Deep Learning - Homework 1

99222 - Frederico Silva, 99326 - Sebastião Carvalho

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Contents

1 Question 1

Medical image classification with linear classifiers and neural networks.

1.1 Question 1.1

1.1.1 Question 1.1 a)

Answer After running the code, the following plot was generated:

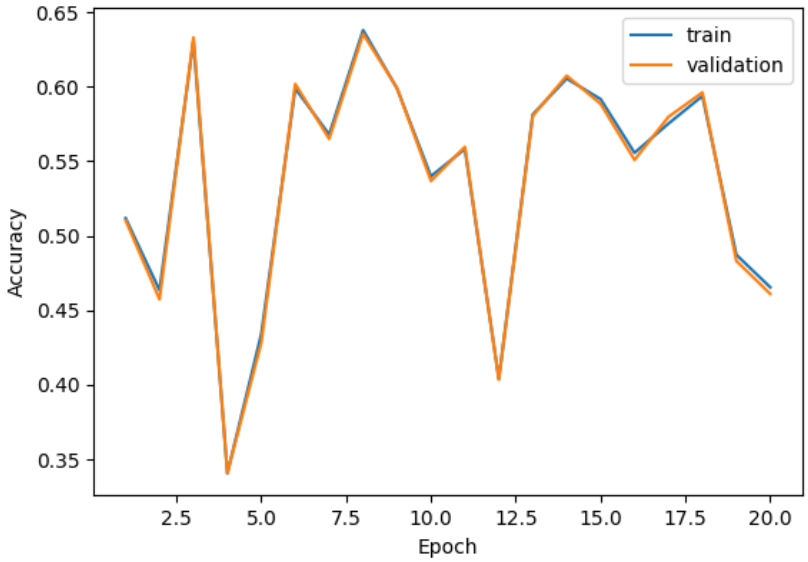

Figure 1: Perceptron Training and Validation Accuracy

The final test accuracy was 0.3422.

1.1.2 Question 1.1 b)

Answer After running the code, the following plots were generated for learning rates $\eta = 0.01$ and $\eta = 0.001$, respectively:

