Si F(x) est une primitive de f(x), on a

$$F(x) = \int f(x) dx \iff f(x) = \frac{d}{dx}F(x)$$

si on pose x = g(t), alors

$$\frac{d}{dx}F(g(t)) = f(g(t)) \cdot g'(t) \iff \int f(g(t)) \cdot g'(t) dt = F(g(t)) + C$$

On a donc

$$\int f(x) dx = \int f(g(t)) \cdot g'(t) dt$$

Ex:

$$\int \frac{1}{\sqrt{x+1}} dx \Longrightarrow t = \sqrt{x}, \quad x = t^2, \quad dx = 2t dt$$

$$\Longrightarrow \int \frac{1}{\sqrt{x+1}} dx = \int \frac{1}{t^2+1} \cdot 2t dt = 2 \int \frac{t}{t^2+1} dt$$

$$= 2 \int \left(1 + \frac{-1}{t^2+1}\right) dt = 2 \cdot \left(t - \ln(t^2+1)\right) + C$$