Learning Robot Skills Through Motion Segmentation and Constraints Extraction

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Abstract—Learning skills while interacting with human users allows robots to work in human environments and become efficient when performing everyday tasks. This paper presents a method of analyzing data obtained from human demonstrations of a task in order to: (a) extract specific constraints for each part of the task (important variables, the frame of reference to be used, and the most suitable controller for reproducing the task); (b) use these task features as continuously embeddable constraints in the learned robot motion (c) properly segment into subtask; The proposed method has been tested on a common kitchen task and its performance was compared against standard control modes.

I. Introduction

Employing robots in human environments draws attention on one hand to developing natural ways of interaction that would empower the user to teach skills to a robot and on the other hand on developing strategies to enable robots to understand the important aspects of the interaction. One such way of teaching a robot is Programming by Demonstration (PbD) [1]. A human can teach a robot how to perform a task by physically guiding the robotic arm throughout the task (kinesthetic teaching). The demonstrated tasks may involve a sequence of basic sub-tasks, with different characteristics. Learning such a complex motion requires determining the specific constraints in each part of the task.

In this work we propose a method for extracting task features based on a notion of variance. We make the assumption that variables that vary substantially within a demonstration but have little variance across trials represent features of the motion that we would like to reproduce. This approach allows us to determine the task constraints as well as perform segmentation without any prior task information. More specifically, this reduces to automatically determining the suitable frame of reference for each part of the task and learning a decomposition between force and position control that is applied in the given reference frame. These represent continuously embeddable constraints that can be used for reproducing the motion. For this we use a cartesian impedance controller and learn a weighting factor between the force and position that modulates these variables' contribution at each time step in the final motion. The approach is time independent and can easily be extended to other variables. An overview is shown in Fig. 1.

Based on the extracted constraints the demonstrated task can be easily segmented into subtasks. The resulted segments can be used by a high-level planner [2] to reorganize the task, or to ask the human for additional demonstrations of a specific task part, and moreover the segments are consistent with the human's mental representation of the task [3].

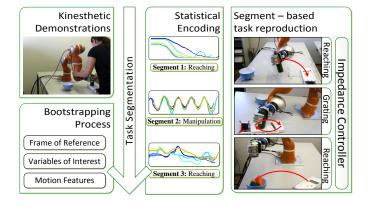


Fig. 1. Our task segmentation and feature extraction approach are part of the bootstrapping process, that comes prior to learning a task model.

A. Related Work

When analyzing a demonstrated task the difficulty consists firstly in accounting for the large variability that may exist between demonstrations and deciding what features of the motion should be reproduced (extracting task constraints), and secondly expressing these features in relation to the objects involved in the task (extracting the suitable frame of reference).

Existing approaches of task representation encode motion constraints in a time dependent manner. In our previous work [4] we proposed a method for extracting time dependant constraints in a manipulation task from demonstrated robot motion. Constraints are extracted with respect to a proposed metric of imitation, and later used for learning a task model. Data is projected into a lower dimensionality latent space and further encoded in a Gaussian Mixture Model (GMM). Gaussian Mixture Regression (GMR) is used to generate an optimal trajectory. A second attempt aimed at encoding the temporal variations in a Hidden Markov Model (HMM) [5]. A more recent approach allows for time independent representation of the dynamics of the recorded motion [6], [7]. In this work we build on these existing approaches by encoding in a time independent manner, not only kinematic constraints, but also constraints with respect to forces applied on objects.

Furthermore, using different reference frames is often embedded as prior task-knowledge. However, objects relevant in a task can be determined using statistical based methods [8], gaze tracking as a prior in learning the task [9], or complex models of observed human behaviors [10].

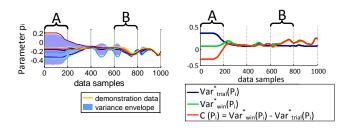


Fig. 2. Example for computing the variance over trials (Var_{trial}^*) and over a time window (Var_{win}^*) for a given parameter p(i). The variance is computed on data projected in the reference frame of two objects involved in the task. Region **A** shows data with large variance over trials, and low variance over a time window (almost constant). Region **B** shows data with little variance across trials (i.e. a feature of this parameter that should be reproduced).

