Nikolaus Hansen. A restart cma evolution strategy with increasing population size. In *Evolutionary Computation, 2005. The 2005 IEEE Congress on*, volume 2, pages 1769–1776. IEEE, 2005.

[9] Richard Ernest Bellman. *Dynamic Programming*. Princeton University Press, Princeton, NJ, 1957.

[10] Felix Berkenkamp, Angela P. Schoellig, and Andreas Krause. Safe Controller Optimization for Quadrotors with Gaussian Processes. In *International Conference on Robotics and Automation (ICRA)*, 2016.

[11] B. Bischoff, D. Nguyen-Tuong, Herke van Hoof, A. McHutchon, Carl Edward Rasmussen, Aaron Knoll, Jochen Peters, and Marc Peter Deisenroth. Policy search for learning robot control using sparse data. In *Robotics and Automation (ICRA), 2014 IEEE International Conference on*, pages 3882–3887. IEEE, 2014.

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[8]IT&E uses an evolutionary illumination algorithm (MAP-Elites [113, 155]) to discover solutions to thousands of problems in a single run; the work by [121] uses a related approach (hindsight experience replay [4]).

[12] Josh Bongard, Victor Zykov, and Hod Lipson. Resilient machines through continuous self-modeling. *Science*, 314 (5802):1118–1121, 2006.

[13] Josh C. Bongard and Rolf Pfeifer. Evolving complete agents using artificial ontogeny. In Fumio Hara and Rolf Pfeifer, editors, *Morpho-functional Machines: The New Species*, pages 237–258, Tokyo, 2003. Springer Japan. ISBN 978-4-431-67869-4.

[14] Craig Boutilier, Richard Dearden, and Moisés Goldszmidt. Stochastic dynamic programming with factored representations. *Artificial Intelligence*, 121(1-2):49–107, 2000.

[15] Douglas A Bristow, Marina Tharayil, and Andrew G Alleyne. A survey of iterative learning control. *IEEE Control Systems*, 26(3):96–114, 2006.

[16] Eric Brochu, Vlad M. Cora, and Nando De Freitas. A tutorial on Bayesian optimization of expensive cost functions, with application to active user modeling and hierarchical reinforcement learning. *arXiv preprint arXiv:1012.2599*, 2010.

[17] Cameron B Browne, Edward Powley, Daniel Whitehouse, Simon M Lucas, Peter I Cowling, Philipp Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis, and Simon Colton. A survey of monte carlo tree search methods. *IEEE Transactions on Computational Intelligence and AI in games*, 4(1):1–43, 2012.

[18] Jonas Buchli, Freek Stulp, Evangelos Theodorou, and Stefan Schaal. Learning variable impedance control. *International Journal of Robotics Research*, 30(7):820–833, 2011.

[19] R. Calandra, A. Seyfarth, J. Peters, and M.P. Deisenroth. Bayesian optimization for learning gaits under uncertainty. *Annals of Mathematics and Artificial Intelligence (AMAI)*, 2015.

[20] S. Calinon. A tutorial on task-parameterized movement learning and retrieval. *Intelligent Service Robotics*, 9(1): 1–29, January 2016. ISSN 1861-2776. doi: 10.1007/s11370-015-0187-9.

[21] S. Calinon, P. Kormushev, and D. G. Caldwell. Compliant skills acquisition and multi-optima policy search with EM-based reinforcement learning. *Robotics and Autonomous Systems*, 61(4):369–379, April 2013.

[22] S. Calinon, D. Bruno, and D. G. Caldwell. A task-parameterized probabilistic model with minimal intervention control. In *Proc. IEEE Intl Conf. on Robotics and Automation (ICRA)*, pages 3339–3344, Hong Kong, China, May-June 2014.

[23] Konstantinos Chatzilygeroudis and Jean-Baptiste Mouret. Using Parameterized Black-Box Priors to Scale Up Model-Based Policy Search for Robotics. In *International Conference on Robotics and Automation (ICRA)*, 2018.

[24] Konstantinos Chatzilygeroudis, Roberto Rama, Rituraj Kaushik, Dorian Goepp, Vassilis Vassiliades, and Jean-Baptiste Mouret. Black-Box Data-efficient Policy Search for Robotics. In *International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2017.

