Fourier series

```
In[ ]:= f[t_] := Cos[2 Pi * t] + Cos[4 Pi * t] / 2
       h[t_] := Cos[2 Pi * t] + Cos[2 Pi * √2 * t]
       Plot[f[t], {t, 0, 6}, PlotLabel → "f(t) = Cos[2 Pi*t] + Cos[4 Pi*t]/2",
        AxesLabel → {"t", "f(t)"}, PlotStyle → Blue, PlotRange → All]

       Plot[h[t], {t, 0, 6}, PlotLabel → "h(t) = Cos[t] + Cos[Sqrt[2]*t]",
        AxesLabel → {"t", "h(t)"}, PlotStyle → Red, PlotRange → All]

       Plot[{f[t], h[t]}, {t, 0, 6},
        PlotLegends → {"f(t) = Cos[2 Pi*t] + Cos[4 Pi*t]/2", "h(t) = Cos[t] + Cos[ √2 *t]"},
        PlotStyle → {Blue, Red}, AxesLabel → {"t", "f(t), h(t)"}, PlotRange → All]
```
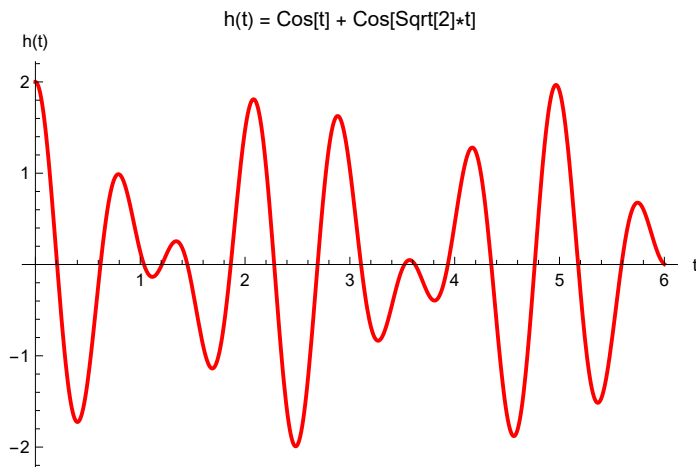
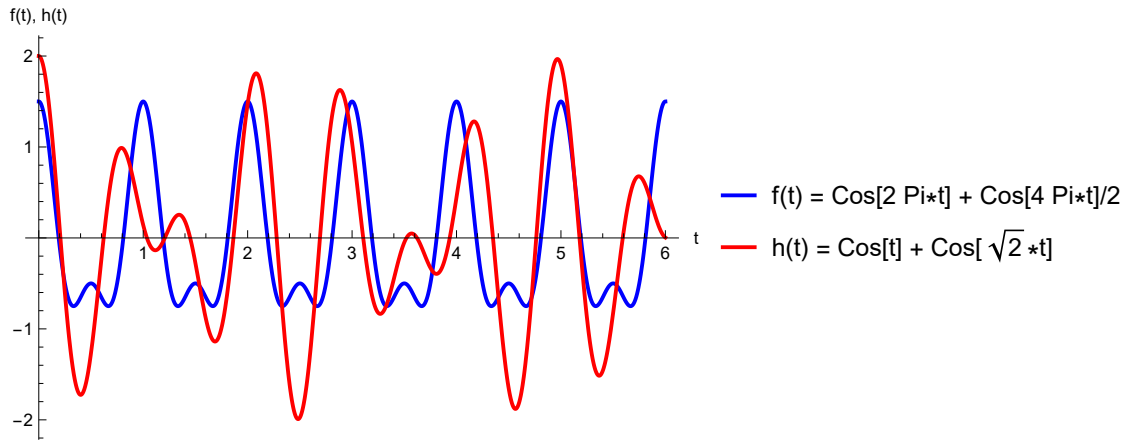Out[ ]=

f(t) = Cos[2 Pi*t] + Cos[4 Pi*t]/2

Out[ ]=

h(t) = Cos[t] + Cos[Sqrt[2]*t]

