Sistema de control por realimentación de estados

Juan Sebastian Casas Barbosa
cod: 20211005031

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \left\{ \begin{array}{l} OS\% \quad 9.5\% \\ t_s = 0{,}74 \ \text{sg} \end{array} \right.$$

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \longrightarrow (s^3+5s^2+4s)X_1(s) = U(s)$$

$$\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

Variables de estado

$x_1 = x_1$
$x_2 = \dot{x}_1$
$x_3 = \dot{x}_2 = \ddot{x}_1$
$\dot{x}_3 = \dddot{x}_1$

Remplazando

$$\dot{x}_3 + 5x_3 + 4x_2 = u \longrightarrow \boxed{\dot{x}_3 = -5x_3 - 4x_2 + u} \quad 1.$$

$$Y(s) = (b_2 s^2 + b_1 s + b_0)X_1(s)$$

$$= (0s^2 + 20s + 100)X_1(s) \longrightarrow (20s+100)X_1(s)$$

$$= 20\dot{x}_1 + 100x_1 \longrightarrow \text{Remplazando variables de estado}$$

$$\boxed{y = 20x_2 + 100x_1} \quad 2.$$

Representación en espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$$

$$0{,}095 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 \longrightarrow \ln(0{,}095) = \ln\left(e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\right)$$

$$-2{,}3534 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \longrightarrow \cancel{+}2{,}3534\left(\sqrt{1-\zeta^2}\right) = \cancel{+}\zeta\pi$$

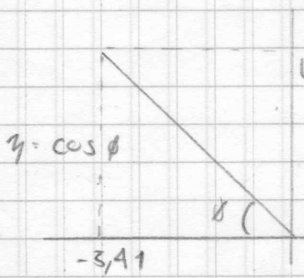
Elevando al cuadrado ambos lados $5.5407(1-\xi^2) = \xi^2\pi^2$

$$5,5407 - 5,5407\xi^2 = \xi^2\pi^2 \longrightarrow 5,5407 = \xi^2\pi^2 + 5,5407\xi^2$$

$$5,5407 = \xi^2(\pi^2 + 5,5407) \longrightarrow \xi^2 = \frac{5,5407}{\pi^2 + 5,5407}$$

$$\xi = \sqrt{\frac{5,5407}{\pi^2 + 5,5407}} \longrightarrow \boxed{\xi = 0,5996}$$

Plano s

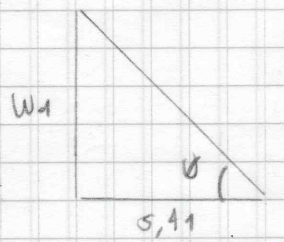


$$s = \sigma + j\omega_d \longrightarrow \sigma = \xi\omega_n$$

$$\phi = \cos^{-1}(0,5996) = 53,16°$$

$$t_s = \frac{4}{\sigma} \longrightarrow 0,74 = \frac{4}{\sigma} \longrightarrow \sigma = \frac{4}{0,74} = 5,405$$

$$\sigma = \xi\omega_n \longrightarrow 5,405 = 0,5996\,\omega_n \longrightarrow \boxed{\omega_n = 9,02\ rad/sg}$$



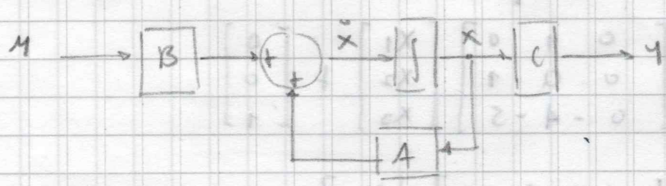
$$\tan\phi = \frac{\omega_d}{3,41}$$

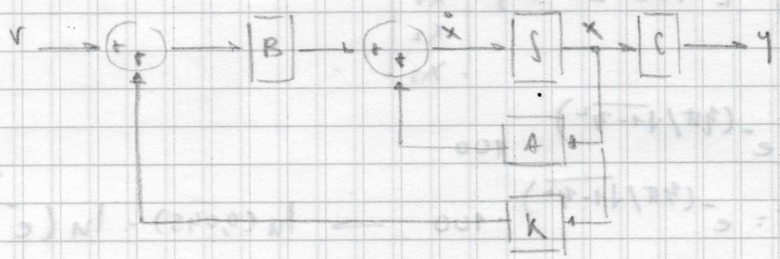
$$\tan^{-1}(53,16)(5,41) = \omega_d \longrightarrow \boxed{\omega_d = 7,2146}$$

- Realimentacion en espacio de estados

$\dot{x} = Ax + Bx$

$y = Cx$



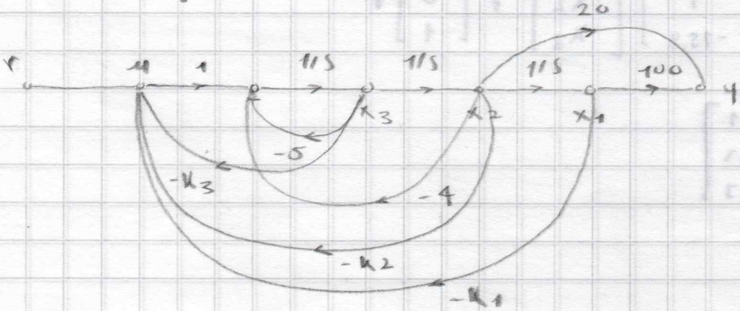


$$\dot{x} = Ax + Bu$$
$$= Ax + B(-Kx + v)$$
$$= Ax - BKx + Bv \longrightarrow \boxed{\dot{x} = (A - B)x + v}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Diagrama de flujo de señal

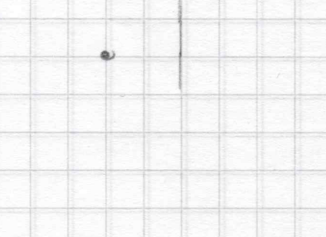


$$\dot{x}_3 = -4x_2 - 5x_3 + u$$
$$= -4x_2 - 5x_3 + [-k_3x_3 - k_2x_2 - k_1x_1] + r$$
$$= -4x_2 - 5x_3 - k_3x_3 - k_2x_2 - k_1x_1 + r$$
$$= -k_1x_1 - (4+k_2)x_2 - (5+k_3)x_3 + r$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -(5+k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\det(sI - A(BK)) = \boxed{s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = 0} \quad 4.$$

↳ Ecuación característica del sistema



$$(s+5{,}4-j7{,}2)(s+5{,}4+j7{,}1)(s+5{,}1) = T$$

$$T \rightarrow s^3 + 15{,}9s^2 + 136{,}21s + 413{,}83 = 0$$

5.

$$s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = s^3 + 15,9\,s^2 + 136,22\,s + 413,83$$

$$(5+k_3)s^2 = 15,9\,s^2 \qquad (4+k_2)s = 136,22\,s$$

$$5+k_3 = 15,9 \qquad 4+k_2 = 136,22 \qquad \boxed{k_1 = 413,83}$$

$$\boxed{k_3 = 10,9} \qquad \boxed{k_2 = 132,22}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413,8 & -136,22 & -15,9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$