[25] Konstantinos Chatzilygeroudis, Vassilis Vassiliades, and Jean-Baptiste Mouret. Reset-free trial-and-error learning for robot damage recovery. *Robotics and Autonomous Systems*, 100:236–250, 2018.

[26] Kurtland Chua, Roberto Calandra, Rowan McAllister, and Sergey Levine. Deep reinforcement learning in a handful of trials using probabilistic dynamics models. *arXiv preprint arXiv:1805.12114*, 2018.

[27] Kamil Ciosek and Shimon Whiteson. Expected policy gradients. In *Thirty-Second AAAI Conference on Artificial Intelligence (AAAI)*, February 2018.

[28] Kamil Ciosek and Shimon Whiteson. Expected Policy Gradients for Reinforcement Learning. *arXiv preprint arXiv:1801.03326*, 2018.

[29] Ignasi Clavera, Anusha Nagabandi, Ronald S Fearing, Pieter Abbeel, Sergey Levine, and Chelsea Finn. Learning to Adapt: Meta-Learning for Model-Based Control. *arXiv preprint arXiv:1803.11347*, 2018.

[30] Debora Clever, Monika Harant, Katja Mombaur, Maximilien Naveau, Olivier Stasse, and Dominik Endres. Cocomopl: A novel approach for humanoid walking generation combining optimal control, movement primitives and learning and its transfer to the real robot hrp-2. *IEEE Robotics and Automation Letters*, 2(2):977–984, 2017.

[31] Antoine Cully, Jeff Clune, Danesh Tarapore, and Jean-Baptiste Mouret. Robots that can adapt like animals. *Nature*, 521(7553):503–507, 2015.

[32] Mark Cutler and Jonathan P How. Efficient reinforcement learning for robots using informative simulated priors. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 2605–2612. IEEE, 2015.

[33] C. Daniel, G. Neumann, O. Kroemer, and J. Peters. Hierarchical relative entropy policy search. *Journal of Machine Learning Research (JMLR)*, pages 1–50, 2016.

[34] Thomas Dean and Keiji Kanazawa. A model for reasoning about persistence and causation. *Computational intelligence*, 5(2):142–150, 1989.

[35] Thomas Degris, Martha White, and Richard S Sutton. Linear off-policy actor-critic. In *International Conference on Machine Learning (ICML)*. Citeseer, 2012.

[36] Marc P Deisenroth and Carl Edward Rasmussen. PILCO: A model-based and data-efficient approach to policy search. In *ICML*, 2011.

[37] Marc P. Deisenroth, Carl Edward Rasmussen, and Dieter Fox. Learning to control a low-cost manipulator using data-efficient reinforcement learning. In *Robotics: Science & Systems (RSS)*, 2011.

[38] Marc Peter Deisenroth, Roberto Calandra, André Seyfarth, and Jan Peters. Toward fast policy search for learning legged locomotion. In *Proc. of IROS*, 2012.

[39] Marc Peter Deisenroth, Gerhard Neumann, and Jan Peters. A Survey on Policy Search for Robotics. *Foundations and Trends in Robotics*, 2(1):1–142, 2013.

[40] Marc Peter Deisenroth, Peter Englert, Jochen Peters, and Dieter Fox. Multi-task policy search for robotics. In *Robotics and Automation (ICRA), 2014 IEEE International Conference on*, pages 3876–3881. IEEE, 2014.

[41] Marc Peter Deisenroth, Dieter Fox, and Carl Edward Rasmussen. Gaussian processes for data-efficient learning in robotics and control. *IEEE Trans. Pattern Anal. Mach. Intell.*, 37(2):408–423, 2015.

[42] Stefan Depeweg, José Miguel Hernández-Lobato, Finale Doshi-Velez, and Steffen Udluft. Learning and policy search in stochastic dynamical systems with bayesian neural networks. In *ICLR*, 2017.

[43] Stefan Depeweg, José Miguel Hernández-Lobato, Finale Doshi-Velez, and Steffen Udluft. Decomposition of uncertainty in bayesian deep learning for efficient and risk-sensitive learning. In *ICML*, 2018.

[44] Andreas Doerr, Christian Daniel, Duy Nguyen-Tuong, Alonso Marco, Stefan Schaal, Toussaint Marc, and Sebastian Trimpe. Optimizing long-term predictions for model-based policy search. In *Conference on Robot Learning*, pages 227–238, 2017.

