Taller espacio de estados
sistema rotacional y traslacional
Pendulo invertido

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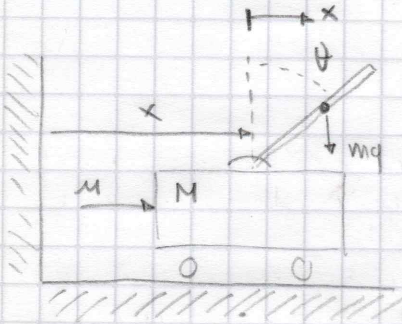

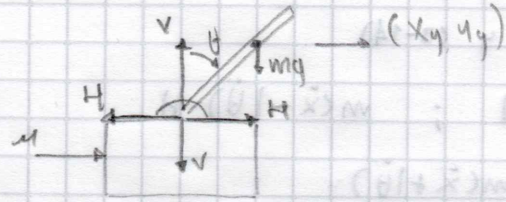

- Ecuacionamiento del sistema

$x_g = x + l\,sen\theta$

$y_g = l\cos\theta$

Movimiento rotacional

$I\ddot{\theta} = Vl\,sen\theta + Hl\cos\theta$ (3-9)

Movimiento horizontal

$m\frac{d^2(x + l\,sen\theta)}{dt^2} = H \rightarrow m\ddot{x} + m\frac{d^2(l\,sen\theta)}{dt^2}$

$= m\ddot{x} + m\frac{d(l\cos(\theta)\dot{\theta})}{dt} \rightarrow m\ddot{x} + ml\frac{d\cos(\theta)\dot{\theta}}{dt}$

$= m\ddot{x} + ml[-sen(\theta)\dot{\theta}\dot{\theta} + \cos(\theta)\ddot{\theta}] \rightarrow m\ddot{x} - ml\,sen(\theta)\dot{\theta}^2 + ml\cos(\theta)$ (3-10)

Movimiento vertical

$m\frac{d^2(l\cos(\theta))}{dt^2} = V - mg$ (3-11)

Movimiento horizontal del carro

$M\ddot{x} = u - H$

- Considerando θ muy pequeño $\begin{cases} sen\,\theta \simeq \theta \\ \cos\theta \simeq 1 \\ \theta\dot{\theta}^2 = 0 \end{cases}$

de (3-9)

$I\ddot{\theta} = Vl\theta - Hl$ (3-13)

de (3-10)

$m\ddot{x} + ml\ddot{\theta} = H$

$m(\ddot{x} + l\ddot{\theta}) = H$ (3-14) de (3-11)

$$m\frac{d^2(l\cos\theta)}{dt^2} = V - mg \quad \longrightarrow \quad 0 = V - mg \quad (3\text{-}15)$$

de (3-12) y (3-14)

$$M\ddot{x} = u - H \quad ; \quad m(\ddot{x} + l\ddot{\theta}) = H$$

$$\therefore \; M\ddot{x} = u - m(\ddot{x} + l\ddot{\theta})$$

$$= u - m\ddot{x} + ml\ddot{\theta} \quad \longrightarrow \quad M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = u$$

$$u = (M+m)\ddot{x} + ml\ddot{\theta} \quad (3\text{-}16)$$

de (3-13), (3-14) y (3-15)

$$I\ddot{\theta} = Vl\theta - Hl \quad ; \quad H = m(\ddot{x} + l\ddot{\theta}) \quad ; \quad 0 = V - mg \longrightarrow V = mg$$

$$\therefore \; I\ddot{\theta} = mg\theta - m(\ddot{x} + l\ddot{\theta})l$$

$$= mgl\theta - ml\ddot{x} - ml^2\ddot{\theta}$$

$$I\ddot{\theta} = mgl\theta - l(m\ddot{x} + ml\ddot{\theta}) \quad \longrightarrow \quad mgl\theta = (I + ml^2)\ddot{\theta} + ml\ddot{x} \quad (3\text{-}17)$$

de (3-16)

