Contents

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- Higher order, multi-level and cascaded modulators

- Exercises
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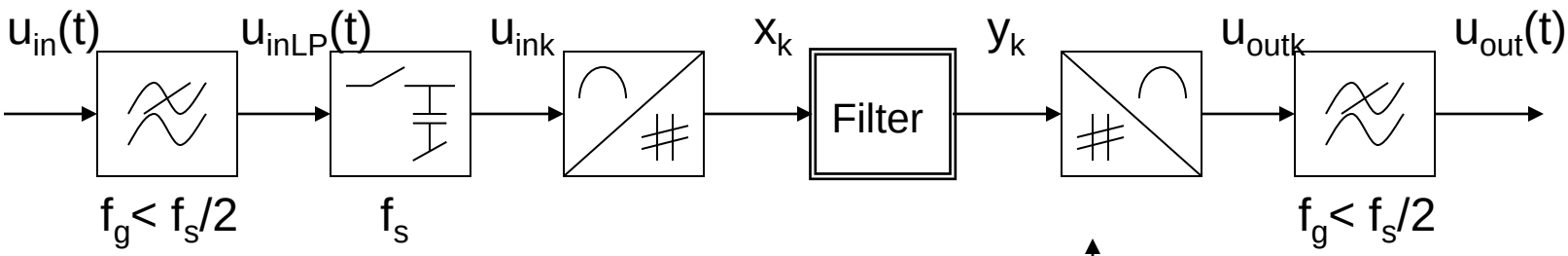
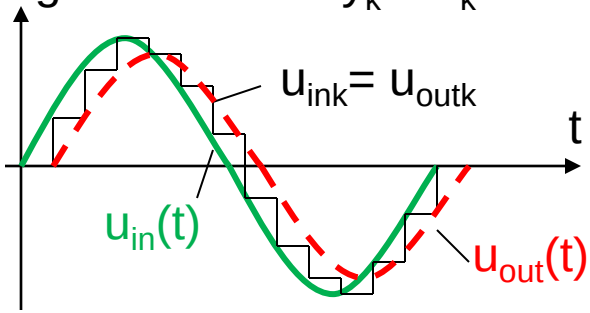
Signal processing chain

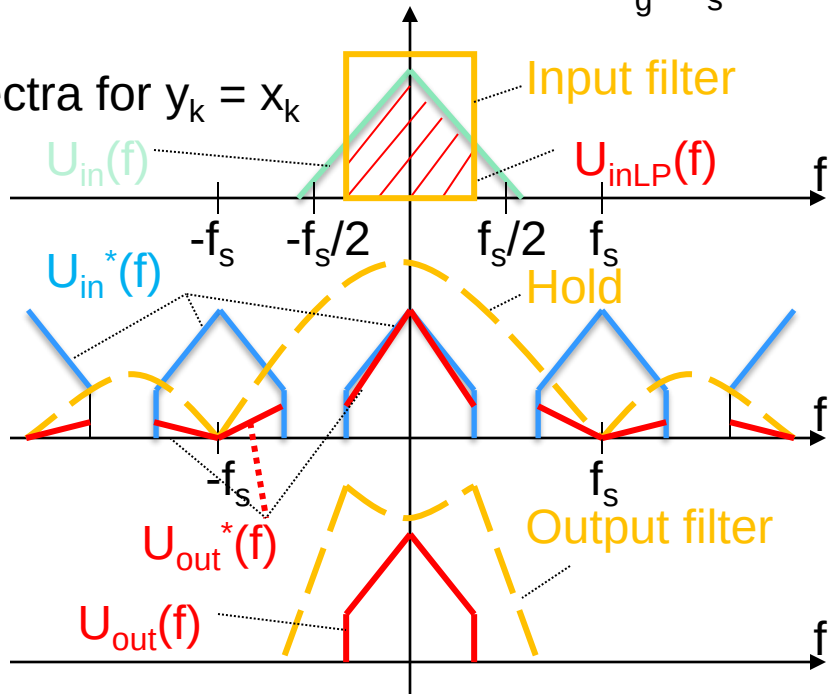


Signal forms for $y_k = x_k$



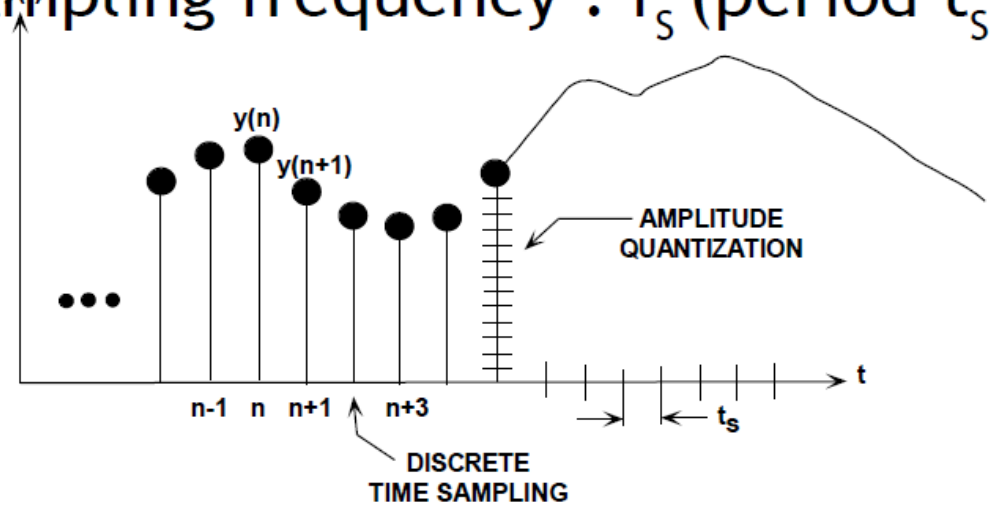
$$U^*_{out}(jf) = \frac{\sin\frac{\pi f}{f_s}}{\frac{\pi f}{f_s}} \frac{1}{f_s} \sum_{k=0}^{\infty} u_{in}(kT_s) e^{-2\pi jkf/f_s}$$

Spectra for $y_k = x_k$



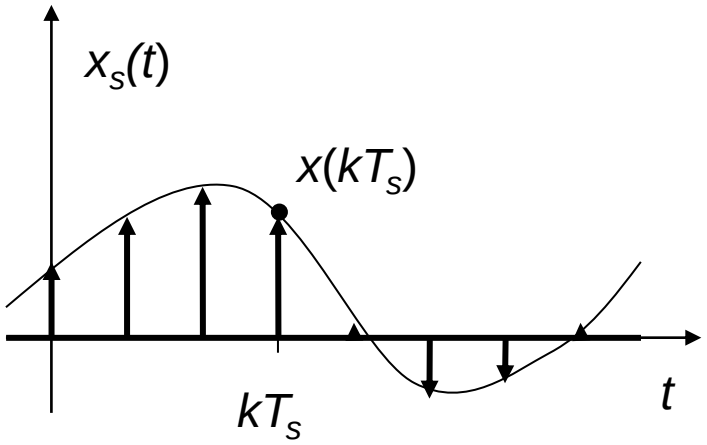
Discretization (quantization) of time

- The value of the signal taken at a particular moment : a sample
  - After sampling, the signal is reduce to a suite of amplitude quantifed numbers taken at regular time frequency
    - Unregular sampling period not used, only a sampling clock.
- Def.: Sampling frequency : f_s (period t_s)



Ideal regularly sampled signal

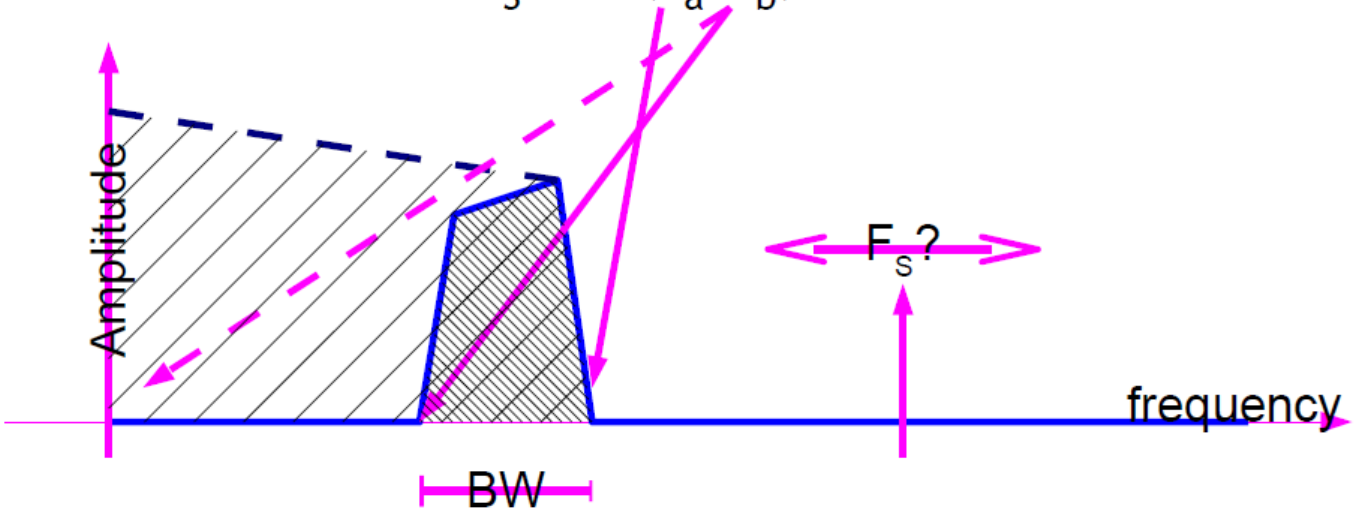
- Sampling a signal in the time domain corresponds to a multiplication by a Dirac Sequence (mathematical notation)



$$x_s(t) = \sum_{k=-\infty}^{+\infty} x(kT_s)\cdot\delta(t-kT_s) = x(t)\cdot\sum_{k=-\infty}^{+\infty}\delta(t-kT_s)$$

