

# Bourbaki vs Pragmatism

## A methodological comparison through the multi-armed bandits problem

Sebastiano Ferraris\*

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In these pages we introduce the well known multi-armed bandits problem and we propose a solution with two different mathematical methodologies: a Bourbakist approach and a pragmatic one. The Bourbakist way is concerned with the mathematical foundation upon which a formal solution is derived in the shape of an axiomatic structure. In contrast, the pragmatist approach aims at finding the shortest path towards a solution, reducing the mathematical formalisms to the bare minimum. The article ends with a critical comparison, where pros and cons of each method are compared.

If you came across this article when searching for an introduction to the multi-armed bandit problem, and not a methodological comparison, please do ignore section 2. The code to create the figures and run a range of algorithms to solve the problem can be found at <https://github.com/SebastianoF/MAB>.

### 1 Multi-armed bandits problem

Consider that you have to repeatedly choose between  $K$  different possibilities, each having a cost and each a possible cash reward. For each possibility the cost is fixed and the reward is drawn from an *unknown* probability distribution.

The problem of finding a strategy to maximise the reward in this setting is named multi armed bandits (MAB) after the situation of playing repeatedly at a row of  $K$  slot machines (or single armed bandit). Given an initial amount of money of \$1000 and a costs of \$1 for each draw, the player has 1000 attempts to balance an exploration phase

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\*sebastiano.ferraris@gmail.com

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when estimating the unknown distributions of each arm, with an exploitation phase, when the acquired knowledge is used for a gain<sup>1</sup>.

The problem can be generalised to clinical or pre-clinical trials, control engineering, mechanical and software testing, stock market investments, behavioural modelling, dynamic pricing, and many more<sup>2</sup>.

In the next section we present a solution *a la Bourbaki*, and in section 3 in a more pragmatic way.

## 2 The Bourbakist perspective

**Definition 2.1.** Let  $(\Omega, \mathcal{A})$  be a  $\sigma$ -algebra defined as a non-empty set  $\Omega$  paired with a subset of its power set  $\mathcal{A}$ , containing the empty set and closed under numerable union and complement set. Let this be called *action space*. Let  $\mathcal{I}_K = \{1, 2, \dots, K\} \subset \mathbb{N}$  be a set of indexes whose generic element  $k$  is called *arm*, by convention. Let  $\mathcal{I}_T = \{1, 2, \dots, T\} \subset \mathbb{N}$  be another set whose elements are called, again conventionally, *time*. Let  $\mathbb{R}_+$  the positive real axis including the zero.

The relationship between the above defined elements are given by a function  $A$  defined as:

$$\begin{aligned} A : \mathcal{I}_T \times \mathcal{A} &\longrightarrow \mathcal{I}_K \\ (t, \omega) &\longmapsto A(t, \omega) = A_t(\omega) \end{aligned}$$

and by a function  $R$ , defined as:

$$\begin{aligned} \mathcal{R} : \mathcal{I}_T \times \mathcal{A} &\longrightarrow \mathbb{R}_+ \\ (t, \omega) &\longmapsto R(t, \omega) = \mathcal{R}_t(\omega) \end{aligned}$$

Let the former be called *action* and the latter be called *reward*. Let

$$\begin{aligned} R : \mathcal{I}_T \times \mathcal{I}_K &\longrightarrow \mathbb{R}_+ \\ (t, \omega) &\longmapsto R(t, k) = R_t(k) \end{aligned}$$

another function, defined as the only possible function making the diagram below commutative.

$$\begin{array}{ccc} \mathcal{A} & \xrightarrow{A_t} & \mathcal{I}_K \\ & \searrow \mathcal{R}_t & \downarrow R_t \\ & & \mathbb{R}_+ \end{array}$$

<sup>1</sup>See Thompson [Tho33] for an early approach where the two arms are two medical treatments, Bellman [Bel56] where the problem is formulated in a Bayesian perspective for two arms, and the more recent Sutton [SB18], chapter 1, for a reinforcement learning perspective.

