Deep Learning - Homework 1

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Medical image classification with linear classifiers and neural networks.

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3 Question 3

- 3.1 Question 3.1
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Answer To demonstrate that the specified Boolean function cannot be computed by a single perceptron, let's consider a simple case where D = 2, A = -1, and B = 1. The function f is defined as:

$$f(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{D} x_i \in [-1, 1], \\ -1 & \text{otherwise} \end{cases}$$

In this setup:

- For x = (+1, +1), the sum $\sum x_i = 2$. Since 2 is not in the range [-1, 1], f(x) = -1.
- For x = (-1, -1), the sum $\sum x_i = -2$. Since -2 is also not in the range [-1, 1], f(x) = -1.
- For x = (-1, +1) or x = (+1, -1), the sum $\sum x_i = 0$. This falls within the range [-1, 1], so f(x) = 1 for these inputs.

The visual representation of the points can be seen in Figure ??. The red points represent the inputs that should be classified as +1 and the blue points represent the inputs that should be classified as -1.

The critical point here is that a single perceptron is fundamentally a linear classifier, which means it can only separate data points using a straight line in the feature space. However, in this example, there is no straight line that can separate these points accordingly in a 2D space to satisfy the function f.

This example thus serves as a counter-example proving that the given function cannot generally be computed with a single perceptron, as it requires a non-linear decision boundary which a single perceptron cannot provide.

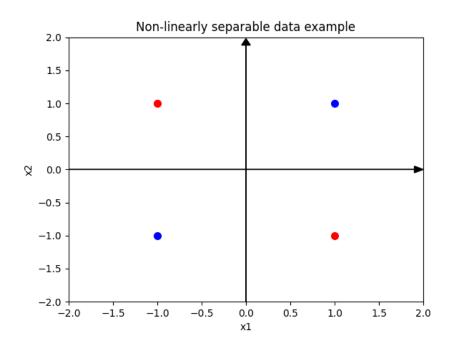


Figure 1: Classification of points using the function f

3.1.2 Question 3.1 b)

3.1.3 Question 3.1 c)