Série 13

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1. Dériver les fonctions suivantes:

(a) $x \mapsto (1+x^2)^7$

(b) $x \mapsto \dfrac{1}{\sqrt[5]{x}}$

(c) $x \mapsto (x^2-1)^2 \cdot \sqrt{x}$

(d) $x \mapsto \dfrac{\sin(x)}{1+\cos(x)}$

(e) $x \mapsto \sin^2(x^3)$

(f) $x \mapsto \dfrac{1}{15}\cos^3(x)\left(3\cos^2(x)-5\right)$

(g) $x \mapsto \dfrac{1}{\sin(x)\cos(x)}$

(h) $x \mapsto \dfrac{\sqrt[3]{x\sqrt{8x}}}{\sqrt[4]{2x^3}}$

(i) $x \mapsto \dfrac{x^2+x+1}{x^2-x+1}$

(j) $x \mapsto \dfrac{ax+b}{cx+d}$

(k) $x \mapsto \dfrac{1}{(1+x^2)\sqrt{1+x^2}}$

(l) $x \mapsto \sqrt{x+\sqrt{x+\sqrt{x}}}$

(m) $x \mapsto \sqrt{x\sqrt{x\sqrt{x}}}$

(n) $x \mapsto \sin(\cos(x))$

(o) $x \mapsto \cot(1-2x)$

(p) $x \mapsto \dfrac{1}{3}\tan^3(x) - \tan(x) + x$

(q) $x \mapsto \arcsin\left(\dfrac{1-x^2}{1+x^2}\right)$

(r) $x \mapsto \left(\arctan\left(\dfrac{\pi}{2}\right)\right)^2$

(s) $x \mapsto \arctan\left(\dfrac{1-x}{1+x}\right)$

(t) $x \mapsto \sin(\arcsin(x))$

(u) $x \mapsto \arcsin(\sin(x))$

(v) $x \mapsto \cos^2\left(\arccos\left(\sqrt{x}\right)\right)$

1.2 : Règles de dérivation

Théorème : 1) $(f(x)+g(x))' = f'(x)+g'(x)$

2) $(\underbrace{\alpha}_{\text{constante}} \cdot f(x))' = \alpha \cdot f'(x)$

Règle du produit 3) $(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Règle du quotient 4) $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$

Si $\dfrac{1}{g(x)} = \dfrac{0\cdot g(x) - 1\cdot g'(x)}{g^2(x)} \Rightarrow \dfrac{-g'(x)}{g^2(x)}$

Règle de la chaîne 5) $(f(g(x)))' = ((f\circ g)(x))' = f'(g(x)) \cdot g'(x)$

Dérivée interne / intérieur

Règle de l'inverse 6) $(f^{-1}(x))' = \dfrac{1}{f'(f^{-1}(x))}$

1.3 : Dérivées des fonctions standards

Théorème :

$f(x)$	$f'(x)$
x^α	$\alpha \cdot x^{\alpha-1}$, $\alpha \in \mathbb{R}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\dfrac{1}{\cos^2(x)} = 1+\tan^2(x)$
$\arcsin(x)$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan(x)$	$\dfrac{1}{1+x^2}$
$\text{arccot}(x)$	$-\dfrac{1}{1+x^2}$
e^x	e^x
a^x	$a^x \cdot \ln(a)$
$\ln(x)$	$\dfrac{1}{x}$
$\log_a(x)$	$\dfrac{1}{x\cdot\ln(a)}$

a) $f(x) = (\underbrace{1+x^2}_{g(x)})^7 = 7\,(1+x^2)^6 + (1+x^2)'$

$= 7\,(1+x^2)^6 + 1' + (x^2)'$

$= 7\,(1+x^2)^6 + 2x$

$= 14x\,(1+x^2)^6$

b) $\dfrac{1}{\sqrt[5]{x}} = \dfrac{1}{x^{\frac{1}{5}}} = \dfrac{(x^{\frac{1}{5}})'}{(x^{\frac{1}{5}})^2} = -\dfrac{\frac{1}{5}\cdot x^{-\frac{4}{5}}}{x^{\frac{2}{5}}} = \dfrac{-\frac{1}{5}}{x^{\frac{6}{5}}} = -\dfrac{1}{5\sqrt[5]{x^6}} = -\dfrac{1}{5x\sqrt[5]{x}}$

$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

c) $f(x) = (x^2-1)^2\cdot\sqrt{x}$

$= ((x^2-1)^2)'\cdot\sqrt{x} + (x^2-1)^2\cdot(\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}})'$

$= 2\cdot(x^2-1)\cdot(x^2-1)'\cdot\sqrt{x} + (x^2-1)^2\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}}$

$= 2(x^2-1)\cdot 2x\cdot\sqrt{x} + \dfrac{(x^2-1)^2}{2\sqrt{x}}$

$= 4x\cdot\sqrt{x}\cdot(x^2-1) + \dfrac{(x^2-1)^2}{2\sqrt{x}}$

$= 4x^3\sqrt{x} - 4x\sqrt{x} + \dfrac{(x^2-1)^2}{2\sqrt{x}}$

$= \dfrac{4x^3\sqrt{x}\cdot 2\sqrt{x} - 4x\sqrt{x}\cdot 2\sqrt{x} + (x^2-1)^2}{2\sqrt{x}}$

$= \dfrac{8x^4 - 8x^2 + x^4 - 2x + 1}{2\sqrt{x}} = \dfrac{9x^4 - 10x^2 + 1}{2\sqrt{x}}$

d) $f(x) = \dfrac{\sin(x)}{1+\cos(x)} = \dfrac{\sin(x)'\cdot(1+\cos(x)) - \sin(x)\cdot(1+\cos(x))'}{(1+\cos(x))^2}$

$= \dfrac{\cos(x)\cdot(1+\cos(x)) - \sin(x)\cdot(-\sin(x))}{(1+\cos(x))^2}$

$= \dfrac{\cos(x) + \overbrace{\cos^2(x) + \sin^2(x)}^{=1}}{\cos^2(x) + 2\cos(x) + 1} = \dfrac{\cancel{\cos(x)+1}}{(\cos(x)+1)^{\cancel{2}\,1}} = \dfrac{1}{\cos(x)+1}$

e) $f(x) = \sin^2(x^3) = (\sin(x^3))^2 = 2\cdot\sin(x^3)\cdot\overbrace{\sin(x^3)'}^{=(\sin(x^3))'\cdot(x^3)'}$

$= 2\sin(x^3)\cdot\cos(x^3)\cdot 3x^2$

f)

$= 2\sin(x^3)\cdot\cos(x^3)\cdot 3x^2$

f)

(f) $x \mapsto \frac{1}{15}\cos^3(x)\left(3\cos^2(x)-5\right)$

$f(x) = \frac{1}{15}\cos^3(x)\left(3\cos^2(x)-5\right)$

$= \left(\frac{1}{15}\cos^3(x)\right)'\cdot\left(3\cos^2(x)-5\right) + \frac{1}{15}\cos^3(x)\cdot\left(3\cos^2(x)-5\right)'$

$\frac{1}{15}\cdot\left(\cos^3(x)\right)'\cdot\left(3\cos^2(x)-5\right)+\frac{1}{15}\cos^3(x)\cdot\underbrace{\left(3\cos^2(x)\right)'}_{=3\cdot(\cos^2(x))'} - \underbrace{5'}_{=0}$

$= 1\cdot\cos^2(x)^{2-1}$

$\frac{1}{15}\cdot 3\cdot\cos^2(x)\cdot\underbrace{(-\sin(x))}_{(\cos(x))'}\cdot\left(3\cos^2(x)-5\right)+\frac{1}{15}\cdot\cos^3(x)\cdot 3\cdot\cos(x)$

$\frac{1}{5}\cos^2(x)(-\sin(x))\cdot\left(3\cos^2(x)-5\right)+\frac{1}{5}\cos^4(x)$

FAux. 2ème try:

(f) $x \mapsto \frac{1}{15}\cos^3(x)\left(3\cos^2(x)-5\right)$

$f(x) = \left(\frac{3\cos^5(x)-5\cos^3(x)}{15}\right)'$

$= \frac{1}{15}\cdot\left(3\cos^5(x)-5\cos^3(x)\right)'$

$= \frac{1}{15}\cdot\left((3\cos^5(x))'-(5\cos^3(x))'\right)$

$= \frac{1}{15}\cdot\left(15\cdot\cos^4(x)\cdot\cos(x)' - 15\cdot\cos^2(x)\cdot\cos(x)'\right)$

$= \cos^4(x)\cdot(-\sin(x)) - \cos^2(x)\cdot(-\sin(x))$

$= -\cos^4(x)\sin(x)+\cos^2(x)\sin(x)$

$= \cos^2(x)\sin(x)\left(\underbrace{1-\cos^2(x)}_{=\sin^2(x)}\right)$

$= \cos^2(x)\sin^3(x)$

(g) $x \mapsto \frac{1}{\sin(x)\cos(x)}$

$f(x) = \frac{1}{\sin(x)\cos(x)} = -\frac{(\sin(x)\cos(x))'}{\sin^2(x)\cos^2(x)} = \frac{(\sin(x))'\cdot\cos(x)+\sin(x)(\cos(x))'}{\sin^2(x)\cos^2(x)}$

$= -\frac{\overbrace{\cos^2(x)-\sin^2(x)}^{\cos^2(x)-\sin^2(x)=\cos(2x)}}{\sin^2(x)\cdot\cos^2(x)} = -\frac{\cos(2x)}{\sin^2(x)\cos^2(x)}$

(h) $x \mapsto \frac{\sqrt[3]{x\sqrt{8x}}}{\sqrt[4]{2x^3}}$

$\overbrace{x'}^{=1}\cdot(8x)^{\frac{1}{2}}+x\cdot\left((8x)^{\frac{1}{2}}\right)' = (8x)^{\frac{1}{2}}+x\cdot\left(\frac{1}{2}\cdot(8x)^{-\frac{1}{2}}+\overbrace{(8x)'}^{=8\cdot(x)'=8}\right.$

$= \sqrt{8x}+x\left(\frac{\sqrt{8x}}{2}+8\right)$

$f(x) = \frac{\sqrt[3]{x\sqrt{8x}}}{\sqrt[4]{2x^3}} = \frac{\left(x\cdot(8x)^{\frac{1}{2}}\right)^{\frac{1}{3}}}{(2x^3)^{\frac{1}{4}}} = \frac{\frac{1}{3}\cdot\left(x\cdot(8x)^{\frac{1}{2}}\right)^{-\frac{2}{3}}\cdot\overbrace{\left(x\cdot(8x)^{\frac{1}{2}}\right)'}}{\frac{1}{4}\cdot(2x^3)^{-\frac{3}{4}}\cdot\underbrace{2\cdot(x^3)'}_{2\cdot 3\cdot x^2 = 6x^2}}$

FAUX → Règle du quotient !!!

2ème try:

(h) $x \mapsto \frac{\sqrt[3]{x\sqrt{8x}}}{\sqrt[4]{2x^3}}$

$f(x) = \frac{\sqrt[3]{x\sqrt{8x}}}{\sqrt[4]{2x^3}} = \frac{\left(x\cdot(8x)^{\frac{1}{2}}\right)^{\frac{1}{3}}}{(2x^3)^{\frac{1}{4}}} = \frac{x^{\frac{1}{3}}\cdot(8x)^{\frac{1}{6}}}{2^{\frac{1}{4}}\cdot x^{\frac{3}{4}}} = \frac{(8x)^{\frac{1}{6}}}{2^{\frac{1}{4}}\cdot x^{\frac{3}{4}}\cdot x^{-\frac{1}{3}}}\;\frac{(8x)^{\frac{1}{6}}}{2^{\frac{1}{4}}\cdot x^{\frac{5}{12}}} = \frac{1}{2^{\frac{1}{4}}\cdot 8^{-\frac{1}{6}}\cdot x^{\frac{5}{12}}\cdot x^{-\frac{1}{6}}} = \frac{1}{2^{\frac{1}{4}}\cdot\underbrace{8^{-\frac{1}{6}}}_{=(2^3)^{-\frac{1}{6}} = 2^{-\frac{1}{2}}}\cdot x^{\frac{1}{4}}} = \frac{1}{2^{\frac{1}{4}}\cdot 2^{-\frac{1}{2}}\cdot x^{\frac{1}{4}}}$

$= \left(\frac{1}{2^{\frac{1}{2}}\cdot x^{\frac{1}{4}}}\right)'$

$= -\frac{\left(\sqrt{2}\cdot x^{\frac{1}{4}}\right)'}{\left(\sqrt{2}\cdot x^{\frac{1}{4}}\right)^2}$ -

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$$= -\frac{(\sqrt{2}\cdot x^{\frac{1}{4}})'}{(\sqrt{2}\cdot x^{\frac{1}{4}})^2} -$$

$$= \frac{\sqrt{2}\cdot\frac{1}{4}\cdot x^{-\frac{3}{4}}}{2\cdot\sqrt{x}} = \frac{\sqrt{2}\cdot\frac{1}{4}}{2\sqrt{x}\cdot\sqrt[4]{x^3}}$$

(i) $x \mapsto \dfrac{x^2+x+1}{x^2-x+1}$

$$f(x) = \frac{x^2+x+1}{x^2-x+1} = \frac{\overbrace{(x^2+x+1)'}^{=2x+1+0}(x^2-x+1) - (x^2+x+1)\overbrace{(x^2-x+1)'}^{2x-1+0}}{(x^2-x+1)^2} = \frac{(2x+1)(x^2-x+1)-(2x-1)(x^2+x+1)}{(x^2-x+1)^2}$$

$$= \frac{\cancel{2x^3}-\cancel{2x^2}+\cancel{2x}+\cancel{x^2}-\cancel{x}+1-\cancel{2x^3}-2x^2-\cancel{2x}+\cancel{x^2}+\cancel{x}+1}{(x^2-x+1)^2}$$

$$= \frac{-2x^2+2}{(x^2-x+1)^2}$$

j) $x \mapsto \dfrac{ax+b}{cx+d}$

$$f'(x) = \frac{(ax+b)'(cx+d)-(ax+b)(cx+d)'}{(cx+d)^2} = \frac{a(cx+d)-c(ax+b)}{(cx+d)^2} = \frac{acx+ad-acx-bc}{(cx+d)^2}$$

$$= \frac{ad-bc}{(cx+d)^2}$$

$(1+x^2)^{\frac{1}{2}} = \left((1+x^2)^{\frac{1}{2}}\right)^{2}\cdot(1+x^2)'$

k) $$f(x) = \frac{1}{(1+x^2)\cdot\sqrt{1+x^2}} = \frac{-\left((1+x^2)\cdot\sqrt{1+x^2}\right)'}{\left((1+x^2)\cdot\sqrt{1+x^2}\right)^2} = -\left(\frac{(1+x^2)'\cdot\sqrt{1+x^2}+(1+x^2)\cdot(\sqrt{1+x^2})'}{(1+x^2)^3}\right)$$

$$= -\left(\frac{2x\sqrt{1+x^2}+(1+x^2)\cdot\left(\frac{1}{\cancel{2}}(1+x^2)^{-\frac{1}{2}}\cdot\cancel{2}x\right)}{//}\right)$$

$$= -\frac{2x\sqrt{1+x^2}+(1+x^2)\cdot\frac{x}{\sqrt{1+x^2}}}{//}$$

$$= -\frac{2x\cdot(\sqrt{1+x^2})^2+(1+x^2)\cdot x}{(1+x^2)^3\cdot\sqrt{1+x^2}} = -\frac{2x(1+x^2)+(1+x^2)\cdot x}{(1+x^2)^{\cancel{3}\,2}\sqrt{1+x^2}}$$