<sup>2</sup>See Bouneffouf [BR19] for a survey with a list of applications of the main algorithms solving the MAB problem.

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for each  $t \in \mathcal{I}$ . Let  $R_t$  be called again *reward*, and the difference between  $\mathcal{R}$  and  $R$  will be clear from the context.

We observe that  $\mathcal{R}$  maps the events of the  $\sigma$ -algebra, while  $R$  maps the corresponding indexes. This is an analogous of the definition of probability respect to the one of random variable and probability density function, for when the real axis is restricted to  $[0, 1] \subset \mathbb{R}$ .

Now consider

$$\begin{aligned} \mathcal{Q} : \mathcal{I}_T \times \mathcal{A} &\longrightarrow \mathbb{R}_+ \\ (t, \omega) &\longmapsto \mathcal{Q}(t, \omega) = Q_t(\omega) \end{aligned}$$

the *estimated reward of the action  $\omega$  up to time  $t$* , for  $\omega = A_t^{-1}(k)$  for a fixed  $k \in \mathcal{I}_K$ , with the corresponding function  $Q : \mathcal{I}_T \times \mathcal{I}_K \rightarrow \mathbb{R}$ . It follows that  $\mathcal{Q}$  is defined as an application of the mean value in a Lebesgue space over  $(\Omega, \mathcal{A})$ , that is now a Borel  $\sigma$ -algebra<sup>3</sup> as:

$$Q_t(\omega) = \mathbb{E}[R_\tau(k) \mid k = A_\tau(\omega), \forall \tau \in \mathcal{I}_t] \quad \omega \in \mathcal{A} \quad t \in \mathcal{I}_T \quad (1)$$

and therefore

$$Q_t(k) = \mathbb{E}[\mathcal{R}_\tau(\omega) \mid \omega = A_\tau^{-1}(k), \forall \tau \in \mathcal{I}_t] \quad t \in \mathcal{I}_T \quad k \in \mathcal{I}_K \quad (2)$$

As  $\mathcal{R}$  and  $R$  did, also  $\mathcal{Q}$  and  $Q$  satisfies the commutativity of a diagram analogous to the one shown above. The notation can be simplified for brevity<sup>4</sup> to:

$$Q_t(k) = \mathbb{E}[R_t(k) \mid A_t = k] \quad (3)$$

where the mean value is for all the time indexes up to  $t$  and where the domain values of  $A_t$  is clear from the context.

**Definition 2.2.** Let the *total reward*  $Q_\infty : \mathcal{I}_K \rightarrow \mathbb{R}$  be the function

$$Q_\infty(k) = \mathbb{E}[\mathcal{Q}_t(\omega) \mid \omega = A_t^{-1}(k), \forall t \in \mathcal{I}_T] \quad k \in \mathcal{I}_K \quad (4)$$

or with the simplified notation as:

$$Q_\infty(k) = \mathbb{E}[R_t(k) \mid A_t = k] \quad (5)$$

So far we have been considering the reward and the total reward for a fixed choice of  $k$ . We can vary  $k \in \mathcal{I}_K$  in function of the time index. So let  $\mathbf{k}$  an element of

$$\mathcal{I}_K^T = \underbrace{\mathcal{I}_K \times \mathcal{I}_K \times \cdots \times \mathcal{I}_K}_{T\text{-times}}$$

<sup>3</sup> For a foundational perspective, see Bourbaki [Bou04a].

<sup>4</sup> The simplified notation is often the only notation appearing in engineering textbooks (e.g. Sutton [SB18]), although this would not allow the reader to understand the subtle formalisation of assigning to an event  $\omega$  its index  $k$ . More tragically the simplified notation makes most of the concepts introduced so far pedantic and irrelevant.

