Question 3

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In this exercise, you will design a multilayer perceptron to compute a Boolean function of D variables, $f : \{-1, +1\}^D \to \{-1, +1\}$, defined as:

(a) Perceptron's Limitations (5 points)

Show that the function above cannot generally be computed with a single perceptron. *Hint: think of a simple counter-example.*

Answer To demonstrate that the specified Boolean function cannot be computed by a single perceptron, let's consider a simple case where $D = 2$, $A = -1$, and $B = 1$. The function f is defined as:

$$f(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{D} x_i \in [-1, 1], \\ -1 & \text{otherwise} \end{cases}$$

In this setup:

- For $x = (+1, +1)$, the sum $\sum x_i = 2$. Since 2 is not in the range [-1, 1], $f(x) = -1$.
- For $x = (-1, -1)$, the sum $\sum x_i = -2$. Since -2 is also not in the range [-1, 1], $f(x) = -1$.
- For $x = (-1, +1)$ or $x = (+1, -1)$, the sum $\sum x_i = 0$. This falls within the range [-1, 1], so $f(x) = 1$ for these inputs.

The visual representation of the points can be seen in Figure 1. The red points represent the inputs that should be classified as $+1$ and the blue points represent the inputs that should be classified as -1.

The critical point here is that a single perceptron is fundamentally a linear classifier, which means it can only separate data points using a straight line in the feature space. However, in this example, there is no straight line that can separate these points accordingly in a 2D space to satisfy the function f.

This example thus serves as a counter-example proving that the given function cannot generally be computed with a single perceptron, as it requires a non-linear decision boundary which a single perceptron cannot provide.

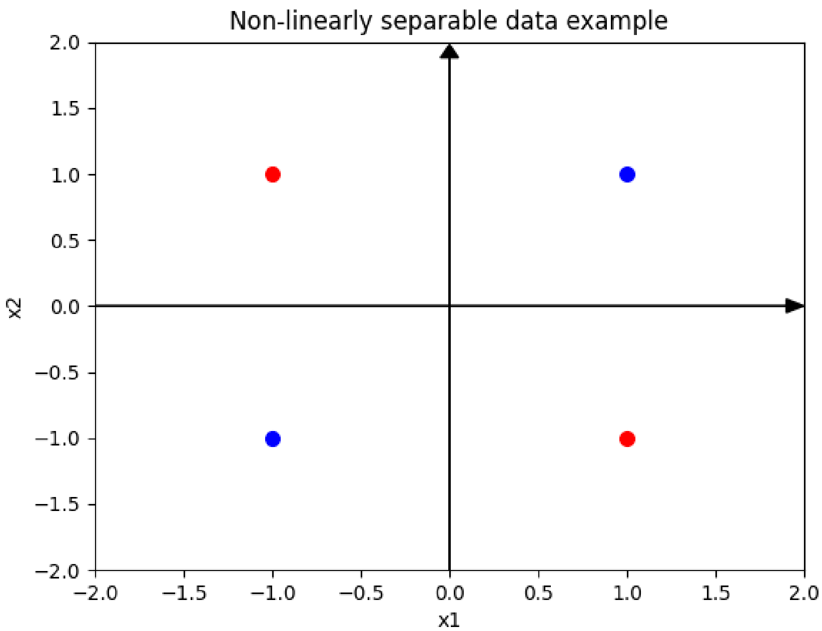

Figure 1: Classification of points using the function f