$$= \frac{(\cancel{1+x^2})(2x+x)}{(1+x^2)^{\cancel{3}\,2}\sqrt{1+x^2}} = -\frac{3x}{(1+x^2)^2\sqrt{1+x^2}}$$

l) $$f(x) = \sqrt{x+\sqrt{x+\sqrt{x}}} = \left(x+(x+x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}\left(x+(x+x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{-\frac{1}{2}} + \left(x+(x+x^{\frac{1}{2}})^{\frac{1}{2}}\right)'$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1+\frac{1}{2}(x+x^{\frac{1}{2}})^{-\frac{1}{2}}\cdot\left((x+x^{\frac{1}{2}})'\right)\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(1+\frac{1}{2\sqrt{x}}\right)\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)\right)$$

$$= \text{———}//\text{———}\cdot\left(1+\frac{2\sqrt{x}+1}{4\sqrt{x^2+x\sqrt{x}}}\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\frac{4\sqrt{x^2+x\sqrt{x}}+2\sqrt{x}+1}{4\sqrt{x^2+x\sqrt{x}}}$$

$$= \frac{4\sqrt{x^2+x\sqrt{x}}+2\sqrt{x}+1}{8\sqrt{(x+\sqrt{x+\sqrt{x}})(x^2+x\sqrt{x})}} = \frac{\text{———}//\text{———}}{8\sqrt{x^3+x^2\sqrt{x}+x^2\sqrt{x+\sqrt{x}}+x\sqrt{x^2+x\sqrt{x}}}}$$

$$= \frac{4\sqrt{x^2+x\sqrt{x}}+2\sqrt{x}+1}{8\sqrt{x^3+x^2\sqrt{x}+x^2\sqrt{x+\sqrt{x}}+x\sqrt{x^2+x\sqrt{x}}}}$$

(m) $x \mapsto \sqrt{x\sqrt{x\sqrt{x}}}$

$$f(x) = \sqrt{x\cdot\sqrt{x\cdot\sqrt{x}}}$$

$$= \left(x\cdot(x\cdot x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$f(x) = \sqrt{x \cdot \sqrt{x \cdot \sqrt{x}}}$

$= \left(x \cdot (x \cdot x^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= x^{\frac{1}{2}} \cdot (x^{\frac{1}{2}} \cdot x^{\frac{1}{4}})^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} = x^{\frac{4}{8}+\frac{2}{8}+\frac{1}{8}} = x^{\frac{7}{8}}$

$\Rightarrow f'(x) = \frac{7}{8} \cdot x^{-\frac{1}{8}} = \frac{7}{8\sqrt[8]{x}}$

n) $f(x) = \sin(\cos(x))$

$f'(x) = (\sin(\cos(x)))' \cdot (\cos(x))'$

$= -\cos(\cos(x)) \cdot \sin(x)$

o) $f(x) = \cot(1-2x)$ $\quad 0 - 2 \cdot 1 = -2$

$f'(x) = -1 - \cot^2(1-2x) \cdot \underbrace{(1-2x)'}$

$= -2 - 2\cot^2(1-2x)$

p) $f(x) = \frac{1}{3}\tan^3(x) - \tan(x) + x$

$f'(x) = \frac{1}{3} \cdot (\tan^3(x))' - (\tan(x))' + x'$

$= \frac{1}{3} \cdot \left(3\tan^2(x) \cdot (1 + \tan^2(x))\right) - 1 - \tan^2(x) + 1$

$= \frac{1}{3}\left(3\tan^2(x) \cdot \tan^4(x)\right) - \tan^2(x)$

$= \tan^6(x) - \tan^2(x)$

q) $f(x) = \arcsin\left(\frac{1-x^2}{1+x^2}\right)$

$f'(x) = \frac{1}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \left(\frac{1-x^2}{1+x^2}\right)'$

$= —〃— \cdot \frac{(1-x^2)' \cdot (1+x^2) - (1-x^2) \cdot (1+x^2)'}{(1+x^2)^2}$

$= —〃— \cdot \frac{-2x(1+x^2) - (1-x^2) \cdot (2x)}{(1+x^2)^2}$

$= —〃— \cdot \frac{-2x - \cancel{2x^3} - 2x + \cancel{2x^3}}{(1+x^2)^2}$

$= -\frac{4x}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2}\right)^2} \cdot (1+x^2)^2}$

$= -\frac{4x}{\sqrt{\frac{(1+x^2)^2 - (1-x^2)^2}{(1+x^2)^2}} \cdot (1+x^2)^2} = -\frac{4x}{\sqrt{\frac{(1+x^2+1-x^2)(1+x^2-1+x^2)}{(1+x^2)^2}} \cdot 〃}$

$= -\frac{4x}{\sqrt{\frac{2 \cdot 2x^2}{(1+x^2)^2}} \cdot (1+x^2)^2}$

$= -\frac{\cancel{4}^{2}x}{\frac{\cancel{2}x^2}{\cancel{1+x^2}} \cdot (1+x^2)^2}$

$= -\frac{2x}{x^2(1+x^2)}$

r) $f(x) = \left(\arctan\left(\frac{\pi}{2}\right)\right)^2$ $\quad = 0 \quad$ (constante$' = 0$)

$f'(x) = \frac{1}{1 + \left(\frac{\pi}{2}\right)^2} \cdot \left(\frac{\pi}{2}\right)'$

$= \frac{1}{\frac{4+\pi^2}{4}} = \frac{4}{4+\pi^2}$

s) $f(x) = \arctan\left(\frac{1-x}{1+x}\right)$

$f'(x) = \frac{1}{1 + \left(\frac{1-x}{1+x}\right)^2} \cdot \frac{(1-x)' \cdot (1+x) - (1-x) \cdot (1+x)'}{(1+x)^2}$

$= \frac{\cancel{(1+x)^2}}{(1+x)^2 + (1-x)^2} \cdot \frac{-(1+x) - (1-x) \cdot 1}{\cancel{(1+x)^2}}$ $\Big\}\; -1 - \cancel{x} - 1 + \cancel{x}$

$$f'(x) = \frac{}{1+\left(\frac{1-x}{1+x}\right)^2} \cdot \frac{}{(1+x)^2}$$

$$= \frac{\cancel{(1+x)^2}}{(1+x)^2+(1-x)^2} \cdot \frac{-(1+x)-(1-x)\cdot 1}{\cancel{(1+x)^2}} \Big\} -1-\cancel{x}-1+\cancel{x}$$

$$= \frac{-2}{(1+x)^2+(1-x)^2} = \dots$$

t) $f(x) = \sin(\arcsin(x)) = x$

$\Rightarrow f'(x) = x' = 1$

u) $f(x) = \arcsin(\sin(x)) = x$

$\Rightarrow f'(x) = x' = 1$

v) $f(x) = \cos^2(\arccos(\sqrt{x}))$

$$= 2 \cdot \cos(\arccos(\sqrt{x})) \cdot (\cos(\arccos(\sqrt{x})))'$$

$$= 2\sqrt{x} \cdot -\sin(\arccos(\sqrt{x})) \cdot (\arccos(\sqrt{x}))'$$

$$= 2\sqrt{x} \cdot -\sin(\arccos(\sqrt{x})) \cdot \left(-\frac{1}{\sqrt{1-(\sqrt{x})^2}}\right) \cdot \sqrt{x}'$$

$$= 2\sqrt{x} \cdot -\sin(\arccos(\sqrt{x})) \cdot -\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$= \frac{\cancel{2}\cancel{\sqrt{x}} \cdot (-\sin(\arccos(\sqrt{x}))) \cdot (-1) \cdot 1 \cdot 1}{\cancel{2} \cdot \sqrt{1-x} \cdot \cancel{\sqrt{x}}}$$

$$= \frac{\overbrace{\sin(\arccos(\sqrt{x}))}^{=\sqrt{1-x}}}{\sqrt{1-x}} = \underline{\underline{1}}$$