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or equivalently a function from  $\mathcal{I}_T$  to  $\mathcal{I}_K$ . Definitions 1 and 3 are so generalised to  $Q_t : \mathcal{I}_K^T \rightarrow \mathbb{R}$  for  $t \in \mathcal{I}_{\leq T}$ , having defined  $\mathcal{I}_{\leq T}$  any interval of positive integers between 1 and  $T$ , and

$$Q_t(\mathbf{k}) = \mathbb{E} [\mathcal{R}_t(\omega) \mid \omega = A_t^{-1}(\mathbf{k}_t), \forall t \in \mathcal{I}_t] \quad \mathbf{k} \in \mathcal{I}_K^T \quad t \in \mathcal{I}_T$$

and therefore

$$Q_\infty(\mathbf{k}) = \mathbb{E} [\mathcal{R}_t(\omega) \mid \omega = A_t^{-1}(\mathbf{k}_t), \forall t \in \mathcal{I}_T] \quad \mathbf{k} \in \mathcal{I}_K^T$$

The mean value computed with a Lebesgue measure, over the Borel space generated as the sets of images <sup>5</sup>  $\mathcal{R}_t(\omega)$  for all  $\omega \in \mathcal{A}$  can be reformulated as:

$$Q_t(\mathbf{k}) = \frac{\sum_{\tau=1}^t R_\tau(\mathbf{k}_\tau) \mathbf{1}_{A_\tau=\mathbf{k}_\tau}}{\sum_{\tau=1}^t \mathbf{1}_{A_\tau=\mathbf{k}_\tau}}$$

where  $\mathbf{1}_{A_\tau=\mathbf{k}_\tau}$  equals to 1 for when the event  $\omega$  corresponding to  $\mathbf{k}_\tau$  is mapped exactly to  $\mathbf{k}_\tau$  through  $A_t$ , and 0 for any other event. Extending the time indexes up to infinity, and to justify the notation introduced above, where we used  $\infty$  for a finite case, we have the following theorem:

**Theorem 2.1.** *Given a ring of infinite cardinality to which the time index  $t$  belongs, and an Hilbert module<sup>6</sup> to which the vector  $\mathbf{k}$  belongs, it follows that*

$$Q_\infty(\mathbf{k}) = \lim_{T \rightarrow \infty} Q_T(\mathbf{k})$$

*Proof.* Direct consequence of the definition of  $Q_\infty$  generalised to the theorem hypothesis' extended structures.  $\square$

We now consider the value  $\hat{k}$  that satisfies

$$\hat{k} = \arg \max_{k \in \mathcal{I}_K} Q_t(k) \quad \forall t \in \mathcal{I}_T$$

for a constant value for each time index, as in the definition of  $Q_t$  given in 2. If we consider the possibility of varying the chosen arm  $k$  across time, and so if we are allowed to compare different images of the function  $A_t$  then  $\hat{\mathbf{k}}$  is defined as

$$\hat{\mathbf{k}} = \arg \max_{\mathbf{k} \in \mathcal{I}_K^T} Q_t(\mathbf{k}) \quad \forall t \in \mathcal{I}_T \quad (6)$$

Under the light of theorem 2.1, and with the given definitions, we can now call the defined vector  $\hat{\mathbf{k}}$  the *solution of the generalised multi-armed bandits problem*.

<sup>5</sup> We consider the definition under the accordance with the axiom of choice as in the ZFC axiomatic set theory, in order to avoid *virages dangereux*. See also Bourbaki [Bou04b] and [TZ82].

<sup>6</sup> An algebraic structure generalising Hilbert vector spaces over the now introduced ring of time indexes. See for example [Bou87].

### 3 The pragmatic perspective

### 4 Discussion

In this article, we considered the example of the multi-armed bandit problem to prove the futility of the Bourbachist approach.

Even if formally correct (excluding typos) and coherent (excluding Goedel<sup>7</sup>), the theory provided in section 2 has the effect of turning a relatively simple problem into a complicated maze of concepts.