Complementary to the constraints extraction topic is that of performing task segmentation which offers the possibility to easily recognize and classify motions. A vast majority of recent works focus on segmenting motion data represented by sets of joint positions or hand positions and orientation retrieved by motion capture systems in the case of human motion and by robots proprioception in the case of robotic motions. However very few works focus on segmenting task data. In this case, for achieving proper reproduction, extra information such as forces, torques or tactile information is necessary.

Current existing approaches for motion segmentation [11] rely on either (1) classification based on existing motion primitives that algorithms use for prior training [12]–[14]; (2) looking for changes in a variable, like zero-crossings [15]; or (3) clustering similar motions by means of unsupervised learning [16]. The downside of these approaches is the need of prior knowledge about the task (which may be poor and incomplete according to real-life situations). Moreover they are sensitive to the variables encoded and raise difficulties when applied to data such as tactile information where a large number of zero crossings may appear and the encoding of motion primitives can be difficult. In the larger scope of human movements, motions performed in a known environment usually follow specific patterns [17] and are specific to given contexts [18] or goals which allows a tree representation of skills [19].

While the topics of extracting task constraints and performing segmentation have been addressed previously, our work proposes a one shot algorithm that extracts all the necessary information from human demonstrations of complex tasks, in a bootstrapping process preceding learning a task model, without requiring any prior information about the type or goal of the motion.

II. EXTRACTING TASK CONSTRAINTS

The proposed method for extracting the task constraints is illustrated on a simple toy example, consisting of an unidimensional measurement of force F estimated at the robot's end effector and the end effector's position P. The dataset $\xi = \{F, P\}$ of length t = 1..k is considered to be recorded over a number of N demonstrations of a task (Fig. 2 (left)). The data is temporally aligned using Dynamic Time Warping (DTW), resulting in a set of length t = 1..M. Considering that in the task are involved 2 objects, for each object o_i ,



Fig. 3. Comparison between the criteria computed for force (F) and position (P) in two different reference frames (R.F.1 and R.F.2) of the toy example.



Fig. 4. Toy example illustrating the choice of reference frame and controller.

 $i=1..N^o$ we compute the set of parameters in the object's frame of reference: $p_i=\{P_i,\,F_i\}$.

For all objects o_i we compute the variance of each parameter k (in this example k=2), over trials (i.e. consecutive demonstrations), $(Var_{trial}(p_{i,k}))$ and averaged over a time window $(Var_{win}(p_{i,k}))$. The size of the time window is chosen arbitrarily as being the shortest time period in which we see noticeable changes in the task flow. The values of the two variances are normalized, obtaining $Var_{trial}^*(p_{i,k})$, $Var_{win}^*(p_{i,k})$. An example is given in Fig. 2 (right).

Further on, we make the assumption that variables that vary substantially within a demonstration, but have little variance across trials represent features of the motion that should be reproduced. Based on this assumption we define a criterion given by the difference between the variance over a time window and that over trials. At each time step the criterion is computed as follows:

$$C(p_{i,k}) = Var_{win}^*(p_{i,k}) - Var_{trial}^*(p_{i,k})$$
 (1)

This criterion allows us to compare in an absolute manner the influence of variables of different types (like force vs. position), see Fig. 3 (left). The aim is to be able to quantify their relevance to the task, so as to give more importance to the variable of interest in the running controller and to adjust the controller when a change occurs. Similarly the criterion can be used for determining the suitable reference frame to be used by the controller, and changing it when necessary.

A. Choice of Reference Frame

Expressing the control variables in the reference frame (RF) attached to the objects involved in the task allows the robot to properly perform the task when the positions of the objects change in the scene. For choosing a frame of reference we compute at each time index t, t=1..M the value of the highest criterion for all the variables considered $\max(C(p_{i,k}))$, expressed in the reference frame of the objects involved in the task. This results in segments of different length for each object, see Fig. 3 (right).

The obtained value $max(C(p_{i,k}))$ is analyzed for each axis, over a time window of arbitrary size (in this case $w_1 = 100$ data samples). We consider that for each time step the reference system is be given by the object o_i for which a corresponding segment of maximum length is found. In this

example there are two changes in the reference frame: for the first 100 data samples the R.F. is given by object o_2 , for the next 200 samples there is a change to o_1 , and for the rest of the motion the R.F. is changed to o_2 , see Fig. 4 (left).

