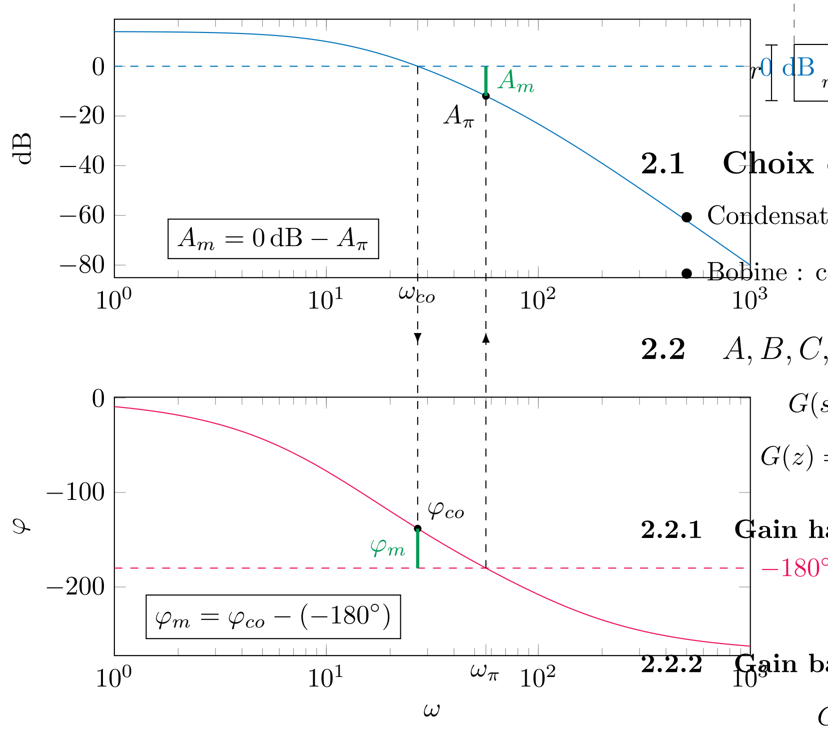
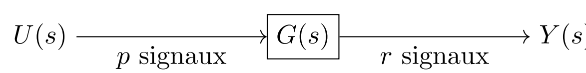
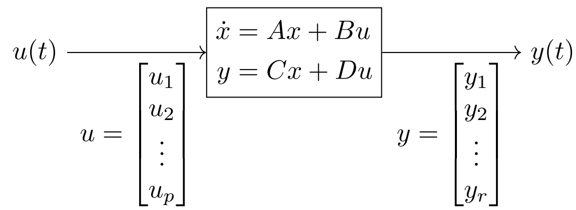
# 1 Fonctions de transfert

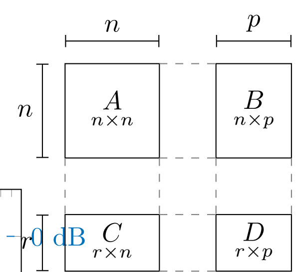
## 1.1 Marge de gain / marge de phase




# 2 Espaces d'états




## 2.1 Choix des variables d'état

- Condensateur : tension
- Bobine : courant

## 2.2 $A, B, C, D \longrightarrow G$

$$G(s) = C(sI - A)^{-1}B + D$$

$$G(z) = C_n(zI - A_n)^{-1}B_n + D_n$$

### 2.2.1 Gain haute fréquence

$$\lim_{s\to\infty} G(s) = D$$

### 2.2.2 Gain basse fréquence

$$G(0) = -CA^{-1}B + D$$

## 2.3 $G \longrightarrow A, B, C, D$

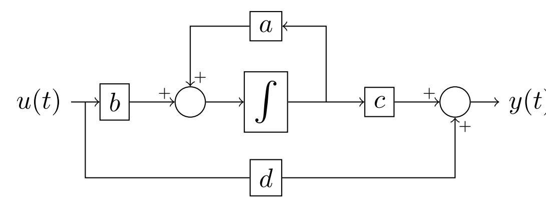
On utilise la forme commandable

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

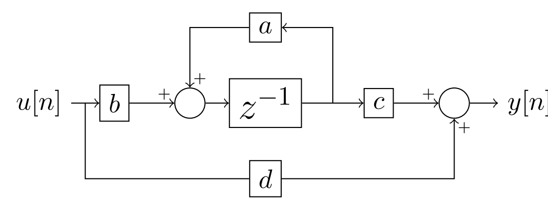
$$\boxed{\begin{array}{ll} A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} & B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ C = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} & D = 0 \end{array}}$$

