Lelchure 30

Quening Models (waiting line Models)

- Single server } Reservation system
- Multiple server }
- finate queue length } Single server: input queue
- infinate queue length } input pop
- finate population
- infinate pop.

$\frac{\lambda}{\mu} < 1$ } · infinate queue length · to be mor efficent.

Balking → If an arrival doesn't joint the system and leave.

Reneging → joins and after some time decides not to cont. and move out of system

Jockeying → waiting the first of few mints at begny.

Queue displane (First in first out)

single server, infinate queu length

$M/M/1 : \text{FIFO}/\infty$ model

M/M/1: M = arrivale passion λ/hr; M = service μ/hr; 1 = no: of server; FIFO = queue length; ∞ = population.

$P_n(t+h) = P_{n-1}(t) \rightarrow$ (Probablity of one arrivle. no server)
$+ P_{(n+1)}(t) \rightarrow$ (one server no; of arrivels.)
$+ P_n(t) \rightarrow$ (no: server, no: of arrivels)

$P_n(t+h) = P_{n-1}(t) \times \lambda h (1-\mu h) + P_{n+1}(t)\, \mu h (1-\lambda h)$
$+ P_n(t)(1-\lambda h)(1-\mu h)$

$= P_{n-1}(t)\lambda h + P_{(n+1)}(t)\mu h + P_n(t)(1-\lambda h-\mu h)$

$$\frac{P_n(t+h) - P_n(t)}{h} = P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\lambda+\mu)$$

$$\lambda P_{n-1} + \mu P_{n+1} = (\lambda+\mu) P_n \quad \text{—— } (1)$$

$$P_0(t+h) = P_1(t)\underbrace{(1-\lambda h)}_{\text{no arrivals}}\mu h + P_0(t)(1-\lambda h)$$

$$\frac{P_0(t+h) - P_0(t)}{h} = P_1(t) \times \mu - P_0(t)\lambda$$

$$\mu P_1 = \lambda P_0 \quad \text{—— } (2)$$

$$P_1 = \lambda/\mu \, P_0$$

$$\lambda P_0 + \mu P_2 = (\lambda+\mu) P_1$$

$$= \lambda P_1 + \mu P_1$$

$$P_2 = \lambda/\mu \, P_1 - (\lambda/\mu)^2 P_0$$

$$P_1 = \rho P_0$$

$$P_2 = \rho P_1 = \rho^2 P_0$$

$$P_3 = \rho P_2 = \rho^3 P_0$$

$$\vdots$$

$$\therefore \; P_0 + \rho P_0 + \rho^2 P_0 = 1$$

$$\rho < 1$$

$$L_s = \sum_{j=0}^{\infty} j P_j = \sum j \rho^j P_0$$

$$= P_0 \rho \sum j \rho^{j-1} = P_0 \rho \sum \frac{d}{d\rho} \rho^j$$

$$= P_0 \rho \frac{d}{d\rho}(1 + \rho + \rho^2 + \dots) \; P_0 \rho \frac{1}{(1-\rho)}$$

$$= \frac{1-\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho}$$

$$L_s = L_q + \text{expected server} \; \frac{\lambda}{\mu}$$

$$L_s = \lambda W_s$$

$$L_q = \lambda W_q$$

Lecture 31

$$\rho = \lambda / \mu$$

$$P(\text{no queue}) = P_0 + P_1 \longrightarrow \rho P_0$$

$$L_s = \frac{\rho}{1-\rho} \qquad W_s = \frac{L_s}{\lambda}$$

$$L_q = L_s - \rho \qquad W_q = \frac{L_q}{\lambda}$$

$$M/M/1 : N/\infty$$

$$P_0 + \rho P_0 + \rho^2 P_0 = 1$$

$$P_0 [1 + \rho + \rho^2 + \dots] = 1$$

$$P_0 \left(\frac{1-\rho^{N+1}}{1-\rho} \right) = 1$$

$$P = \frac{1-\rho}{1-\rho^{N+1}}$$

$$P_n = \rho^n P_0$$

$$L_s = \sum_{n=0}^{N} n P_n = \sum_{n=0}^{N} n \rho^n P_0$$

$$= P_0 \rho \sum_{n=0}^{N} n \rho^{n-1}$$

$$= P_0 \rho \frac{d}{d\rho} \left(\frac{1-\rho^{N+1}}{1-\rho} \right)$$

$$= P_0 \rho \left\{ \frac{1-\rho\,(N+1)\rho^N + (1-\rho^{N+1})}{(1-\rho)^2} \right\}$$

$$= \frac{P_0 \rho}{(1-\rho)^2} \left[1 - \rho^{N+1} - (N+1)\rho^N (1-\rho) \right]$$

$$= \frac{P_0 \rho}{(1-\rho)^2} \left[1 + N\rho^{N+1} - N \cdot \rho^N \right]$$

$$L_s = L_q + \frac{\lambda}{\mu} eff$$

$$L_s = \lambda W_s$$

$$L_q = \lambda W_q$$

$$\lambda eff = \lambda (1 - P_N)$$

$M/M/C : \infty/\infty$ model.

$$\rho/c = \lambda/_{c\mu} < 1$$

$$\lambda_n = \lambda$$

$$\mu_n = n\mu \quad n < c$$

$$= c\mu \quad n \geq c$$

$$P_n = \rho^n P_0$$

$$= \frac{\lambda^n}{\mu_2 \mu_n \mu} P_0 = \frac{\lambda}{\mu}^n \frac{1}{n!} P_0 \quad n < c$$

$$P_n = \frac{\lambda^n}{\mu_2 \mu_3 \mu_4 \mu_n \mu_n \mu} = \frac{\lambda^n}{c! \mu^n c^{n-c}} P_0 \quad n \geq c$$

$$\sum_{n=0}^{c-1} \frac{\rho^n P_0}{n!} + \sum_{n=c}^{\infty} \frac{\rho^n}{c! c^{n-c}} P_0 = 1$$

$$P_0 = \frac{1}{\left(\sum_0^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1 - \rho/c}\right)\right)}$$