Proceedings of the 30th Conference
on Decision and Control
Brighton, England • December 1991

EXCITING TRAJECTORIES FOR THE IDENTIFICATION OF BASE INERTIAL PARAMETERS OF ROBOTS

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Abstract

A common way to identify the inertial parameters of robots is to use a linear model in relation to the parameters and standard least squares (L.S.) techniques. This paper presents a method to generate exciting identification trajectories in order to minimize the effect of noise and error modelling on the L.S. solution. Using non linear optimization techniques, the condition number of a matrix W obtained from the energy model is minimized and the scaling of its terms is carried out. An example of a 3 degree of freedom robot is presented.

1. Introduction

Recently, Gautier, Khalil, [1] and Sheu, Walker, [2], have proposed a new algorithm to identify the inertial parameters of robots. The identification model is called the energy model because it is based on the energy theorem. It is linear in relation to the inertial parameters. It is a function of the joint positions and velocities, and doesn't require calculating or measuring the accelerations.
In [3, ..., 7], we have presented symbolic and numerical methods to obtain a minimum set of inertial parameters, also called a set of base parameters [8, 9, 10]. These parameters constitute the identifiable inertial parameters.

By sampling the linear minimum model on a trajectory $(q, \dot{q})$, base parameters X can be estimated as the L.S. solution of a full rank overdetermined linear system , $Y = W X + \rho$ [1].
The problem treated in this paper is to find a trajectory which minimizes the effect of noise measurements and error modelling on the L.S. solution $\hat{X}$.

The sensitivity of the estimated parameters $\hat{X}$ to errors on Y or W can be measured by the condition number of W provided that W is well equilibrated [11].
In this paper non linear optimization techniques are used to find a trajectory which minimizes the condition number of $W(q, \dot{q})$ and which scales its rows and columns .

At first the algorithm finds a sequence of optimum points $(q, \dot{q})$. Then a continuous trajectory is defined on these optimum points using fifth order polynomial functions.
The joint positions, velocities and accelerations constraints are taken into account.

Previous work

Several authors [12, ..., 15], had the idea of using special test motions based on moving only one or two axes at a time.
Each test allows the identification of a small number of parameters at a time. There is no study about the difference to the global L.S. solution in the presence of perturbations.
Armstrong [16, 17, 18] has developed a method to minimize a condition number of an excitation matrix computed from the inverse dynamic model. The degrees of freedom are the points of a sequence of acceleration $\ddot{q}$. Then $\dot{q}$ and q are obtained by numerical integration. This is a non linear path optimization problem which can be solved by the Lagrangian technique. It requires a large number of variables and joint constraints are difficult to satisfy. Finding an optimal trajectory of 300 points for a 3 degree of freedom robot using a gradient method, requires 40 hours of VAX time.

2. The identification model

Details about this section can be found in [1, 3, 4, 6, 7].
The system to be considered is an open loop structure mechanism. The description of the system will be carried out by the use of the modified Denavit and Hartenberg notation [19, 20].
The system is composed of n joints and n+1 links, link 0 is the base while link n is the end effector. A coordinate frame j is assigned fixed with respect to link j.
The ten standard inertial parameters of link j are composed of :
- XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j representing the inertia matrix elements of link j about the origin of frame j, referred to frame j,
- MX_j, MY_j, MZ_j the first moments of link j about the origin of frame j, referred to frame j, and
- M_j the mass of link j.

The 10n standard parameters of the robot can be reduced to b base parameters by eliminating some of them and by grouping some others.
We define X as the bx1 vector containing the base (identifiable) parameters.
Different dynamic models linear in the inertial parameters can be obtained from Newton-Euler or Lagrangian formulation and from the energy theorem.
In this paper the energy model is used because it is easy to calculate and it doesn't depend on the joint accelerations.
From the energy theorem comes:

$$y = \Delta H \tag{1}$$

where :

$$y = \int_{t_a}^{t_b} T^T \dot{q}\, dt \tag{2}$$

$$\Delta H = H(t_b) - H(t_a) \tag{3}$$

where :
T is the nx1 vector of joint torques or forces not derived from a potential,

q , $\dot{q}$ are the nx1 vectors of joint positions and velocities respectively,
H is the total energy function, (the Hamiltonian), of the system, $H = E + U$,

