Dion

Q1

c) $x^2 + \frac{1}{2}x + \frac{1}{8} = 0 \Leftrightarrow 8x^2 + 4x + 1 = 0$

$\Delta = 4^2 - 4 \cdot 8 \cdot 1 = 16 - 32 = -16 \Rightarrow \emptyset$

d) $(2x-5)^2 - (x-6)^2 = 80 \Leftrightarrow 4x^2 - 20x + 25 - x^2 + 12x - 36 - 80 = 0$

$\Leftrightarrow 3x^2 - 8x - 91 = 0$

$\Delta = 8^2 - 4 \cdot 3 \cdot (-91) = 1156 = 34^2 \Rightarrow x = \frac{8 \pm 34}{6} = \begin{cases} 7 \\ -\frac{13}{3} \end{cases}$

$S = \left\{-\frac{13}{3} ; 7\right\}$ → aucune contrainte sur notation

e) $\frac{x}{2x-3} - \frac{1}{2x} = \frac{3}{4x-6}$ $\quad \mathbb{D}: \begin{array}{l} x \neq \frac{3}{2} \\ x \neq 0 \\ x \neq \frac{3}{2} \end{array} \Rightarrow \mathbb{D} = \mathbb{R} \setminus \left\{0 ; \frac{3}{2}\right\}$

$\Leftrightarrow \frac{2x^2 - (2x-3) - 3x}{2x(2x-3)} = 0 \Leftrightarrow 2x^2 - 2x - 3x + 3 = 0$

$\Leftrightarrow \cancel{2x^2 - 5x + 3} = 0 \Leftrightarrow 2x(x-1) - 3(x-1) = 0$

$\Leftrightarrow (2x-3)(x-1) = 0 \Rightarrow \begin{array}{l} x_1 = \frac{3}{2} \notin \mathbb{D} \\ x_2 = 1 \in \mathbb{D} \end{array} \Rightarrow S = \{1\}$

Q2

c) $\frac{x^2+x}{48} - \frac{48}{x^2+x} = 4,8$ $\quad \mathbb{D} = \mathbb{R} \setminus \{0, -1\}$

$\Leftrightarrow (x^2+x)(x^2+x) - 48^2 = 4,8 \cdot 48 \cdot (x^2+x)$

$\Leftrightarrow (x^2+x)^2 - 230,4\,(x^2+x) - 2304 = 0$

Soit $y = (x^2+x)$: $y^2 - \frac{2304}{10}y - 2304 = 0$

$\Delta = \left(\frac{2304}{10}\right)^2 - 4 \cdot 1 \cdot (-230,4) = \frac{5400576}{100}$

$y = \frac{\frac{2304}{10} \pm \frac{96}{10}\sqrt{586}}{2} = \frac{2304}{20} \pm \frac{96}{20}\sqrt{586}$

Soit $(x^2+x) = y$: $\frac{y}{48} - \frac{48}{y} = 4,8 \Rightarrow$ Soit $\frac{y}{48} = t$

$\Rightarrow t - \frac{1}{t} = \frac{24}{5} \Leftrightarrow \frac{5t^2 - 5 = 24t}{5t} \Leftrightarrow 5t^2 - 24t - 5 = 0$

$\Leftrightarrow 5t^2 - 25t + t - 5 = 0 \Leftrightarrow 5t(t-5) + (t-5) \Leftrightarrow (5t+1)(t-5) = 0$

$\Rightarrow t_1 = -\frac{1}{5},\ t_2 = 5 \Rightarrow y_1 = -\frac{48}{5},\ y_2 = 240$

$\Rightarrow x(x+1) = y \Rightarrow$ ① $x^2 + 2x + \frac{48}{5} = 0 \Leftrightarrow 5x^2 + 10x + 48 = 0 \Rightarrow$ irrationnel

② $x^2 + 2x - 240 = 0 \Rightarrow \begin{array}{l} x_1 = -16 \in \mathbb{D} \\ x_2 = 15 \in \mathbb{D} \end{array}$

