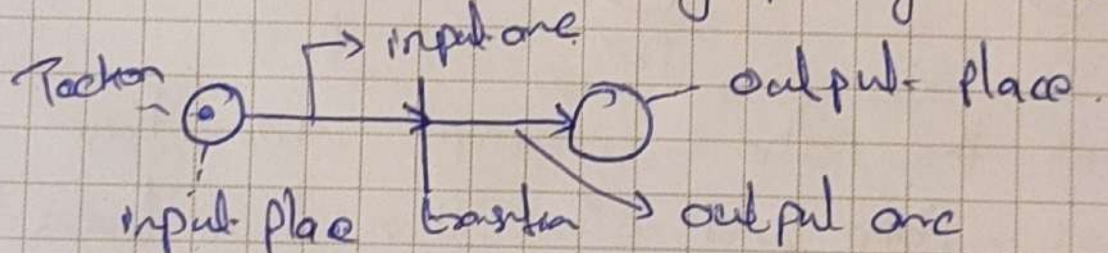
Continues time Markov chain

Stochastic Petri Nets

Formal and graphical appealing for Modelling system concurrely.

- Petri Net → Pripartie directed graph with 2 nodes → places and transitions.
- Arcs exist between places and transition but no arcs is present between two places or two transmitions.
- Places represent conditions within the system and are graphically denoted by circles.
- Transitions representals events occurring in the system that change in the conditions and are denoted graphically as bars



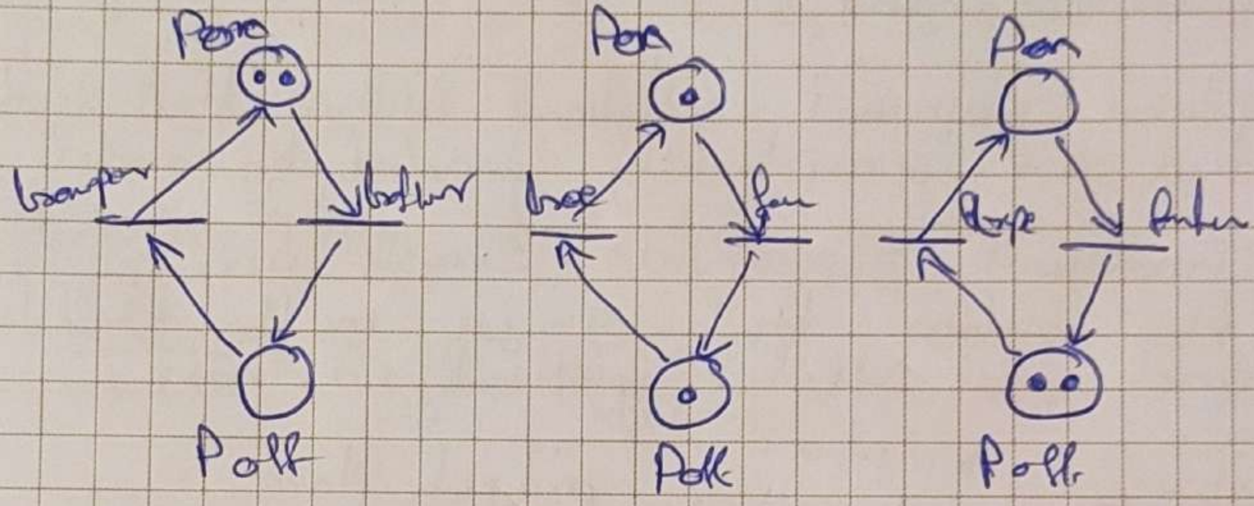
Definations of components of petri nets.

- input places → The set of places that are connected to the transition through input arcs.
- Input arcs → arcs drawn from place to transmitions. The represent the conditions that need to be satisfied for the events.
- output place → set of places to which output arcs exist from transition.

- output arcs → directed arcs drawn from transition to places.
- Tokens → The dots associated with place. A place containing token indicate that condition is active.

Example - 1

Enabling and firing of transitions.



So, in the above example, we have 2 places, Pone and off. Then two transitions Trepair and Tfailure.

So we will se each condition one by one. In the initial condition Pon has two tokens. As a result Tfailure transition will be enabled. At the same time Poff has no tokens and therefore transition Trepair will not be enable

When Tfailure is enable, the required tokens will be remove from Ton and required no off tokens will be deposited on Toff so here Pon already has on two tokens and one will be removed and one token will be deposited in Toff.

