Espacio de estados
sistema rotacional #2

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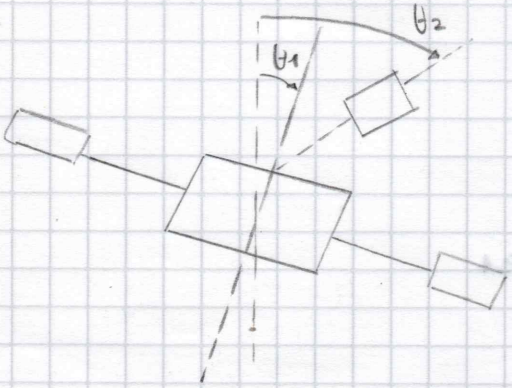
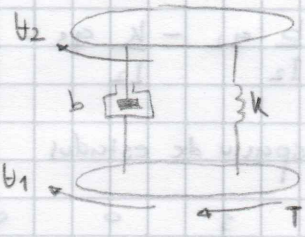


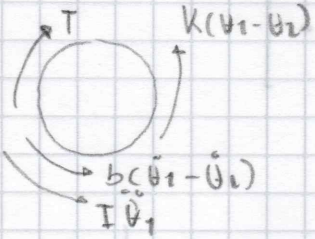
θ_2: Sensor
θ_1: cuerpo

- Hallar expresión en espacio de estados del sistema
- Representandolo como un sistema rotacional



- Diagramas de cuerpo libre

Para θ_1



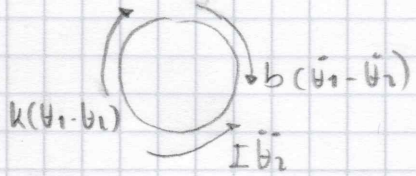
$$\sum F$$

$$T - K(\theta_1-\theta_2) - b(\dot{\theta}_1-\dot{\theta}_2) - I_1\ddot{\theta}_1 = 0$$

$$\frac{T}{I_1} - \frac{K}{I_1}\theta_1 + \frac{K}{I_1}\theta_2 - \frac{b}{I_1}\dot{\theta}_1 + \frac{b}{I_1}\dot{\theta}_2 = \ddot{\theta}_1$$

$$\ddot{\theta}_1 = \frac{T}{I_1} - \frac{K}{I_1}\theta_1 - \frac{b}{I_1}\dot{\theta}_1 + \frac{K}{I_1}\theta_2 + \frac{b}{I_1}\dot{\theta}_2 \quad (1)$$

Para θ_2



$$\sum F$$

$$k_1(\theta_1-\theta_2) + b(\dot{\theta}_1-\dot{\theta}_2) - I_2\ddot{\theta}_2 = 0$$

$$\frac{K_1}{I_2}(\theta_1-\theta_2) + \frac{b}{I_2}(\dot{\theta}_1-\dot{\theta}_2) = \ddot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{K}{I_2}\theta_1 + \frac{b}{I_2}\dot{\theta}_1 - \frac{K}{I_2}\theta_2 - \frac{b}{I_2}\dot{\theta}_2 \quad (2)$$

Variables de estado

$$q_1 = \theta_1 \qquad q_3 = \theta_2$$
$$q_2 = \dot{q}_1 = \dot{\theta}_1 \qquad q_4 = \dot{q}_3 = \dot{\theta}_2$$
$$\dot{q}_2 = \ddot{\theta}_1 \qquad \dot{q}_4 = \ddot{\theta}_2$$

Remplazando en (1) y (2)

$$\dot{q}_2 = \frac{T}{I_1} - \frac{k}{I_1} q_1 - \frac{b}{I_1} q_2 + \frac{k}{I_1} q_3 + \frac{b}{I_1} q_4$$

$$\dot{q}_4 = \frac{k}{I_2} q_1 + \frac{b}{I_2} q_2 - \frac{k}{I_2} q_3 - \frac{b}{I_2} q_4$$

Representación en espacio de estados

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{I_1} & \frac{-b}{I_1} & \frac{k}{I_1} & \frac{b}{I_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{I_2} & \frac{b}{I_2} & \frac{-k}{I_2} & \frac{-b}{I_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I_1} \\ 0 \\ 0 \end{bmatrix} [T]$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$