[45] Miguel Duarte, Jorge Gomes, Sancho Moura Oliveira, and Anders Lyhne Christensen. Evolution of repertoire-based

control for robots with complex locomotor systems. *IEEE Transactions on Evolutionary Computation*, 2017.

[46] SM Ali Eslami, Danilo Jimenez Rezende, Frederic Besse, Fabio Viola, Ari S Morcos, Marta Garnelo, Avraham Ruderman, Andrei A Rusu, Ivo Danihelka, Karol Gregor, et al. Neural scene representation and rendering. *Science*, 360 (6394):1204–1210, 2018.

[47] Matthias Feurer, Jost Tobias Springenberg, and Frank Hutter. Initializing bayesian hyperparameter optimization via meta-learning. In *AAAI*, pages 1128–1135, 2015.

[48] Peggy Fidelman and Peter Stone. Learning ball acquisition on a physical robot. In *International Symposium on Robotics and Automation (ISRA)*, 2004.

[49] Chelsea Finn, Pieter Abbeel, and Sergey Levine. Model-agnostic meta-learning for fast adaptation of deep networks. In *International Conference on Machine Learning*, pages 1126–1135, 2017.

[50] Yarin Gal and Zoubin Ghahramani. Dropout as a Bayesian approximation: Representing model uncertainty in deep learning. In *International Conference on Machine Learning (ICML)*, pages 1050–1059, 2016.

[51] Yarin Gal, Rowan Thomas McAllister, and Carl Edward Rasmussen. Improving PILCO with Bayesian neural network dynamics models. In *Data-Efficient Machine Learning workshop*, volume 951, page 2016, 2016.

[52] Carlos E Garcia, David M Prett, and Manfred Morari. Model predictive control: theory and practice—a survey. *Automatica*, 25(3):335–348, 1989.

[53] Jacob R Gardner, Matt J Kusner, Zhixiang Eddie Xu, Kilian Q Weinberger, and John P Cunningham. Bayesian Optimization with Inequality Constraints. In *ICML*, pages 937–945, 2014.

[54] F. Guenter, M. Hersch, S. Calinon, and A. Billard. Reinforcement learning for imitating constrained reaching movements. *Advanced Robotics, Special Issue on Imitative Robots*, 21(13):1521–1544, 2007.

[55] David Ha and Jürgen Schmidhuber. World models. *arXiv preprint arXiv:1803.10122*, 2018.

[56] Tuomas Haarnoja, Vitchyr Pong, Aurick Zhou, Murtaza Dalal, Pieter Abbeel, and Sergey Levine. Composable Deep Reinforcement Learning for Robotic Manipulation. In *Proceedings of ICRA*, 2018.

[57] Nikolaus Hansen. *The CMA Evolution Strategy: A Comparing Review*. Springer, 2006. ISBN 978-3-540-32494-2. doi: 10.1007/3-540-32494-1_4.

[58] Nikolaus Hansen. Benchmarking a BI-population CMA-ES on the BBOB-2009 noisy testbed. In *Proc. of GECCO*, pages 2397–2402. ACM, 2009.

[59] Nikolaus Hansen and Andreas Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary computation*, 9(2):159–195, 2001.

[60] Nikolaus Hansen, André SP Niederberger, Lino Guzzella, and Petros Koumoutsakos. A method for handling uncertainty in evolutionary optimization with an application to feedback control of combustion. *IEEE Trans. on Evolutionary Computation*, 13(1):180–197, 2009.

[61] Nicolas Heess, Srinivasan Sriram, Jay Lemmon, Josh Merel, Greg Wayne, Yuval Tassa, Tom Erez, Ziyu Wang, Ali Eslami, Martin Riedmiller, et al. Emergence of locomotion behaviours in rich environments. *arXiv preprint arXiv:1707.02286*, 2017.

[62] Philipp Hennig and Christian J Schuler. Entropy search for information-efficient global optimization. *Journal of Machine Learning Research*, 13(Jun):1809–1837, 2012.

[63] Juan Camilo Gamboa Higuera, David Meger, and Gregory Dudek. Synthesizing neural network controllers with probabilistic model based reinforcement learning. *arXiv preprint arXiv:1803.02291*, 2018.