$\Rightarrow S = \{-16, 15\}$

e) $\sqrt{x-4}+3 = x-\sqrt{x-4}$ $\qquad \mathbb{D} = [4;+\infty[$

$\Rightarrow x-4+6\sqrt{x-4}+9 = x^2-2x\sqrt{x-4}+(x-4) \Leftrightarrow -2x+9-x^2 = (-2x-6)\sqrt{x-4}$

$\Leftrightarrow x^2+2x-9 = (x+3)\sqrt{x-4} \Rightarrow x^4+2x^3-9x^2+4x^2-18x+81 = (x+3)^2(x-4)$

$\Leftrightarrow x^4+2x^3-5x^2-18x+81 = x^3+2x^2-15x-36$

$\Leftrightarrow x^4+x^3-7x^2-3x+117 = 0$ Chercher à rassembler les termes avant de résoudre

$2\sqrt{x-4} = x-3 \Rightarrow 4(x-4) = (x-3)^2 \Leftrightarrow 4x-16-x^2+6x-9 = 0$

$\Leftrightarrow x^2-10x+25 = 0 \Leftrightarrow (x-5)(x-5) = 0 \Rightarrow x = 5 \in \mathbb{D}$

$\Rightarrow S = \{5\}$ attention, manque vérification

g) $\sqrt{1+x\sqrt{1+8x}} = x+1$ $\qquad \mathbb{D}$: ① $x > -\frac{1}{8}$ ② $x \geq 0$ $\Rightarrow \mathbb{D} = \mathbb{R}_+$

$\Rightarrow 1+x\sqrt{1+8x} = x^2+2x+1 \Leftrightarrow \cancel{x^2+x(2-\sqrt{1+8x}}$

$\Leftrightarrow x^2+2x = x\sqrt{1+8x} \Rightarrow x^4+4x^3+4x^2 = x^2(1+8x)$

$\Leftrightarrow x^4-4x^3+3x^2 = 0 \Leftrightarrow x^4-x^3-3x^3+3x^2 = 0$

$\Leftrightarrow x^3(x-1)-3x^2(x-1) = 0 \Leftrightarrow (x^3-3x^2)(x-1) = 0$

$\Leftrightarrow x^2(x-3)(x-1) = 0 \Rightarrow \underset{\in \mathbb{D}}{x_1 = 0}, \underset{\in \mathbb{D}}{x_2 = 3}, \underset{\in \mathbb{D}}{x_3 = 1}$

$\Rightarrow S = \{0;1;3\}$ attention manque vérification

i) $x^{\ln(x^3)} = \frac{e^5}{x^{14}}$ $\qquad \mathbb{D}$: $x \neq 0$, $x > 0$ $\Rightarrow \mathbb{D} = \mathbb{R}_+^*$

$\Leftrightarrow x^{\ln(x^3)}$ ↳ $\frac{\log_x x^3}{\log_x e} = \frac{3}{\log_x e} \Rightarrow x^{\frac{3}{\log_x e}} = \frac{e^5}{x^{14}}$ compliqué

$\Rightarrow \ln\ldots = \ln\ldots \Leftrightarrow \ln(x^3)\cdot\ln x = \ln e^5 - \ln x^{14}$

$\Leftrightarrow 3\cdot\ln^2 x = 5 - 14\ln x \Leftrightarrow \ln x(3\ln x+14)-5 = 0$

$\Leftrightarrow 3\ln^2 x+14\ln x-5 = 0$ soit $\ln x = u$: $\Rightarrow 3u^2+14u-5 = 0$

$\Leftrightarrow 3u^2+15u-u-5 = 0 \Leftrightarrow u(3u-1)+5(3u-1) = 0$

$\Leftrightarrow (u+5)(3u-1) = 0 \Rightarrow u_1 = -5, u_2 = \frac{1}{3}$

$\Rightarrow \begin{cases} \ln x = -5 \Rightarrow x = \frac{1}{e^5} \in \mathbb{D} \\ \ln x = \frac{1}{3} \Rightarrow x = \sqrt[3]{e} \in \mathbb{D} \end{cases}$