2. Dériver les fonctions suivantes:

(a) $x \mapsto \ln\left(\sqrt{\frac{1-x}{1+x}}\right)$

(b) $x \mapsto \ln(\sin(x))$

(c) $x \mapsto \sin(\ln(x))$

(d) $x \mapsto \ln(7x \cdot \ln(\ln(4x))) - \ln(x)$

(e) $x \mapsto e^{-1/x^2}$

(f) $x \mapsto e^{1/\ln(x)}$

(g) $x \mapsto e^{\ln(x)+\ln(\sqrt{x})+\ln(\sqrt[3]{x})}$

(h) $x \mapsto x^{\sin(x)}$

(i) $x \mapsto x^{1/x}$

a) $f(x) = \ln\left(\sqrt{\frac{1-x}{1+x}}\right)$

$$f'(x) = \frac{1}{\sqrt{\frac{1-x}{1+x}}} \cdot \left(\sqrt{\frac{1-x}{1+x}}\right)'$$

$$= \text{——} \text{〃} \text{——} \cdot \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \left(\frac{1-x}{1+x}\right)'$$

3. (a) Déterminer l'équation de la droite de pente 5 tangente à la parabole $y = x^2 - x + 2$.
(b) Déterminer l'équation de la normale à la courbe $x^2 + 2x - 3y = 6$ au point $x_0 = 2$.
(c) Déterminer l'équation des tangentes à la parabole $y = -x^2 + 2x - 2$ qui passent par l'origine.

a) $f(x) = 5x + b$ tangeant à $p(x) = x^2 - x + 2$

$\Rightarrow$ trouver pts sur $p(x)$ où pente $= 5$

$\Rightarrow p(x)' = (x^2)' - x' + 2 = 2x - 1$ $\Big\}$ $2x - 1 = 5$, $\underline{\underline{x = 3}}$ $\rightsquigarrow P(3; ?)$

$\Rightarrow p(3) = 9 - 3 + 2 = 8 \Rightarrow P(3; 8)$

$\Rightarrow \in f(x)$: $8 = 5 \cdot 3 + b$

$8 = 15 + b$ $\Rightarrow f(x) = \underline{\underline{5x - 7}}$

$b = -7$

3. (a) Déterminer l'équation de la droite de pente 5 tangente à la parabole $y = x^2 - x + 2$.
(b) Déterminer l'équation de la normale à la courbe $x^2 + 2x - 3y = 6$ au point $x_0 = 2$.
(c) Déterminer l'équation des tangentes à la parabole $y = -x^2 + 2x - 2$ qui passent par l'origine.

b) Courbe : $x^2+2x-3y=6$

$x^2+2x-6=3y$

$\Rightarrow f(x)=\frac{1}{3}x^2+\frac{2}{3}x-2$

$f(2)=\frac{1}{3}\cdot 4+\frac{2}{3}\cdot 2-2$

$=\frac{4}{3}+\frac{4}{3}-2$

$=\frac{8}{3}-2=\frac{8-6}{3}=\frac{2}{3}$ $\rightarrow P(2;\frac{2}{3})$

$f'(x)=\frac{1}{3}\cdot 2\cdot x+\frac{2}{3}\cdot 1-0=\frac{2}{3}x+\frac{2}{3}$

pente en $x_0=2$: $\frac{2}{3}\cdot 2+\frac{2}{3}=\frac{6}{3}=\underline{\underline{2}}$

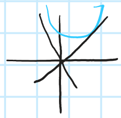
Pente de la normale : $-\frac{1}{2}$

$\Rightarrow n(2)=\frac{2}{3}=-\frac{1}{2}\cdot 2+b$

$\Leftrightarrow \frac{2}{3}+1=b$

$b=\frac{5}{3}$ $\Rightarrow n(x)=-\frac{1}{2}x+\frac{5}{3}$

3. (a) Déterminer l'équation de la droite de pente 5 tangente à la parabole $y=x^2-x+2$.
(b) Déterminer l'équation de la normale à la courbe $x^2+2x-3y=6$ au point $x_0=2$.
(c) Déterminer l'équation des tangentes à la parabole $y=-x^2+2x-2$ qui passent par l'origine.



c) $p(x)=-x^2+2x-2$ / $p(x)'=-1\cdot 2x+2\cdot 1=-2x+2$

t_1, t_2: $b=0 \Rightarrow f_{1,2}(x)=ax$

Reste à trouver

Égaler equ: $ax=-x^2+2x-2$

$x^2+x(a-2)+2=0$

$\Delta=b^2-4ac=0$

$=a^2-4a+4-4\cdot 1\cdot 2$

$=a^2-4a-4=0$

$\Delta=16-4\cdot(-4)\cdot 1$

$=32$

$\Rightarrow a_1=\frac{4+\sqrt{32}}{2}=\frac{4+4\sqrt{2}}{2}=2+2\sqrt{2}$

$a_2=\frac{4-\sqrt{32}}{2}=2-2\sqrt{2}$

Juste mais utiliser dérivée :

$\Rightarrow$ $\boxed{t_{x_0}: \ y=f'(x_0)(x-x_0)+f(x_0)}$

$(0;0)$: $0=f'(x_0)(0-x_0)+f(x_0)$

$-f(x_0)=-x_0\cdot f'(x_0)$

$-f(x_0)=-x_0\cdot(-2x_0+2)$

$f(x_0)=-2x_0^2+2x_0$

$-x_0^2+\cancel{2x_0}-2=-2x_0^2\cancel{+2x_0}$

$x_0^2-2=0$

$x_0^2=2 \Rightarrow x_0=\pm\sqrt{2}$ = Pts de contact

$\Rightarrow a_1=-2\sqrt{2}+2$

$a_2=2\sqrt{2}+2$

4. Sous quel angle α les courbes suivantes se coupent–elles? Calculer $\tan(\alpha)$ sans évaluer α.

(a) $y=\sin(x)$ et $y=\cos(x)$,

(b) $y=\frac{1}{x}$ et $y=\sqrt{x}$.

a) $f(x)=\sin(x)$, $g(x)=\cos(x)$

intersection : $\sin(x)=\cos(x)$

$\frac{\sin(x)}{\cos(x)}=1$ $\Rightarrow x=\frac{\pi}{4}$ $\Rightarrow$ Pcroisement $=(\frac{\pi}{4};?)$

$\tan(x)=1$

$f'(x)=\cos(x)$, $g'(x)=-\sin(x)$

$\Rightarrow$ $a_1=\cos(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$, $a_2=-\sin(\frac{\pi}{4})=-\frac{\sqrt{2}}{2}$

$\Rightarrow$ $a_1=\tan(\alpha_1)$, $a_2=\tan(\alpha_2)$

$\alpha_1=\arctan(\frac{\sqrt{2}}{2})$ $\alpha_2=\arctan(-\frac{\sqrt{2}}{2})$

$\alpha_1=35{,}26°$ $\alpha_2=-35{,}26$

Angle total : $2\cdot 35{,}26°=\underline{\underline{70{,}53°}}$

$\alpha_1 = \arctan(\frac{\sqrt{2}}{2})$ $\qquad$ $\alpha_2 = \arctan(-\frac{\sqrt{2}}{2})$

$\alpha_1 = 35{,}26°$ $\qquad$ $\alpha_2 = -35{,}26$

Angle total : $2 \cdot 35{,}26° = \underline{\underline{70{,}53°}}$

b) $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$

V : $\frac{1}{x} = \sqrt{x}$

$\frac{1}{x^2} = x$

$x^3 = 1$

$\underline{x = 1}$

$f'(x) = -\frac{1}{x^2}$

$g'(x) = (x^{\frac{1}{2}})' = \frac{1}{2\sqrt{x}}$

$\Rightarrow a_1 = \frac{-1}{1} = -1$

$a_2 = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

$\Rightarrow \alpha_1 = \arctan(-1) = -45°$

$\alpha_2 = \arctan(\frac{1}{2}) = 26{,}57°$

$\Rightarrow$ Angle total $= -45 - 26{,}57 = -71{,}57°$

Série 14

mercredi, 26 mars 2025 11:19

1. Etudier les fonctions suivantes et dessiner leur graphe:

(a) $f : x \mapsto \dfrac{x^2 - 12x + 27}{x^2 - 4x + 5}$ (c) $f : x \mapsto \dfrac{x^3}{6(x-2)}$