Out[•]=

f(t), h(t)

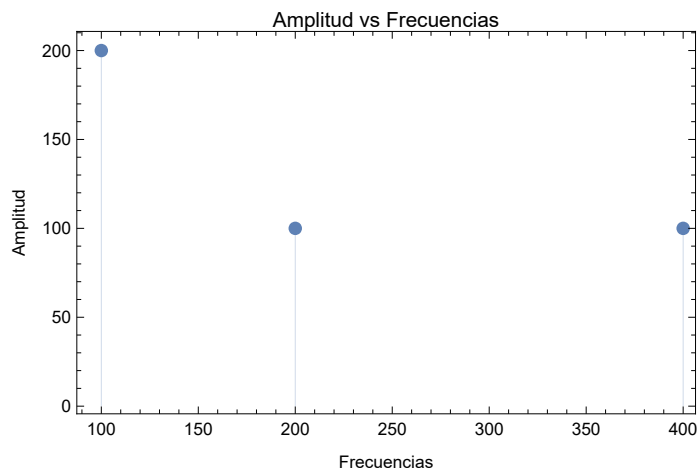— f(t) = Cos[2 Pi*t] + Cos[4 Pi*t]/2

— h(t) = Cos[t] + Cos[$\sqrt{2}$ *t]
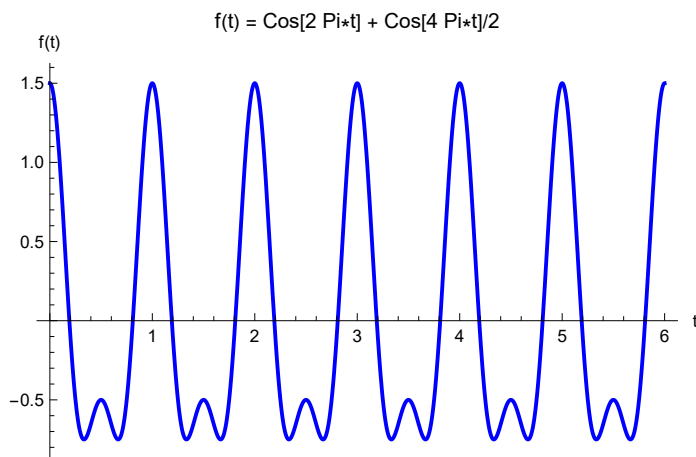
Superposition of harmonic functions

In[•]:=
```
amplitudes = {200, 100, 100};
frecuencias = {100, 200, 400};

ListPlot[Transpose[{frecuencias, amplitudes}],
 Filling → Axis, PlotStyle → PointSize[Large], Frame → True,
 FrameLabel → {"Frecuencias", "Amplitud"}, PlotLabel → "Amplitud vs Frecuencias"]
```
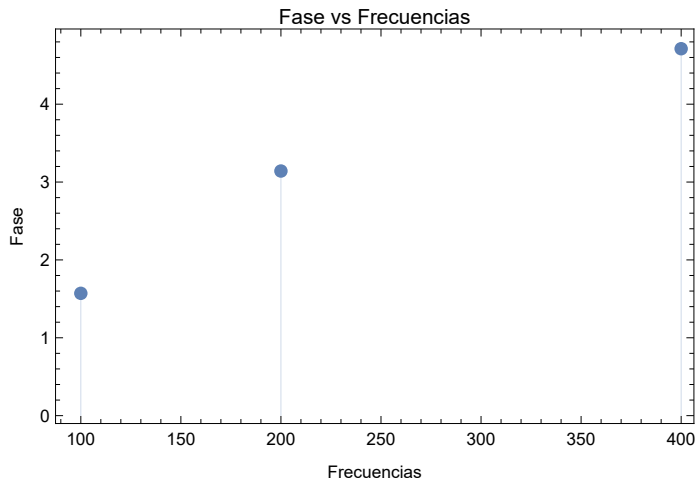
Out[•]=