$E(q, \dot{q})$ is the kinetic energy, $U(q)$ is the potential energy.
From the linearity of the energy with respect to the inertial parameters comes:

$$H = h\, X = \sum_{i=1}^{b} h_i X_i \tag{4}$$

$$\Delta H = [h(t_b) - h(t_a)]\, X = \Delta h\ X \quad (5)$$

where :

h is a 1xb row matrix composed of the partial derivatives h_i

$$h_i = \frac{\partial H}{\partial X_i} \quad (6)$$

The identification model is the energy difference equation :

$$y = \int_{t_a}^{t_b} T^T \dot{q}\, dt = \Delta h\ (q, \dot{q})\, X \quad (7)$$

Let us define $(q, \dot{q})_{ab(i)}$ as a sample of $(q, \dot{q})$ at times $(t_a, t_b)_{(i)}$.

$(q, \dot{q})_{a(i)} = [q(t_{a(i)}), \dot{q}(t_{a(i)})]$, $(q, \dot{q})_{ab(i)} = [(q, \dot{q})_{a(i)}, (q, \dot{q})_{b(i)}]$

From a sequence of r samples $(q, \dot{q})_{ab(i)}$, i=1, ...,r, we can obtain r samples of the energy difference equation.

$$\Delta H(i) = \Delta h(i)\, X \quad (8)$$

We define the rxb matrix **W**, $r \geq b$ and the rx1 vector **Y** :

$$Y = \begin{bmatrix} y(1) \\ \cdots \\ y(r) \end{bmatrix}, \quad W = \begin{bmatrix} \Delta h(1) \\ \cdots \\ \Delta h(r) \end{bmatrix} \quad (9)$$

$$y(i) = \int_{t_{a(i)}}^{t_{b(i)}} T^T \dot{q}\, dt \quad , \quad \Delta h(i) = h((q, \dot{q})_{b(i)}) - h((q, \dot{q})_{a(i)})$$

We obtain an overdetermined linear system :

$$Y(T, \dot{q}) = W(q, \dot{q})\, X + \rho \quad (10)$$

where ρ is the vector of errors.

A good choice of $(q, \dot{q})_{ab(i)}$, and the use of the base parameters ensures that **W** is a full rank matrix.

The base parameters can be identified using the least squares solution $\hat{X}$ of the linear system (10).

$$\hat{X} = \underset{X}{\text{Arg.min}}\ \|\rho\|^2$$

The sequential L.S. solution using the efficient numerical algorithm developed by Bierman [21] is well suited for on line identification, [1, 22].

The most efficient method to calculate the batch L.S. solution by direct method uses QR and SVD decompositions of **W** [11]. Moreover, these factorizations provide efficient tools for practical analysis of L.S. problems [23, 24].

3. Perturbation bounds for the L.S. solution

In practical application **Y** and **W** are perturbated by noise measurements on Γ, $q, \dot{q}$ and by error modelling.

In this section we study the effect of these perturbations on the L.S. solution of (10).

Let $\hat{X}+\delta\hat{X}$ be the L.S. solution of the perturbated system :

$$Y + \delta Y = (W + \delta W)\, X + \rho \quad (11)$$

In the case where r=b, simple bounds for $\delta\hat{X}$ are given by [25] :

$$\frac{\|\delta\hat{X}\|}{\|\hat{X}\|} \leq \text{Cond}(W) \frac{\|\delta Y\|}{\|Y\|} \quad \text{with } \delta W = 0 \quad (12)$$

$$\frac{\|\delta\hat{X}\|}{\|\hat{X}+\delta\hat{X}\|} \leq \text{Cond}(W) \frac{\|\delta W\|}{\|W\|} \quad \text{with } \delta Y = 0 \quad (13)$$

In the case where r>b, general relations are given in [23, Sect. 9].