Nyquist criterion

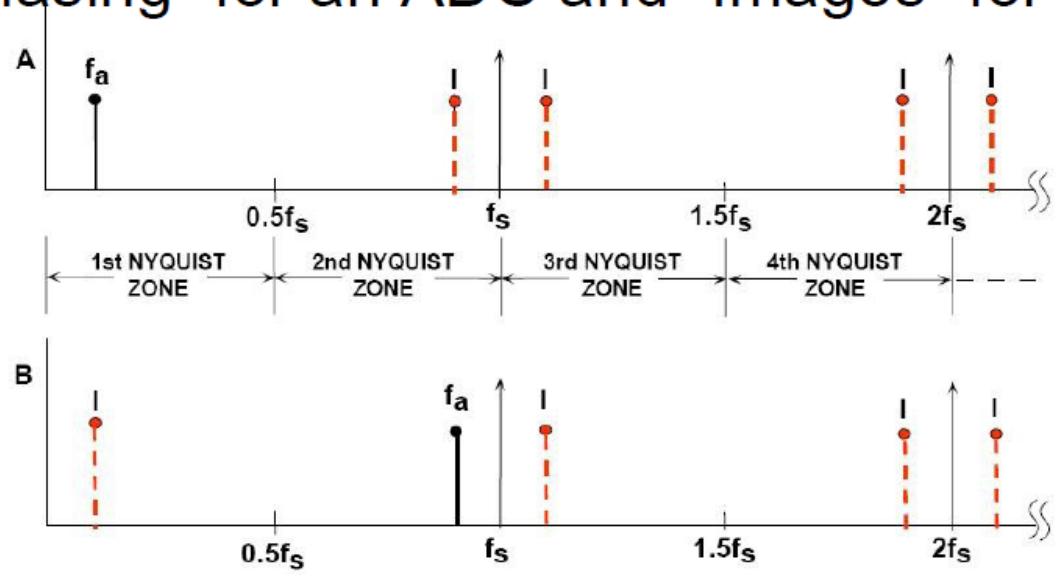
- Sampling frequency at least twice the *Signal BANDWITH* : $f_S > 2(f_a - f_b)$



- A converter used with a fS = or just ≥ 2fa is assumed to make a « Nyquist conversion »

Aliasing

- Sampling a signal causes a repetition of the signal spectrum around f_s and multiples of f_s
  - Repetition in Frequency Domain are called "Aliasing" for an ADC and "Images" for a DAC



Analog Signal f_a Sampled @ f_s Using Ideal Sampler Has Images (Aliases) at $|\pm Kf_s \pm f_a|$, K = 1, 2, 3, . . .

Signal to noise ratio

- Quant. Noise RMS is assumed = $V_{LSB}/\sqrt{12}$
- RMS of SinFS = $1/\sqrt{2} \cdot V_{LSB} \cdot 2^N/2$

$$SNR = 20\log\left(\frac{S_{RMS}}{N_{RMS}}\right) = 20\log\sqrt{\frac{12}{8}} + 20\log 2^N$$

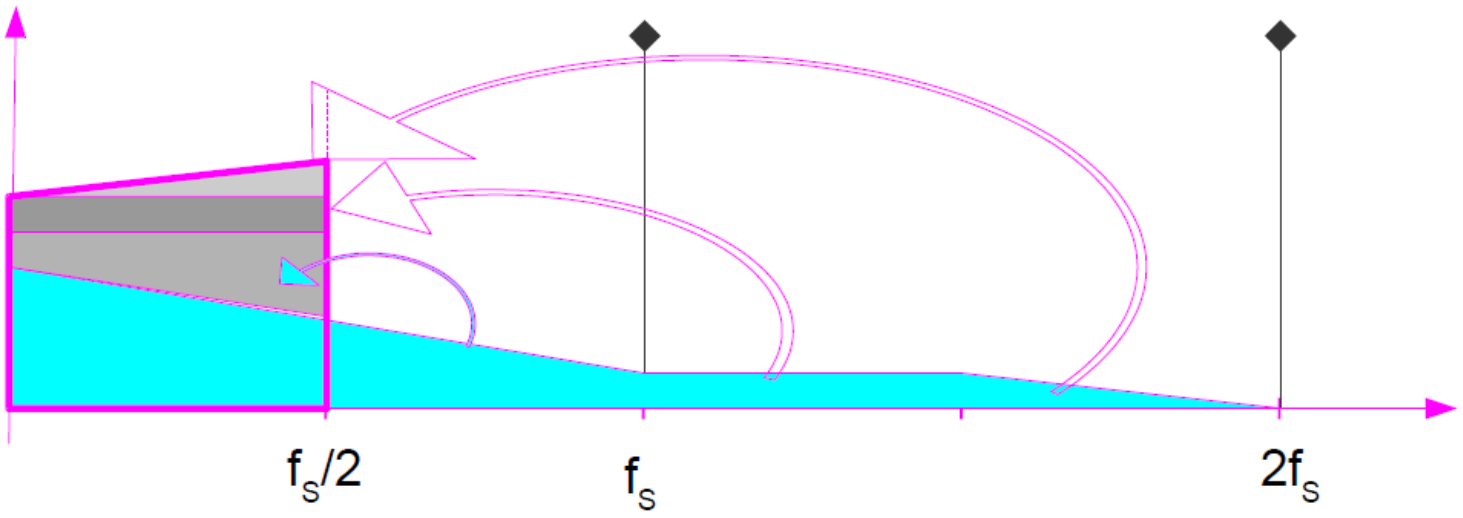
$$SNR = N \cdot 6.02\,dB + 1.76\,dB$$

[fig 2.39]

- All quantiz Noise is folded in Nyquist zone 1 (by aliasing !)

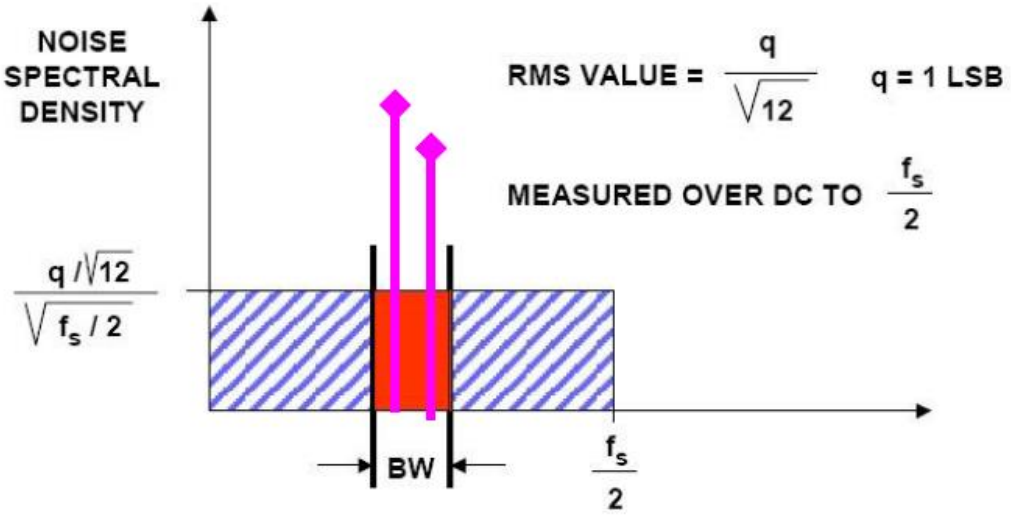
Wide band aliasing

- Wide band *noise or signal* will be folded in the « base band » (Nyquist zone 1 : [0 … $f_s/2$])



Process gain

- Process gain is the benefit when the signal don't use all the base band [0 .. $f_s/2$]



Quantization Noise Spectrum

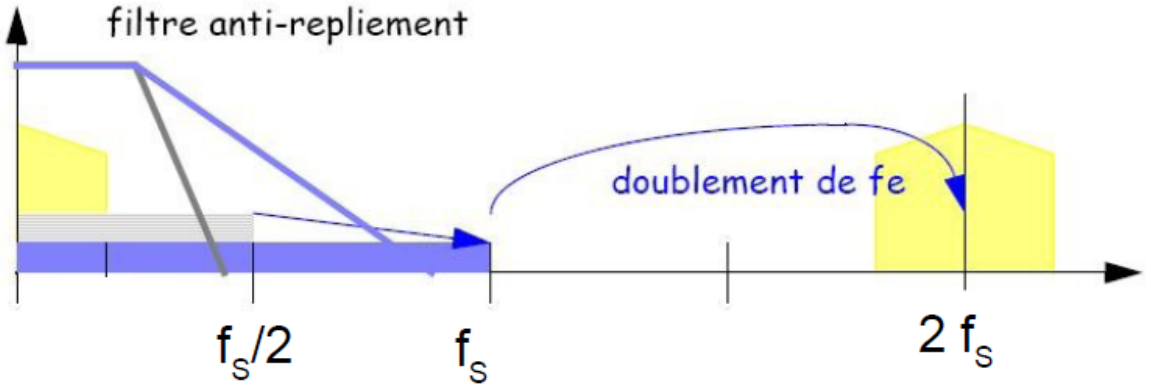
Oversampling

Oversamling is a convenient way to reduce noise - if BW of the signal effectivly filtered! -

- Oversampling ratio **OSR** : M time the Nyquist sample frequency ($f_s/2$)
- The SNR of a oversampled system looks like:

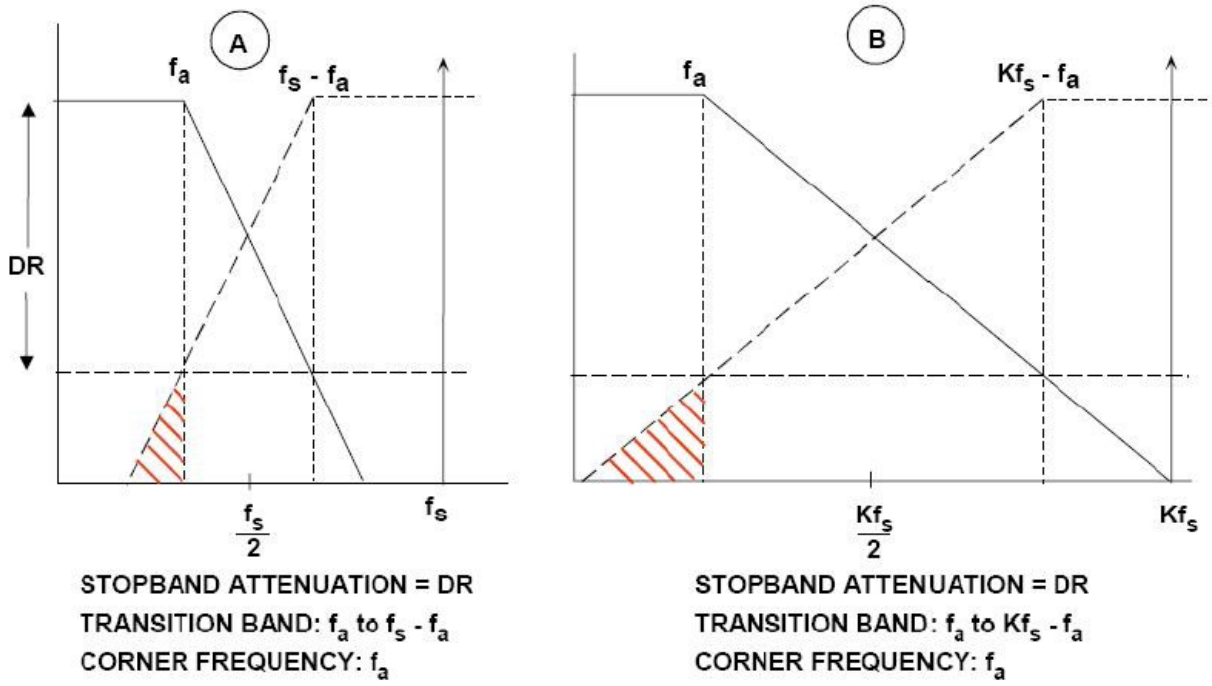
$$SNR = 1.76 + 6N + 10 \cdot \log(OSR)$$

- SNR better of +3dB each time f_s double !