Amplitud vs Frecuencias

In[ ]:= **fases = {Pi / 2, Pi, 3 Pi / 2};**

**ListPlot[Transpose[{frecuencias, fases}], Filling → Axis, PlotStyle → PointSize[Large],
Frame → True, FrameLabel → {"Frecuencias", "Fase"}, PlotLabel → "Fase vs Frecuencias"]**

Out[ ]=

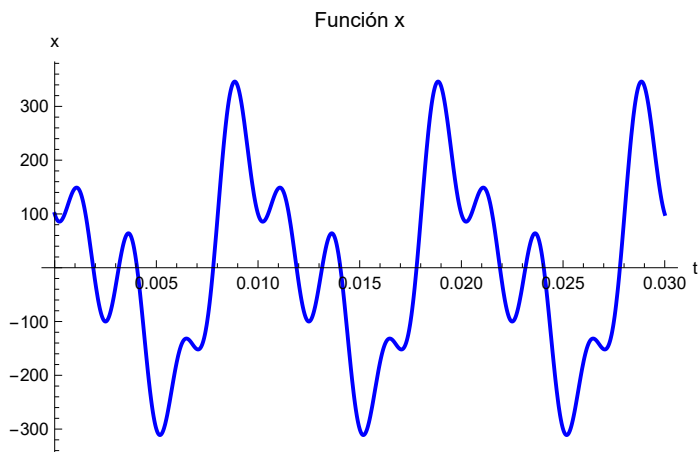*In[ ]:=* **x[t_] :=
200 Sin[2 Pi * 100 * t + Pi / 2] + 100 Sin[2 Pi * 200 * t + Pi] + 100 Sin[2 Pi * 400 * t + 3 Pi / 2]**

**Plot[x[t], {t, 0, 0.03}, PlotLabel → "Función x",
AxesLabel → {"t", "x"}, PlotStyle → Blue, PlotRange → All]**

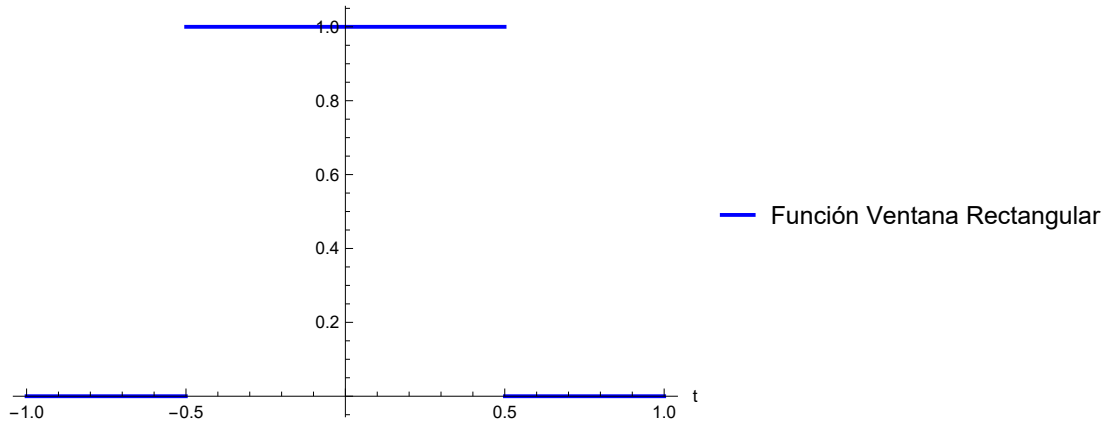*Out[ ]=*

Serie de Fourier Función Par

In[ ]:= **x[t_] := Piecewise[{{DirichletWindow[t], -0.5 < t < 0.5}}]**
Plot[{x[t]}, {t, -1, 1}, PlotStyle → {Blue},
PlotLegends → {"Función Ventana Rectangular"}, AxesLabel → {"t"}, PlotRange → All]