$\|.\|$ is a p vector norm or its subordinate p matrix norm.

Cond(**W**) is the condition number of **W** , w.r.t the p norm.

Relations (12) and (13) show that the condition number is a quantity which measures the sensitivity of the solution $\hat{X}$ to errors in **W** or **Y**. So it is very important that Cond(**W**) be as small as possible before computing $\hat{X}$.

Cond(**W**) in the p norm is given by :

$$\text{Cond}_p(W) = \|W\|_p\, \|W^+\|_p \quad (14)$$

where W^+ is the pseudo inverse of **W**.

It can be seen that :

$$\text{Cond}(W) \geq 1 \quad (15)$$

The usual p norms are the 1-norm, the 2-norm and the ∞-norm, but any two condition numbers in two different norms are equivalent. Thus, if a matrix is well conditioned in a norm, it is well conditioned in all the other norms, depending on some constants [24, p.26].

The 2-norm condition number is easy to calculate using the singular value decomposition (SVD) of **W** [11, 23, 24 26] :

$$\text{Cond}_2(W) = \frac{\sigma_1}{\sigma_b} \quad (16)$$

σ_1 is the largest singular value,

σ_b is the smallest singular value.

The SVD of **W** is given by :

$$U^T W = S\, V^T \quad (17)$$

with :

$$S = \begin{bmatrix} \Sigma \\ 0 \end{bmatrix}$$

U is a rxr orthogonal matrix,

V is a bxb orthogonal matrix,

Σ is a bxb diagonal matrix whose elements are the singular values of **W** in nonincreasing order ($\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_b > 0$).

In the 2-norm, the minimum condition is obtained for an orthogonal matrix :

$\text{Cond}(W) = 1 \Leftrightarrow W^T W = I$, **I** is the identity matrix.

*It may be noted that the bounds given by relations (12), (13) should be meaningless, unless the errors δW_{ij} on the elements W_{ij} of the matrix **W** are all about the same size, that is to say **W** must be scaled before computing $\hat{X}$, [11, p. I-11].*

4. The optimization problem

The problem is to find a trajectory $(q, \dot{q})$ that gives a matrix $W(q, \dot{q})$ well conditioned and well equilibrated.

The trajectory is defined by a sequence of points $(q, \dot{q})_{ab(i)}$, i=1, ...,r and $r \geq b$, corresponding to samples of $(q, \dot{q})$ at times $t_a(i)$ and $t_b(i)$.

Cost function

The cost function f can be defined as:

$$f(q, \dot{q}) = \lambda_1\, \text{Cond}(W(q, \dot{q})) + \lambda_2\, S(q, \dot{q}) \quad (18)$$

λ_1 and λ_2 are two weighting scalars and S is the measure of the equilibrium.

When **W** is computed highly accurately, the absolute error in an element is proportional to its size and the strategy of equilibrium reduces to scale **W** so that all its elements are of the same order of magnitude [11, p.11.2].

Thus S can be defined as:

$$S = \frac{|W_{ij}|_{max}}{|W_{ij}|_{min}} \quad (19)$$

$|W_{ij}|_{max}$ and $|W_{ij}|_{min}$ are the maximum and minimum absolute values of the elements W_{ij} of W, and $|W_{ij}|_{min} \neq 0$.

The criterion f is a non-linear function of $(q, \dot{q})_{ab(i)}$, i=1, ...,r . Taking into account the joint positions and velocities constraints, the problem is a non linear optimization problem with constraints. It can be formulated as follows :

Find $(q, \dot{q})_{ab(i)}$, i=1, ...,r which minimizes $f(q, \dot{q})$ under the constraints:

$$q_{Ij} \leq q_j \leq q_{Fj} \text{ and } |\dot{q}_j| \leq \dot{q}_{Mj} \text{ , } j = 1, ..., n \quad (20)$$

where:

q_{Ij} and q_{Fj} define the position limits of joint j, and $\dot{q}_{Mj}$ is the maximum absolute value of the velocity of joint j .