## 2.4 $G(s)/G(z) \longleftrightarrow A, B, C, D$

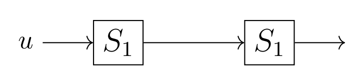
$$\boxed{d + \frac{b\cdot c}{s-a}} \qquad \boxed{\begin{array}{ll} A = \begin{bmatrix} a \end{bmatrix} & B = \begin{bmatrix} b \end{bmatrix} \\ C = \begin{bmatrix} c \end{bmatrix} & D = \begin{bmatrix} d \end{bmatrix} \end{array}}$$




$$\boxed{d + \frac{b\cdot c}{z-a}} \qquad \boxed{\begin{array}{ll} A_n = \begin{bmatrix} a \end{bmatrix} & B_n = \begin{bmatrix} b \end{bmatrix} \\ C_n = \begin{bmatrix} c \end{bmatrix} & D_n = \begin{bmatrix} d \end{bmatrix} \end{array}}$$




## 2.5 Mise en cascade

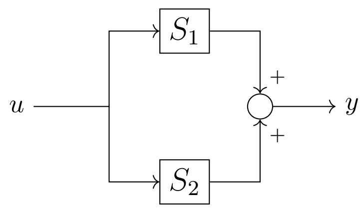



$S_{tot} = S_2(s)\cdot S_1(s)$ ordre important

$$A_{tot} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \qquad D_{tot} = D_2 D_1$$

## 2.6 Mise en parallèle

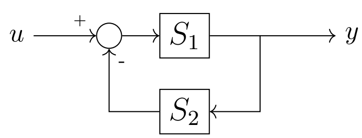



$$S_{tot}(s) = S_1(s) + S_2(s)$$

$$A_{tot} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \qquad D_{tot} = D_1 + D_2$$

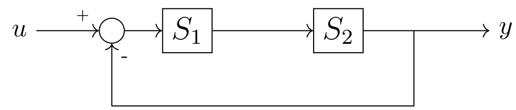
### 2.6.1 Mise en contre-réaction 1




$$A_{tot} = \begin{bmatrix} A_1 - B_1D_2(I - D_1D_2)^{-1}C_1 & -B_1(C_2 - D_2D_1C_2) \\ B_2(I - D_1D_2)^{-1}C_1 & A_2 - B_2(I - D_1D_2)^{-1}D_1C_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 - B_1D_2ND_1 \\ B_2ND_1 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} (I - D_1D_2)^{-1}C_1 & -(I - D_1D_2)^{-1}D_1C_2 \end{bmatrix} \qquad D_{tot} = (I - D_1D_2)^{-1}D_1$$

### 2.6.2 Mise en contre-réaction 2




$$S_{tot}(s) = (I + S_1(s)S_2(s))^{-1}\, S_1(s)$$

$$A_{tot} = \begin{bmatrix} A_1 & 0 \\ B_2C_1 & A_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 \\ B_2D_1 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} D_2C_1 & C_2 \end{bmatrix} \qquad D_{tot} = D_1D_2$$

## 2.7 Commandabilité

$$\boxed{P_c = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}}$$

Pour des systèmes monoentrée :

$$\det(P_c) \neq 0 \longrightarrow \text{Commandable}$$

Pour des systèmes multi-entrées (généralisation) :

$$\text{rang}(P_c) == n \longrightarrow \text{Commandable}$$

Faire une permutation avec $T$ ne change pas la commandabilité du système. La nouvelle matrice $\tilde{P_c}$ est donnée par $T^{-1}P_c$

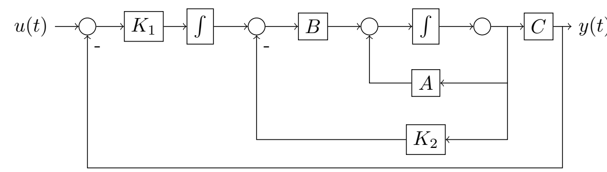
## 2.8 Observabilité

$$\boxed{P_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}}$$

$$\boxed{\text{rang}(P_0) = n \longrightarrow \text{Observable}}$$

En monosortie on peut utiliser $\det(P_0) \neq 0 \longrightarrow$ Observable

## 2.9 Action intégrale sur la commande




$$\hat{A} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

## 2.10 Placement de pôles

Il faut que le polynôme caractéristique de la boucle fermée (par exemple $sI - A_{bf} = sI - (A - BK)$) corresponde aux pôles que l'ont souhaite

$$\det(sI - A_{bf}) = (s - p_1)(s - p_2)\cdots(s - p_n)$$