B. Choice of relevant variables

For determining relevant task variables, we analyze for each axis the criterion obtained at point A using a time window of arbitrary size (in this case $w_2=100$). Similarly to extracting the reference frame, we consider the relevant variable in each time window to be the one that has a corresponding segment of maximum length in that interval. In the toy example, there are several changes between position and force as variables of interest, as shown in Fig. 4 (right).

Determining the variable of interest at each time step allows us to compute a weighting factor λ that balances the contribution of the force and position according to the relevance determined above. Therefore, for each axis the value of λ is given by the difference between the criterion computed for position and the one computed for force

$$\lambda = C(P_i) - C(F_i) \tag{2}$$

Considering a simple spring model of the system, in which the damping and acceleration terms are ignored, given by $F_{ext} = K_d \cdot \tilde{x}$, where $\tilde{x} = x - x_d$, the normalized λ parameter can be seen as a weighting factor for a base stiffness K_b , such that $K_d = K_b \cdot \lambda$. Therefore we can use a simple impedance controller for reproducing the motion whith the factors described above representing continuously emdeddable constraints.

III. CONSTRAINT BASED SEGMENTATION

We perform segmentation based on the change in the extracted task constraints. We consider that the points in which either the reference frame or the variables of interest change is a segmentation points in the task (e.g. force sensed at the end effector might be relevant in the first task segment while after the segmentation point, end effector's position and orientation could become more relevant). Segmenting and interpreting the data in a stochastic manner allows regenerating the motion according to the measures determined to be important as well as finding optimal control strategies with respect to the variables of interest.

Whenever the reference object changes, a segmentation point is created. This results in a first set of segmentation points ψ_1 . In the toy example there are two such segmentation points $\psi_1 = \{\psi_1(1), \psi_1(2)\}$, at 100 and 300 data samples.

A set ψ_2 of segmentation points is created for every change in the control mode. In the toy example described above, there are 3 segmentation points corresponding to the change of the variable of interest. The first two points $\psi_2(1), \psi_2(2)$ are identical to the segmentation points found by the change in the reference frame $\psi_1(1), \psi_1(2)$. The last point determined $\psi_2(3)$ marks a change from a force-based part of the task to a position based part, as seen in Fig. 4 (right). The final task segmentation points are given by the vector θ , consisting of

$$\theta = \psi_1 \cup \psi_2 \tag{3}$$

IV. LEARNING THE TASK MODEL

We aim to encode the task in a time independent manner, which is important for properly reproducing the task in changing conditions. Particularly for segments where position was determined as important (e.g. in reaching motions), we analyze the behavior near the segmentation point. If this segmentation point concludes a position control sequence, then the object with the associated reference frame becomes an attractor for the system (i.e. all demonstrated trajectories converge to this point). We thus consider this segment to be a discrete motion that can be modeled using a non-linear, timeindependent dynamical system (SEDS) [20], which learns a mapping between a state variable $\xi(P)$ and it's first derivative: $\xi = f(\xi)$. The function f is estimated in SEDS by using a mixture of K Gaussian. The model is learned through maximization of likelihood under stability constraints. This ensures that the learned motion follows the original dynamics of the demonstrated motion, it is stable at the target and robust to perturbations and inaccuracies, being able to adapt online to changes in the environment.

Secondly, for segments of the task in which the force is more important that the position, we use GMM to learn a joint distribution of the variables F and \dot{P} using a model comprising a mixture of K Gaussian components, such that: $p(F,\dot{P})=\sum_{i=1}^K(\alpha_i p(F,\dot{P};\mu^i,\Sigma^i))$, where α_i represent the prior of the i-th Gaussian component, and μ^i and Σ^i represent the mean and covariance matrix for Gaussian i, whose parameters are learned through Expectation-Maximization (EM) algorithm. GMR is used for predicting the force to be applied based on the current position: $E\{p(F|\dot{P})\}$.