[64] A. Ijspeert, J. Nakanishi, P Pastor, H. Hoffmann, and S. Schaal. Dynamical Movement Primitives: Learning attractor models for motor behaviors. *Neural Computation*, 25(2):328–373, 2013.

[65] A. J. Ijspeert, J. Nakanishi, and S. Schaal. Movement imitation with nonlinear dynamical systems in humanoid robots. In *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, 2002.

[66] Auke J Ijspeert, Jun Nakanishi, and Stefan Schaal. Learning attractor landscapes for learning motor primitives. In *Advances in neural information processing systems*, pages 1547–1554, 2003.

[67] Auke Jan Ijspeert. Central pattern generators for locomotion control in animals and robots: a review. *Neural networks*, 21(4):642–653, 2008.

[68] Verne T Inman, Howard D Eberhart, et al. The major determinants in normal and pathological gait. *JBJS*, 35(3):543–558, 1953.

[69] David H Jacobson and David Q Mayne. Differential dynamic programming. 1970.

[70] Yaochu Jin and Jürgen Branke. Evolutionary optimization in uncertain environments-a survey. *IEEE Trans. on Evolutionary Computation*, 9(3):303–317, 2005.

[71] Rico Jonschkowski and Oliver Brock. Learning state representations with robotic priors. *Autonomous Robots*, 39(3):407–428, 2015.

[72] Simon J Julier and Jeffrey K Uhlmann. Unscented filtering and nonlinear estimation. *Proceedings of the IEEE*, 92(3):401–422, 2004.

[73] Leslie Pack Kaelbling, Michael L Littman, and Andrew W Moore. Reinforcement learning: A survey. *Journal of artificial intelligence research*, 4:237–285, 1996.

[74] M. Kalakrishnan, L. Righetti, P. Pastor, and S. Schaal. Learning force control policies for compliant manipulation. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2011)*, 2011.

[75] Rudolph Emil Kalman. A new approach to linear filtering and prediction problems. *Journal of basic Engineering*, 82 (1):35–45, 1960.

[76] Kirthevasan Kandasamy, Jeff Schneider, and Barnabás Póczos. High dimensional bayesian optimisation and bandits via additive models. In *International Conference on Machine Learning*, pages 295–304, 2015.

[77] Sertac Karaman and Emilio Frazzoli. Sampling-based algorithms for optimal motion planning. *The international journal of robotics research*, 30(7):846–894, 2011.

[78] Jonathan Ko, Daniel J Klein, Dieter Fox, and Dirk Haehnel. Gaussian processes and reinforcement learning for identification and control of an autonomous blimp. In *International Conference on Robotics and Automation (ICRA)*, pages 742–747. IEEE, 2007.

[79] Jens Kober and Jan Peters. Learning motor primitives for robotics. In *Robotics and Automation, 2009. ICRA'09. IEEE International Conference on*, pages 2112–2118. IEEE, 2009.

[80] Jens Kober, J. Andrew Bagnell, and Jan Peters. Reinforcement learning in robotics: A survey. *The International Journal of Robotics Research*, 32(11):1238–1274, 2013. doi: 10.1177/0278364913495721.

[81] Jonas Koenemann, Andrea Del Prete, Yuval Tassa, Emanuel Todorov, Olivier Stasse, Maren Bennewitz, and

Nicolas Mansard. Whole-body model-predictive control applied to the HRP-2 humanoid. In *Intelligent Robots and Systems (IROS), 2015 IEEE/RSJ International Conference on*, pages 3346–3351. IEEE, 2015.

[82] Nate Kohl and Peter Stone. Policy gradient reinforcement learning for fast quadrupedal locomotion. In *Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on*, volume 3, pages 2619–2624. IEEE, 2004.

[83] Sylvain Koos and J-B Mouret. Online discovery of locomotion modes for wheel-legged hybrid robots: A transferability-based approach. In *Proc. of CLAWAR*, pages 70–77. World Scientific, 2012.

[84] Sylvain Koos, Antoine Cully, and Jean-Baptiste Mouret. Fast damage recovery in robotics with the t-resilience algorithm. *The International Journal of Robotics Research*, 32(14):1700–1723, 2013.

[85] Sylvain Koos, Jean-Baptiste Mouret, and Stéphane Doncieux. The transferability approach: Crossing the reality gap in evolutionary robotics. *IEEE Transactions on Evolutionary Computation*, 17(1):122–145, 2013.

