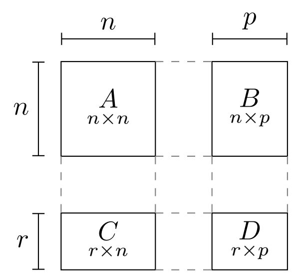
1 Espaces d'états

$$U(s) \xrightarrow[p \text{ signaux}]{} \boxed{G(s)} \xrightarrow[r \text{ signaux}]{} Y(s)$$

$$u(t) \longrightarrow \boxed{\begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}} \longrightarrow y(t)$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}$$



1.1 Choix des variables d'état

- Condensateur : tension
- Bobine : courant

1.2 $A, B, C, D \longrightarrow G$

$$G(s) = C(sI - A)^{-1}B + D$$

$$G(z) = C_n(zI - A_n)^{-1}B_n + D_n$$

1.2.1 Gain haute fréquence

$$\lim_{s\to\infty} G(s) = D$$

1.2.2 Gain basse fréquence

$$G(0) = -CA^{-1}B + D$$

1.3 $G \longrightarrow A, B, C, D$

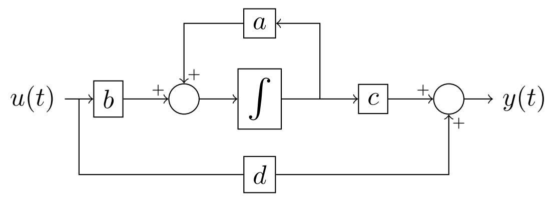
On utilise la forme commandable

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

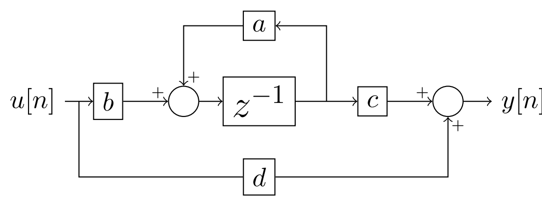
$$\boxed{\begin{array}{ll} A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} & B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ C = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} & D = 0 \end{array}}$$

1.4 $G(s)/G(z) \longleftrightarrow A, B, C, D$

$$\boxed{d + \frac{b \cdot c}{s - a}} \qquad \boxed{\begin{array}{ll} A = [a] & B = [b] \\ C = [c] & D = [d] \end{array}}$$



$$\boxed{d + \frac{b \cdot c}{z - a}} \qquad \boxed{\begin{array}{ll} A_n = [a] & B_n = [b] \\ C_n = [c] & D_n = [d] \end{array}}$$



1.5 Mise en cascade

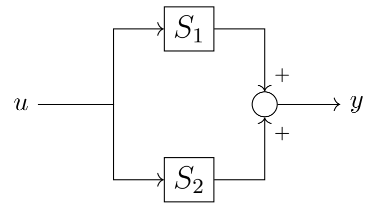
$$u \longrightarrow \boxed{S_1} \longrightarrow \boxed{S_1} \longrightarrow y$$

$$S_{tot} = S_2(s) \cdot S_1(s) \qquad \text{ordre important}$$

$$A_{tot} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \qquad D_{tot} = D_2 D_1$$

1.6 Mise en parallèle

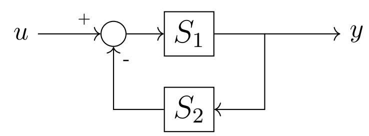


$$S_{tot}(s) = S_1(s) + S_2(s)$$

$$A_{tot} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \qquad D_{tot} = D_1 + D_2$$

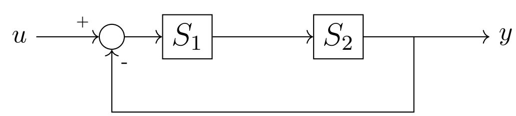
1.6.1 Mise en contre-réaction 1



$$A_{tot} = \begin{bmatrix} A_1 - B_1 D_2 (I - D_1 D_2)^{-1} C_1 & -B_1 (C_2 - D_2 D_1 C_2) \\ B_2 (I - D_1 D_2)^{-1} C_1 & A_2 - B_2 (I - D_1 D_2)^{-1} D_1 C_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 - B_1 D_2 N D_1 \\ B_2 N D_1 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} (I - D_1 D_2)^{-1} C_1 & -(I - D_1 D_2)^{-1} D_1 C_2 \end{bmatrix} \qquad D_{tot} = (I - D_1 D_2)^{-1} D_1$$