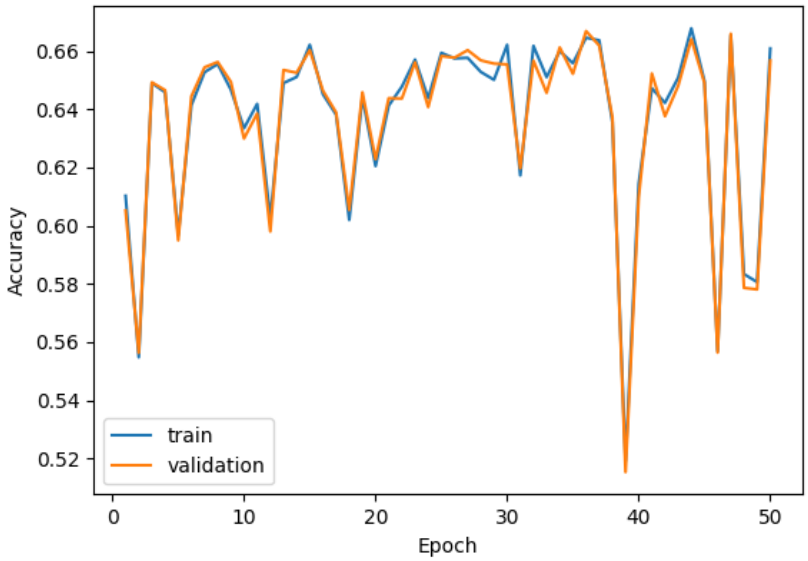

Figure 2: Logistic Regression Accuracy with Learning Rate $\eta = 0.01$

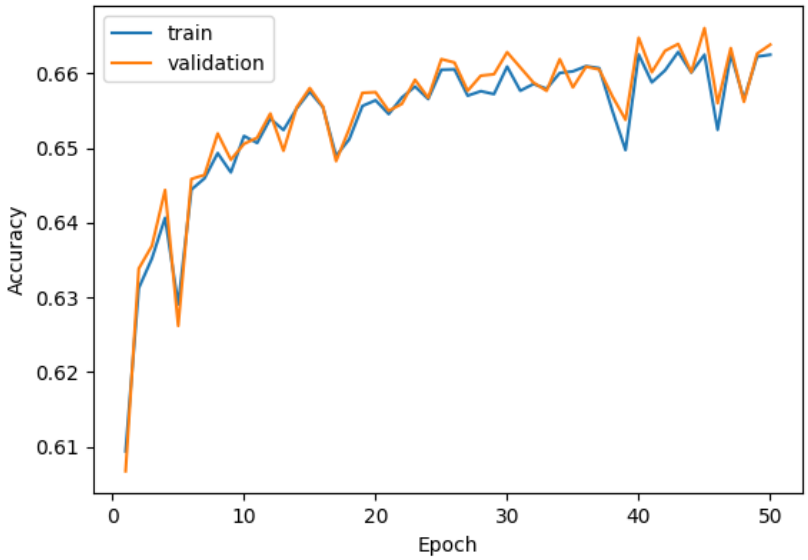

Figure 3: Logistic Regression Accuracy with Learning Rate $\eta = 0.001$

The final test accuracies were 0.5784 and 0.5936 for $\eta = 0.01$ and $\eta = 0.001$, respectively.

1.2 Question 1.2

1.2.1 Question 1.2 a)

1.2.2 Question 1.2 b)

2 Question 2

Medical image classification with an autodiff toolkit.

2.1 Question 2.1

2.2 Question 2.2

2.2.1 Question 2.2 a)

2.2.2 Question 2.2 b)

2.2.3 Question 2.2 c)

3 Question 3

3.1 Question 3.1

3.1.1 Question 3.1 a)

Answer To demonstrate that the specified Boolean function cannot be computed by a single perceptron, let's consider a simple case where $D = 2$, $A = -1$, and $B = 1$. The function f is defined as:

$$f(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{D} x_i \in [-1, 1], \\ -1 & \text{otherwise} \end{cases}$$

In this setup:

- For $x = (+1, +1)$, the sum $\sum x_i = 2$. Since 2 is not in the range [-1, 1], $f(x) = -1$.
- For $x = (-1, -1)$, the sum $\sum x_i = -2$. Since -2 is also not in the range [-1, 1], $f(x) = -1$.
- For $x = (-1, +1)$ or $x = (+1, -1)$, the sum $\sum x_i = 0$. This falls within the range [-1, 1], so $f(x) = 1$ for these inputs.

The visual representation of the points can be seen in Figure 4. The red points represent the inputs that should be classified as $+1$ and the blue points represent the inputs that should be classified as -1.

The critical point here is that a single perceptron is fundamentally a linear classifier, which means it can only separate data points using a straight line in the feature space. However, in this example, there is no straight line that can separate these points accordingly in a 2D space, to satisfy the function f.

This example thus serves as a counter-example proving that the given function cannot generally be computed with a single perceptron, as it requires a non-linear decision boundary which a single perceptron cannot provide.

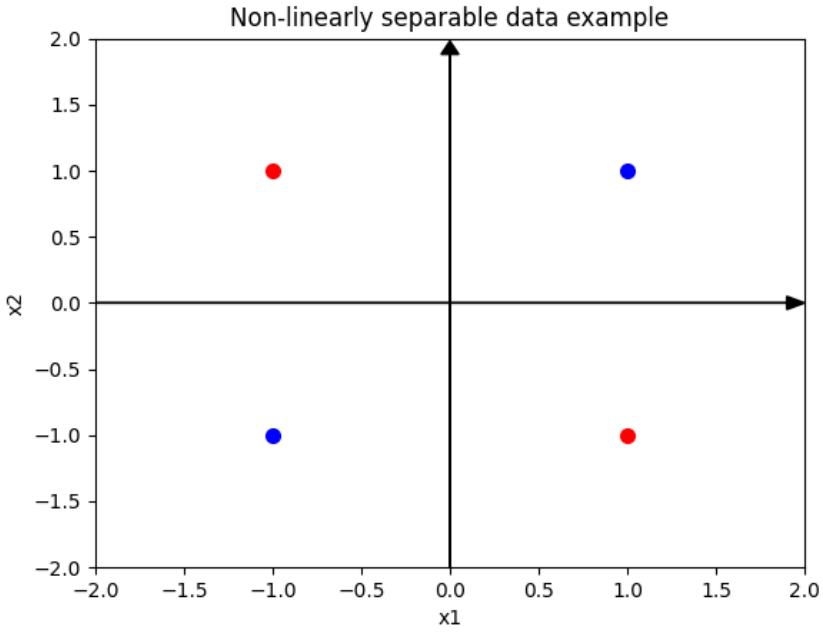

Figure 4: Classification of points using the function f

3.1.2 Question 3.1 b)

3.1.3 Question 3.1 c)