1 Fondamentali

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2 Potenziale ed Energia

$$U = \frac{1}{2} \iiint \rho(\mathbf{x}) V(\mathbf{x}) d\nu$$
$$U = \frac{1}{8\pi} \iiint |E|^2 \, d\nu$$

3 Funzione di Green

Generica:

$$V(\mathbf{x}) = \iiint \rho(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') + \frac{1}{4\pi} \oiint \left(G(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} - V(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right) dS(\mathbf{x}')$$

Condizioni Dirichlet:

$$G_D(\mathbf{x}, \mathbf{x}') = 0 \qquad \text{per } \mathbf{x}' \in S$$
$$V(\mathbf{x}) = \iiint \rho(\mathbf{x}') G_D(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') - \frac{1}{4\pi} \oiint V(\mathbf{x}') \frac{\partial G_D(\mathbf{x}, \mathbf{x}')}{\partial n'} dS(\mathbf{x}')$$

Condizioni Neumann:

$$\frac{\partial G_N(\mathbf{x}, \mathbf{x}')}{\partial n'} = -\frac{4\pi}{S} \qquad \text{per } \mathbf{x}' \in S$$
$$V(\mathbf{x}) = \langle V \rangle + \iiint \rho(\mathbf{x}') G_N(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') + \frac{1}{4\pi} \oiint G_N(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} dS(\mathbf{x}')$$

Piano:
Sfera:

$$G(\mathbf{x}, \mathbf{x}') = \frac{1}{|\mathbf{x} - \mathbf{x}'|} - \frac{a}{x' \left| \mathbf{x} - \frac{a^2}{x'^2} \mathbf{x}' \right|}$$
$$\left. \frac{\partial G}{\partial n'} \right|_{x'=a} = -\frac{x^2 - a^2}{a(x^2 + a^2 - 2ax\cos\gamma)^{3/2}}$$

4 Laplaciano

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V(r,\phi,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \theta^2}$$

5 Armoniche sferiche

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