[86] Vikash Kumar, Emanuel Todorov, and Sergey Levine. Optimal control with learned local models: Application to dexterous manipulation. In *Robotics and Automation (ICRA), 2016 IEEE International Conference on*, pages 378–383. IEEE, 2016.

[87] Visak C.V. Kumar, Sehoon Ha, and Katsu Yamane. Improving Model-Based Balance Controllers using Reinforcement Learning and Adaptive Sampling. In *Proceedings of ICRA*, 2018.

[88] Andras Kupcsik, Marc Peter Deisenroth, Jan Peters, Ai Poh Loh, Prahlad Vadakkepat, and Gerhard Neumann. Model-based contextual policy search for data-efficient generalization of robot skills. *Artificial Intelligence*, 247:415–439, 2017.

[89] Harold J Kushner. A new method of locating the maximum point of an arbitrary multipeak curve in the presence of noise. *Journal of Basic Engineering*, 86(1):97–106, 1964.

[90] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *Nature*, 521(7553):436–444, 2015.

[91] Gilwoo Lee, Siddhartha S Srinivasa, and Matthew T Mason. Gp-ilqg: Data-driven robust optimal control for uncertain nonlinear dynamical systems. *arXiv preprint arXiv:1705.05344*, 2017.

[92] Jay H Lee, Kwang S Lee, and Won C Kim. Model-based iterative learning control with a quadratic criterion for time-varying linear systems. *Automatica*, 36(5):641–657, 2000.

[93] Kwang S Lee, In-Shik Chin, Hyuk J Lee, and Jay H Lee. Model predictive control technique combined with iterative learning for batch processes. *AIChE Journal*, 45(10):2175–2187, 1999.

[94] Timothée Lesort, Mathieu Seurin, Xinrui Li, Natalia Díaz Rodríguez, and David Filliat. Unsupervised state representation learning with robotic priors: a robustness benchmark. *arXiv preprint arXiv:1709.05185*, 2017.

[95] Timothée Lesort, Natalia Díaz-Rodríguez, Jean-François Goudou, and David Filliat. State representation learning for control: An overview. *arXiv preprint arXiv:1802.04181*, 2018.

[96] Sergey Levine and Pieter Abbeel. Learning neural network policies with guided policy search under unknown dynamics. In *Proc. of NIPS*, pages 1071–1079, 2014.

[97] Sergey Levine and Vladlen Koltun. Guided policy search. In *International Conference on Machine Learning (ICML)*, 2013.

[98] Sergey Levine, Chelsea Finn, Trevor Darrell, and Pieter Abbeel. End-to-end training of deep visuomotor policies. *Journal of Machine Learning Research*, 17(39):1–40, 2016.

[99] Timothy P Lillicrap, Jonathan J Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. In *International Conference on Learning Representations (ICLR)*, 2016.

[100] D. J. Lizotte, T. Wang, M. H Bowling, and D. Schuurmans. Automatic gait optimization with gaussian process regression. In *Proc. of IJCAI*, 2007.

[101] Ryan Lober, Vincent Padois, and Olivier Sigaud. Efficient reinforcement learning for humanoid whole-body control. In *Humanoid Robots (Humanoids), 2016 IEEE-RAS 16th International Conference on*, pages 684–689. IEEE, 2016.

[102] Ryan Lober, Jorhabib Eljaik, Gabriele Nava, Stefano Dafarra, Francesco Romano, Daniele Pucci, Silvio Traversaro, Francesco Nori, Olivier Sigaud, and Vincent Padois. Optimizing task feasibility using model-free policy search and model-based whole-body control. 2017.

[103] Perla Maiolino, Marco Maggiali, Giorgio Cannata, Giorgio Metta, and Lorenzo Natale. A flexible and robust large scale capacitive tactile system for robots. *IEEE Sensors Journal*, 13(10):3910–3917, 2013.

[104] Alonso Marco, Felix Berkenkamp, Philipp Hennig, Angela P. Schoellig, Andreas Krause, Stefan Schaal, and Sebastian Trimpe. Virtual vs. Real: Trading Off Simulations and Physical Experiments in Reinforcement Learning with Bayesian Optimization. In *International Conference on Robotics and Automation (ICRA)*, 2017.