Oversampling

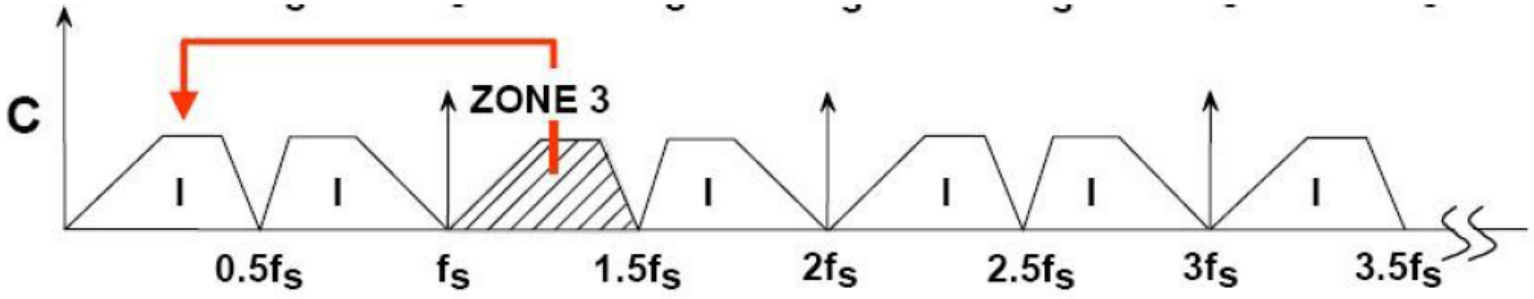
- Oversamling is a convenient way to reduce complexity of the antialiasing filter



Oversampling Relaxes Requirements on Baseband Antialiasing Filter

Undersampling

- Possible when signal is BW-limited (=> fS can be smaller than fb)



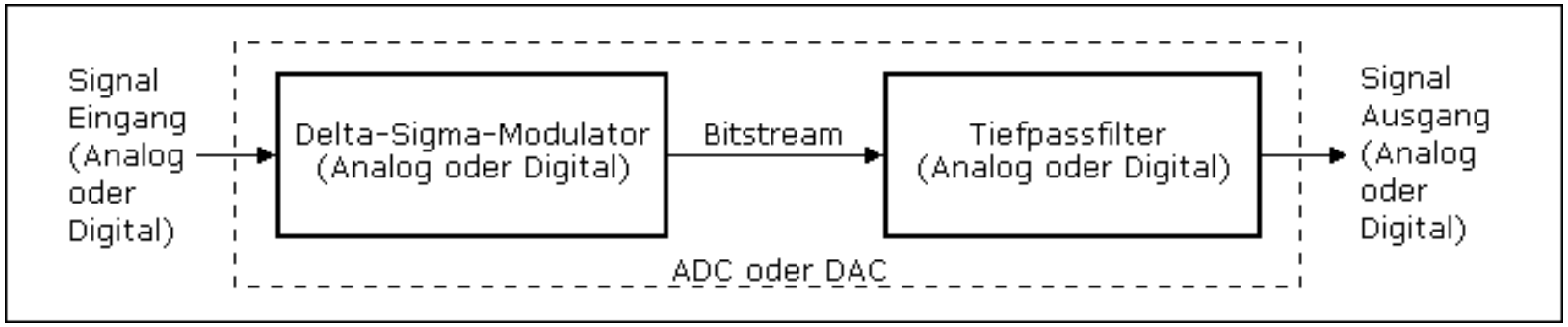
Undersampling and Frequency Translation Between Nyquist Zones

- Usual in communication systems (modulated signals) : undersampling process is *analog to demodulation !*
- But all ADC are not suited for undersampling : Caution analog BW, dynamic performances, etc.

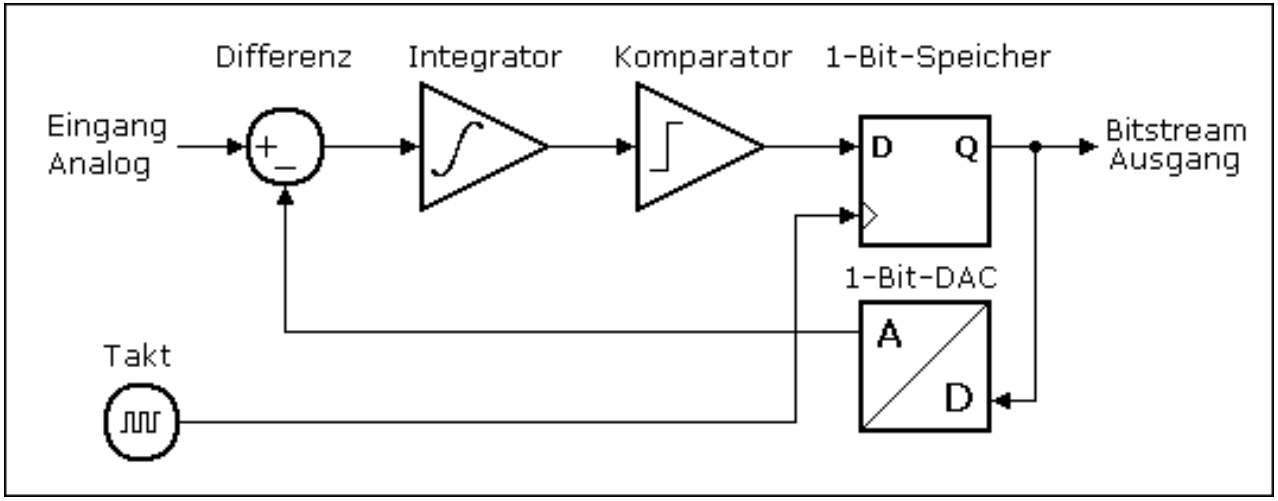
Effect of clock jitter on conversion noise

We have a 20bit audio SAR ADC with a sampling clock rate of f_s = 48ksps. How much rms jitter of the ADC clock signal (suppose $f_{CLK} = 24 \cdot f_s$) can be admitted, so that the conversion error remains below one LSB for sinusoidal full scale input (±1V) signals of frequency f = 4kHz?