(b) $f : x \mapsto \sqrt{x^2(9 - x^2)}$ (d) $f : x \mapsto 2\cos(x) + \sin(2x)$

a) $f(x) = \dfrac{x^2-12x+27}{x^2-4x+5}$

- $D = \mathbb{R}$
- Zéro : $x^2-12x+27=0$

  $(x-3)(x-9)=0$

  $x_1 = 3,\ x_2 = 9$
- AV : $x^2-4x+5=0$ $(x-4)(x-1)$

  $\Delta = 16-4\cdot5 = -4$

  $\Rightarrow \varnothing \rightarrow$ Pas d'as.
- Parité : $f(-x) = \dfrac{x^2+12x+27}{x^2+4x+5} \neq -f(x) \neq f(x)$

  $\hookrightarrow$ Aucune
- As non vert : $x^2-12x+27$ | x^2-4x+5

  $-(x^2-4x+5)$ | 1

  $-8x+5$

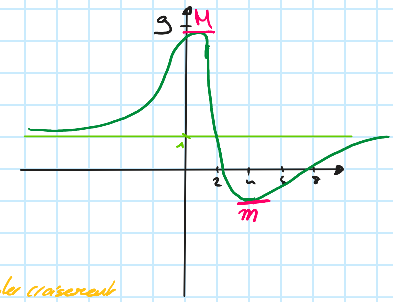
  $\Rightarrow \dfrac{x^2-12x+27}{x^2-4x+5} = 1 + \dfrac{-8x+5}{x^2-4x+5}$

Calculer correctement

Asymptote : égaler à zéro

- Signe : $f(x) = \dfrac{(x-3)(x-9)}{x^2-4x+5}$

x		3		9		$\mathbb{R}$
	+	0	−	0	+	



Dérivée : $f'(x) = \left(\dfrac{x^2-12x+27}{x^2-4x+5}\right)' = \dfrac{(x^2-12x+27)'(x^2-4x+5)-(x^2-12x+27)(x^2-4x+5)'}{(x^2-4x+5)^2}$

$= \dfrac{(2x-12)(x^2-4x+5)-(x^2-12x+27)(2x-4)}{(x^2-4x+5)^2}$

$= \dfrac{\cancel{2x^3}-8x^2+10x-12x^2+48x-60-\cancel{2x^3}+24x^2-54x+4x^2-48x+108}{(x^2-4x+5)^2}$

$= \dfrac{8x^2-44x+48}{(x^2-4x+5)^2}$

$\Rightarrow f'(x)=0 \iff 8x^2-44x+48=0$

$x_1 = 4,\ x_2 = \frac{3}{2}$

x		$3/2$		4	
$f'(x)$	+	0	−	0	+
$f(x)$	$\nearrow$	$\overrightarrow{M}$	$\searrow$	$\overrightarrow{m}$	$\nearrow$

$M : f\left(\frac{3}{2}\right) = \dfrac{\left(\frac{3}{2}\right)^2-12\cdot\left(\frac{3}{2}\right)+27}{\left(\frac{3}{2}\right)^2-4\cdot\frac{3}{2}+5} = \dfrac{\frac{9}{4}-18+27}{\frac{9}{4}-6+5} = \dfrac{\frac{45}{4}}{\frac{5}{4}} = \frac{45}{4}\cdot\frac{4}{5} = 9$

$\Rightarrow M\left(\frac{3}{2}; 9\right)$

$m : f(4) = \dfrac{16-48+27}{16-16+5} = -\frac{5}{5} = -1 \Rightarrow m(4;-1)$

b) $f(x) = \sqrt{x^2(9-x^2)}$

$= x\sqrt{9-x^2}$

$= x\sqrt{(3-x)(3+x)}$

$D = x > -3,\ x < 3$

$\Rightarrow [-3;3]$

Parité : $f(-x) = \sqrt{x^2(9-x^2)}$

$\Rightarrow f(x) = f(-x) \Rightarrow$ paire (sym $y \rightarrow$ étude $[0;3]$)

zéro : $0 = x\sqrt{(3-x)(3+x)} \Rightarrow 0 = x^2(3-x)(3+x)$

$\Rightarrow x_1 = 0,\ x_2 = 3,\ x_3 = -3$

AV : /

A NV : /

Dérivée : $f(x)' = \left(\sqrt{x^2(9-x^2)}\right)'$

$1 \quad (9x^2-x^4))'$

A NV : /

Dérivée : $f(x)' = \left(\sqrt{x^2(9-x^2)}\right)'$

$$= \frac{1}{2\sqrt{x^2(9-x^2)}} \cdot (9x^2 - x^4))'$$

$$= \frac{1}{2\sqrt{9x^2 - x^4}} \cdot (9x^2 - x^4)'$$

$$= \frac{1}{\cancel{2}|x|\sqrt{9x^2-x^4}} \cdot \overset{9}{\cancel{18}}x - \overset{2}{\cancel{4}}x^3$$

$$= \frac{9x - 2x^3}{|x| \cdot \sqrt{9x^2 - x^4}}$$

$\Rightarrow \quad f'(x) = \frac{9x - 2x^3}{|x| \cdot \sqrt{9x^2-x^4}} = 0$

$x(9 - 2x^2) = 0$

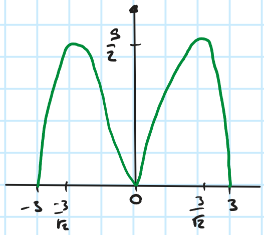
$x_1 = 0 \qquad x_2 : 2x^2 = 9$

$x = \sqrt{9/2} \quad \pm\frac{3}{\sqrt{2}}$

x	-3		$-\frac{3}{\sqrt{2}}$		0		$3/\sqrt{2}$		3
$f'(x)$		+	0	−	0	+	0	−	
x		↗	M	↘	m	↗	M_1	↘	

$m(0;0)$

$M_1\left(\frac{3}{\sqrt{2}}; \frac{9}{2}\right)$



$f\left(\frac{3}{\sqrt{2}}\right) : \sqrt{\left(\frac{3}{\sqrt{2}}\right)^2\left(9 - \left(\frac{3}{\sqrt{2}}\right)^2\right)}$

$= \sqrt{\frac{9}{2}\left(9 - \frac{9}{2}\right)} = \sqrt{\frac{9}{2}\left(\frac{9}{2}\right)} = \sqrt{\left(\frac{9}{2}\right)^2} = \frac{9}{2}$

c) $f(x) = \frac{x^3}{6(x-2)}$

- $D : \mathbb{R} \setminus \{2\}$
- Parité : $\frac{-x^3}{6(-x-2)} \rightarrow \varnothing$
- zéro : $x = 0$
- AV : $6(x-2) = 0$

  $x = 2$

- AO :

$$\begin{array}{l|l} x^3 & 6x - 12 \\ -(x^3 - 2x^2) & \frac{x^2}{6} + \frac{x}{3} + \frac{2}{3} \\ \quad 2x^2 & \\ -(2x^2 - 4x) & \\ \quad 4x & \\ -(4x - 8) & \\ \quad 8 & \end{array}$$

$\Rightarrow \frac{x}{6x-12} = \underbrace{\frac{x^2+2x+4}{6}}_{\text{parabole} = \text{A.NV.}} + \frac{8}{6x-12}$ } Croisement : $\frac{8}{6x-12} = 0 \Rightarrow \varnothing$