The pragmatic approach of section 3 show that there is no need of taking a functional perspective and to link it to several méchanemas developed in entirely different contexts. These are not just irrelevant in reaching a solution, they are also suffocating any possible creativity when facing a slightly different problem. If in doubt on this point the reader to continue the formalisation for the case where the unknown distributions are not fixed over time, to see how many pages and new definitions and diagrams are required. We also challenge the reader to implement the code to solve the problem having only the functional and algebraic definitions at hand rather than relying on the matrix point of view. None of the given definitions in the Bourbaki approach had provided any hints on how to solve the problem numerically, as, in conformity with the Bourbaki style, no examples had been provided.

#### Other examples

Despite many, and all of them more experienced than the author, have express their negative view about the Bourbakist's mathematics<sup>8</sup>, this approach is still widespread if not predominant across the mathematical community.

There are in fact numerous examples of practical problems, whose pragmatic approach had been ruined by an overformalisation<sup>9</sup> similar to the short one here proposed. They are useful examples, in particular to anyone who may believe, reading this article, that the author had been overzealous in writing section 2, to give on purpose a negative light upon this the methodology.

The most notable one is the optimal transport (OT) theory. From being a pragmatic methodology of solving a class of optimisation problem, it had become a 500 pages book underpinned by a great amount of measure theory and Lebesgue spaces, perfectly irrelevant to solve any instance of an optimal transport problem. Comparing one of the original optimal transport theory presentation by Hitchcock [Hit41] and the formalised one by Villani [Vil03] is possible to see the extent of the damage. The first one is easy to read, understand, implement and possibly extend in several directions by anyone having

<sup>7</sup> To this regard, in the article *the ignorance of Bourbaki* [Mat92], Mathias noticed that the attempt of grounding the whole corpus of mathematics in an axiomatic sense have happened after, and in a conscious effort of ignoring the Goedel incompleteness theorems.

<sup>8</sup> Amongst the many: Arnold [Arn98], De Finetti [DFN08], Lockhart [Loc09], the already cited Mathias and its follow up [Mat98], Velupillai [Vel12]

<sup>9</sup> Or *assiomattisation*, as Bruno de Finetti [DFN08] would have said playing on the Italian word *matti*, meaning crazy.

an highschool mathematical education. The second one is a seemingly uncreated maze of unassailable interlinked concepts, requiring few years of academic studies only to grasp the first few pages, with no advantages in facing the problem.

A second notable example of the Bourbaki effect, is in the domain of medical image registration. Here the aim is to solve the problem of finding the non-rigid deformation or metamorphosis between anatomies. The problem originates from the anatomical studies of shapes growth by D’Arcy Thompson [d<sup>+</sup>42] and pragmatically extended amongst others in Modersitzki [Mod04]. The Bourbaki over-formalized branch can be found in works like [You10], where we have to wait for 12 chapters before seeing what had motivated the formal mathematical theory developed until there. In this case too, there is no explanation of what are the advantages of the axiomatic approach respect to the pragmatic one, appeared half a century earlier.

The reader may say, for this specific case, that the pragmatic approach [Mod04] does not use neither diffeomorphisms nor reproducing Kernel Hilbert spaces. And this is true, although it is not proved that these two mathematical devices can provide more accurate results than their pragmatic counterparts when implemented in practice. Instead it is true that they are computationally slower.

### The Bourbaki pattern

Other than the branches of mathematics mentioned above, there are others that have been touched by Bourbachism, such as algebraic topology, fuzzy logic and topological data analysis. Their detachment from the practical set of problems that had them originated, had turned them into axiomatic buildings.

There is in fact a pattern in the Bourbaki method: a problem that had been solved by 50 years or more, often in a very elegant way, is transformed into a list of axioms and theorems where no echo of the original problem that motivates the theory has to be found. Examples and counterexamples are removed, every reference to the history of the theory’s development is eradicated too.