Three measures of the condition number were tested:

i) the 2-norm condition number using the SVD decomposition of W, (Eq.16) calculated by a program from the IMSL software package [27],

ii) an estimate of the 2-norm condition from the QR decomposition of W with column pivoting [11, 23, 27],

iii) a condition of the matrix $A = W^T W$ in the Frobenius norm, given by :

$$Cond_F(A) = \|A\|_F \|A^{-1}\|_F \quad (21)$$

$$\|A\|_F = \left(\sum_{i=1}^{b} \sum_{j=1}^{b} A_{ij}^2 \right)^{1/2} \quad (22)$$

Although Similar results were obtained in the three cases the last measure gives the best convergence rate.

Constraints

In order to transform the problem into another simpler one without constraints, the following change of variables can be used [28]:

$$q_{Ij} \leq q_j \leq q_{Fj} \Leftrightarrow q_j = q_{Ij} + (q_{Fj} - q_{Ij}) \sin^2 x_k \text{ , } j = 1, ..., n$$

$$|\dot{q}_j| \leq \dot{q}_{Mj} \Leftrightarrow \dot{q}_j = \dot{q}_{Mj} \sin x_k \text{ , } j = 1, ..., n$$

The n_d new variables x_k are not constrained anymore.

Optimization method

Symbolic expressions of the derivatives of the criterion cannot be obtained for this problem. The problem is also characterized by a large number of degrees of freedom n_d.

To reduce the number of points, every sample $(q, \dot{q})_{(i)}$ is used twice, such that :

$$(q, \dot{q})_{a(i+1)} = (q, \dot{q})_{b(i)}$$

To obtain r rows of W needs :

$$n_d = (r+1)\, 2n \text{ , } r \geq b \quad (23_a)$$

In many industrial robots where the first axis is rotational and vertical, W is independent of q_1, thus equation (23_a) reduces to :

$$n_d = (r+1)\,(2n-1) \text{ , } r \geq b \quad (23_b)$$

For the 3 d.o.f robot given in the example there are 15 base parameters. Thus using (23_b) we find $n_d \geq 80$.

The numerical method developed by Powell [28, 29] and implemented in the program VA04AD of the Harwell software package [30] is adequate for this problem.

It is a gradient conjugate type method which approaches the conjugate directions by an iterative procedure.

The n_d values of x_k, k=1, ..., n_d, are initialized by normal random sequences and converge to n_d optimum values which correspond to (r+1) optimum pairs $(q_j, \dot{q}_j)$ for joints j, j=1, ..., n, within joint positions and velocities limits.

5. Interpolation

As a result of the optimization procedure we can obtain a set of optimum points $(q, \dot{q})_{(i)}$. A continuous and smooth trajectory is calculated by interpolating a line between these points, assuming zero initial and final accelerations and using a fifth order polynomial [7].

The following notations are used :

t is the current time,

$[t_{a(i)}, t_{b(i)}]$ defines an interval (i),

u(i) is the relative time on an interval (i) :

$u(i) = t - t_{a(i)}$, $u(i) \in [0, u_{f(i)}]$,

$u_{f(i)} = t_{b(i)} - t_{a(i)}$.

$u_{f(i)}$ is the time necessary to move from $(q, \dot{q})_{a(i)}$ to $(q, \dot{q})_{b(i)}$ and is calculated in the following section.

For any interval (i) and for each joint j the following equations can be defined (subscripts j and i are omitted):

$$q(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 \quad (24)$$

$$\dot{q}(u) = a_1 + 2 a_2 u + 3 a_3 u^2 + 4 a_4 u^3 + 5 a_5 u^4 \quad (25)$$

$$\ddot{q}(u) = 2 a_2 + 6 a_3 u + 12 a_4 u^2 + 20 a_5 u^3 \quad (26)$$

Calculation of the coefficients a_i

The coefficients a_i are determined to satisfy the following initial and final conditions :

$$q(0)=q_a,\ q(u_f)=q_b,\ \dot{q}(0)=\dot{q}_a,\ \dot{q}(u_f)=\dot{q}_b,\ \ddot{q}(0)=\ddot{q}(u_f)=0 \quad (27)$$