[105] Takamitsu Matsubara, Sang-Ho Hyon, and Jun Morimoto. Learning parametric dynamic movement primitives from multiple demonstrations. *Neural Networks*, 24(5):493–500, 2011. doi: 10.1016/j.neunet.2011.02.004.

[106] David Mayne. A second-order gradient method for determining optimal trajectories of non-linear discrete-time systems. *International Journal of Control*, 3(1):85–95, 1966.

[107] Brad L Miller and David E Goldberg. Genetic algorithms, selection schemes, and the varying effects of noise. *Evolutionary Computation*, 4(2):113–131, 1996.

[108] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, Georg Ostrovski, et al. Human-level control through deep reinforcement learning. *Nature*, 518(7540):529, 2015.

[109] Volodymyr Mnih, Adria Puigdomenech Badia, Mehdi Mirza, Alex Graves, Timothy Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous methods for deep reinforcement learning. In *International Conference on Machine Learning (ICML)*, pages 1928–1937, 2016.

[110] William Montgomery, Anurag Ajay, Chelsea Finn, Pieter Abbeel, and Sergey Levine. Reset-free guided policy search: efficient deep reinforcement learning with stochastic initial states. In *International Conference on Robotics and Automation (ICRA)*, pages 3373–3380. IEEE, 2017.

[111] Kevin L Moore, Mohammed Dahleh, and SP Bhattacharyya. Iterative learning control: A survey and new results. *Journal of Field Robotics*, 9(5):563–594, 1992.

[112] Jean-Baptiste Mouret. Micro-data learning: The other end of the spectrum. *ERCIM News*, (107):2, 2016.

[113] Jean-Baptiste Mouret and Jeff Clune. Illuminating search spaces by mapping elites. *arXiv preprint arXiv:1504.04909*, 2015.

[114] John A Musick and Colin J Limpus. Habitat utilization and

migration in juvenile sea turtles. *The biology of sea turtles*, 1:137–163, 1997.

[115] Andrew Y Ng and Michael Jordan. PEGASUS: a policy search method for large MDPs and POMDPs. In *Proc. of Uncertainty in Artificial Intelligence*, pages 406–415. Morgan Kaufmann, 2000.

[116] Andrew Y Ng, Adam Coates, Mark Diel, Varun Ganapathi, Jamie Schulte, Ben Tse, Eric Berger, and Eric Liang. Autonomous inverted helicopter flight via reinforcement learning. In *Experimental Robotics IX*, pages 363–372. Springer, 2006.

[117] Junhyuk Oh, Xiaoxiao Guo, Honglak Lee, Richard L Lewis, and Satinder Singh. Action-conditional video prediction using deep networks in atari games. In *Advances in Neural Information Processing Systems*, pages 2863–2871, 2015.

[118] Vaios Papaspyros, Konstantinos Chatzilygeroudis, Vassilis Vassiliades, and Jean-Baptiste Mouret. Safety-aware robot damage recovery using constrained bayesian optimization and simulated priors. In *BayesOpt'16: Proceedings of the International Workshop "Bayesian Optimization: Black-box Optimization and Beyond" at NIPS*, 2016.

[119] Supratik Paul, Konstantinos Chatzilygeroudis, Kamil Ciosek, Jean-Baptiste Mouret, Michael A. Osborne, and Shimon Whiteson. Alternating Optimisation and Quadrature for Robust Control. In *Thirty-Second AAAI Conference on Artificial Intelligence (AAAI)*, 2018.

[120] Rémi Pautrat, Konstantinos Chatzilygeroudis, and Jean-Baptiste Mouret. Bayesian optimization with automatic prior selection for data-efficient direct policy search. In *International Conference on Robotics and Automation (ICRA)*., 2018.

[121] Xue Bin Peng, Marcin Andrychowicz, Wojciech Zaremba, and Pieter Abbeel. Sim-to-real transfer of robotic control with dynamics randomization. *arXiv preprint arXiv:1710.06537*, 2017.

[122] Tu-Hoa Pham, Giovanni De Magistris, and Ryuki Tachibana. OptLayer-Practical Constrained Optimization for Deep Reinforcement Learning in the Real World. In *Proceedings of ICRA*, 2018.

[123] Tony Pinville, Sylvain Koos, Jean-Baptiste Mouret, and Stéphane Doncieux. How to promote generalisation in evolutionary robotics: the progab approach. In *Proceedings of the 13th annual conference on Genetic and evolutionary computation GECCO*, pages 259–266. ACM, 2011.