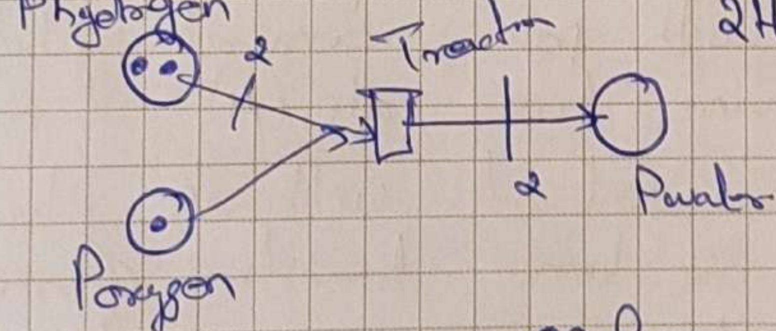
Now Ton and Toff has one Token each. This condition will enable both Trepair and Tfailure.

Now if Tfailure fires, Then one token will be removed from Pon and on token will be deposited in Poff. Now Pon has 0 tokens and poff has 2 tokens.

If Trepair was fired instead of Tfailure then one token will be removed from Poff and one token will be deposited in Pon. Now Pon has 2 tokens and Poff has 0 tokens.

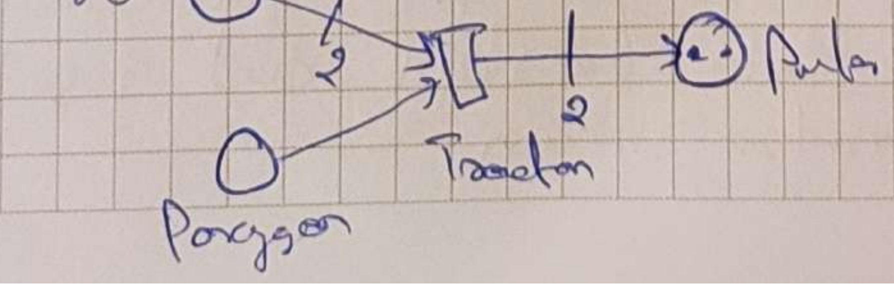
When Poff has 2 tokens and Pon has 0 token, Trepair will be enabled and fired Therefore one token will be removed from Toff and one token will be deposited in Ton.

Example -2



$2H_2 + O_2 \rightleftharpoons 2H_2O$

on firing

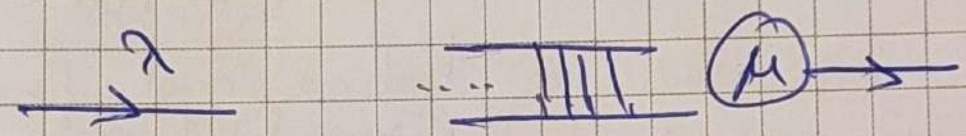


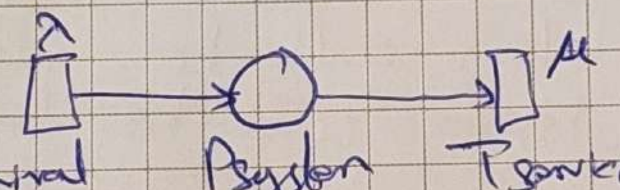
We have 3 Places, Phydrogen, Poxygen and Pwater and transition Treaction

When the transition Treaction is enabled and fired because of the multiplicity 2 on the input arc from Phydrogen two tokens will be removed and because of multiplicity 1 on the input arc Poxygen one token will be removed.

The output arc from transition Treaction to output place Pwater contains 2 tokens and so 2 tokens will be deposited in Pwater.