$\Rightarrow S = \{\frac{1}{e^5}; \sqrt[3]{e}\}$

j) $e^{3x}+4e^{2x}-e^{x}=0$, $e^{x}=y$, $\mathbb{D}=\mathbb{R}$

$\Rightarrow y^{3}+4y^{2}-y=0 \Leftrightarrow y(y^{2}+4y-1)=0$

$\Rightarrow y=0$ ou $y^{2}+4y-1=0$

$e^{x}=0$ $\qquad \Delta=16-4\cdot1(-1)=\cancel{12}\ 20=(2\sqrt{5})^{2}$

$\varnothing$

$\Rightarrow y=\dfrac{-4\pm2\sqrt{5}}{2}=\begin{cases}-2+\sqrt{5}\\-2-\sqrt{5}\end{cases}$

$e^{x}=-2+\sqrt{5}\Rightarrow x=\ln(-2+\sqrt{5})$

$\downarrow$ >0

$e^{x}=-2-\sqrt{5}\Rightarrow x=\ln(<0)$?

$\Rightarrow\varnothing$

$\Rightarrow S=\{\ln(\sqrt{5}-2)\}$

! singleton 1

h) $\log_{x}10+\log_{10x}10+\log_{100x}10=0$ $\qquad \mathbb{D}=\cancel{]0,+\infty[\cup]}$

$=]0,\frac{1}{100}[\cup]\frac{1}{100},\frac{1}{10}[\cup]\frac{1}{10},1[\cup]1,+\infty[$

$\Rightarrow\dfrac{\log10}{\log x}+\dfrac{\log10}{\log10x}+\dfrac{\log10}{\log100x}=0\Leftrightarrow\dfrac{1}{\log x}+\dfrac{1}{1+\log x}+\dfrac{1}{2+\log x}=0$

Soit $\log x=a\Rightarrow\dfrac{1}{a}+\dfrac{1}{1+a}+\dfrac{1}{2+a}=0\Leftrightarrow\underbrace{\dfrac{1}{a}+\dfrac{a}{a(1+a)}+\dfrac{a}{a(2+a)}=0}_{\text{inutile}}$

$\Leftrightarrow\dfrac{(1+a)(2+a)+(2+a)a+a(1+a)}{a(1+a)(2+a)}=0$

$\Leftrightarrow\dfrac{2+3a+a^{2}+2a+a^{2}+a+a^{2}}{a(1+a)(2+a)}=0\Leftrightarrow\dfrac{3a^{2}+6a+2}{a(1+a)(2+a)}=0$

$\Rightarrow 3a^{2}+6a+2=0\Leftrightarrow\cancel{3a(a+1)+2(a+1)=0\Leftrightarrow(3a+2)(a+1)=0}$

mauvais car $3\cdot2\neq3+2=6$

$\Rightarrow\cancel{a=-\frac{2}{3}}$ ou $\cancel{a=-1}$

$\downarrow$ div.

$\Rightarrow\cancel{\log x=-\frac{2}{3}}$

$\Rightarrow 3a^{2}+6a+2=0$, $\Delta=36-4\cdot3\cdot2=12=(2\sqrt{3})^{2}$

$\Rightarrow a=\dfrac{-6\pm2\sqrt{3}}{6}=\begin{cases}-1+\frac{1}{3}\sqrt{3}=\dfrac{-3+\sqrt{3}}{3}\\-1-\frac{1}{3}\sqrt{3}=\dfrac{-3-\sqrt{3}}{3}\end{cases}$

$\Rightarrow\log x=\dfrac{-3\pm\sqrt{3}}{3}\Rightarrow x=10^{\frac{-3\pm\sqrt{3}}{3}}\in\mathbb{D}$