$8 \neq 0$

• Signe :

x		0		2	
	+	0	−	‖	+

$$f'(x) = \frac{(x^3)' \cdot (6x-12) - x^3(6(x-2))'}{36(x-2)^2}$$

$$= \frac{3x^2(6x-12) - x^3 \cdot 6(\overset{1}{x'} - \overset{0}{2'})}{36(x-2)^2}$$

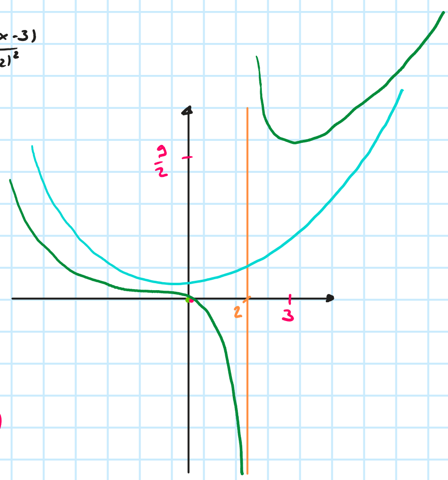
$$= \frac{18x^3 - 36x^2 - 6x^3}{\text{— // —}} = \frac{12x^3 - 36x^2}{\text{— // —}} = \frac{\cancel{12}x^2(x-3)}{\underset{3}{\cancel{36}}(x-2)^2}$$

$$= \frac{x^2(x-3)}{3(x-2)^2}$$

$\Rightarrow f'(x) = 0 \Rightarrow x^2(x-3) = 0$

$x_1 = 0, \; x_2 = 3$

x		0		2		3	
$f'(x)$	−	0	−	//	−	0	+
f	↘	S	↘		↘	m	↗



m : $f(3) = \frac{27}{6} = \frac{9}{2} \Rightarrow m\left(3; \frac{9}{2}\right)$

S : $f(0) = 0 \rightarrow S(0;0)$

d) $f(x) = 2\cos(x) + \sin(2x) = 2\cos x\,(1 + \sin x)$

- $\mathbb{D}: \mathbb{R}$, périodique $[0; 2\pi]$

- $f(0) : 2 \cdot 1 + 0 = 2$

- zéros : $0 = 2\cos(x) + \underbrace{\sin(2x)}_{= 2\sin(x)\cos(x)}$

$0 = 2\cos(x)\left(\sin(x) + 1\right)$

$2\cos x = 0 \qquad \sin(x) + 1 = 0$

$\cos x = 0 \qquad \sin(x) = -1$

$x = \frac{\pi}{2} \qquad x = \frac{3\pi}{2}$

- Signe $\frac{\pi}{2} \quad \frac{3\pi}{2}$

Exercice 2

2. Déterminer l'équation de la tangente à la courbe $y = \dfrac{\ln(3x)}{x}$ passant par l'origine.

$y = \dfrac{\ln(3x)}{x} = f(x)$

$$f'(x) = \frac{\ln(3x)' \cdot x - \ln(3x) \cdot x'}{x^2} = \frac{\ln(3x)' \cdot 3x' \cdot x - \ln(3x) \cdot x'}{x^2} = \frac{\frac{1}{\cancel{3x}} \cdot \cancel{3x} - \ln(3x)}{x^2} = \frac{1 - \ln(3x)}{x^2} = \frac{1 - \ln(3) - \ln(x)}{x^2}$$

$\Rightarrow \boxed{t_{x_0} : y = f'(x_0)(x - x_0) + f(x_0)}$

$\Rightarrow t_{x_0} : y = f'(x_0)(x - x_0) + f(x_0)$

$y = \dfrac{1 - \ln(3x_0)}{x_0^2}(x - x_0) + \dfrac{\ln(3x_0)}{x_0}$

$P(0;0) \in t$: $0 = \dfrac{1 - \ln(3) - \ln(x_0)}{x_0^{\cancel{2}}}(-\cancel{x_0}) + \dfrac{\ln(3) + \ln(x_0)}{x_0}$

$0 = -\dfrac{1 - \ln(3) - \ln(x_0)}{x_0} + \dfrac{\ln(3) + \ln(x_0)}{x_0}$

$0 = \dfrac{\ln(3) + \ln(x_0) - 1 + \ln(3) + \ln(x_0)}{x_0}$

$0 = \dfrac{2\ln(3) + 2\ln(x_0) - 1}{x_0}$

$0 = 2\ln(3x_0) - 1$

$\frac{1}{2} = \ln(3x_0) \qquad \Rightarrow \left(f\left(\frac{\sqrt{e}}{3}\right) = \dfrac{\ln(\sqrt{e}) \cdot 3}{\sqrt{e}} = \dfrac{\frac{1}{2} \cdot 3}{\sqrt{e}} = \dfrac{3}{2\sqrt{e}} \right)$

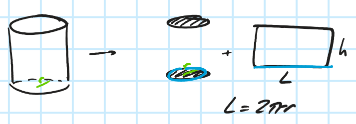
$e^{\frac{1}{2}} = 3x_0$

$\frac{\sqrt{e}}{3} = x_0$

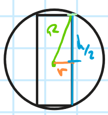
$\Rightarrow$ Pente en x_0 : $f'\left(\frac{\sqrt{e}}{3}\right) = \dfrac{1 - \ln\left(3\frac{\sqrt{e}}{3}\right)}{\left(\frac{\sqrt{e}}{3}\right)^2} = \dfrac{1 - \frac{1}{2}}{\frac{e}{9}} = \dfrac{\frac{1}{2}}{e} \cdot 9 = \dfrac{9}{2e}$

$\Rightarrow t(x) : \frac{9}{2e} \cdot x$

3. Un cylindre d'aire latérale maximale est inscrit dans une sphère de rayon R. Quelle est la hauteur du cylindre?



$L = 2\pi r$



$\Rightarrow R^2 = r^2 + \frac{h^2}{4} \Rightarrow r = \sqrt{R^2 - h^2/4}$

$h \to x$

$A(h) = 2\pi r h = 2\pi h\sqrt{R^2 - \frac{h^2}{4}} = 2\pi h\sqrt{\frac{4R^2 - h^2}{4}}$

$= \pi x\sqrt{4R^2 - x^2} = \pi x \cdot (4R^2 - x^2)^{\frac{1}{2}}$

$$A(h) = 2\pi r h = 2\pi h\sqrt{R^2 - \frac{h^2}{4}} = 2\pi h\sqrt{\frac{4R^2 - h^2}{4}}$$

$$= \pi x\sqrt{4R^2 - x^2} = \pi x \cdot (4R^2 - x^2)^{\frac{1}{2}}$$

$$A'(h) = (\pi x)' \cdot \sqrt{4R^2 - x^2} + \pi x\left(\sqrt{4R^2 - x^2}\right)'$$

$$= \pi\sqrt{4R^2 - x^2} + \pi x\left(\frac{1}{2\sqrt{4R^2 - x^2}} \cdot (-2x)\right)$$

$$= \pi\sqrt{4R^2 - x^2} - \frac{2\pi x^2}{2\sqrt{4R^2 - x^2}}$$

$$= \frac{2\pi(4R^2 - x^2) - 2\pi x^2}{2\sqrt{4R^2 - x^2}} = \frac{8\pi R^2 - 4\pi x^2}{2\sqrt{4R^2 - x^2}}$$

$$= 2\pi \frac{2R^2 - x^2}{\sqrt{4R^2 - x^2}}$$

$$\Rightarrow A'(x) = 0 \qquad \frac{2\pi(2R^2 - x^2)}{\sqrt{4R^2 - x^2}} = 0$$

$$2\pi(2R^2 - x^2) = 0$$

$$2R^2 - x^2 = 0$$

$$x^2 = 2R^2$$

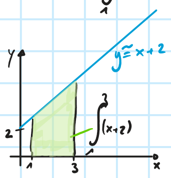
$$x = R\sqrt{2}$$

Série 17

samedi, 3 mai 2025 16:28

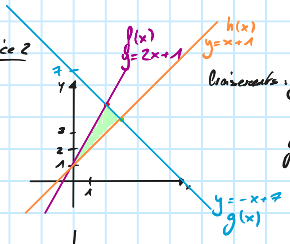
Exercice 1

$$\int_1^3 (x+2)\,dx = \int_1^3 x\,dx + \int_1^3 2 = \int_1^3 x\,dx + 2\cdot\int_1^3 1$$