V. VALIDATION

This approach was validated on a common kitchen task, grating vegetables, using a KUKA Light Weight Robot (LWR) with 7 degrees of freedom (DOFs) and a Barrett hand with 4 DOFs. Two objects were involved in the task: a grater and a bowl. The task consisted of: reaching from the initial position to the slicer, a repetitive grating motion, a reaching motion from the slicer to a trashing container. The vegetable was held using a power grasp. Due to the fact that the Barrett hand is not back-drivable (the motion of the fingers cannot be demonstrated kinesthetically), the task demonstration started after the vegetable was in place (i.e. the grasping and releasing motions were not part of the task demonstration).

The variability of the task consists in: (1) starting each demonstration from a different initial position of the robot, and placing the objects in different positions in the reachable space of the robot (we recorded data for 3 different positions of the objects, placed on average 30, 45 and 65 cm apart from the initial position); (2) using vegetables of different sizes and types (we recorded data for 3 types of vegetables (carrots, zukinis and cucumbers)). The vegetables varied in length, from a minimum of 10 cm for a carrot to a maximum of 35 cm for a cucumber, and with about 2 cm in diameter); The variability of the manipulated object effected the grasp, the force applied by the user when providing demonstrations and the duration of the demonstration. The task lasted until the vegetable was fully grated; (3) inerrant user variability between demonstrations.

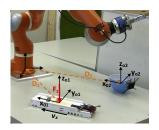






Fig. 5. The importance of choosing the suitable RF when the objects' positions change.

A number of N=18 task demonstrations were performed. We collected a data set, sampled at 100Hz: $\xi=\{x(P_x,P_y,P_z),F(F_x,F_y,F_z),x_o\}$, where $x\in R^3$ represents the robot's end effector cartesian position (the end effector's orientation is considered constant throughout the task); $F\in R^3$ represents the vector of external forces sensed at the end effector; $x^o\in R^3$ represents the cartesian position of each $o_i, i=1..N^o$ objects involved in the scene, recorded using a vision tracking system. The objects were static during the task, but their initial position could change, see Fig. 5.

The set of segmentation points $\theta = [\psi_1(1) \, \psi_2(1)]$ has two values for this task, determining 3 distinct segments: (1) a position-weighted controller expressed in RF 1, (2) an force-weighted controller in RF 1, corresponding to the grating subtask and (3) another force-weighted controller in RF 2, for the final part of the task.

We performed two different performance assessments. First we tested whether the automatic segmentation of the task and the extraction of RF was correct and led to a correct reproduction when positioning of objects was changed. The robot regenerated the complete sequence and managed to complete the overall task comprising of the 3 segments and when the objects were located in arbitrary positions, none of which were seen during training. Second, to validate whether the model had correctly extracted the dimension onto which to provide either force or position control, we contrasted the obtained controller with a pure position and a fixed impedance control modes, during the grating part of the task.

For evaluating the framework we compared the proposed approach with standard control modes: a position controller and an impedance controller with fixed stiffness values. For these two control modes, 5 different demonstrations were provided $(D_i, i=1..5)$, using gravity compensation mode (gcmp) and robot's execution was evaluated during motion replays $(R_i, i=1..5)$ in the different setups: position control (pos) and impedance control (imp). The performance under these control modes was compared to the developed approach (aimp). Several replays were performed for each demonstrated motion. We constantly compensate for the decrease of the vegetable height by 2 cm. Each group of 1 demonstration followed by 5 replays were performed on the same vegetable. A single vegetable type was used, and the task was demonstrated using 5 passes over the grating surface during each trial.

For all the trials we measured: the original and final weight of the vegetable $(w_{init}, w_{fin}[g])$; the original and final height $(h_{init}, h_{fin}[cm])$. The original values were measured before

the demonstration was performed, while the final values were measured at the end of the last replay round. For each round of demonstration and replay we measured the weight of the grated part $(\Delta w[g])$ with a precision of $\pm 1g$ and counted the number of successful passes (SP).