$\Rightarrow S=\left\{10^{\frac{-3\pm\sqrt{3}}{3}}\right\}$

j) $e^{3x} + 4^{e^{2x}} - e^{x} = 0 \quad \mathbb{D} = \mathbb{R}$

$\Leftrightarrow (e^{x})^3 - e^{x} + 4^{e^{2x}} = 0$ Soit $e^{x} = y$

$\searrow 4^{(e^x)^2}$

$\Rightarrow y^3 - y + 4^{2y} = 0 \Leftrightarrow \cancel{y(y^2-1) = -4^{2y}}$

$\Rightarrow \cancel{\ln y (y^2-1) = 2y \ln -4}$

$\Leftrightarrow y(1-y^2) = 4^{2y} \Rightarrow \ln y(1-y^2) = 2y \cdot \ln 4$

$\Leftrightarrow$

h) $\log_x 10 + \log_{10x} 10 + \log_{100x} 10 = 0 \quad \mathbb{D} : \mathbb{R}_+^*$

$\log_{10x} 10 \to \dfrac{\log_x 10}{\log_x 10x} \to \log_x 10 + 1$

$\log_{100x} 10 \to \dfrac{\log_x 10}{\log_x 100x} \to \log_x 100 + 1$

Soit $\log_x 10 = y$

$\Rightarrow y + \dfrac{y}{y+1} + \dfrac{y}{y^2+1} = 0 \Leftrightarrow y(y+1)^2 + y(y+1) + y = 0$

$\Leftrightarrow \cancel{y^3} + 2y^2 + \cancel{1} + y^2 + 2y = 0 \Leftrightarrow y^3 + 3y^2 + 3y + 1 = 0$

$\Leftrightarrow \cancel{y^2(y+3)} \Leftrightarrow \cancel{y^2(y+3)} +$

1) $x^{\log_{10}(x^2)+\log_{10}(x)+1} = 10^{14} \Leftrightarrow x^{\log_{10}(10x^3)} = 10^{14}$ $\qquad D = \mathbb{R}_+^*$

$\Rightarrow \log_{10}(10x^3)\cdot\log_{10}x = 14 \Leftrightarrow (1+3\cdot\log_{10}x)\cdot\log_{10}x = 14$

soit $t=\log_{10}x$: $\Rightarrow t+3t^2-14=0 \Leftrightarrow (3t+7)(t-2)=0$

$\Rightarrow \cancel{(3t+\log_{10}x} \; (3\cdot\log_{10}x+7)(\log_{10}x-2)=0$

$\Rightarrow \log_{10}x = -\frac{7}{3}$ ou $\log_{10}x = 2$

$\Rightarrow x = 10^{-\frac{7}{3}} \in D \qquad \Rightarrow x = 10^2 = 100 \in D$

$\Rightarrow \boxed{S=\{100, 10^{-\frac{7}{3}}\}}$

Q3 2) $8x^2+10x>3 \Leftrightarrow 8x^2+10x-3>0 \Leftrightarrow (2x+3)(4x-1)>0$

$\Rightarrow S =]-\infty; -\frac{3}{2}[\cup]\frac{1}{4};+\infty[$

c) $\frac{2x-3}{3x+2}\geq 1 \Leftrightarrow 2x-3 \geq \cancel{3x+2} \Leftrightarrow \frac{2x-3-3x-2}{3x+2}\geq 0 \Leftrightarrow \frac{-x-5}{3x+2}\geq 0$

potentielle div. par 0

x		-5		$-\frac{2}{3}$	
	$-$	0	$+$	$\|$	$-$

$\Rightarrow S=[-5;-\frac{2}{3}[$

e) $\frac{5x+8}{2x-3} > \frac{3-x}{x+1} \Leftrightarrow \frac{(5x+8)(x+1)-(3-x)(2x-3)}{(2x-3)(x+1)}>0 \Leftrightarrow \frac{7x^2+4x+17}{(2x-3)(x+1)}>0$