From (24, ..., 27) we obtain :

$$a_0 = q_a$$

$$a_1 = \dot{q}_a$$

$$a_2 = 0$$

$$a_3 = 10\, A\, u_f^{-3} - (6\dot{q}_a + 4\dot{q}_b)\, u_f^{-2}$$

$$a_4 = -15\, A\, u_f^{-4} + (8\dot{q}_a + 7\dot{q}_b)\, u_f^{-3}$$

$$a_5 = 6\, A\, u_f^{-5} - 3\,(\dot{q}_a + \dot{q}_b)\, u_f^{-4} \quad (28)$$

where : $A = q_b - q_a$

Calculation of u_f

To minimize the identification time, u_f is calculated so that maximum velocity or maximum acceleration is reached on one joint and velocity and acceleration constraints (Eq. 20 and Eq.29) are also satisfied.

The acceleration constraint of joint j is given as:

$$|\ddot{q}_j| \leq \ddot{q}_{Mj} \quad (29)$$

This is a univariate non-linear optimization problem which is solved using a quadratic interpolation method [30, VD04AD].

After this, the calculated position trajectory has to be checked to see that it doesn't violate the position constraints.

If it does, a new sequence of $(q, \dot{q})_{ab(i)}$ must be determined using position and velocity constraints smaller than q_I, q_F, $\dot{q}_M$ respectively. This is because extrema of q are increasing functions of u_f while extrema of $\dot{q}$ and $\ddot{q}$ are decreasing functions of u_f [7].

6. Application

The method was simulated on a 3 d.o.f. robot similar to the first three d.o.f of the PUMA 560 robot, Fig. 1.
The geometric parameters of the robot are given in table 1.
The symbolic expressions of the functions h_i are automatically computed using the SYMORO software package [31]. They are also easy to calculate numerically [7].

Figure 1: 3 degree of freedom robot

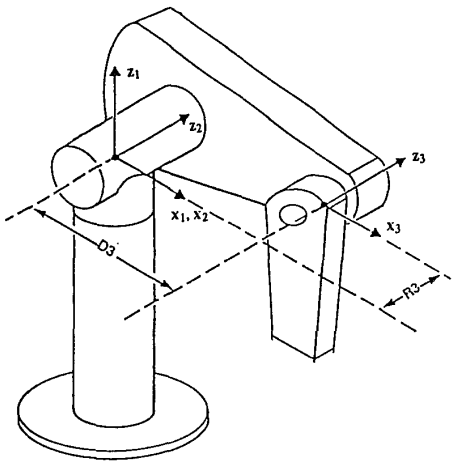


Table 1: The geometric parameters of the 3 d.o.f robot.

j	σ_j	α_j	d_j	θ_j	r_j
1	0	0	0	θ_1	0
2	0	-90	0	θ_2	0
3	0	0	D3=0.5m	θ_3	R3=0.2m

Influence of the number of equations

Cond(W) has been calculated using random sequences $(q, \dot{q})_{ab(i)}$ for different values of r. Minimum, maximum and mean values over 1000 samples are reported in table (2) and on fig(2).
It can be seen that Cond(W) decreases with r, but doesn't vary any more for r≥50. Table (2) gives optimization results with respect to r. For r≥50, the optimization fails because of the great number of variables n_d. For r=30 the condition number reduces to 11 due to optimization, thus r=30 can be considered as a good compromise between the difficulty of the optimization and the minimum value of Cond(W).