1.6.2 Mise en contre-réaction 2



$$S_{tot}(s) = (I + S_1(s) S_2(s))^{-1} S_1(s)$$

$$A_{tot} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \qquad B_{tot} = \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix}$$

$$C_{tot} = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \qquad D_{tot} = D_1 D_2$$

1.7 Commandabilité

$$\boxed{P_c = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}}$$

Pour des systèmes SISO :

$$\det(P_c) \neq 0 \longrightarrow \text{Commandable}$$

Pour des systèmes MIMO (généralisation) :

$$\text{rang}(P_c) == n \longrightarrow \text{Commandable}$$

Faire une permutation avec T ne change pas la commandabilité du système. La nouvelle matrice $\tilde{P}_c$ est donnée par $T^{-1}P_c$

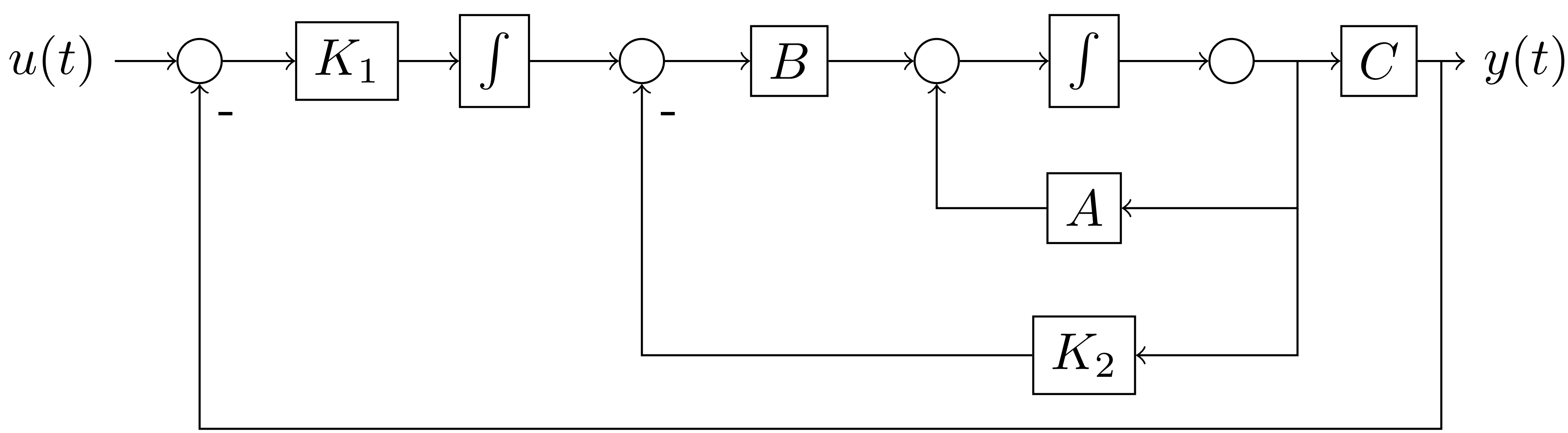
1.8 Observabilité

$$\boxed{P_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}}$$

$$\boxed{\text{rang}(P_0) = n \longrightarrow \text{Observable}}$$

En monosortie on peut utiliser $\det(P_0) \neq 0 \longrightarrow$ Observable

1.9 Action intégrale sur la commande



$$\hat{A} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

1.10 Placement de pôles

Il faut que le polynôme caractéristique de la boucle fermée (par exemple $sI - A_{bf} = sI - (A - BK)$) corresponde aux pôles que l'ont souhaite

$$\det(sI - A_{bf}) = (s - p_1)(s - p_2) \cdots (s - p_n)$$