We evaluated the task performance with respect to the following computed measures: (1) $w_{ratio}[\%]$ the ratio of the grated vegetable ($w_{grated} = \sum \Delta w$) as a percentage of the initial weight. Note that the value of the $w_{init}-w_{grated}$ is often different than the final weight (w_{fin}) as the vegetable might break in the grating process. The broken part is not accounted for in the grated weight (Δw), but is reflected in a lower final weight; (2) $h_{ratio}[\%]$ the percentage of the vegetable length being grated ($h_{init}-h_{fin}$) with respect to the initial length; (3) $SP_{ratio}[\%]$ the percentage of successful passes (SP) out of the total demonstrated passes. Results are shown in Table I.

Using a standard position controller (Trials 1 - 5) for replaying the motion aims good results in a very low number of cases: mean (M) = 12% and standard deviation (SD) = 12%

	1									
Type	Controller	Winit [g]	Δw [g]	w_{fin} [g]	Wratio [%]	h_{init} [cm]	h _{fin} [cm]	hratio [%]	SP	SP _{ratio} [%]
Trial 1										
D1 R1 R2 R3 R4 R5	gcmp pos imp imp imp imp	100	4 1 2 3 4 7	60	21.00	14.5	8.4	42.06	5 1 3 2 4 4	100 20 60 40 80 80
Trial 2										
D2 R1 R2 R3	gcmp pos imp imp	74	7 2 2 5	48	21.62	11.5	7.4	35.65	5 1 2 3	100 20 40 60
Trial 3										
D3 R1 R2 R3	gcmp pos imp imp	74	9 1 7 6	43	31.08	10.0	6.5	35.00	5 1 4 4	100 20 80 80
Trial 4										
D4 R1 R2 R3 R4 R5	gcmp pos imp imp imp imp	90	6 0 5 3 1	55	17.78	13.0	7.5	42.30	5 0 4 2 1	100 0 80 40 20 20
Trial 5										
D5 R1 R2 R3 R4 R5	gcmp pos imp imp imp imp	83	6 0 2 1 1 5	52	18.07	13.2	9.7	26.92	5 0 2 1 1 4	100 0 40 20 20 80
				_	Trial 6					
D _N R1 R2 R3 R4	aimp aimp aimp aimp	92	7 4 5 8 9	56	35.86	13.5	7	48.15	5 4 4 5 5	100 80 80 100 100
TAB	TABLE I. EVALUATION OF THE CONTROL MODES. FOR TRIALS 1 - 5									

TABLE I. EVALUATION OF THE CONTROL MODES. FOR TRIALS 1 - 5 The demonstrated motion D_i (provided using the robot's gravity compensation mode (gcmp)) was compared, with a standard position control mode (pos) , and with an impedance controller with fixed stiffness (imp) . Trial 6, illustrates the performance of the proposed controller, learned from the $D_N=18$ demonstrations (aimp) .

10.95 successful passes, while the amount of vegetable grated is bellow one gram per trial (M=0.80g, SD=0.83). When replaying the recorded motion using an impedance controller the number of successful passes increases (M=52.5%, SD=25.16). These results are compared against the proposed approach, See Table I, Trial 6. The overall performance was better with respect to the amount of grated vegetable, and the number of successful passes.

VI. CONCLUSIONS

The proposed approach for performing motion segmentation takes advantage of the existing variance in the demonstrated data, and develops a criterion for performing segmentation based on the assumption that variables that change very little throughout a set of sequential demonstrations (thus proving that the user was coherent in that part of the task), but have large variability within a demonstration (proving that the variable becomes important in only a given region of the task) are the feature of the task that should be learned.

This criterion allows us to compare different measures (like position and force) and modulate their contribution to the controller used in reproducing the motion, by using a weighting factor that adapts the robot's stiffness. Also weighting the relative importance of each of the task variables when expressed in the reference system of the objects involved in the task we can determine the suitable reference frame to be used in each segment. Finally a set of segmentation points are obtained by splitting the motion whenever a change in the reference frame or in the variables on interest occurs. The approach was validated on a common kitchen task grating vegetables, achieving good generalization results.

The advantages of using this segmentation and feature extraction method are firstly decreasing the task complexity by focusing on learning just the variables that are important for each region of the task (i.e. encode just end effector position for a reaching motion vs. accounting for position and force in manipulation sub-tasks) and secondly achieving efficient generalization when the position of the objects is changed.

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