Figure 2: Effect of the number of equations

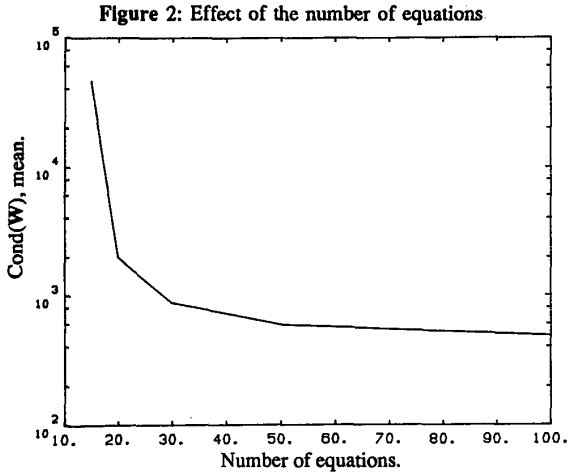


Table 2 : Effect of the number of equations r

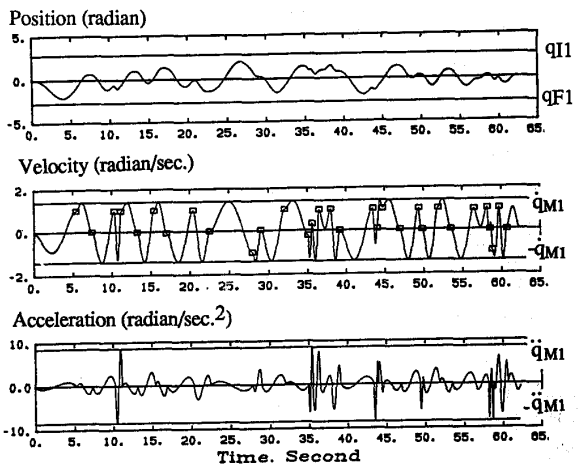
r	Cond(W)			
	Random sequences			O. S.
	Minimum	Mean	Maximum	
15	1.0715D+03	4.5912D+04	1.9596D+06	53
20	5.7771D+02	2.0200D+03	1.2265D+04	30
30	4.0940D+02	8.8067D+02	2.1569D+03	11
50	3.3677D+02	5.9512D+02	1.2228D+03	
100	3.3811D+02	4.8474D+02	8.7330D+02	
200	3.4180D+02	4.4097D+02	5.9686D+02	
300	3.4909D+02	4.2746D+02	5.2962D+02	
500	3.5887D+02	4.2081D+02	4.9306D+02	

O.S.: Optimum sequence

Exciting trajectories

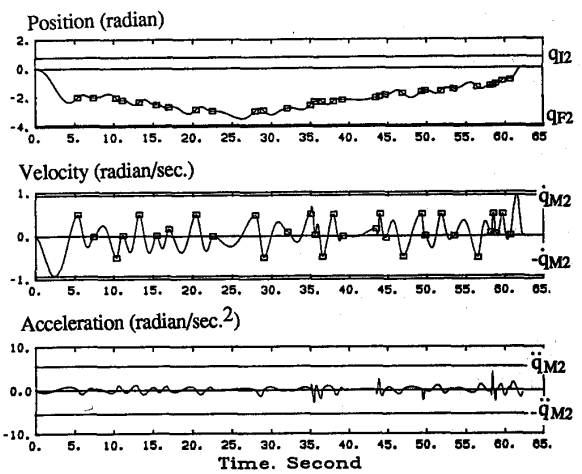
Trajectories are given in fig. (3, 4, 5).