Remarkably, also Einstein had noted that *Since the mathematicians have invaded the theory of relativity, I do not understand it myself any more* [Sch51]. And it is difficult to find what are the new results that the Bourbaki formalisation have added to the theory of relativity.

In fact, part of the Bourbaki pattern is that there is no added value in the formalised version of the proposed theory. As it is also true that no new results used in practice can be found in the work by Bourbaki and followers.

There is only one notable exception to this rule that we could find. The work Mochizuki [Moc12] is claimed to have solved the *abc conjecture*. Worth of a pantomime, according to the mathematicians<sup>10</sup> who had tried to follow the passages, “the proof is too impenetrable to be understood”.

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<sup>10</sup>Nature paper Castelvechi [Cas15].

## Why is Bourbaki still around?

What the examples proposed in these pages intend to show, from a qualitative and empirical evaluation, is that the over-axiomatic version of a theory has several negative aspects respect to its counterpart.

These are: Bourbaki formalisation has an exhaustive cognitive load, increases the difficulties that a generalisation of a simple change in the initial problem setting may require. It is by no mean close to the formalisation that is required to be implemented in modern programming language. Bourbaki does not provide any additional insight and intuition over the problem that originated the theory. The reader get used to work on useless details and pedantic discussions about notations, rather than solving the problem.

Nonetheless, these pages are not written to accuse Bourbaki. In fact, if all so negative, then the Bourbaki approach would not be still around any more. In reality, this approach is not only predominant in the universities. It is also greatly valued, prized, and appreciated by academics<sup>11</sup>.

It seems that the over-axiomatization of a theory developed outside academia by mathematicians and scientist chasing practical problems is worth, every recognitions, including the greatest of all, the Fields Medal. Highly valued and cited uber-bourbachists works are for example Grothendieck [Gro11] and the four volumes of Mochizuki [Moc12]<sup>12</sup>.

We have at this point to admit that there are several advantages of the Bourbachism that can be found, and that would justify the fact that this approach is still considered.

*The outcome of the Bourbaki approach is inaccessible to neophytes*<sup>13</sup>, and it allows the existence of professors of mathematics who can not code or do not have any ability to solve any problem in practice. This seems to be the main reason that justifies the continuation of the Bourbachism, as neophytes usually are the people who have to decide where the public money goes and professors the ones who receives it.

*The Bourbaki approach is good for the ego.* The pleasure of owning an elegant notebook filled with mathematical formulae written with a well thought after handwriting is a most sublime one. Unfortunately, after empirical evidence we can assure that this very same pleasure is a great obstacle in acquiring knowledge. The notebook owner, when facing a problem, will inevitably tend to abandon any scientific method and will bend the problem to the behold solution.

*Bourbachism detaches completely mathematics from reality*, so it makes impossible to have any objective measurement of the quality or value of the work produced, besides the readers' opinion. This is a very unscientific feature, and again a very useful one for

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<sup>11</sup> To this regard, the active reader is welcome to copy-paste section 2 and to extend it, in the Bourbakist style, into an article whose title could be something like *The multi-armed bandits for the working mathematician*, to see how this would be appreciated by the mathematical community.

<sup>12</sup> It may be interesting to notice the similarity between this work and the outcome of the randomized generator of maths paper at [thatmathematics.com/mathgen/](https://thatmathematics.com/mathgen/).

<sup>13</sup> An attempt of turning an over-formalised Bourbakist theory into something accessible for the layman, can be found in Villani [Vil12]. Interestingly enough, it seems that the only way the author could popularize his theory was by omitting the definitions and proofs while leaving only the main mathematical formulas alongside some autobiographical events. The result is even less accessible than the original work, proving that mathematicians can also be successful surrealist artists.

## 4 Discussion

whoever must defeat more skilled competitors within the academic environment.

The advocates of the superiority of pure mathematics<sup>14</sup> have even arrived at accusing the mathematics that has anything to do with reality of being more prone of making mistakes<sup>15</sup>. We can be reassured by the fact that under every circumstance, the reality is adamant to persist in being what it is, and it is difficult to imagine anyone advocating for detaching medicine and engineering from reality.