$$= \frac{9}{2} - \frac{1}{2} + 2\cdot(3-1) = 4+4 = \underline{\underline{8}}$$

Exercice 2

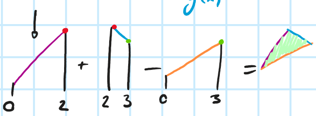


Croisements : $f(x)/g(x)$: $2x+1 = -x+7$

$3x = 6 \Rightarrow x = 2$

$g(x)/h(x)$: $x+1 = -x+7$

$2x = 6 \Rightarrow x = 3$



$$\Rightarrow \int_0^2 (2x+1)\,dx + \int_2^3 (-x+7)\,dx - \int_0^3 (x+1)\,dx$$

$$= 2\cdot\int_0^2 x\,dx + \int_0^2 1\,dx - \int_2^3 x\,dx + 7\cdot\int_2^3 1\,dx - \int_0^3 x\,dx - \int_0^2 1\,dx$$

$$= 2\cdot 2 + 2 - \frac{5}{2} + 7 - \frac{9}{2} - 3 = 4 + 2 - 2{,}5 + 7 - 4{,}5 - 3$$

$$= \underline{\underline{3}}$$

3. Calculer les intégrales suivantes: ?

(a) $\int_0^{2\pi} \sin(4x)\cdot\cos(3x)\,dx$

(b) $\int_0^{2\pi} \sin^2(5x)\,dx$

(c) $\int_{-1}^{3} \sqrt{3+2x-x^2}\,dx$

b)

c)

a) $\int_0^{2\pi} \sin(4x)\cdot\cos(3x)\,dx$

4. (a) Nous avons vu: $\int_0^b x^2\,dx = \frac{b^3}{3}$. En déduire $\int_0^a \sqrt{x}\,dx$.

(b) Déterminer l'aire comprise entre les courbes:

i. $y = -x+10$ et $y = -\frac{1}{2}x^2 + 4x + 2$

ii. $y = x^2$ et $y = \sqrt{x}$

a) $\int_0^b x^2\,dx = \frac{b^3}{3} \Rightarrow \int_0^a x^{1/2} = \frac{a^{\frac{1}{2}+1}}{\frac{1}{2}} = 2\cdot a^{2/3}$?

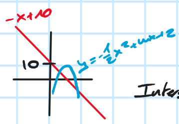
$= 2\sqrt[3]{a^2}$

b) i) $y = -x+10$ et $y = -\frac{1}{2}x^2 + 4x + 2$

$\hookrightarrow S = \left(\frac{-4}{-\frac{2}{2}}\right) = 4$

$f(4) = -\frac{1}{2}\cdot 16 + 16 + 2$

$= -8 + 16 + 2 = 10 \Rightarrow S(4;10)$



Intersection : $-x+10 = -\frac{1}{2}x^2 + 4x + 2$

$0 = \frac{1}{2}x^2 + 4x - 8$

$\Delta = 16 + 4\cdot 8\cdot\frac{1}{2}$

$= 16 + \frac{32}{2} = 32$

$x_1 = \frac{-4 \pm \sqrt{32}}{1} = -4 \pm \sqrt{2\cdot 16}$

$= -4 \pm 4\sqrt{2}$

Série 18

mercredi, 21 mai 2025 10:16

1. Calculer les intégrales suivantes:

(a) $\int \frac{1}{x^2}\,dx$	(h) $\int (t-x)\,dt$	(o) $\int \tan(3x)\,dx$
(b) $\int \frac{x^2-5\sqrt{x}+2\sqrt[3]{x^2}}{\sqrt[4]{x^5}}\,dx$	(i) $\int \sin(3x+5)\,dx$	(p) $\int \frac{x}{3+2x^2}\,dx$
(c) $\int \sqrt[4]{4x+5}\,dx$	(j) $\int \frac{x}{1+x^4}\,dx$	(q) $\int \frac{dx}{x\ln(x)}$
(d) $\int \frac{4}{\sqrt[3]{2x-1}}\,dx$	(k) $\int \frac{x^9}{\sqrt{1-x^{20}}}\,dx$	(r) $\int x^7 e^{x^8}\,dx$
(e) $\int \frac{x}{\sqrt{a^2+x^2}}\,dx$	(l) $\int \frac{x^7}{(1+x^8)^3}\,dx$	(s) $\int x e^{x^2} e^{e^{x^2}}\,dx$
(f) $\int \frac{1}{a^2+b^2x^2}\,dx$	(m) $\int (\tan(x)+\tan^3(x))\,dx$	(t) $\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx$
(g) $\int (t-x)\,dx$	(n) $\int \frac{1}{x^2-2x+5}\,dx$	(u) $\int \frac{e^x}{e^x+1}\,dx$

a) $\int \frac{1}{x^2} = \int x^{-2} = x^{-1} \iff (x^{-1})' = -1 \cdot x^{-2} = -\frac{1}{x^2}$

$\hookrightarrow -x^{-1} = -\frac{1}{x} + C$ ✓

b) $\int \frac{x^2 - 5\sqrt{x} + 2\sqrt[3]{x^2}}{\sqrt[4]{x^5}}\,dx$

$\int \frac{x^2 - 5x^{1/2} + 2x^{2/3}}{x^{5/4}}\,dx = \int \frac{x^2}{x^{5/4}}\,dx - 5\int \frac{x^{1/2}}{x^{5/4}} + 2\int \frac{x^{2/3}}{x^{5/4}}$

$= \frac{4}{7}\int x^{3/4}\,dx - 5\int x^{-3/4}\,dx + 2\int x^{-7/12}\,dx$

$x^{7/4\,\prime} = \frac{7}{4}\cdot x^{3/4}$

$= \frac{4}{7}\int \underbrace{\frac{7}{4}x^{3/4}}\,dx - 5\cdot 4\int \frac{1}{4}x^{-3/4}\,dx \cdot 2\cdot\frac{12}{5}\int \frac{5}{12}x^{-7/12}$

$x^{7/4\,\prime} = \frac{7}{4}\cdot x^{3/4}$ $\quad x^{1/4\,\prime} = \frac{1}{4}\cdot x^{-3/4}$ $\quad x^{5/12\,\prime} = \frac{5}{12}\cdot x^{-7/12}$

$= \frac{4}{7}\cdot x^{7/4} - 20\cdot x^{1/4} + \frac{24}{5}\cdot x^{5/12} + C$ ✓

c) (c) $\int \sqrt[4]{4x+5}\,dx$

$\int \sqrt[4]{4x+5}\,dx = \int (4x+5)^{1/4} = \frac{4}{5}\cdot\frac{1}{4}\int \frac{5}{4}\cdot 4\,(4x+5)^{1/4} = \frac{1}{5}\int 5(4x+5)^{1/4} = \frac{(4x+5)^{5/4}}{5} + C$ ✓

$(4x+5)^{5/4\,\prime} = \frac{5}{\cancel{4}}\cdot(4x+5)^{1/4}\cdot\cancel{4} = 5(4x+5)^{1/4}$

(d) $\int \frac{4}{\sqrt[3]{2x-1}}\,dx$

$-\frac{1}{3}+1 = \frac{-1+3}{3} = \frac{2}{3}$

$\int \frac{4}{\sqrt[3]{2x-1}}\,dx = \int 4(2x-1)^{-1/3}\,dx = 4\int (2x-1)^{-\frac{1}{3}} = \frac{1}{2}\cdot 4\cdot\frac{3}{2}\int \frac{2}{3}(2x-1)^{-\frac{1}{3}}\cdot 2$