$D = \mathbb{R}\setminus\{\frac{3}{2}; -1\}$

$\cancel{7x^2+4x+17 : \Delta = 4^2-4\cdot7\cdot17 = <0}$

$\Rightarrow \cancel{\emptyset \text{ (aucuns zéros)}}$

on cherche pos ou nég

$7x^2+4x+17>0,\ x\in D$ et $(2x-3)(x+1)>0 \Rightarrow]-\infty;-1[\cup]\frac{3}{2};+\infty[$

x		-1		$3/2$	
	$+$		$-$		$+$

$\Rightarrow$ cas positifs = intersection = $]-\infty;-1[\cup]\frac{3}{2};+\infty[$

négatif: $7x^2+4x+17<0 \Rightarrow \emptyset$

$\Rightarrow S =]-\infty;-1[\cup]\frac{3}{2};+\infty[$

f) $2x^4-4x^2+1<0$, soit $y=x^2 \Rightarrow 2y^2-4y+1<0$

$\Delta = 16-4\cdot 2\cdot 1 = 8 = (2\sqrt{2})^2 \Rightarrow y = \frac{4\pm 2\sqrt{2}}{4} = 1\pm\frac{1}{2}\sqrt{2} = x^2$

$\Rightarrow x = \pm\sqrt{1\pm\frac{1}{2}\sqrt{2}}$, $\Delta > 0 \Rightarrow 2y^2-4y+1 = 2\left(x+\frac{-4+2\sqrt{2}}{4}\right)\left(x+\frac{-4-2\sqrt{2}}{4}\right)$

$\Rightarrow$ strictement négative : ~~entre zéros~~ $\Rightarrow$ ~~$S =]1-\frac{1}{2}\sqrt{2}\,;\,1+\frac{1}{2}\sqrt{2}[$~~

quatre sol. $\Rightarrow$ différent

↳ ① ② ③ ④ ⑤

$-\sqrt{1+\frac{1}{2}\sqrt{2}}$ $\quad -\sqrt{1-\frac{1}{2}\sqrt{2}}$ $\quad \sqrt{1-\frac{1}{2}\sqrt{2}}$ $\quad \sqrt{1+\frac{1}{2}\sqrt{2}}$ $\quad x$

On prend des valeurs dans chaque intervalle puis test

①: $-2 \Rightarrow 2\cdot(-2)^4-4\cdot(-2)^2+1<0 \Leftrightarrow 32-16+1=17 \overset{?}{<} 0$ non

②: $-1 \Rightarrow 2\cdot(-1)^4-4\cdot(-1)^2+1<0 \Leftrightarrow 2-4+1=-1 \overset{?}{<} 0$ oui

③: $1 \Rightarrow 2-4+1<0 \Leftrightarrow -1 \overset{?}{<} 0$ oui

④: $0 \Rightarrow 1<0$ non

⑤: $2 \Rightarrow 2\cdot 2^4-2^2\cdot 2^2+1<0 \Leftrightarrow 2^5-2^4+1 \overset{?}{<} 0$ non

$\Rightarrow S =]-\sqrt{1+\frac{1}{2}\sqrt{2}}\,;\,-\sqrt{1-\frac{1}{2}\sqrt{2}}[\,\cup\,]\sqrt{1-\frac{1}{2}\sqrt{2}}\,;\,\sqrt{1+\frac{1}{2}\sqrt{2}}[$

g) $\underbrace{(x^2-x+1)}_{①}\underbrace{(-x^2+x+1)}_{②}\underbrace{(x^2+x-1)}_{③}>0$

①: $\Delta<0 \Rightarrow$ aucun zéro
$a>0 \Rightarrow$ positif } strictement positifs

Ainsi, on souhaite ①·② même signe que ③ $\begin{cases} \oplus\cdot\oplus \\ \ominus\cdot\ominus \end{cases}$

①·②: positifs: ①, $x\in\mathbb{R}$

②: $\Delta = 1^2-4\cdot(-1)\cdot 1 = 5$

$x = \frac{-1\pm\sqrt{5}}{-2} = \frac{1}{2}\mp\frac{\sqrt{5}}{2}$, forme 

$\Rightarrow]\frac{1}{2}-\frac{\sqrt{5}}{2}\,;\,\frac{1}{2}+\frac{\sqrt{5}}{2}[$

$\Rightarrow ①\cdot② = \cap =]\frac{1}{2}-\frac{\sqrt{5}}{2}\,;\,\frac{1}{2}+\frac{\sqrt{5}}{2}[$

négatifs: ①, strictement positif

②: $]-\infty\,;\,\frac{1}{2}-\frac{\sqrt{5}}{2}[\,\cup\,]\frac{1}{2}+\frac{\sqrt{5}}{2}\,;\,\infty[$