[124] Athanasios S Polydoros and Lazaros Nalpantidis. Survey of Model-Based Reinforcement Learning: Applications on Robotics. *Journal of Intelligent & Robotic Systems*, pages 1–21, 2017.

[125] Justin K Pugh, Lisa B Soros, and Kenneth O Stanley. Quality diversity: A new frontier for evolutionary computation. *Frontiers in Robotics and AI*, 3:40, 2016.

[126] Jeffrey Queisser and Jochen Steil. Bootstrapping of Parameterized Skills Through Hybrid Optimization in Task and Policy Spaces. *Frontiers in Robotics and AI*, 2018.

[127] Aravind Rajeswaran, Sarvjeet Ghotra, Balaraman Ravindran, and Sergey Levine. Epopt: Learning robust neural network policies using model ensembles. *arXiv preprint arXiv:1610.01283*, 2016.

[128] Carl Edward Rasmussen and Christopher KI Williams. *Gaussian processes for machine learning*, volume 1. MIT press Cambridge, 2006.

[129] John Rieffel and Jean-Baptiste Mouret. Soft tensegrity robots. *Soft Robotics*, 2018.

[130] Paul Rolland, Jonathan Scarlett, Ilija Bogunovic, and Volkan Cevher. High-dimensional bayesian optimization via additive models with overlapping groups. *arXiv preprint arXiv:1802.07028*, 2018.

[131] Steindór Sæmundsson, Katja Hofmann, and Marc Peter Deisenroth. Meta reinforcement learning with latent variable gaussian processes. *arXiv preprint arXiv:1803.07551*, 2018.

[132] Joseph Salini, Vincent Padois, and Philippe Bidaud. Synthesis of complex humanoid whole-body behavior: a focus on sequencing and tasks transitions. In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pages 1283–1290. IEEE, 2011.

[133] Matteo Saveriano, Yuchao Yin, Pietro Falco, and Dongheui Lee. Data-efficient control policy search using residual dynamics learning. In *Proc. of IROS*, 2017.

[134] John Schulman, Sergey Levine, Pieter Abbeel, Michael Jordan, and Philipp Moritz. Trust region policy optimization. In *International Conference on Machine Learning (ICML)*, pages 1889–1897, 2015.

[135] Bobak Shahriari, Kevin Swersky, Ziyu Wang, Ryan P Adams, and Nando de Freitas. Taking the human out of the loop: A review of bayesian optimization. *Proceedings of the IEEE*, 104(1):148–175, 2016.

[136] Bruno Siciliano and Oussama Khatib. *Springer handbook of robotics*. Springer, 2 edition, 2016.

[137] David Silver, Guy Lever, Nicolas Heess, Thomas Degris, Daan Wierstra, and Martin Riedmiller. Deterministic policy gradient algorithms. In *International Conference on Machine Learning (ICML)*, pages 387–395, 2014.

[138] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. Mastering the game of go without human knowledge. *Nature*, 550(7676): 354, 2017.

[139] Karl Sims. Evolving virtual creatures. In *Computer Graphics, Annual Conference Series, (SIGGRAPH 1994 Proceedings)*, pages 15–22, July 1994.

[140] Jonathan Spitz, Karim Bouyarmane, Serena Ivaldi, and Jean-Baptiste Mouret. Trial-and-error learning of repulsors for humanoid qp-based whole-body control. In *IEEE-RAS International Conference on Humanoid Robots*, pages 468–475, 2017.

[141] Mark W. Spong and Daniel J. Block. The pendubot: A mechatronic system for control research and education. In *Proc. of Decision and Control*, 1995.

[142] Kenneth O. Stanley and Risto Miikkulainen. Evolving neural networks through augmenting topologies. *Evolutionary Computation*, 10(2):99–127, 2002.

[143] Freek Stulp and Olivier Sigaud. Policy improvement: Between black-box optimization and episodic reinforcement learning. In *Journées Francophones Planification, Décision, et Apprentissage pour la conduite de systèmes*, 2013.

[144] Freek Stulp, Evangelos Theodorou, and Stefan Schaal. Reinforcement learning with sequences of motion primitives for robust manipulation. *IEEE Transactions on Robotics*, 28(6):1360–1370, 2012.

