Taller espacio de estados
Sistema rotacional

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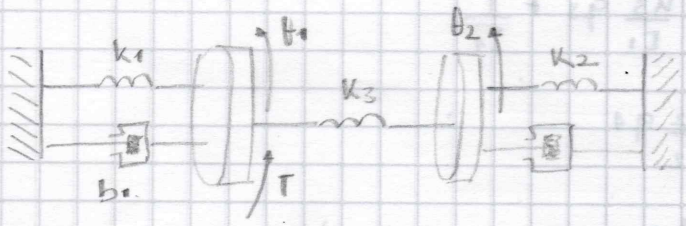
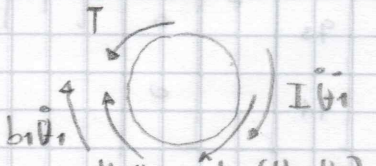


Diagramas de cuerpo libre

Para θ_1



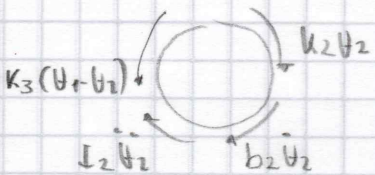
$$\sum F$$

$$-T + b_1\dot{\theta}_1 + k_1\theta_1 + k_3(\theta_1-\theta_2) + I\ddot{\theta}_1 = 0$$

$$\ddot{\theta}_1 = -\frac{k_1\theta_1}{I_1} - \frac{b_1\dot{\theta}_1}{I_1} - \frac{k_3\theta_1}{I_1} + \frac{k_3\theta_2}{I_1} + \frac{I}{I_1}$$

$$\ddot{\theta}_1 - \theta_1\frac{(k_1+k_3)}{I_1} - \frac{b_1}{I_1}\dot{\theta}_1 + \frac{k_3}{I_1}\theta_2 + \frac{I}{I_1} \quad (1)$$

Para θ_2



$$\sum F.$$

$$-k_3(\theta_1-\theta_2) + k_2\theta_2 + b_2\dot{\theta}_2 + I_2\ddot{\theta}_2 = 0$$

$$\ddot{\theta}_2 = \frac{k_3}{I_2}(\theta_1-\theta_1) - \frac{k_2}{I_2}\theta_2 - \frac{b_2}{I_2}\dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{k_3}{I_2}\theta_1 - \frac{k_3}{I_2}\theta_2 - \frac{k_2}{I_2}\theta_1 - \frac{b_2}{I_2}\dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{k_3}{I_2}\theta_1 - \theta_1\left(\frac{k_2+k_3}{I_2}\right) - \frac{b_2}{I_2}\dot{\theta}_1 \quad (2)$$

Variables de estado

$$q_1 = \theta_1$$
$$q_2 = \dot{q}_1 = \dot{\theta}_1$$
$$\dot{q}_2 = \ddot{\theta}_1$$

$$q_3 = \theta_1$$
$$q_4 = \dot{q}_3 = \dot{\theta}_1$$
$$\dot{q}_4 = \ddot{\theta}_1$$

Remplazando en (1) y (7)

$$\dot{q}_2 = -q_1\left(\frac{k_1+k_3}{I_1}\right) - \frac{b_1}{I_1} q_2 + \frac{k_3}{I_1} q_3 + \frac{I}{I_1}$$

$$\dot{q}_4 = \frac{k_3}{I_2} q_1 - q_3\left(\frac{k_2+k_3}{I_2}\right) - \frac{b_2}{I_2} q_4$$

Representación en espacio de estados

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(k_1+k_3)}{I_1} & -\frac{b_1}{I_1} & \frac{k_3}{I_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_3}{I_2} & 0 & -\frac{(k_2+k_3)}{I_2} & -\frac{b_2}{I_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I_1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} I \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

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