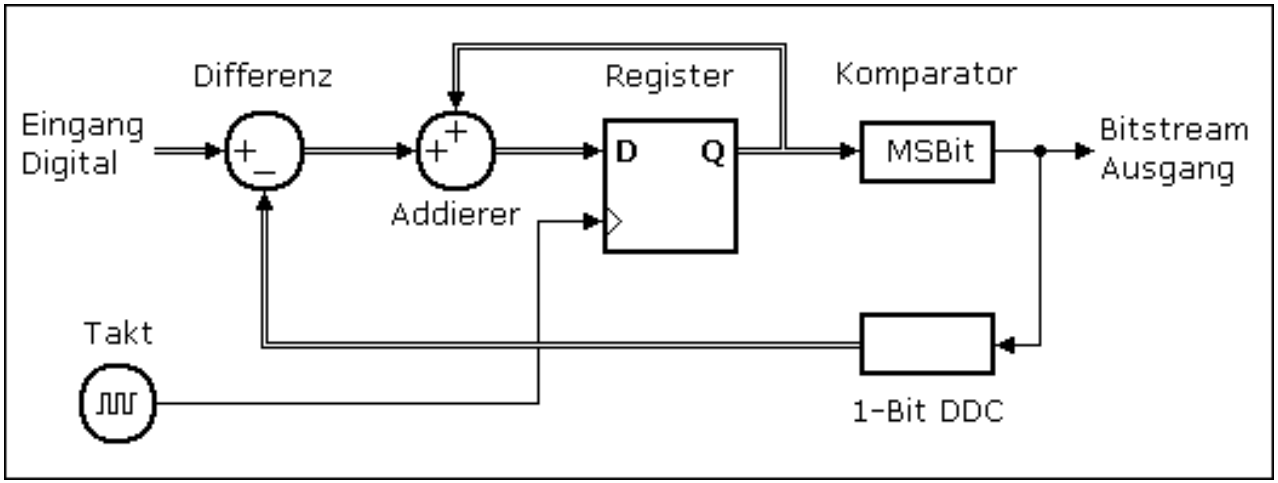
Δ-Σ-Conversion



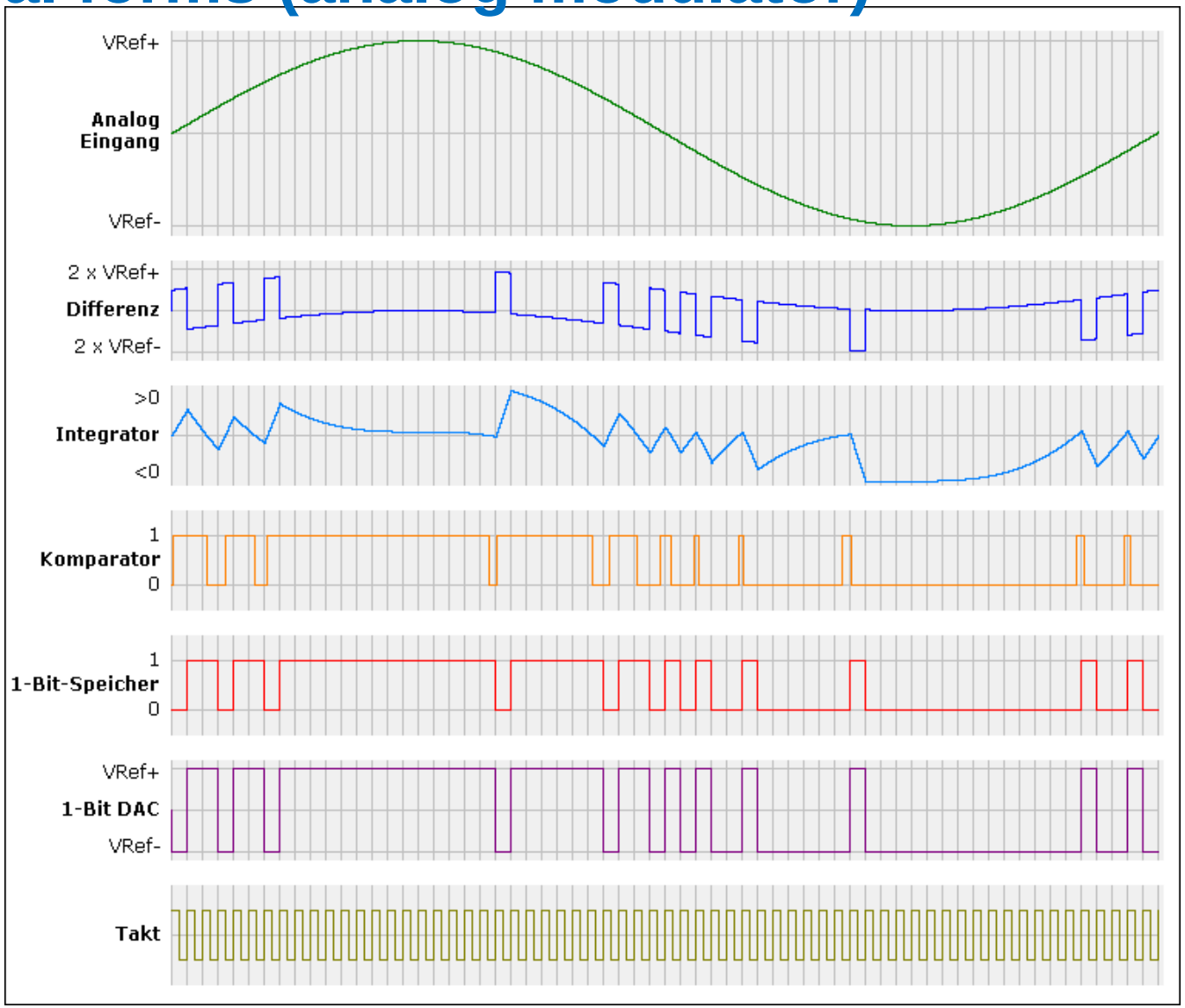
Analog first order modulator



Digital first order modulator

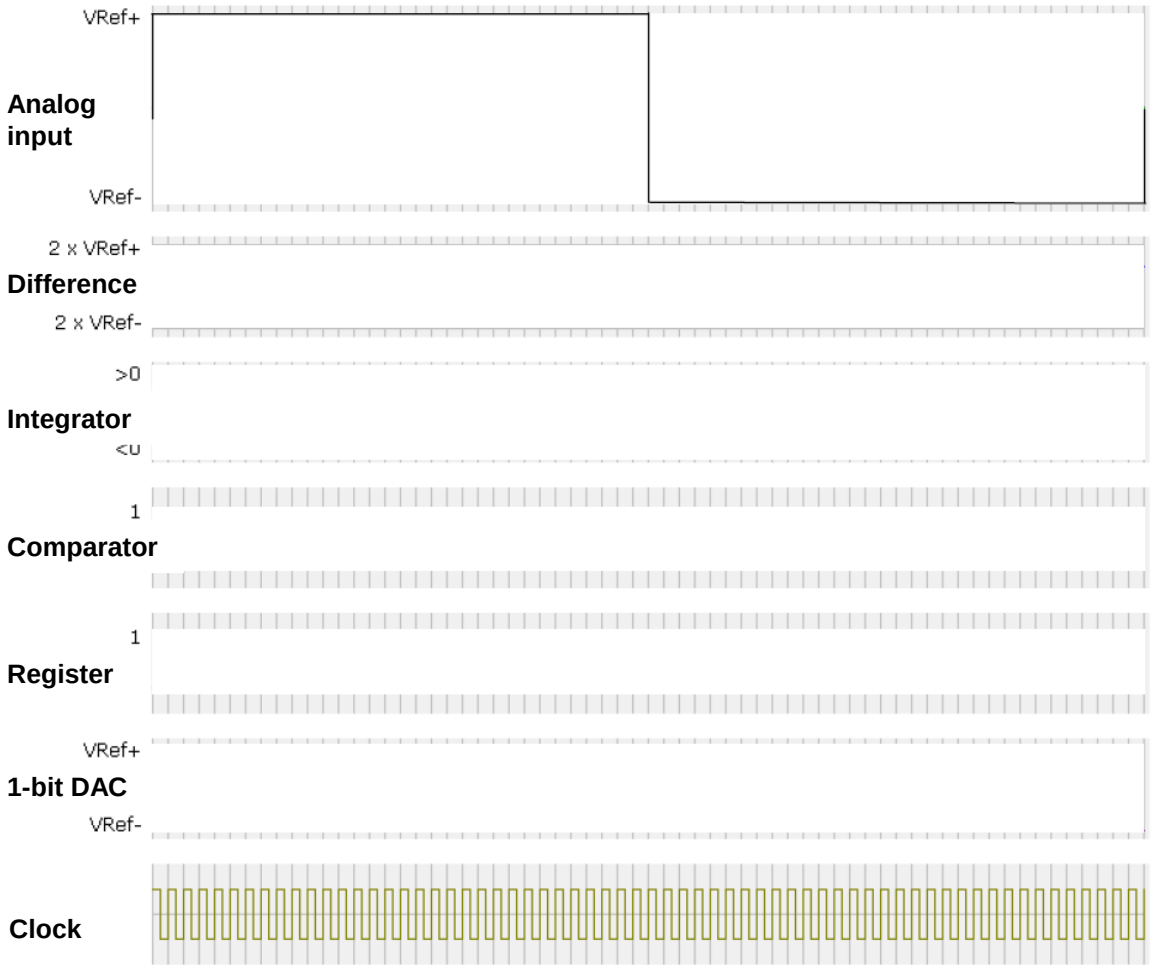


Signal forms (analog modulator)



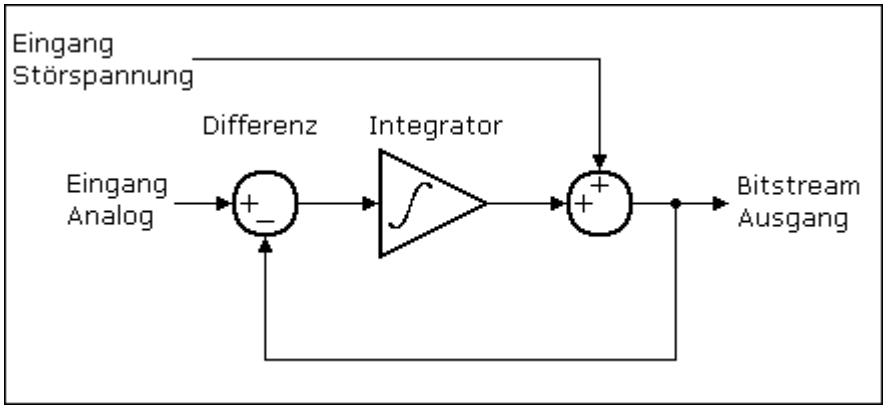
Signal forms

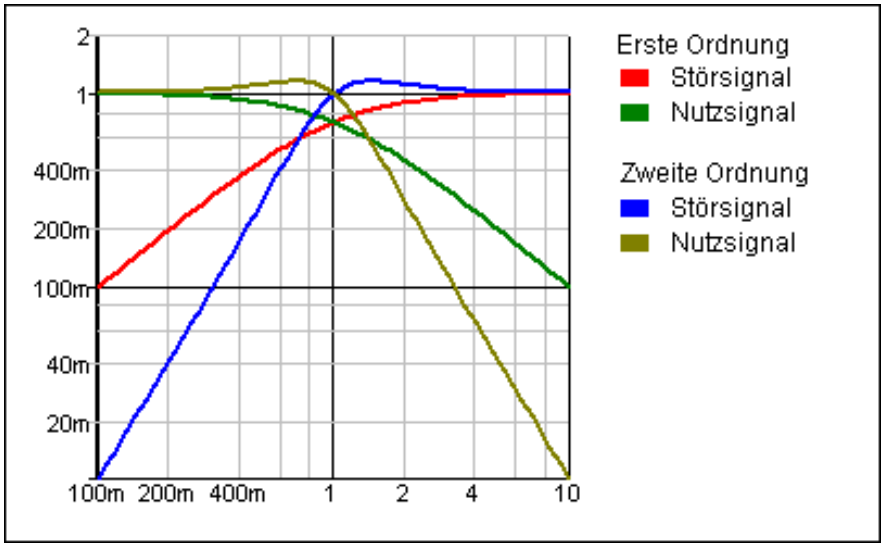
Trace the forms of signals in an analog first order modulator for a rectangular input signal:



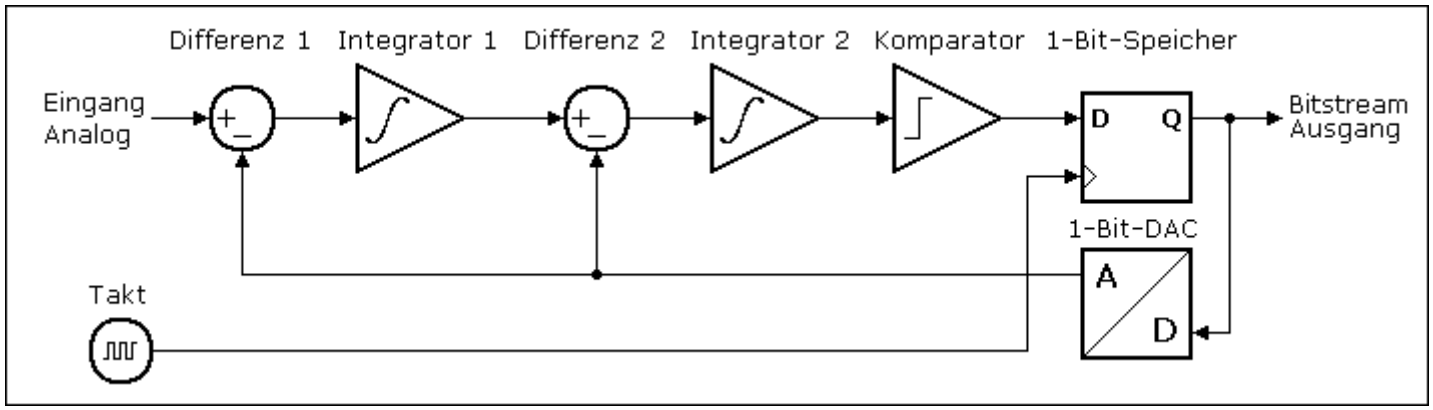
Retrace the bitstream after decimation by 8.