$= 3\int \frac{4}{3}(2x-1)^{-1/3} = 3(2x-1)^{2/3} + C$

$= \frac{2}{3}(2x-1)^{-1/3}\cdot 2 = \frac{4}{3}(2x-1)^{-1/3}\cdot 3$

$= 4(2x-1)^{-1/3} = \frac{4}{3\sqrt[3]{(2x-1)}}$

(e) $\int \frac{x}{\sqrt{a^2+x^2}}\,dx$

$-\frac{1}{2}+1 = \frac{1}{2}$

$\int \frac{x}{\sqrt{a^2+x^2}}\,dx = \int x\cdot\frac{1}{\sqrt{a^2+x^2}}\,dx \;\frac{1}{2}\int 2x\cdot\frac{1}{2}(a^2+x^2)^{-\frac{1}{2}}$

$= (a^2+x^2)^{1/2} = \sqrt{a^2+x^2} + C$

f) (f) $\int \frac{1}{a^2+b^2x^2}\,dx$

$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{1}{ab}\arctan\left(\frac{bx}{a}\right) + C$

g) (g) $\int (t-x)\,dx$

$\int (t-x)\,dx = \int t\,dx - \int x\,dx = tx - \frac{x^2}{2} + C$

h) $\int (t-x)\,dt = \int t\,dt - \int x\,dt = \frac{t^2}{2} - tx + C$

i) $\int \sin(3x+5)\,dx = \frac{1}{3}\int 3\sin(3x+5)\,dx$

$= -\frac{1}{3}\cos(3x+5) + C$

j) $\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int 2x\cdot\frac{1}{1^2+(x^2)^2}\,dx \Rightarrow \frac{1}{2}\arctan(x^2) + C$

$F = \arctan(x^2)$

j) $\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int 2x \cdot \frac{1}{1^2+(x^2)^2}\,dx \Rightarrow \frac{1}{2}\arctan(x^2)+C$

$F = \arctan(x^2)$

k) $\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx = \frac{1}{5}\int 5x^4 \cdot \frac{1}{\sqrt{1-(x^5)^2}} \Rightarrow \frac{1}{5}\arcsin(x^5)+C$

$F(x) = \arcsin(x^5)$

l) $\int \frac{x^2}{(1+x^3)^3}\,dx = \frac{1}{8}\cdot(-\frac{1}{2})\int 8x\cdot(-2)(1+x^3)^{-3}$ $\quad -3+1=-2$ $\quad \int(1+x^3)^{-3} = -\frac{1}{2}\int -2(1+x^3)^{-3}\cdot 9 = -\frac{1}{18}(1+x^3)^{-2}$

$= -\frac{1}{\underset{16}{144}}\cdot(\;+x^3)^{-2}$?

m) $\int \tan x + \tan^3 x\,dx = -\ln|\cos(x)| + \int(\tan x)^3\,dx$?

n) $\int \frac{1}{\underbrace{x^2-2x+5}}\,dx = \int\frac{1}{4+(x-1)^2} = \int\frac{1}{4(1+\underbrace{\frac{(x-1)^2}{4}}_{=(\frac{x-1}{2})^2})} = \frac{1}{4}\int\frac{1}{1+(\frac{x-1}{2})^2} = \frac{1}{4}\cdot\arctan(\frac{x-1}{2})\cdot 2 = \frac{1}{2}\arctan(\frac{x-1}{2})+C$

$= x^2-2x+1+4$
$(x-1)^2$

$\arctan(\frac{x}{2}-1)' = \frac{1}{1+(\frac{x-1}{2})^2}\cdot\frac{x'}{2} = {}'' \cdot \frac{1}{2}$

o) $\int \tan(3x)\,dx = \frac{-\ln|\cos 3x|}{3}+C$

p) $\int\frac{x}{3+2x^2}\,dx = \frac{1}{4}\int\frac{x\cdot 4}{3+2x^2} = \frac{\ln(2x^2+3)}{4}+C$

$= \int\frac{f'(x)}{f(x)}$

$(3+2x^2)' = 4x$

q) $\int\frac{dx}{x\ln(x)} = \int\frac{1}{x\ln(x)}\,dx = \int\frac{1}{x}\cdot\frac{1}{\ln(x)}\,dx$?

$f = \ln|x|$ $\quad = \int$

IPP

$-\int f$
$x\cdot\int$

$\boxed{\Rightarrow \int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx}$ $\quad f = \text{intégr}$, $g = \text{dériv}$

$\hookrightarrow f = \text{dériv}, \; g = \text{intégr}$

Ex : 1) $\int xe^x\,dx = xe^x - \int 1\cdot e^x\,dx$ ← Pratique

$= xe^x - e^x + C$

r) $\frac{1}{8}\int 8 x^7 e^{x^8} \leftarrow f(g(x)),\; g(x) = x^8$

$\to g'(x) = x^{8\,\prime} = \underbrace{8x^7}$

Manque 8 pour avoir $\int f(g(x))\cdot g'(x) = F(g(x))$

$= \frac{1}{8}\cdot e^{x^8}+C$

s) $\frac{1}{2}\int 2x e^{x^2} e^{e^{x^2}} \to f(g(x)),\; g(x) = e^{x^2}$

$\} f(g(x)),\; g'(x) = e^{x^2\,\prime} = \underbrace{e^{x^2}\cdot 2x}$

$= \frac{1}{2}e^{e^{x^2}}+C$

t) $\int\frac{\sin(x)}{(1-\cos x)^2}\,dx =$?

u) $\int\frac{e^x}{\underbrace{e^x+1}_{(e^x+1)'=e^x}}\,dx = \int\frac{f'(x)}{f(x)} = \ln|e^x+1|+C$

2. Calculer

(a) $\int_1^5 \frac{4}{3x+2}\,dx$ $\qquad$ (b) $\int_4^5 \frac{3x+5}{7-2x}\,dx$

a) $\int\frac{4}{3x+2}\,dx = 4\cdot\frac{1}{3}\int\frac{3\cdot 1}{3x+2} = \frac{4}{3}\ln|3x+2|$

$(3x+2)' = 3$

$\Rightarrow \int_1^5\frac{4}{3x+2} = \left[\frac{4}{3}\ln|3x+2|\right]_1^5 = \frac{4}{3}\ln|17| - \frac{4}{3}\ln|5|$

$= \frac{4}{3}(\ln|17| - \ln|5|) = \frac{4}{3}\ln|\frac{17}{5}| = 1{,}005$

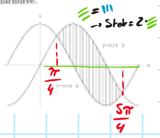
b) $\int_4^5\frac{3x+5}{7-2x}\,dx =$

$= \frac{4}{3}(\ln|17| - \ln|8|) = \underline{\underline{\frac{4}{3}\ln|\frac{17}{8}|}} = 1{,}005$

b) $\int_4^5 \frac{3x+5}{7-2x}\,dx =$

$F = \int \frac{3x+5}{7-2x} \int \frac{3x+5}{-2x+7} = \; ?$

3. Calculer l'aire de la surface hachurée.



Intersection: $\cos x = \sin x$

$1 = \frac{\sin x}{\cos x}$

$1 = \tan(x) \Rightarrow x = \frac{\pi}{4}, \; \frac{\pi}{4} + \pi \Rightarrow \frac{\pi}{4}, \frac{5\pi}{4}$

$\Rightarrow \text{Aire} = \int_{\pi/4}^{5\pi/4} \sin(x) - \int_{\pi/4}^{5\pi/4} \cos(x)$

$= [-\cos x]_{\pi/4}^{5\pi/4} - [\sin x]_{\pi/4}^{5\pi/4} = \; . \; . \; . \; = \frac{-4\sqrt{2}}{2} = -2\sqrt{2} = 2\sqrt{2}$

4. Calculer l'aire de la surface comprise entre les courbes $y^2 = 18x - 72$ et $y^2 = 6x$.

$y^2 = 18x - 72 \Rightarrow y = \sqrt{18x-72}$

$y^2 = 6x \Rightarrow y = \sqrt{6x}$