Figure 3 : Exciting trajectory. Axis 1



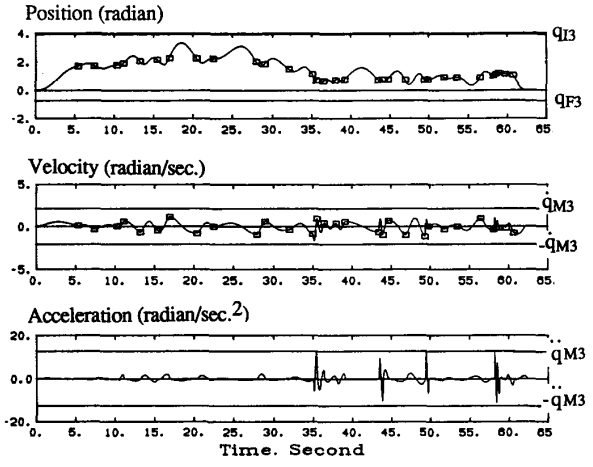
□ Optimal points

Figure 4 : Exciting trajectory. Axis 2



□ Optimal points

Figure 5 : Exciting trajectory. Axis 3



□ Optimal points

To obtain a matrix **W** (30x15), the optimization algorithm provides 155 values of x_k corresponding to 31 points $\dot{q}_1$ and to 31 pairs $(q_j, \dot{q}_j)$ for joint 2 and 3.
The initial values of the random points give a condition number $Cond(W) \approx 10^3$ and a scaling $S \approx 10^5$, and converge to optimum points which give $Cond(W) \approx 10$ and $S \approx 175$.

The optimization required about 30 minutes of VAX 8700 CPU time (6 Mips, 1.2 MFlops, test LINPACK 64 bits).
The position q_1 has no effect on **W** , therefore it was taken using a uniform random sequence of 31 points between q_{I1} and q_{F1}.

Starting point $(q, \dot{q})_{a(0)}$ and end point $(q, \dot{q})_{b(r+1)}$ corresponding to $q = \dot{q} = 0$ were added.

Identification

Tables (3,...,6) compare the results of the identification using a $(q, \dot{q})$ random trajectory with $Cond(W) \approx 10^3$, and using the optimum trajectory.

X is the vector of the base parameters and $\hat{X}$ is the vector of estimated parameters.
The influence of noise is tested. The noise is supposed a zero mean gaussian random sequence whose standard deviation σ_ρ was equal to k/4 times the maximum absolute value of **Y** to generate δY or k/4 times $|q_F - q_I|$ and k/4 times $\dot{q}_M$ to generate δW.

Tables (3 and 4) give the comparison with respect to the 2-norm.
Table (3) shows that the relation (12) is significant and table (4) shows that the condition number is not sensitive to perturbations.
Tables (5 and 6) give the comparison with respect to the components of $\hat{X}$.
Standard deviations σ_{Xi} were computed using SVD factorization of **W** and assuming that $\delta W = 0$ and that ρ was a zero mean noise with a covariance matrix :

$$C_{\rho\rho} = E(\rho\, \rho^T) = \sigma_\rho^2\, I$$

where E is the expectation operator.
Then the covariance matrix of the estimation error is given by [23]:

$$C_{XX} = E([X-\hat{X}]\,[X-\hat{X}]^T) = \sigma_\rho^2\,[W^T W]^{-1} = \sigma_\rho^2\, V\, \Sigma^{-2}\, V^T,$$

$$\sigma_{Xi}^2 = C_{XXii}$$

With k=0.01, the confidence interval $(2\sigma_{Xi})$ shows that some components of $\hat{X}$ (in bold type in table 6), are not significant when **W** is ill-conditioned.

Table 3.

$\delta W=0$						
k	$\frac{\|\delta Y\|}{\|Y\|}$	$\frac{\|\delta X\|}{\|X\|}$	K_R	$\frac{\|\delta Y\|}{\|Y\|}$	$\frac{\|\delta X\|}{\|X\|}$	K_R
0.001	$5.35\ 10^{-4}$	$4.7\ 10^{-4}$	12.7	$6.48\ 10^{-4}$	0.018	37
0.01	0.0047	0.007	7.6	0.0071	0.161	45
0.1	0.05	0.059	9.5	0.0649	1.796	39
K=11.16, S=175				K=1020, S=8.9 10^4		

$$K = Cond(W),\ K_R = Cond(W)\,\frac{\|\delta Y\|\,\|X\|}{\|Y\|\,\|\delta X\|}$$

Table 4.