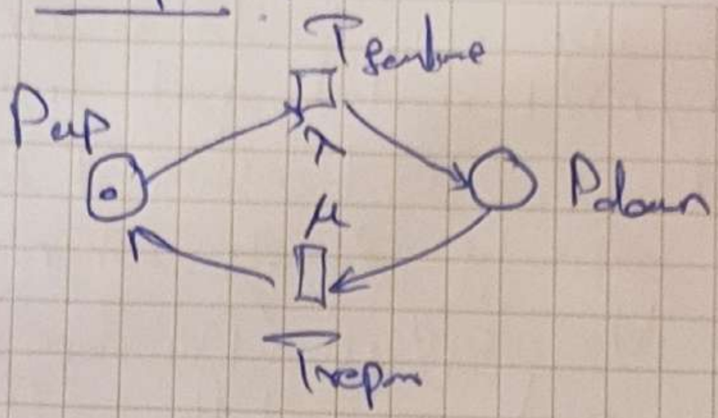
Example - 3

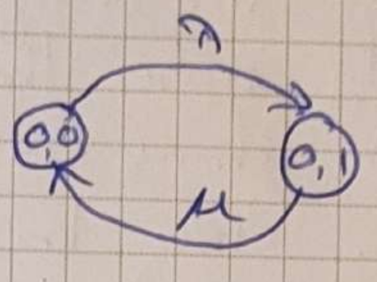




The M/M/1 model describe a single-server queue with poisson arrival rate (λ) and exponential service time rate (μ). It can be modeled as a Petri net with one place (Psystem) and 2 time transition (Tarrival, Tservice) using tokens. The no. of tokens in P represents queue length. The reachability graph shows state transitions and resembles the system Markov chain.

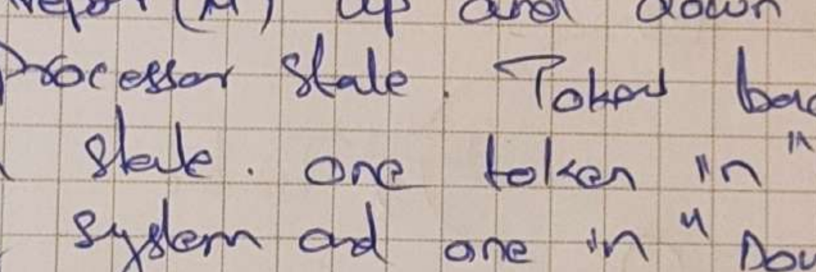
Example - 4





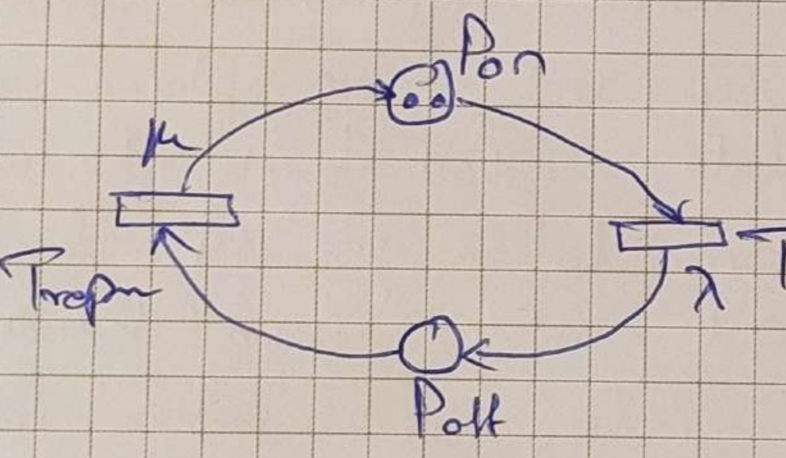
This example uses a two state stochastic petri net to model a repairable processor system with exponential failure (λ) and repair (μ) up and down place represent the processor state. Tokens track the system state. one token in "up" for a working system and one in "Down" for a failed system. Transition model failure and repair, moving tokens between places transition and is equivalent to the systems Markov chain.

Example 5 : M/M/1/5



An M/M/1/5 queuing system limits the number of Jobs in the system at jobs in the system to B. We can build a GSPN based on the M/M/1 model but with an additional instantaneous transition ("t loss") to enforce the capacity limits when b tokens are in the system (Psystem), "t loss" removes and immediately adds back B tokens, preventing more than B jobs. The reachability graph shows 0–B possible tokens, and transition rates based on arrival (λ) and service (μ) rates.

Example - 6



Two places (Pon, Poff) and 2 transitions (failure, repair) Model components failure and repairs (exponential distribution with rates λ and μ).

Tokens represent components state (on/off).

Reachability graph shows marking (token in each place and transition rate ($\lambda, 2\lambda, \mu$)

reward rate indicate system availability (1 for working system, 0 for failure).