Out[ ]=

*In[ ]:=* **y[t_] = FullSimplify[FourierSeries[x[t], t, 30]]**

Out[ ]=

$0.159155 + 0.305212 \cos[t] + 0.267849 \cos[2t] + 0.211675 \cos[3t] +$
$0.144719 \cos[4t] + 0.0761998 \cos[5t] + 0.0149733 \cos[6t] - 0.0319022 \cos[7t] -$
$0.0602244 \cos[8t] - 0.0691461 \cos[9t] - 0.061047 \cos[10t] - 0.0408328 \cos[11t] -$
$0.0148235 \cos[12t] + 0.0105346 \cos[13t] + 0.029875 \cos[14t] + 0.03981 \cos[15t] +$
$0.0393653 \cos[16t] + 0.0299019 \cos[17t] + 0.0145757 \cos[18t] -$
$0.00251804 \cos[19t] - 0.0173167 \cos[20t] - 0.0266682 \cos[21t] - 0.028937 \cos[22t] -$
$0.0242317 \cos[23t] - 0.014233 \cos[24t] - 0.00168887 \cos[25t] + 0.0102879 \cos[26t] +$
$0.018952 \cos[27t] + 0.0225229 \cos[28t] + 0.0205232 \cos[29t] + 0.0137995 \cos[30t]$

In[ ]:= **Plot[{y[t]}, {t, -1, 1}, PlotStyle → {Red},**
PlotLegends → {"Serie de Fourier Ventana Rectangular"},
AxesLabel → {"t"}, PlotRange → All]

Out[ ]=
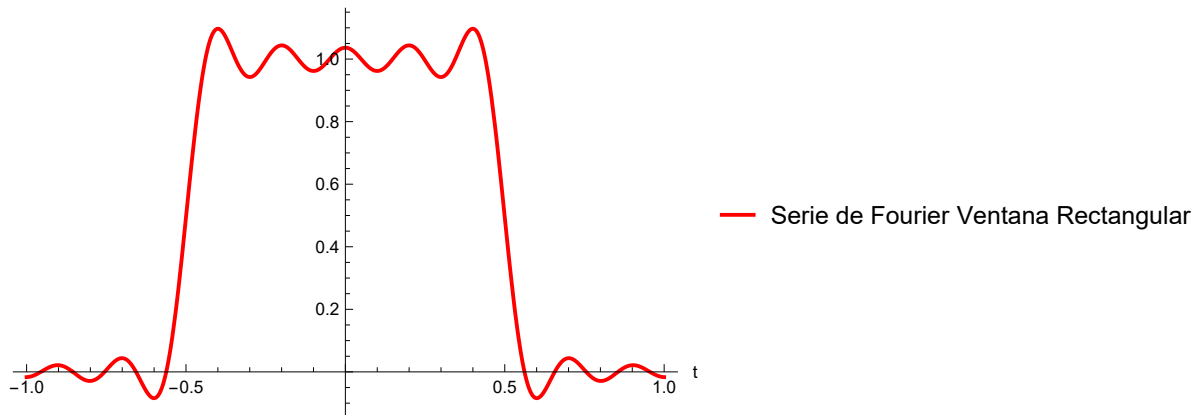
In[ ]:= `Plot[{x[t], y[t]}, {t, -1, 1}, PlotStyle → {Blue, Red},`
    `PlotLegends → {"Función Ventana Rectangular", "Serie de Fourier (nMax términos)"},`
    `Filling → {1 → {2}}, AxesLabel → {"t", "f(t)"}, PlotRange → All]`

