1 Fondamentali

$$foo$$

2 Potenziale ed Energia

$$\begin{aligned}
U &= \frac{1}{2}\int \rho(\mathbf{x})V(\mathbf{x})d\nu(x) \\
U &= \frac{1}{8\pi}\int |E|^2\, d\nu
\end{aligned}$$

3 Funzione di Green

Generica:

$$\begin{aligned}
V(\mathbf{x}) = &\int \rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d\nu(\mathbf{x}') \\
&+ \frac{1}{4\pi}\oint \left(G(\mathbf{x},\mathbf{x}')\frac{\partial V(\mathbf{x}')}{\partial n'} - V(\mathbf{x}')\frac{\partial G(\mathbf{x},\mathbf{x}')}{\partial n'} \right) dS(\mathbf{x}')
\end{aligned}$$

Condizioni Dirichlet:

$$\begin{aligned}
&G_D(\mathbf{x},\mathbf{x}') = 0 \qquad \text{per } \mathbf{x}' \in S \\
&V(\mathbf{x}) = \int \rho(\mathbf{x}')G_D(\mathbf{x},\mathbf{x}')d\nu(\mathbf{x}') - \frac{1}{4\pi}\oint V(\mathbf{x}')\frac{\partial G_D(\mathbf{x},\mathbf{x}')}{\partial n'}dS(\mathbf{x}')
\end{aligned}$$

Condizioni Neumann:

$$\begin{aligned}
&\frac{\partial G_N(\mathbf{x},\mathbf{x}')}{\partial n'} = -\frac{4\pi}{S} \qquad \text{per } \mathbf{x}' \in S \\
&V(\mathbf{x}) = \langle V \rangle + \int \rho(\mathbf{x}')G_N(\mathbf{x},\mathbf{x}')d\nu(\mathbf{x}') + \frac{1}{4\pi}\oint G_N(\mathbf{x},\mathbf{x}')\frac{\partial V(\mathbf{x}')}{\partial n'}dS(\mathbf{x}')
\end{aligned}$$

Piano:
Sfera:

$$\begin{aligned}
G(\mathbf{x},\mathbf{x}') &= \frac{1}{|\mathbf{x}-\mathbf{x}'|} - \frac{a}{x'\left|\mathbf{x} - \frac{a^2}{x'^2}\mathbf{x}'\right|} \\
\left.\frac{\partial G}{\partial n'}\right|_{x'=a} &= -\frac{x^2-a^2}{a(x^2+a^2-2ax\cos(\gamma))^{3/2}}
\end{aligned}$$

4 Laplaciano

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\nabla^2 V(r,\phi,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin(\theta)^2}\frac{\partial^2 V}{\partial \theta^2}$$

5 Polinomi di Legendre

$$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + B_l r^{-(l+1)})P_l(\cos(\theta))$$

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$$

$$\int_{-1}^{1} P_{l'}(x)P_l(x)dx = \frac{2}{2l+1}\delta_{l'l}$$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \tfrac{1}{2}(3x^2-1)$$
$$P_3(x) = \tfrac{1}{2}(5x^3-3x)$$
$$P_4(x) = \tfrac{1}{8}(35x^4-30x^2+3)$$

6 Armoniche sferiche

$$V(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}(A_{lm} r^l + B_{lm} r^{-(l+1)})Y_{lm}(\theta,\phi)$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos(\theta))e^{im\phi}$$

$$Y_{l,-m}(\theta,\phi) = (-1)^m \overline{Y_{lm}(\theta,\phi)}$$

$$Y_{00}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos(\theta)$$
$$Y_{11}(\theta,\phi) = -\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{i\phi}$$

$$Y_{20}(\theta,\phi) = \sqrt{\frac{5}{4\pi}}(\tfrac{3}{2}\cos(\theta)^2 - \tfrac{1}{2})$$
$$Y_{21}(\theta,\phi) = -\sqrt{\frac{15}{8\pi}}\sin(\theta)\cos(\theta)e^{i\phi}$$
$$Y_{22}(\theta,\phi) = \sqrt{\frac{15}{32\pi}}\sin(\theta)^2 e^{2i\phi}$$

$$Y_{30}(\theta,\phi) = \sqrt{\frac{7}{4\pi}}(\tfrac{5}{2}\cos(\theta)^3 - \tfrac{3}{2}\cos(\theta))$$
$$Y_{31}(\theta,\phi) = -\sqrt{\frac{21}{64\pi}}\sin(\theta)(5\cos(\theta) - 1)e^{i\phi}$$
$$Y_{32}(\theta,\phi) = \sqrt{\frac{105}{32\pi}}\sin(\theta)^2\cos(\theta)e^{2i\phi}$$
$$Y_{33}(\theta,\phi) = -\sqrt{\frac{35}{64\pi}}\sin(\theta)^3 e^{3i\phi}$$