1 Fondamentali

$$foo$$

2 Potenziale ed Energia

$$U = \frac{1}{2} \int \rho(\mathbf{x}) V(\mathbf{x}) d\nu(x)$$
$$U = \frac{1}{8\pi} \int |E|^2 \, d\nu$$

3 Funzione di Green

Generica:

$$V(\mathbf{x}) = \int \rho(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') + \frac{1}{4\pi} \oint \left(G(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} - V(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right) dS(\mathbf{x}')$$

Condizioni Dirichlet:

$$G_D(\mathbf{x}, \mathbf{x}') = 0 \qquad \text{per } \mathbf{x}' \in S$$
$$V(\mathbf{x}) = \int \rho(\mathbf{x}') G_D(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') - \frac{1}{4\pi} \oint V(\mathbf{x}') \frac{\partial G_D(\mathbf{x}, \mathbf{x}')}{\partial n'} dS(\mathbf{x}')$$

Condizioni Neumann:

$$\frac{\partial G_N(\mathbf{x}, \mathbf{x}')}{\partial n'} = -\frac{4\pi}{S} \qquad \text{per } \mathbf{x}' \in S$$
$$V(\mathbf{x}) = \langle V \rangle + \int \rho(\mathbf{x}') G_N(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') + \frac{1}{4\pi} \oint G_N(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} dS(\mathbf{x}')$$

Piano:
Sfera:

$$G(\mathbf{x}, \mathbf{x}') = \frac{1}{|\mathbf{x} - \mathbf{x}'|} - \frac{a}{x' \left| \mathbf{x} - \frac{a^2}{x'^2} \mathbf{x}' \right|}$$
$$\left. \frac{\partial G}{\partial n'} \right|_{x'=a} = -\frac{x^2 - a^2}{a(x^2 + a^2 - 2ax\cos(\gamma))^{3/2}}$$

4 Laplaciano

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\nabla^2 V(r,\phi,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$
$$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin(\theta)^2}\frac{\partial^2 V}{\partial \theta^2}$$

5 Polinomi di Legendre

$$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + B_l r^{-(l+1)})P_l(\cos(\theta))$$

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$$

$$\int_{-1}^{1} P_{l'}(x)P_l(x)dx = \frac{2}{2l+1}\delta_{l'l}$$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \tfrac{1}{2}(3x^2-1)$$
$$P_3(x) = \tfrac{1}{2}(5x^3-3x)$$
$$P_4(x) = \tfrac{1}{8}(35x^4-30x^2+3)$$

6 Armoniche sferiche

$$V(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}(A_{lm}r^l + B_{lm}r^{-(l+1)})Y_{lm}(\theta,\phi)$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos(\theta))e^{im\phi}$$

$$Y_{l,-m}(\theta,\phi) = (-1)^m \overline{Y_{lm}(\theta,\phi)}$$

$$\begin{aligned} P_l^m(x) &= (-1)^m (1-x^2)^{m/2} \frac{\partial^m}{\partial x^m} P_l(x) \\ &= (-1)^m (1-x^2)^{m/2} \frac{\partial^{l+m}}{\partial x^{l+m}} (x^2-1)^l \end{aligned}$$

$$Y_{00}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$$

$$\begin{aligned} Y_{10}(\theta,\phi) &= \sqrt{\frac{3}{4\pi}} \cos(\theta) \\ Y_{11}(\theta,\phi) &= -\sqrt{\frac{3}{8\pi}} \sin(\theta) e^{i\phi} \end{aligned}$$

$$\begin{aligned} Y_{20}(\theta,\phi) &= \sqrt{\frac{5}{4\pi}} (\tfrac{3}{2}\cos(\theta)^2 - \tfrac{1}{2}) \\ Y_{21}(\theta,\phi) &= -\sqrt{\frac{15}{8\pi}} \sin(\theta)\cos(\theta) e^{i\phi} \\ Y_{22}(\theta,\phi) &= \sqrt{\frac{15}{32\pi}} \sin(\theta)^2 e^{2i\phi} \end{aligned}$$

$$\begin{aligned} Y_{30}(\theta,\phi) &= \sqrt{\frac{7}{4\pi}} (\tfrac{5}{2}\cos(\theta)^3 - \tfrac{3}{2}\cos(\theta)) \\ Y_{31}(\theta,\phi) &= -\sqrt{\frac{21}{64\pi}} \sin(\theta)(5\cos(\theta)^2 - 1) e^{i\phi} \\ Y_{32}(\theta,\phi) &= \sqrt{\frac{105}{32\pi}} \sin(\theta)^2 \cos(\theta) e^{2i\phi} \\ Y_{33}(\theta,\phi) &= -\sqrt{\frac{35}{64\pi}} \sin(\theta)^3 e^{3i\phi} \end{aligned}$$