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{(m+M)}$$

remplazando en (3-17)

$$mgl\theta = (I + ml^2)\ddot{\theta} + ml\left(\frac{u - ml\ddot{\theta}}{(m+M)}\right)$$

$$= I\ddot{\theta} + ml^2\ddot{\theta} + \frac{mlu}{(m+M)} - \frac{m^2l^2\ddot{\theta}}{(m+M)}$$

$$= \frac{I\ddot{\theta}(m+M) - m^2l^2\ddot{\theta}}{(m+M)} + \frac{(m+M)ml^2\ddot{\theta} + mlu}{(m+M)}$$

$$= \frac{I\ddot{\theta}(m+M) - m^2l^2\ddot{\theta} + (m+M)ml^2\ddot{\theta} + mlu}{(m+M)}$$

$$= \frac{I\ddot{\theta}m + I\ddot{\theta}M - \cancel{m^2l^2\ddot{\theta}} + \cancel{m^2l^2\ddot{\theta}} + mMl^2\ddot{\theta} + mlu}{(m+M)}$$

$$mgl\theta - \frac{mlu}{(m+M)} = \ddot{\theta}\,\frac{(Im + IM + mMl^2)}{(m+M)}$$

$$\ddot{\theta} = \frac{mgl\theta\,(m+M)}{Im + IM + mMl^2} - \frac{ulm}{Im + IM + mMl^2}$$

$$\ddot{\theta} = \left(\frac{(m+M)}{I(m+M) + mMl^2}\right) mgl\theta - \frac{ulm}{I(m+M) + mMl^2} \qquad (1)$$

de (3-16)

$$\ddot{\theta} = \frac{u - (M+m)\ddot{x}}{ml}$$

remplazando en (3-17)

$$mgl\theta = (I + ml^2)\left(\frac{u - (M+m)\ddot{x}}{ml}\right) + ml\ddot{x}$$

$$= \frac{Iu}{ml} - \frac{I(M+m)\ddot{x}}{ml} + \frac{ml^2u}{ml} - \frac{ml^2(M+m)\ddot{x}}{ml} + ml\ddot{x}$$

$$= \frac{Iu + ml^2u}{ml} - \frac{I(M+m)\ddot{x} - ml^2(M+m)\ddot{x}}{ml} + ml\ddot{x}$$

$$= u\,\frac{(I + ml^2)}{ml} - \ddot{x}\,\frac{((m+M)I + (m+M)ml^2)}{ml} + ml\ddot{x}$$

$$= u\,\frac{(I + ml^2)}{ml} - \ddot{x}\,\frac{((m+M)(I + ml^2))}{ml} + ml\ddot{x}$$

$$\ddot{x} = -m^2l^2g\theta\left(\frac{1}{I(m+M) + Mml^2}\right) + u\left(\frac{I + ml^2}{I(m+M) + Mml^2}\right) \qquad (2)$$

Variables de estado

$q_1 = x$ $\qquad$ $q_3 = \theta$

$q_2 = \dot{q}_1 = \dot{x}$ $\qquad$ $q_4 = \dot{q}_3 = \dot{\theta}$

$\dot{q}_2 = \ddot{x}$ $\qquad$ $\dot{q}_4 = \ddot{\theta}$

Remplazando en ecuaciones (1) y (2)

$$\dot{q}_2 = -\frac{m^2l^2g}{I(m+M)+Mml^2}\,q_3 + u\left(\frac{I+ml^2}{I(m+M)+Mml^2}\right)$$

$$\dot{q}_4 = \left(\frac{mgl(m+M)}{I(m+M)+mMl^2}\right)q_3 - u\,\frac{ml}{I(m+M)+mMl^2}$$

Representación en espacio de estados

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & -\frac{m^2l^2g}{I(m+M)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \frac{mgl(m+M)}{I(m+M)+mMl^2} & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{I+ml^2}{I(m+M)+mMl^2}\\ 0\\ \frac{ml}{I(m+M)+mMl^2}\end{bmatrix}\begin{bmatrix}u\end{bmatrix}$$

$$\begin{bmatrix}x\\ \theta\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix}$$