Out[ ]=
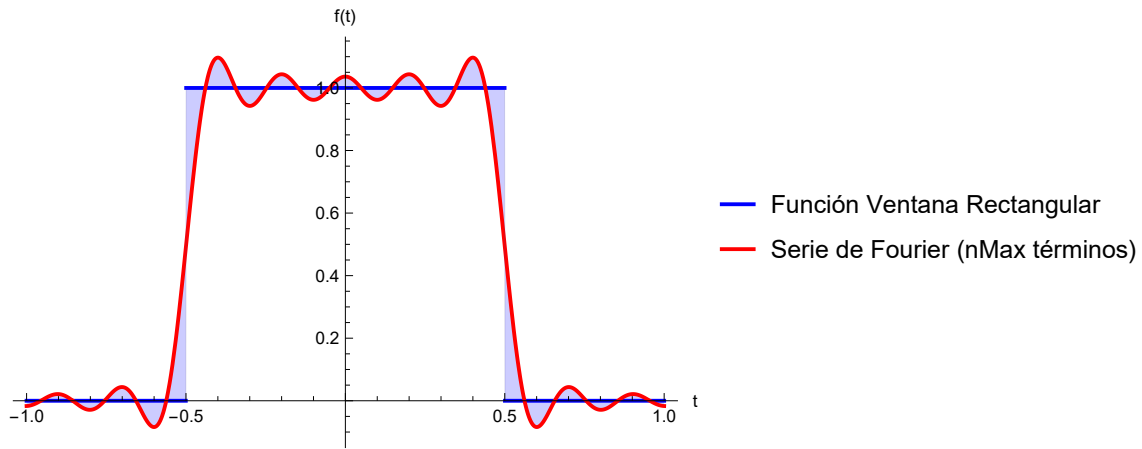
Fourier Series Odd Function

In[ ]:= `x[t_] := Piecewise[{{HeavisideTheta[t + 1], 0 < t < 1}}] +`
    `Piecewise[{{-1 * HeavisideTheta[t - 1], 1 < t < 2}}] + Piecewise[{{0, 0 < t}, {0, t > 2}}];`
    `Plot[{x[t]}, {t, -1, 3}, PlotLegends → {"Función Impar"},`
    `PlotStyle → {Blue}, AxesLabel → {"Tiempo", "Amplitud"}, PlotRange → Full]`

Out[ ]=

In[ ]:= `y[t_] = FullSimplify[FourierSeries[x[t], t, 30]]`

Out[ ]=
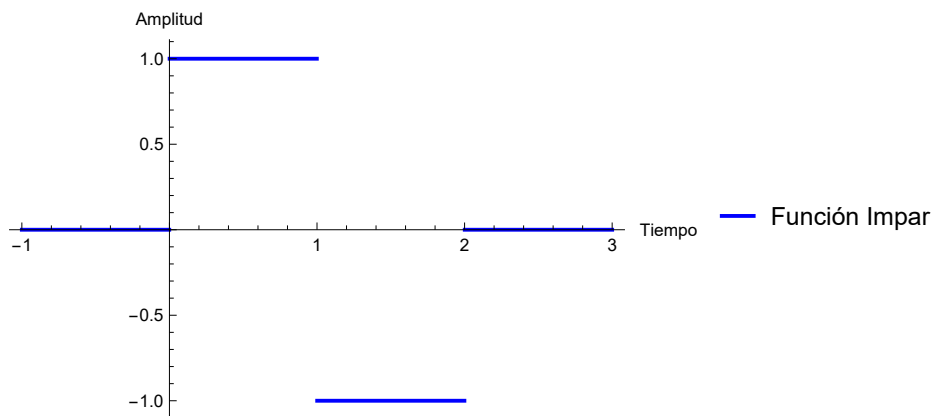
$$-\frac{i\left(-1+e^{i}\right)^2 e^{i(-2+t)}}{2\pi} - \frac{i\left(-1+e^{3i}\right)^2 e^{3i(-2+t)}}{6\pi} - \frac{i\left(-1+e^{4i}\right)^2 e^{4i(-2+t)}}{8\pi} - \frac{i\left(-1+e^{5i}\right)^2 e^{5i(-2+t)}}{10\pi} -$$