Dynamic behavior of the modulator



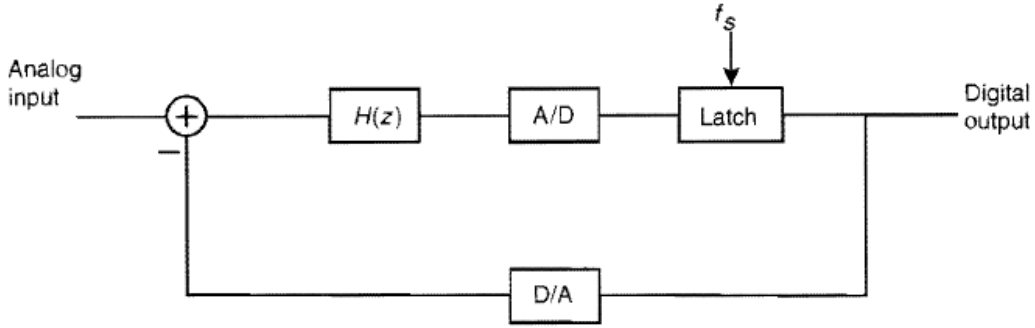


Second order modulator transfer functions

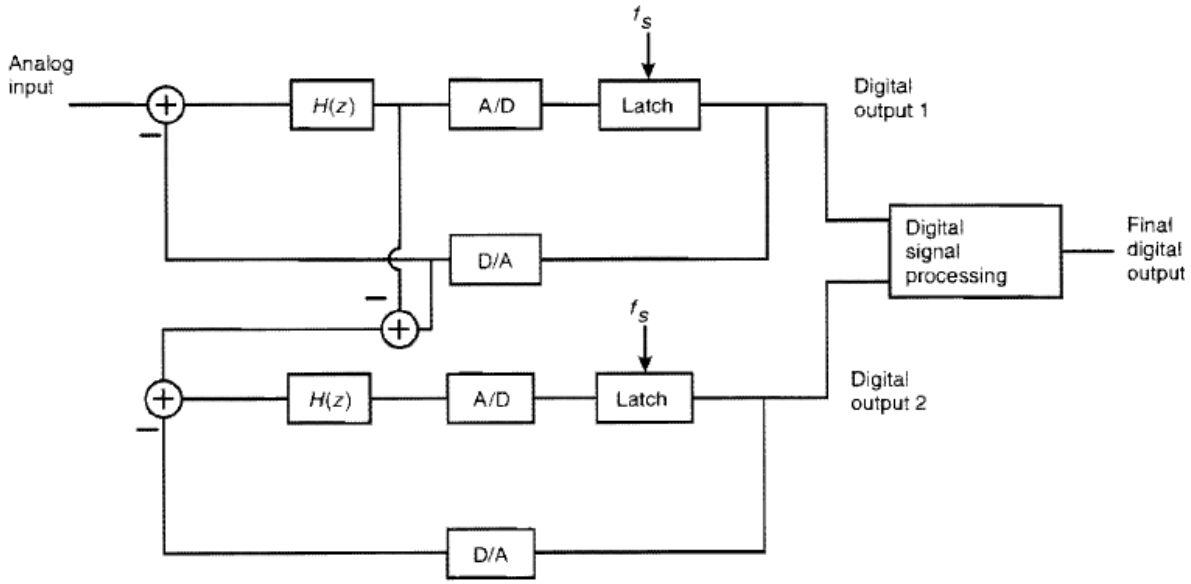


- Determine the equivalent continuous transfer functions of a second order delta-sigma modulator, for the signal and for the noise.
- Represent the corresponding amplitude responses in a Bode diagram.

Cascade of two first order demodulators

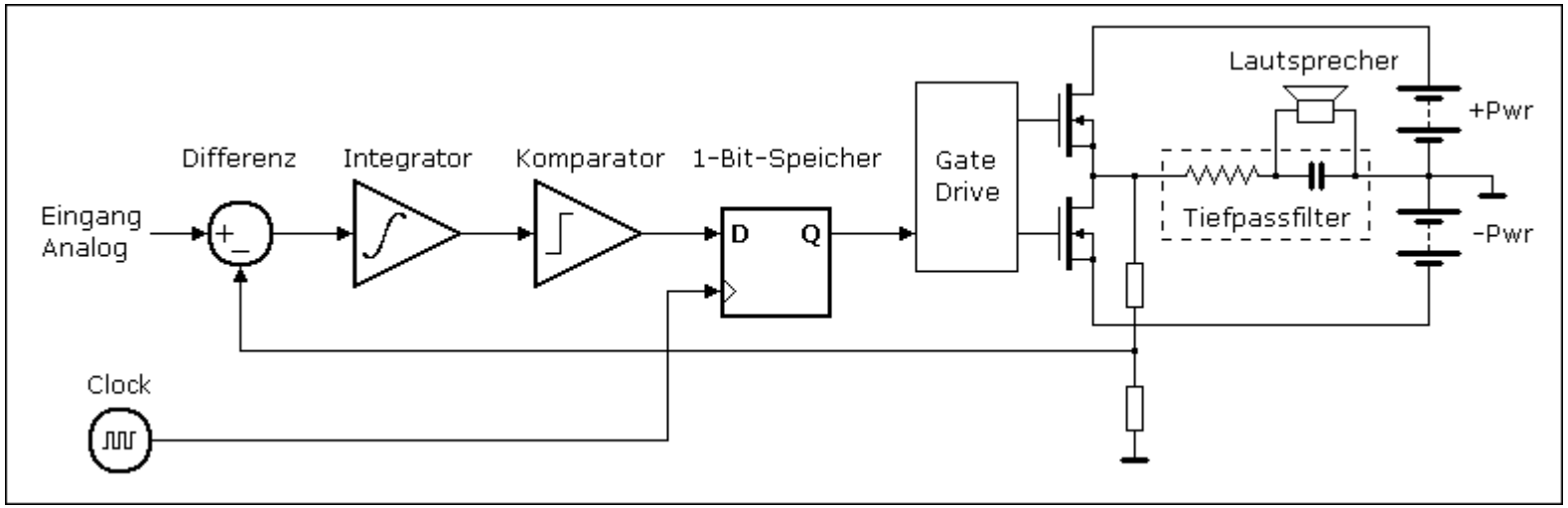


Block diagram of single-loop modulator.



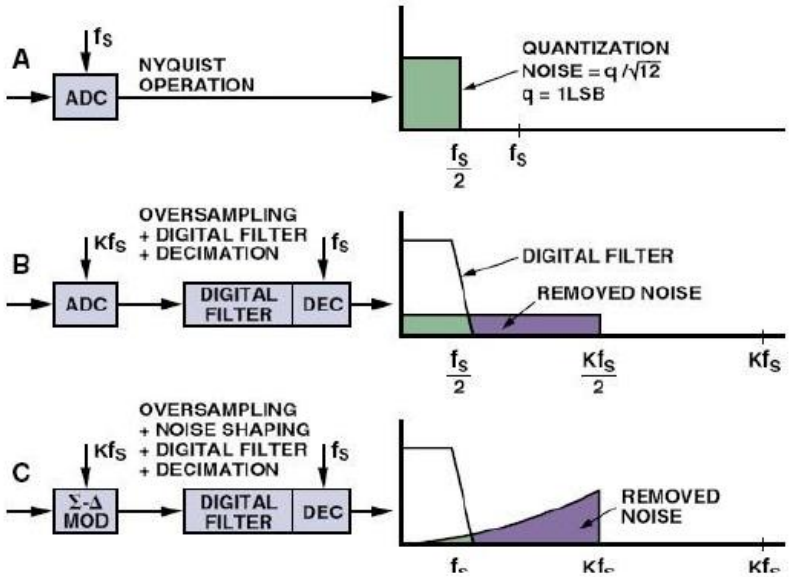
Block diagram of cascaded modulator.

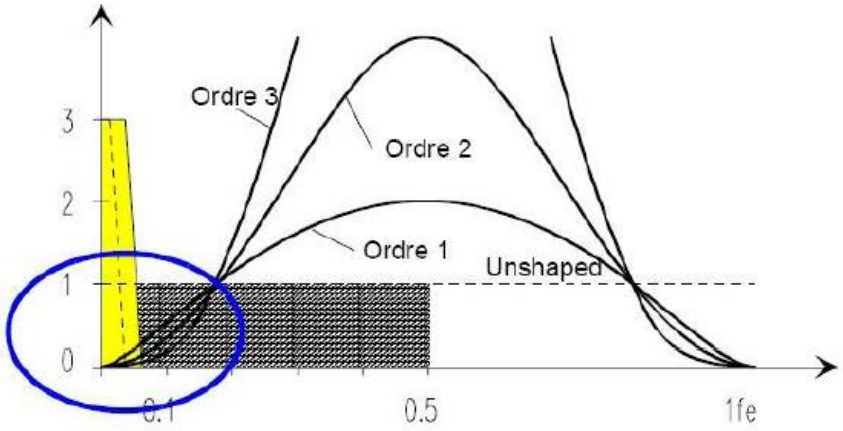
Power amplifier with digital output stage



Oversampling and delta-sigma modulation

- Why S-D converters are much better than simple oversampled systems?
  - SD modulation use Oversampling (OSR : 32..512...) and add a noiseshaping:



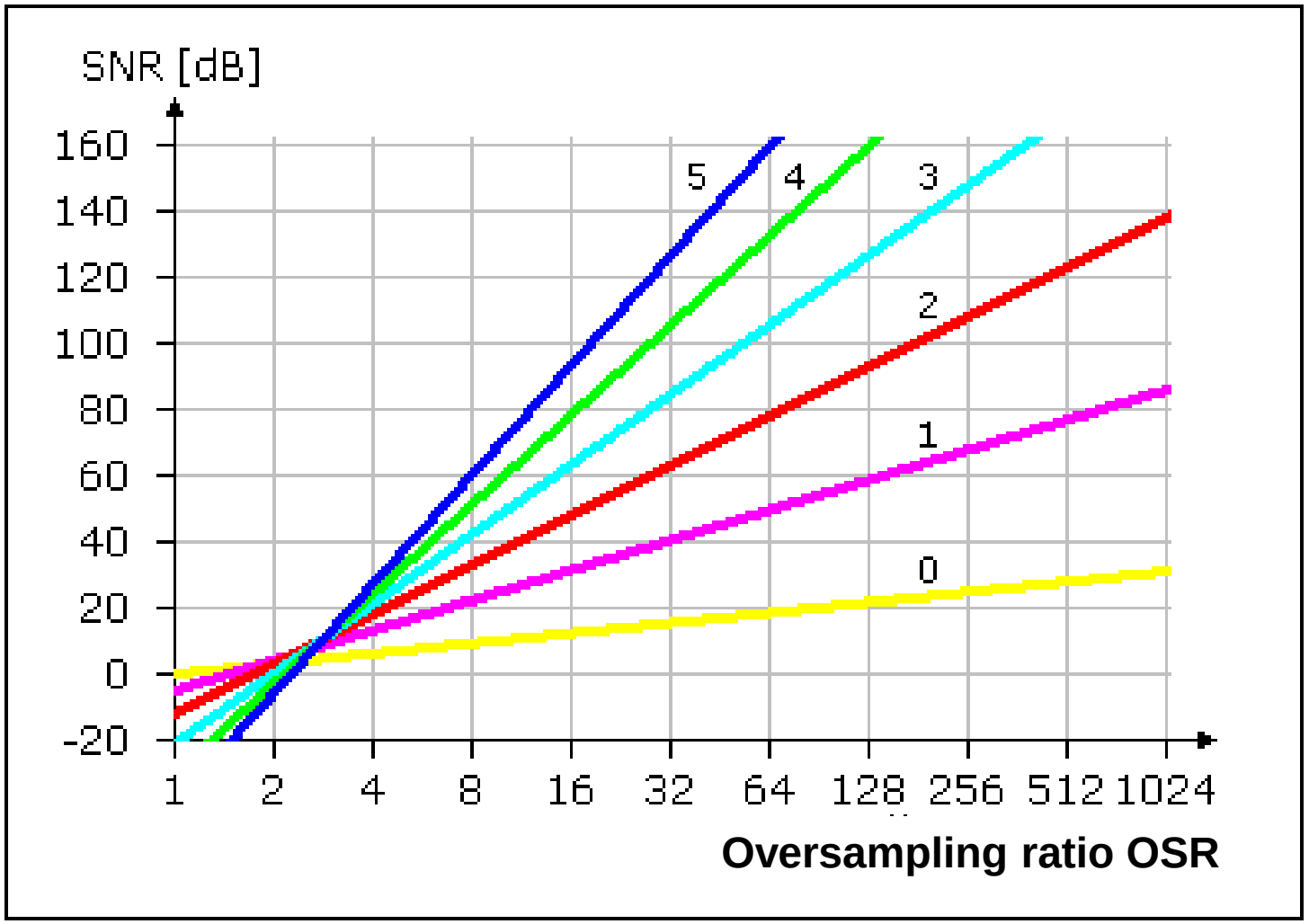


Influence of the order of delta-sigma modulation

- SNR in Function of the order L & OSR :

Order of modulator	1	2	3	L
Transfer N(z)/E(z)	$1-z^{-1}$	$(1-z^{-1})^2$	$(1-z^{-1})^3$	$(1-z^{-1})^L$
Base band noise n_0 (Δ = quant-isation step)	$\frac{\Delta\pi}{6}\sqrt{\left(\frac{1}{OSR}\right)^3}$	$\frac{\Delta\pi^2}{\sqrt{60}}\sqrt{\left(\frac{1}{OSR}\right)^5}$	$\frac{\Delta\pi^3}{\sqrt{84}}\sqrt{\left(\frac{1}{OSR}\right)^7}$	$\frac{\Delta\pi^L}{\sqrt{12(2L+1)}}\sqrt{\left(\frac{1}{OSR}\right)^{2L+1}}$
S/N	$\frac{9}{(2\pi)^2\left(\frac{1}{OSR}\right)^3}$	$\frac{15}{(2\pi)^4\left(\frac{1}{OSR}\right)^5}$	$\frac{21}{(2\pi)^6\left(\frac{1}{OSR}\right)^7}$	$\frac{3(2L+1)}{(2\pi)^{2L}\left(\frac{1}{OSR}\right)^{2L+1}}$

Signal to noise ratio versus oversampling ratio

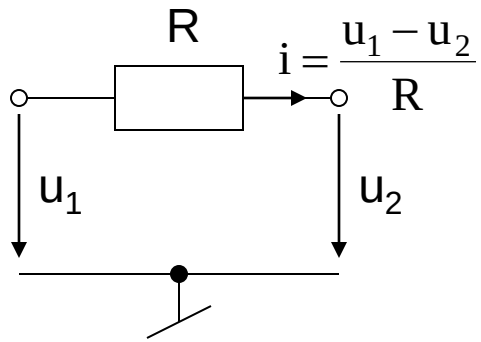


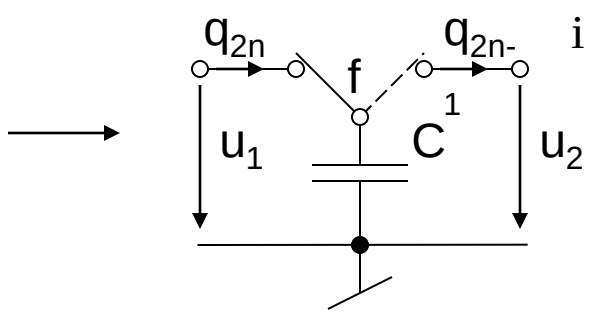
Signal to noise and oversampling ratios

- What is the maximum signal-to-noise ratio of a 16bit ADC? Suppose a sinusoidal input signal, and that the quantisation noise dominates all other noises. The noise is considered as white noise.
- Which resolution, expressed as number of bits (ENOB), can have an ADC with a second order delta-sigma modulator, operating at an oversampling ratio of 128?
- Consider a delta-sigma ADC with ENOB = 16bits and an input signal bandwidth of 1kHz. The oversampling ratio is 128. Which order is required for an analog anti-aliasing filter?

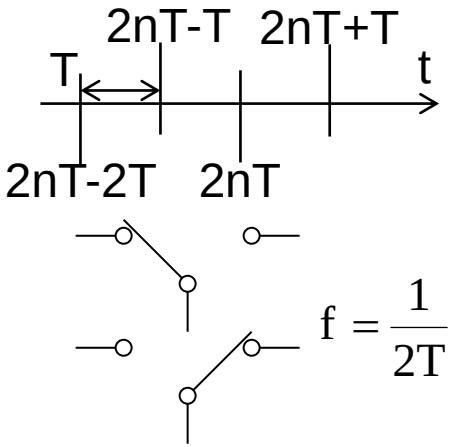
From the resistor to the switched capacitor

In integrated circuits, replacement of area consuming resistors with switched capacitors.





$i \cong Cf(u_1 - u_2)$



Definition of commutation intervals:

even: $2nT - T \le t \le 2nT$

odd: $2nT - 2T \le t \le 2nT - T$

Charge transport

in the even interval: $q_{2n} = Cu_1(2nT) - Cu_2(2nT - T)$

in the odd interval: $q_{2n-1} = Cu_1(2nT - 2T) - Cu_2(2nT - T)$

if u_1 and u_2 are constant, $q_{2n} = q_{2n-1} = q = C(u_1 - u_2)$

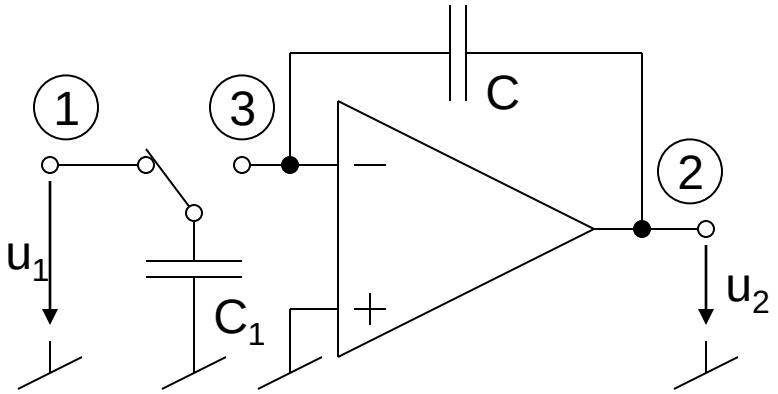
Average of current $I = \frac{q}{2T} = \frac{C}{2T}(u_1 - u_2) = Cf(u_1 - u_2)$

Integrator with switched capacitor

Analysis method:

1) Number nodes (here 1, 2, 3)

2) Establish charge equilibria of inner nodes (here only 3)



- even circuit:

$$C\left(u_2(2nT)-u_3(2nT)\right)=C\left(u_2(2nT-T)-u_3(2nT-T)\right)$$

$$u_3\equiv 0 \quad\Rightarrow\quad u_2(2nT)=u_2(2nT-T)$$

- odd circuit:

$$C\left(u_2(2nT-T)-u_3(2nT-T)\right)+C_1\left(0-u_3(2nT-T)\right)=$$

$$=C\left(u_2(2nT-2T)-u_3(2nT-2T)\right)+C_1\left(0-u_1(2nT-2T)\right)$$

$$u_3\equiv 0 \quad\Rightarrow\quad Cu_2(2nT-T)=Cu_2(2nT-2T)-C_1u_1(2nT-2T)$$

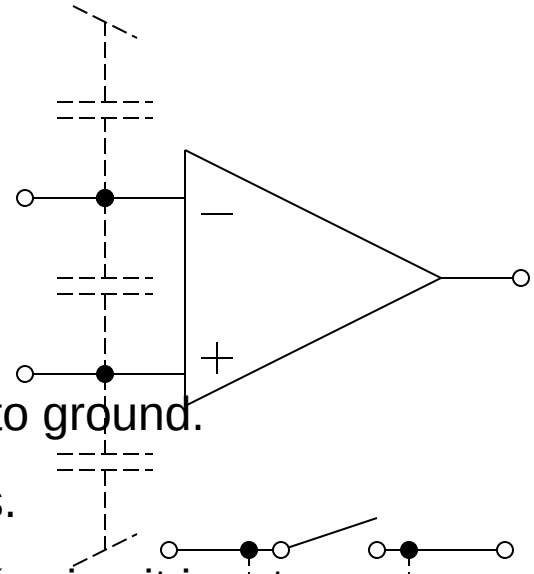
3) Put in relation the start of an odd with the end of an even interval

$$Cu_2(2nT)=Cu_2(2nT-2T)-C_1u_1(2nT-2T)$$

4) z-transform

$$CU_2(z)=CU_2(z)z^{-1}-C_1U_1(z)z^{-1} \quad\Rightarrow\quad \frac{U_2(z)}{U_1(z)}=-\frac{C_1}{C}\frac{1}{z-1}$$

Parasitic capacitances

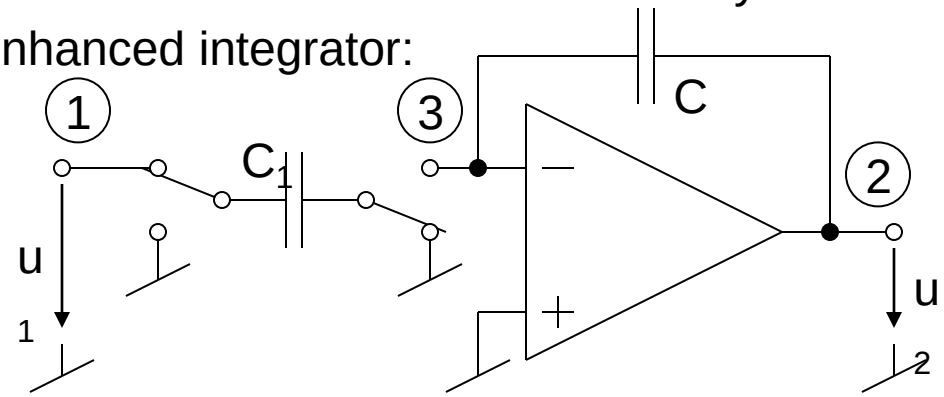


Parasitic capacitances appear in all circuits.