$\delta Y=0$						
k	$\frac{\|\delta Y\|}{\|Y\|}$	$\frac{\|\delta X\|}{\|X\|}$	K_p	$\frac{\|\delta Y\|}{\|Y\|}$	$\frac{\|\delta X\|}{\|X\|}$	K_p
0.001	0.005	0.0041	11.16	0.00053	0.013	1019
0.01	0.055	0.0384	11.17	0.004	0.1853	1020
0.1	0.577	0.4691	13.29	0.056	2.0129	858
K=11.16, S=175				K=1020, S=8.9 10^4		

$K = Cond(W),\ K_p = Cond(W+\delta W)$

Table 5.

Cond(W)=11.16					
	X_i	$\hat{X}_i$	$X_i - \hat{X}_i$	$\frac{X_i - \hat{X}_i}{X_i}$	σ_{Xi}
ZZR_1	5.0186	5.0251	-0.0065	-0.0013	0.0281
XXR_2	-2.0500	-2.0436	-0.0064	0.0031	0.0290
XY_2	0.7000	0.6848	0.0152	0.0218	0.0209
XZR_2	-1.0700	-1.0767	0.0067	-0.0063	0.0148
YZ_2	0.6500	0.6473	0.0027	0.0041	0.0191
ZZR_2	6.5500	6.4994	0.0506	0.0077	0.0388
MXR_2	4.3000	4.3118	-0.0118	-0.0027	0.0066
MY_2	0.6000	0.6065	-0.0065	-0.0109	0.0069
XXR_3	0.7634	0.7906	-0.0272	-0.0356	0.0301
XY_3	0.6872	0.7038	-0.0166	-0.0242	0.0253
XZ_3	0.5500	0.5427	0.0073	0.0133	0.0135
YZ_3	-0.6000	-0.6056	0.0056	-0.0093	0.0140
ZZ_3	0.9646	0.9758	-0.0112	-0.0116	0.0134
MX_3	0.5280	0.5281	-0.0001	-0.0002	0.0075
MY_3	1.1400	1.1278	0.0122	0.0107	0.0064
$\delta W=0$, $\frac{\|\delta Y\|}{\|Y\|} = 4.7\ 10^{-3}$					

Table 6.

Cond(W)=1.02 10^3					
	X_i	$\hat{X}_i$	$X_i-\hat{X}_i$	$\frac{X_i-\hat{X}_i}{X_i}$	σXi
ZZR_1	5.0186	5.3447	-0.3261	-0.0650	0.4688
XXR_2	-2.0500	-2.2357	0.1857	-0.0906	0.4544
XY_2	0.7000	0.5842	0.1158	0.1654	**0.4379**
XZR_2	-1.0700	-2.2074	1.1374	-1.0630	0.8755
YZ_2	0.6500	0.5710	0.0790	0.1215	**0.5830**
ZZR_2	6.5500	7.5616	-1.0116	-0.1544	1.5245
MXR_2	4.3000	4.3138	-0.0138	-0.0032	0.0159
MY_2	0.6000	0.5967	0.0033	0.0056	0.0075
XXR_3	0.7634	0.5788	0.1846	0.2418	**0.4319**
XY_3	0.6872	0.6953	-0.0081	-0.0118	0.1364
XZ_3	0.5500	0.5252	0.0248	0.0450	0.1346
YZ_3	-0.6000	-0.7103	0.1103	-0.1839	0.2298
ZZ_3	0.9646	0.9366	0.0280	0.0290	0.0600
MX_3	0.5280	0.5351	-0.0071	-0.0135	0.0087
MY_3	1.1400	1.1516	-0.0116	-0.0102	0.0167
$\delta W=0$, $\frac{\|\delta Y\|}{\|Y\|} = 7\ 10^{-3}$					

7. Conclusion

Using the linear model and the least squares technique for inertial parameters identification without choosing the trajectories carefully may lead to a poor estimation of the parameters.
To overcome this difficulty, we have proposed a new method of generating trajectories using the energy model. The generated trajectories give acceptable small condition number and a well equilibrated design matrix. Identification results are improved when the number of equations increases, but no more than twice the number of unknown parameters.
The calculated trajectory can be applied to a real robot because it respects the joint position, velocitiy and acceleration limits.
These good simulation results are encouraging for future experimental work.

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