$$\frac{i\left(-1+e^{7i}\right)^2 e^{7i(-2+t)}}{14\pi} - \frac{i\left(-1+e^{9i}\right)^2 e^{9i(-2+t)}}{18\pi} - \frac{i\left(-1+e^{11i}\right)^2 e^{11i(-2+t)}}{22\pi} -$$

$$\frac{i\left(-1+e^{13i}\right)^2 e^{13i(-2+t)}}{26\pi} - \frac{i\left(-1+e^{15i}\right)^2 e^{15i(-2+t)}}{30\pi} - \frac{i\left(-1+e^{17i}\right)^2 e^{17i(-2+t)}}{34\pi} -$$

$$\frac{i\left(-1+e^{19i}\right)^2 e^{19i(-2+t)}}{38\pi} - \frac{i\left(-1+e^{20i}\right)^2 e^{20i(-2+t)}}{40\pi} - \frac{i\left(-1+e^{21i}\right)^2 e^{21i(-2+t)}}{42\pi} -$$

$$\frac{i\left(-1+e^{22i}\right)^2 e^{22i(-2+t)}}{44\pi} - \frac{i\left(-1+e^{23i}\right)^2 e^{23i(-2+t)}}{46\pi} - \frac{i\left(-1+e^{24i}\right)^2 e^{24i(-2+t)}}{48\pi} -$$

$$\frac{i\left(-1+e^{25i}\right)^2 e^{25i(-2+t)}}{50\pi} - \frac{i\left(-1+e^{26i}\right)^2 e^{26i(-2+t)}}{52\pi} - \frac{i\left(-1+e^{27i}\right)^2 e^{27i(-2+t)}}{54\pi} -$$

$$\frac{i\left(-1+e^{28i}\right)^2 e^{28i(-2+t)}}{56\pi} - \frac{i\left(-1+e^{29i}\right)^2 e^{29i(-2+t)}}{58\pi} - \frac{i\left(-1+e^{30i}\right)^2 e^{30i(-2+t)}}{60\pi} +$$

$$\frac{i\left(-1+e^{i}\right)^2 e^{-it}}{2\pi} + \frac{i\left(-1+e^{2i}\right)^2 e^{-2it}}{4\pi} + \frac{i\left(-1+e^{3i}\right)^2 e^{-3it}}{6\pi} + \frac{i\left(-1+e^{4i}\right)^2 e^{-4it}}{8\pi} +$$

$$\frac{i\left(-1+e^{5i}\right)^2 e^{-5it}}{10\pi} + \frac{i\left(-1+e^{6i}\right)^2 e^{-6it}}{12\pi} + \frac{i\left(-1+e^{7i}\right)^2 e^{-7it}}{14\pi} + \frac{i\left(-1+e^{8i}\right)^2 e^{-8it}}{16\pi} +$$

$$\frac{i\left(-1+e^{9i}\right)^2 e^{-9it}}{18\pi} + \frac{i\left(-1+e^{10i}\right)^2 e^{-10it}}{20\pi} + \frac{i\left(-1+e^{11i}\right)^2 e^{-11it}}{22\pi} + \frac{i\left(-1+e^{12i}\right)^2 e^{-12it}}{24\pi} +$$

$$\frac{i\left(-1+e^{13i}\right)^2 e^{-13it}}{26\pi} + \frac{i\left(-1+e^{14i}\right)^2 e^{-14it}}{28\pi} + \frac{i\left(-1+e^{15i}\right)^2 e^{-15it}}{30\pi} + \frac{i\left(-1+e^{16i}\right)^2 e^{-16it}}{32\pi} +$$

$$\frac{i\left(-1+e^{17i}\right)^2 e^{-17it}}{34\pi} + \frac{i\left(-1+e^{18i}\right)^2 e^{-18it}}{36\pi} + \frac{i\left(-1+e^{19i}\right)^2 e^{-19it}}{38\pi} + \frac{i\left(-1+e^{20i}\right)^2 e^{-20it}}{40\pi} +$$