Circuits that are insensitive to parasitic capacitances, have the following 4 properties:

- Operational amplifiers have one input connected to ground.
- There is no node that connects only to capacitors.
- No commutations occur between a voltage node (a circuit input or an output of an operational amplifier) and virtual ground.
- There is no node that connects only to switches.

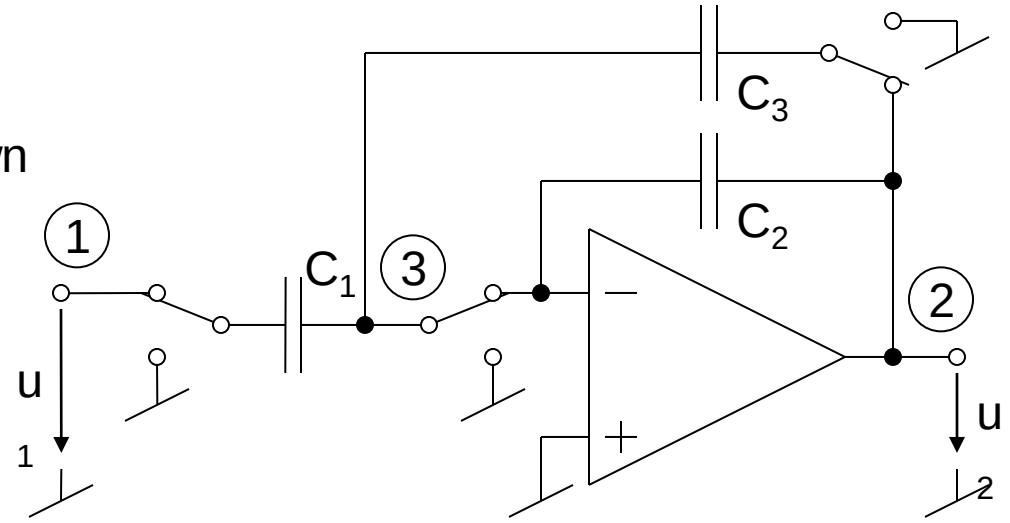
Enhanced integrator:



$$\frac{U_2(z)}{U_1(z)} = +\frac{C_1}{C}\frac{1}{z-1}$$

First order low pass filter

Position of switches shown is for the even interval.



- even circuit:

$$C_1\left(u_{1,2nT}-u_{3,2nT}\right)+\left(C_2+C_3\right)\left(u_{2,2nT}-u_{3,2nT}\right)=$$
$$=C_1\left(0-u_{3,2nT-T}\right)+C_2\left(u_{2,2nT-T}-0\right)+C_3\left(0-u_{3,2nT-T}\right)$$

$$u_{3,2nT}=u_{3,2nT-T}\equiv 0 \quad\Rightarrow\quad C_1u_{1,2nT}+\left(C_2+C_3\right)u_{2,2nT}=C_2u_{2,2nT-T}$$

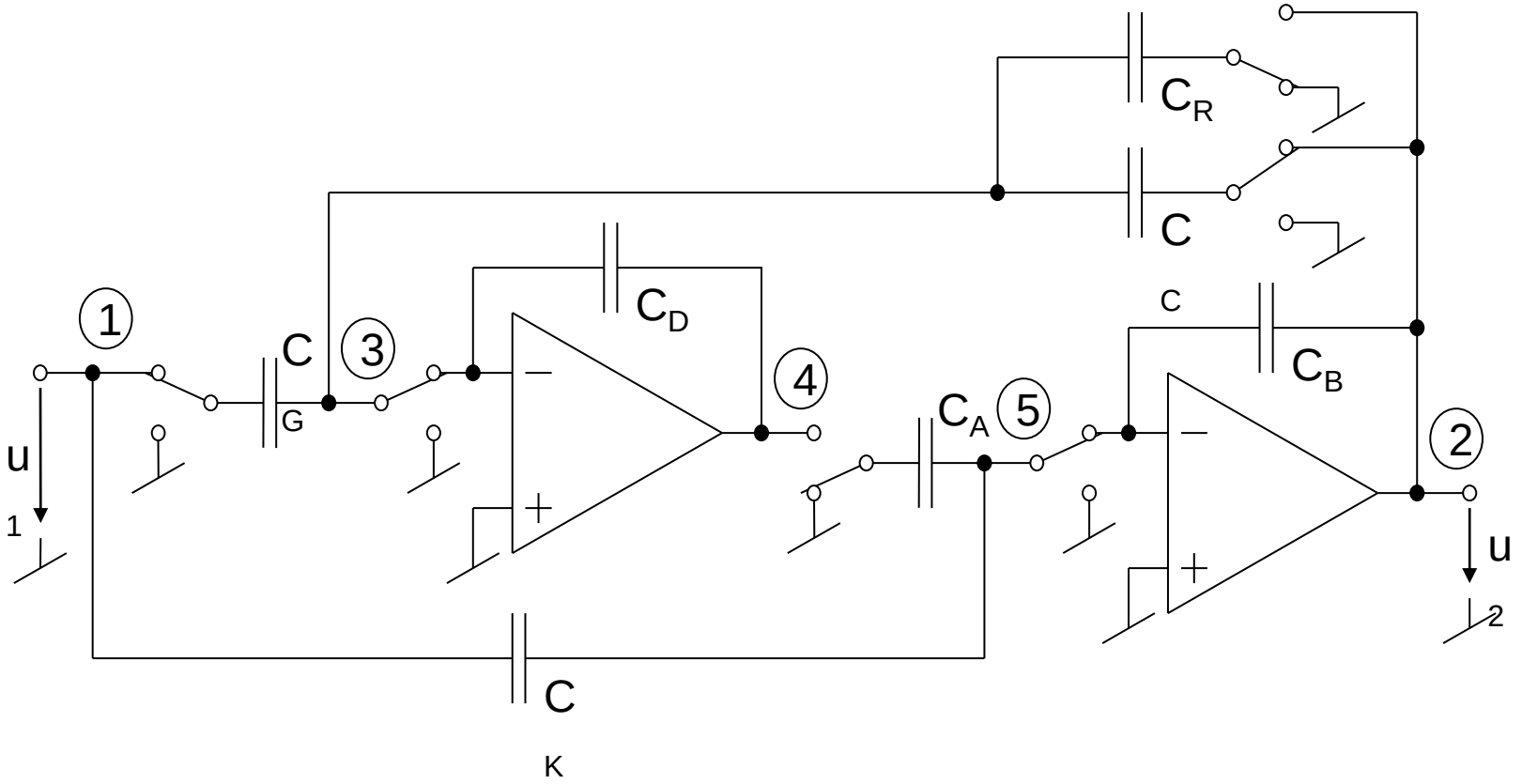
- odd circuit:

$$u_{2,2nT-T}=u_{2,2nT-2T}$$
$$C_1U_1(z)+\left(C_2+C_3\right)U_2(z)=C_2U_2(z)z^{-1}$$

- transfer function:

$$\frac{U_2(z)}{U_1(z)}=\frac{C_1}{C_2z^{-1}-C_2-C_3}$$

Second order low pass filter (1)



Position of switches shown is for the even interval.

Second order low pass filter (2)

-Even circuit:

Node ③

$$C_G\left(u_{1,2nT}-u_{3,2nT}\right)+C_D\left(u_{4,2nT}-u_{3,2nT}\right)+C_C\left(u_{2,2nT}-u_{3,2nT}\right)+C_R\left(0-u_{3,2nT}\right)=$$
$$=C_G\left(0-u_{3,2nT-T}\right)+C_D\left(u_{4,2nT-T}-u_{3,2nT-T}\right)+C_C\left(0-u_{3,2nT-T}\right)+C_R\left(u_{2,2nT-T}-0\right)$$

$$u_{3,2nT}=u_{3,2nT-T}\equiv 0 \quad\Rightarrow\quad C_G u_{1,2nT}+C_D u_{4,2nT}+C_C u_{2,2nT}=C_D u_{4,2nT-T}+C_R u_{2,2nT-T}$$

Node ⑤

$$C_A\left(0-u_{5,2nT}\right)+C_B\left(u_{2,2nT}-0\right)+C_K\left(u_{1,2nT}-u_{5,2nT}\right)=$$
$$=C_A\left(u_{4,2nT-T}-0\right)+C_B\left(u_{2,2nT-T}-u_{5,2nT-T}\right)+C_K\left(u_{1,2nT-T}-0\right)$$

$$u_{5,2nT}=u_{5,2nT-T}\equiv 0 \quad\Rightarrow\quad C_B u_{2,2nT}+C_K u_{1,2nT}=C_A u_{4,2nT-T}+C_B u_{2,2nT-T}+C_K u_{1,2nT-T}$$

- Odd circuit: $u_{2,2nT-T}=u_{2,2nT-2T}$, $u_{4,2nT-T}=u_{4,2nT-2T}$

$$\text{Assumption}: \quad u_{1,2nT-T}=u_{1,2nT-2T} \quad (\text{S \& H in front of the filter})$$

- Transfer function:

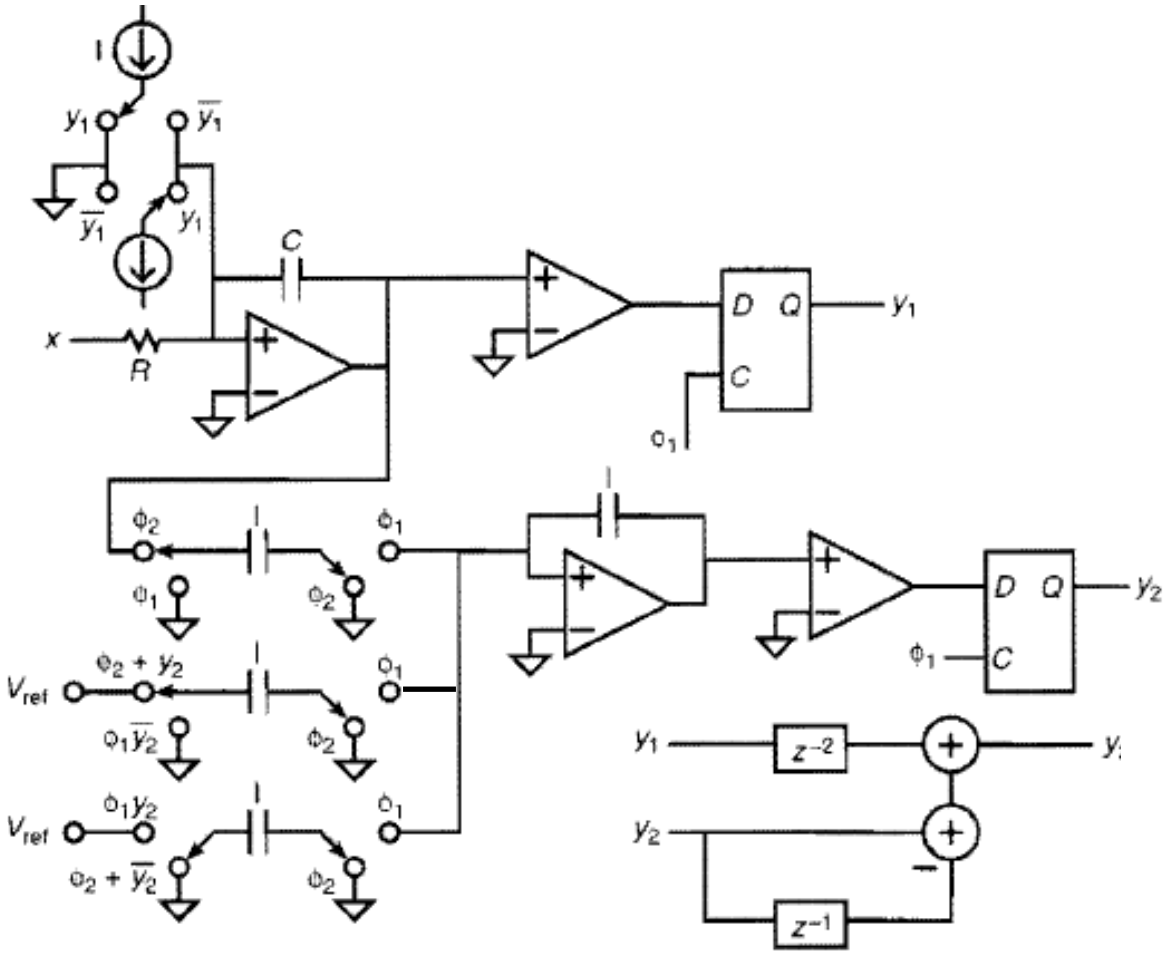
place ⑤ in ③ to eliminate $u_4 \Rightarrow$

$$\frac{U_2(z)}{U_1(z)}=-\frac{C_K}{C_B}\,\frac{z^2+\left(\frac{C_A C_G}{C_D C_K}-2\right)z+1}{z^2+\left(\frac{C_A C_C}{C_B C_D}-2\right)z+1-\frac{C_A C_R}{C_B C_D}}$$

Second order high pass filters

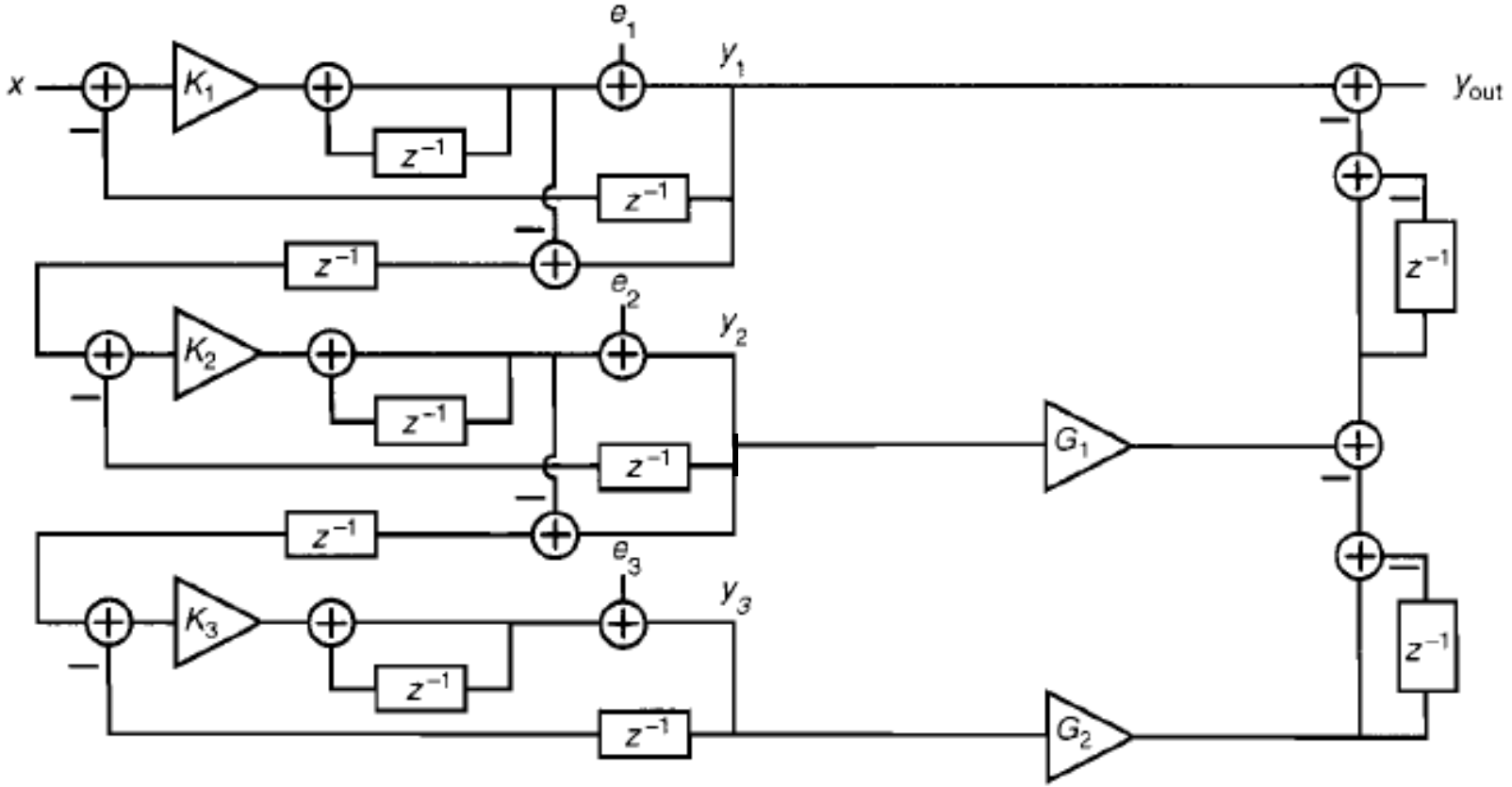
- Propose circuits for second order switched capacitor high-pass filters.

Cascaded second order modulator circuit



A (1-1) cascade, with continuous-time first modulator.

Cascaded third order modulator structure



- Develop the difference equation relating yout to x, to e1, e2 and e3.
- Estimate the frequency response of yout excited by x, and the noise spectrum of yout, assuming white noise inputs e1, e2, e3.
- Represent your results in graphs.

Cascaded third order modulator circuit

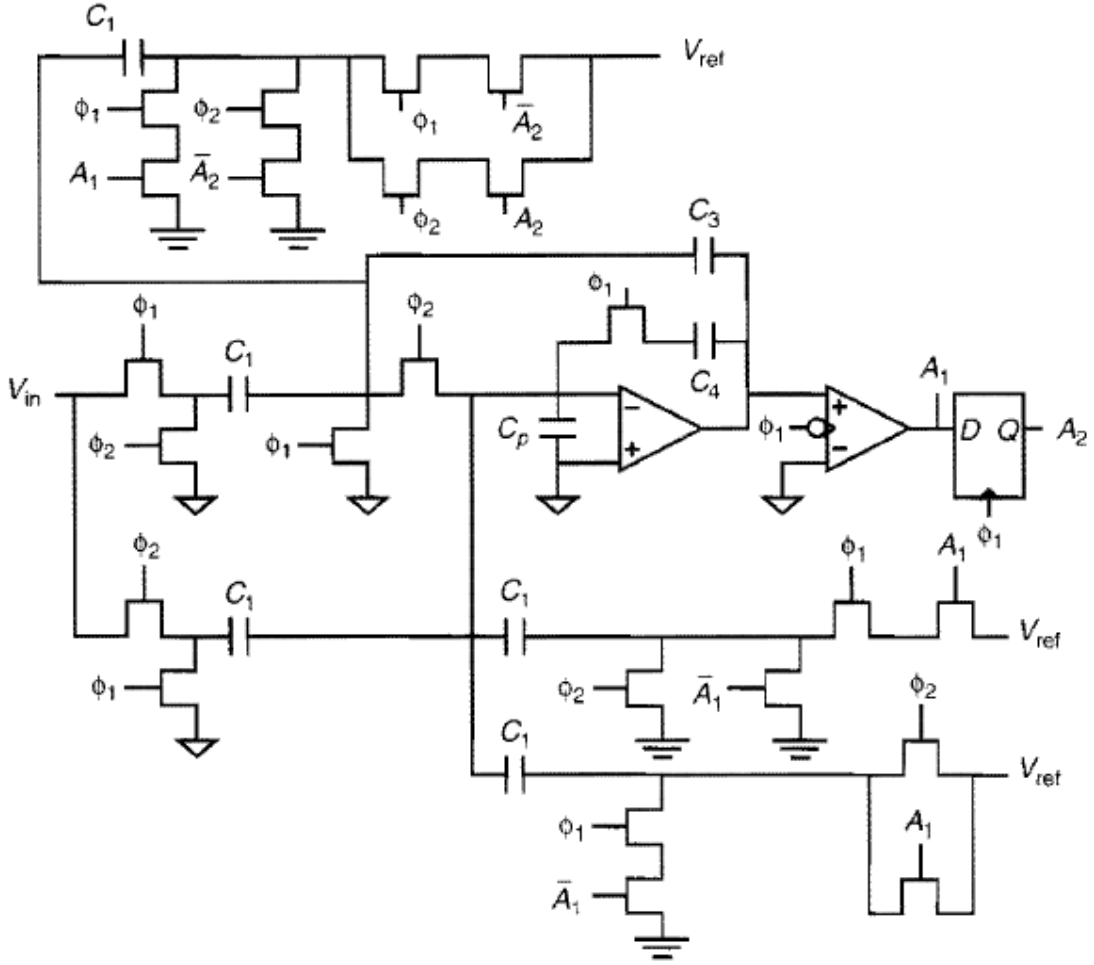


Figure 6.10 First modulator in third-order (1–1–1) cascade.

- Explain the operation of this circuit: Divide it into functional blocks. Draw a chronogram of its operation, with non-overlapping antiphase clocks Φ1, Φ2, and signals A1, A2 and their inverses.

References

U. Beis: *Eine Einführung in Delta-Sigma-Wandler* , http://www.beis.de/Elektronik/DeltaSigma/DeltaSigma_D.html

This reference was used as a basis for the present presentation, several illustrations are taken from it.

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