[145] Freek Stulp, Gennaro Raiola, Antoine Hoarau, Serena Ivaldi, and Olivier Sigaud. Learning compact parameterized skills with a single regression. In *IEEE-RAS International Conference on Humanoid Robots*, 2013.

[146] Masashi Sugiyama, Ichiro Takeuchi, Taiji Suzuki, Takafumi Kanamori, Hirotaka Hachiya, and Daisuke Okanohara. Least-squares conditional density estimation. *IEICE Transactions on Information and Systems*, 93(3):583–594, 2010.

[147] Richard S Sutton. Dyna, an integrated architecture for learning, planning, and reacting. *ACM SIGART Bulletin*, 2(4):160–163, 1991.

[148] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*, volume 1. MIT press Cambridge, 1998.

[149] Richard S Sutton, David A McAllester, Satinder P Singh, and Yishay Mansour. Policy gradient methods for reinforcement learning with function approximation. In *Advances in neural information processing systems (NIPS)*, pages 1057–1063, 2000.

[150] Voot Tangkaratt, Syogo Mori, Tingting Zhao, Jun Morimoto, and Masashi Sugiyama. Model-based policy gradients with parameter-based exploration by least-squares conditional density estimation. *Neural Networks*, 57:128–140, 2014.

[151] Emanuel Todorov and Weiwei Li. A generalized iterative lqg method for locally-optimal feedback control of constrained nonlinear stochastic systems. In *American Control Conference*, pages 300–306. IEEE, 2005.

[152] Nikolaos G Tsagarakis, Giorgio Metta, Giulio Sandini, David Vernon, Ricardo Beira, Francesco Becchi, Ludovic Righetti, Jose Santos-Victor, Auke Jan Ijspeert, Maria Chiara Carrozza, et al. icub: the design and realization of an open humanoid platform for cognitive and neuroscience research. *Advanced Robotics*, 21(10):1151–1175, 2007.

[153] Shigeyoshi Tsutsui and Ashish Ghosh. Genetic algorithms with a robust solution searching scheme. *IEEE transactions on Evolutionary Computation*, 1(3):201–208, 1997.

[154] Harm Van Seijen, Hado Van Hasselt, Shimon Whiteson, and Marco Wiering. A theoretical and empirical analysis of expected sarsa. In *Symposium on Adaptive Dynamic Programming and Reinforcement Learning (ADPRL)*, pages 177–184. IEEE, 2009.

[155] Vassilis Vassiliades, Konstantinos Chatzilygeroudis, and Jean-Baptiste Mouret. Using centroidal Voronoi tessellations to scale up the multi-dimensional archive of phenotypic elites algorithm. *IEEE Transactions on Evolutionary Computation*, 2017.

[156] Youqing Wang, Donghua Zhou, and Furong Gao. Iterative learning model predictive control for multi-phase batch processes. *Journal of Process Control*, 18(6):543–557, 2008.

[157] Ziyu Wang, Frank Hutter, Masrour Zoghi, David Matheson, and Nando de Feitas. Bayesian optimization in a billion dimensions via random embeddings. *Journal of Artificial Intelligence Research*, 55:361–387, 2016.

[158] Pawel Wawrzynski. Learning to control a 6-degree-of-freedom walking robot. In *EUROCON*, 2007.

[159] Aaron Wilson, Alan Fern, and Prasad Tadepalli. Using trajectory data to improve bayesian optimization for reinforcement learning. *JMLR*, 15(1):253–282, 2014.

[160] Tingfan Wu and Javier Movellan. Semi-parametric Gaussian process for robot system identification. In *Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on*, pages 725–731. IEEE, 2012.

[161] Wenhao Yu, C Karen Liu, and Greg Turk. Preparing for the unknown: Learning a universal policy with online system identification. *arXiv preprint arXiv:1702.02453*, 2017.

[162] Tianhao Zhang, Gregory Kahn, Sergey Levine, and Pieter Abbeel. Learning deep control policies for autonomous aerial vehicles with mpc-guided policy search. In *Robotics and Automation (ICRA), 2016 IEEE International Conference on*, pages 528–535. IEEE, 2016.

[163] Shaojun Zhu, Andrew Kimmel, Kostas E. Bekris, and Abdeslam Boularias. Fast Model Identification via Physics Engines for Data-Efficient Policy Search. In *IJCAI*, 2018.