$$\frac{i\left(-1+e^{21i}\right)^2 e^{-21it}}{42\pi} + \frac{i\left(-1+e^{22i}\right)^2 e^{-22it}}{44\pi} + \frac{i\left(-1+e^{23i}\right)^2 e^{-23it}}{46\pi} + \frac{i\left(-1+e^{24i}\right)^2 e^{-24it}}{48\pi} +$$

$$\frac{i\left(-1+e^{25i}\right)^2 e^{-25it}}{50\pi} + \frac{i\left(-1+e^{26i}\right)^2 e^{-26it}}{52\pi} + \frac{i\left(-1+e^{27i}\right)^2 e^{-27it}}{54\pi} + \frac{i\left(-1+e^{28i}\right)^2 e^{-28it}}{56\pi} +$$

$$\frac{i\left(-1+e^{29i}\right)^2 e^{-29it}}{58\pi} + \frac{i\left(-1+e^{30i}\right)^2 e^{-30it}}{60\pi} + \frac{i\,e^{2i(-1+t)}\sin[1]^2}{\pi} + \frac{i\,e^{6i(-1+t)}\sin[3]^2}{3\pi} +$$

$$\frac{i\,e^{8i(-1+t)}\sin[4]^2}{4\pi} + \frac{\left(-1+e^{10i}\right)e^{5i(-3+2t)}\sin[5]}{10\pi} + \frac{\left(-1+e^{12i}\right)e^{6i(-3+2t)}\sin[6]}{12\pi} +$$

$$\frac{\left(-1+e^{14i}\right)e^{7i(-3+2t)}\sin[7]}{14\pi} + \frac{\left(-1+e^{16i}\right)e^{8i(-3+2t)}\sin[8]}{16\pi} + \frac{\left(-1+e^{18i}\right)e^{9i(-3+2t)}\sin[9]}{18\pi}$$

```
In[ ]:= Plot[{y[t]}, {t, -1, 3}, PlotStyle → {Red},
         PlotLegends → {"Serie de Fourier Función Impar"}, AxesLabel → {"t"}, PlotRange → All]
```
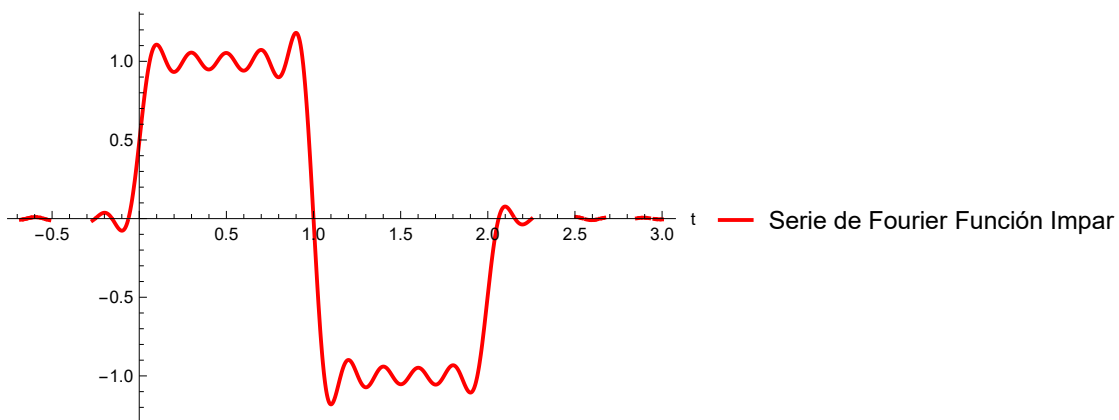
Out[ ]=


```
In[ ]:= Plot[{x[t], y[t]}, {t, -1, 3}, PlotStyle → {Blue, Red},
         PlotLegends → {"Función Impar", "Serie de Fourier (nMax términos)"},
         Filling → {1 → {2}}, AxesLabel → {"t", "f(t)"}, PlotRange → All]
```

Out[ ]=