Méthode de substitution (Analyse)

Si $F(x)$ est une primitive de $f(x)$, on a :

$$F(x) = \int f(x)\,dx \quad \Leftrightarrow \quad f(x) = \frac{d}{dx}F(x)$$

Si on pose $x = g(t)$, alors :

$$\frac{d}{dx}F(g(t)) = f(g(t)) \cdot g'(t) \Rightarrow \int f(g(t)) \cdot g'(t)\,dt = F(g(t)) + C$$

$$\boxed{\int f(x)\,dx = \int f(g(t)) \cdot g'(t)\,dt}$$

Exemple

$$\int \frac{1}{\sqrt{x}+1}\,dx \Rightarrow t = \sqrt{x} \Rightarrow x = t^2 \Rightarrow dx = 2t\,dt$$

$$\int \frac{1}{t+1} \cdot 2t\,dt = 2\int \frac{t}{t+1}\,dt = 2\int \left(1 + \frac{t-1}{t+1}\right)dt$$

$$= 2\left(t - \ln(t+1)\right) + C$$