*The Bourbaki inspired work are more generalizable.* While this appear to be true at first glance, we claim that on the contrary a Bourbaki theory is less generalisable. Simply adding a new or different assumption to the problem settings would require to re-write almost from scratch all the axiomatic building to include the new input in a generalisable manner. The pragmatic way provides handle on the problem that are easy to be tuned or re-adapted to a slightly different one.

*Bourbachism binds the concepts in formal structures avoiding paradoxes and counterexamples.* This is a very valid point in favour of the Bourbakist method, as the concerns of mathematics with counterexample have led to various discoveries, from Galois theory, to Fractals, and the search for counterexample is a valuable tool to have a better understanding when exploring the limits of mathematical knowledge<sup>16</sup>. Although, the problem of the Bourbaki approach is the over-concern with counterexamples originating from theoretical considerations not related in any way to the sought solution. In section 2 we attained an algorithm that solves the problem, having never faced pathological counterexamples despite not relying on  $\sigma$ -algebras, Borel spaces, Lebesgue measures or even without explicit use of functions. What we came across were only some practical malices of the craft that are never learned by whoever is limiting themselves to the Bourbaki presentation of the problem.

About this last point in particular, there is a specific case where over-concerned researchers had been mislead by misquoted counterexamples: the medical imaging paper by Lorenzi [LP13] claims that there is no bijective correspondence between the space of the tangent vector fields and the one of their integral curves. This claim is made after citing a very peculiar counterexample found by Milnor on the tangent space of the circle. Interestingly enough the conditions for the counterexample to happen are never met in the applications presented in the paper. The considered case is of  $\mathbb{R}^3$ , whose tangent space is  $\mathbb{R}^3$  itself and the bijective correspondence is guaranteed by the theorem of existence and uniqueness for ODEs.

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<sup>14</sup> It is worth noticing that the academic dichotomy pure/applied maths is something that can not be found in mathematics before Bourbaki. I wonder if anyone could place any of the work of Archimedes, Gauss, Euler and any other great names of pre-modern mathematics in the category of pure or applied maths.

<sup>15</sup> Gros [GST19] had found how even professional mathematicians can be mislead by reality. And they had leveraged on this most surprising fact, for advocating to increase the detachment. After concluding that “[...] we can’t reason in a totally abstract manner”, instead of suggesting to take into account the reality in the mathematical practice they suggested a move towards the opposite direction: “We have to detach ourselves from our non-mathematical intuition” [Sum19].

<sup>16</sup> For example Procesi [Pro77], Mandelbrot [Man83] and Gelbaum [GO03].



## 5 Conclusion

The comparison proposed in this paper aims at showing how important is the mathematical approach in achieving a possible solution of a problem. It starts showing an academic over-formalised method, that, even if brief compared to what the Bourbaki approach usually generates, it leads the reader into a maze without the centre, or a formal theory whose value can not be established in term of how it solves a problem, and therefore by any objective criteria.

With the pragmatic way applied to the multi-armed bandits, an algorithmic solution had been proposed, implemented and tested. All this without calling into play Borel or Lebesgue.

In the discussion we challenged the usefulness of the Bourbaki approach outside its circle, and we suggest that it may be potentially harmful, as leading mathematics towards something useless for anyone besides who has to justify his academic role. The Bourbaki generalisations are not practical and the counterexamples that they wish to avoid do not arise in practice. The mindset of the Bourbaki educated student can even obstaculate the pathway leading to a solution of a practical problem.

Despite we are convinced that students of the generations to come will be obliged to learn the Bourbaki sophistications to get a degree, and to unlearn them as quickly as possible to be of any use to society, this paper hopes to show that the Bourbaki method is around for academic reasons, and it is not what mathematical practice is about or should be, which is the application of the scientific method to understand the reality and is to find the algorithm that better solves a given problem..

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