

1 Fondamentali

$$f_{00}$$

2 Potenziale ed Energia

$$U = \frac{1}{2} \iiint \rho(\mathbf{x}) V(\mathbf{x}) d\nu$$

$$U = \frac{1}{8\pi} \iiint |E|^2 d\nu$$

3 Funzione di Green

Generica:

$$V(\mathbf{x}) = \iiint \rho(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') + \frac{1}{4\pi} \oint \left(G(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} - V(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right) dS(\mathbf{x}')$$

Condizioni Dirichlet:

$$G_D(\mathbf{x}, \mathbf{x}') = 0 \quad \text{per } \mathbf{x}' \in S$$

$$V(\mathbf{x}) = \iiint \rho(\mathbf{x}') G_D(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') - \frac{1}{4\pi} \oint V(\mathbf{x}') \frac{\partial G_D(\mathbf{x}, \mathbf{x}')}{\partial n'} dS(\mathbf{x}')$$

Condizioni Neumann:

$$\frac{\partial G_N(\mathbf{x}, \mathbf{x}')}{\partial n'} = -\frac{4\pi}{S} \quad \text{per } \mathbf{x}' \in S$$

$$V(\mathbf{x}) = \langle V \rangle + \iiint \rho(\mathbf{x}') G_N(\mathbf{x}, \mathbf{x}') d\nu(\mathbf{x}') + \frac{1}{4\pi} \oint G_N(\mathbf{x}, \mathbf{x}') \frac{\partial V(\mathbf{x}')}{\partial n'} dS(\mathbf{x}')$$

Piano:

Sfera:

$$G(\mathbf{x}, \mathbf{x}') = \frac{1}{|\mathbf{x} - \mathbf{x}'|} - \frac{a}{x' |\mathbf{x} - \frac{a^2}{x'^2} \mathbf{x}'|}$$

$$\frac{\partial G}{\partial n'} \Big|_{x'=a} = -\frac{x^2 - a^2}{a(x^2 + a^2 - 2ax \cos \gamma)^{3/2}}$$

4 Laplaciano

$$\begin{aligned}\nabla^2 V(x, y, z) &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \\ \nabla^2 V(r, \phi, z) &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \\ \nabla^2 V(r, \theta, \phi) &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}\end{aligned}$$

5 Polinomi di Legendre

$$V(r, \theta) = \sum_{l=0}^{\infty} (A_l r^l + B_l r^{-(l+1)}) P_l(\cos \theta)$$

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$$

$$\int_{-1}^1 P_{l'}(x) P_l(x) dx = \frac{2}{2l+1} \delta_{l'l}$$

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

6 Armoniche sferiche

$$V(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l (A_{lm} r^l + B_{lm} r^{-(l+1)}) Y_{lm}(\theta, \phi)$$

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos(\theta)) e^{im\phi}$$

$$Y_{l,-m}(\theta, \phi) = (-1)^m \overline{Y_{lm}(\theta, \phi)}$$

$$Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10}(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos(\theta)$$

$$Y_{11}(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} \sin(\theta) e^{i\phi}$$

$$Y_{20}(\theta, \phi) = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2(\theta) - \frac{1}{2} \right)$$

$$Y_{21}(\theta, \phi) = -\sqrt{\frac{15}{8\pi}} \sin(\theta) \cos(\theta) e^{i\phi}$$

$$Y_{22}(\theta, \phi) = \sqrt{\frac{15}{32\pi}} \sin^2(\theta) e^{2i\phi}$$

$$Y_{30}(\theta, \phi) = \sqrt{\frac{7}{4\pi}} \left(\frac{5}{2} \cos^3(\theta) - \frac{3}{2} \cos(\theta) \right)$$

$$Y_{31}(\theta, \phi) = -\sqrt{\frac{21}{64\pi}} \sin(\theta) (5 \cos^2(\theta) - 1) e^{i\phi}$$

$$Y_{32}(\theta, \phi) = \sqrt{\frac{105}{32\pi}} \sin^2(\theta) \cos(\theta) e^{2i\phi}$$

$$Y_{33}(\theta, \phi) = -\sqrt{\frac{35}{64\pi}} \sin^3(\theta) e^{3i\phi}$$