$\Rightarrow ①\cdot② = \cap =]-\infty\,;\,\frac{1}{2}-\frac{\sqrt{5}}{2}[\,\cup\,]\frac{1}{2}+\frac{\sqrt{5}}{2}\,;\,\infty[$

③: $\Delta = 1^2-4\cdot 1(-1) = 5$

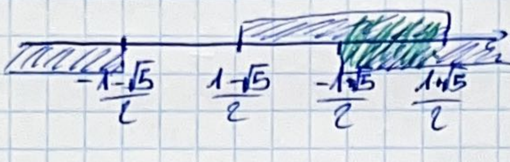
$x = \frac{-1\pm\sqrt{5}}{2}$, forme: 

pos.: $]-\infty\,;\,\frac{-1-\sqrt{5}}{2}[\,\cup\,]\frac{-1+\sqrt{5}}{2}\,;\,+\infty[$

nég.: $]\frac{-1-\sqrt{5}}{2}\,;\,\frac{-1+\sqrt{5}}{2}[$

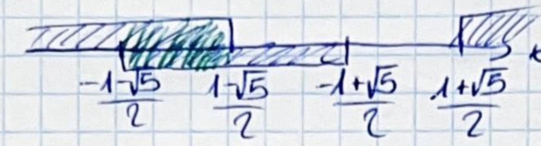
résol. valide pour ①·② et ③ ⇒ intersection

Ainsi, ⊕·⊕ :



$\Rightarrow \left]\frac{-1+\sqrt{5}}{2} ; \frac{1+\sqrt{5}}{2}\right[$

⊖·⊖ :



$\Rightarrow \left]\frac{-1-\sqrt{5}}{2} ; \frac{1-\sqrt{5}}{2}\right[$

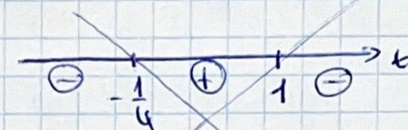
Ainsi, les deux étant valides, union $\Rightarrow \left]\frac{-1-\sqrt{5}}{2} ; \frac{1-\sqrt{5}}{2}\right[\cup \left]\frac{-1+\sqrt{5}}{2} ; \frac{1+\sqrt{5}}{2}\right[$

Cas 2) $ax^2+x+a-\frac{3}{4}=0$, $\Delta = 1^2-4\cdot a\cdot\left(a-\frac{3}{4}\right)<0$ (pas de solutions)

$= 1-4a^2+3a<0$

$\Leftrightarrow -4a^2+3a+1<0 \Leftrightarrow -4a^2+4a-a+1<0 \Leftrightarrow -4a(a-1)-(a-1)<0$

$\Leftrightarrow (a-1)(-4a-1)<0$



$\Rightarrow$ aucune ~~solution si~~ $\cancel{a\in\left]-\infty;-\frac{1}{4}\right[\cup\left]1,+\infty\right[}$ faux car dans ce cas ⊖·⊖ ⇒ ⊕

Soit $a>1$ soit $a<-\frac{1}{4}$ ⇒ disjonction $a\in\left]-\infty;-\frac{1}{4}\right[\vee \left]1;+\infty\right[$

b) $x^2+(a+3)x+a^2+3=0$, $\Delta=(a+3)^2-4\cdot1(a^2+3)<0$

$= a^2+6a+9-4a^2-12<0$

$\Leftrightarrow -3a^2+6a-3<0 \Leftrightarrow -3(a^2-2a+1)<0$

$\Leftrightarrow -3(a-1)^2<0 \Rightarrow a\in\mathbb{R}\setminus\{1\}$

↳ $0 \overset{?}{<} 0$, non