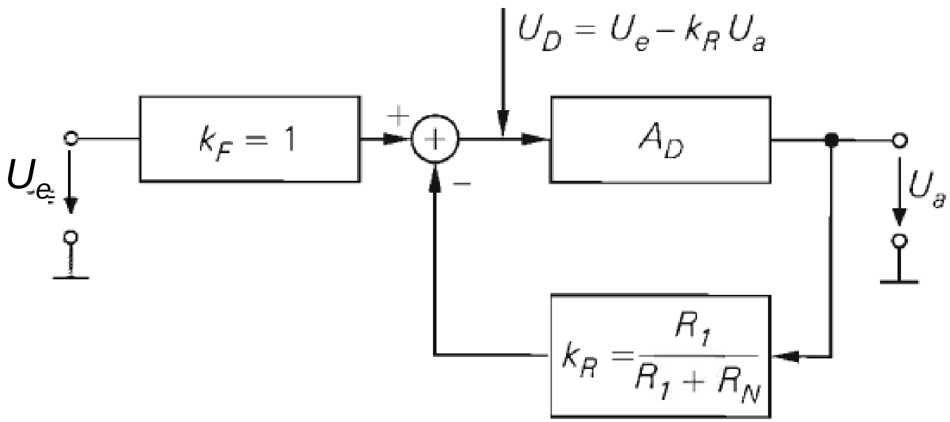
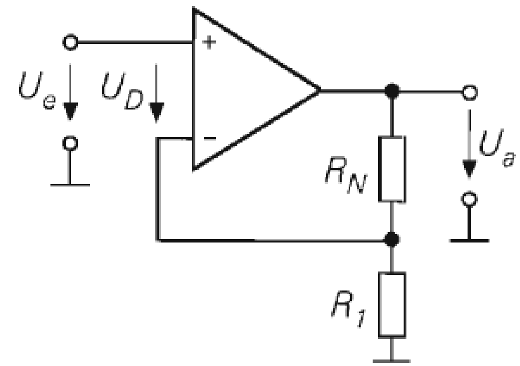
Control loop representation of non-inverting amplifier



Feedback loop model



Non-inverting ampilfier

$$U_a = A_D U_D = A_D\left(U_P - k_R U_a\right)$$

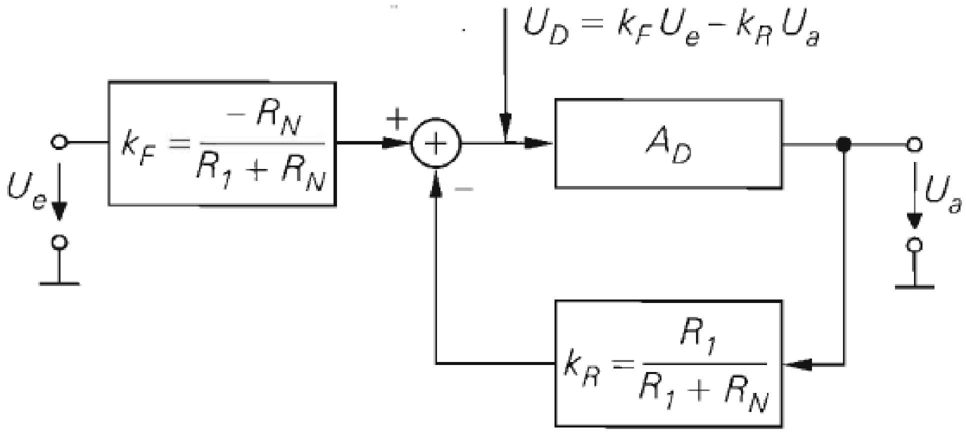
$$A = \frac{U_a}{U_e} = \frac{A_D}{1 + k_R A_D} \cong \frac{1}{k_R} = 1 + \frac{R_N}{R_1}$$

The simplification is valid if the loop gain $g = k_R A_D$ is very high.

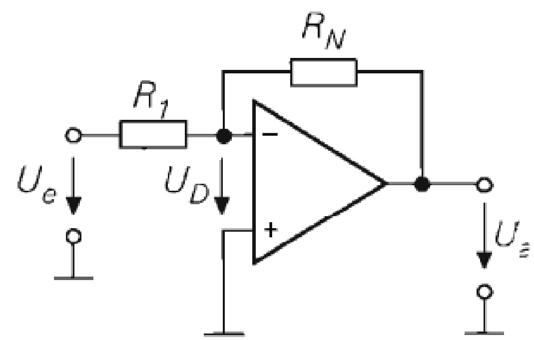
Distinguish four gain definitions:

- open loop gain A_D
- closed loop input → output gain A
- feedback loop gain g
- feedback factor k_R

Control loop representation of inverting amplifier



Feedback loop model



Inverting ampilfier

$$U_a = A_D U_D = A_D\left(k_F U_e - k_R U_a\right)$$

$$A = \frac{U_a}{U_e} = \frac{k_F A_D}{1 + k_R A_D} \cong \frac{k_F}{k_R} = -\frac{R_N}{R_1}$$

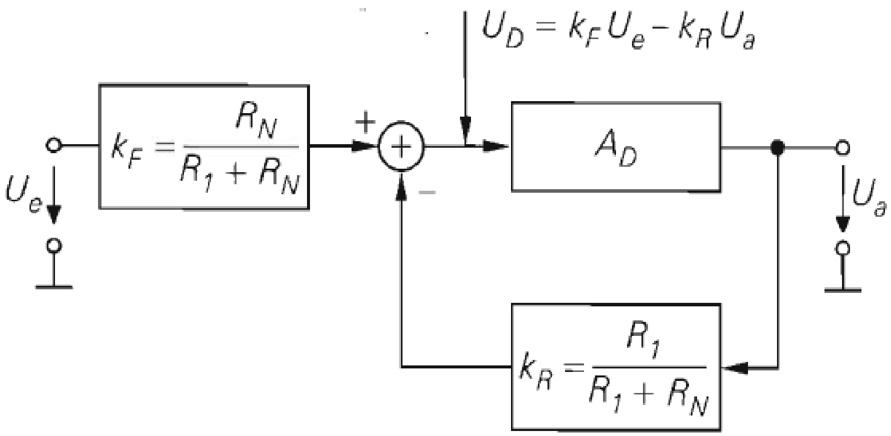
For an ideal amplifier, both inputs are at virtual ground, and the node law yields: $\frac{U_e}{R_1} + \frac{U_a}{R_N} = 0$

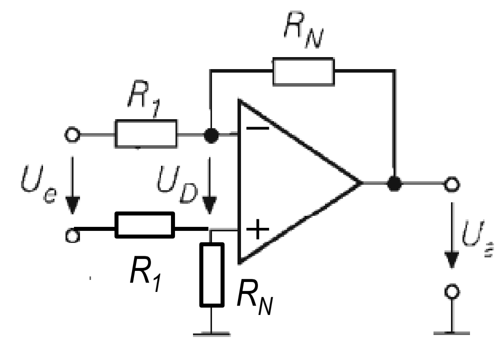
If A_D is not infinite: $0 = \frac{U_e + U_D}{R_1} + \frac{U_a + U_D}{R_N} \Rightarrow 0 = R_N\left(A_D U_e + U_a\right) + R_1 U_a\left(A_D + 1\right)$

$$A = \frac{U_a}{U_e} = -\frac{R_N A_D}{R_1 A_D + R_N + R_1} = k_F \frac{A_D}{1 + k_R A_D}$$

Control loop representation of differential amplifier

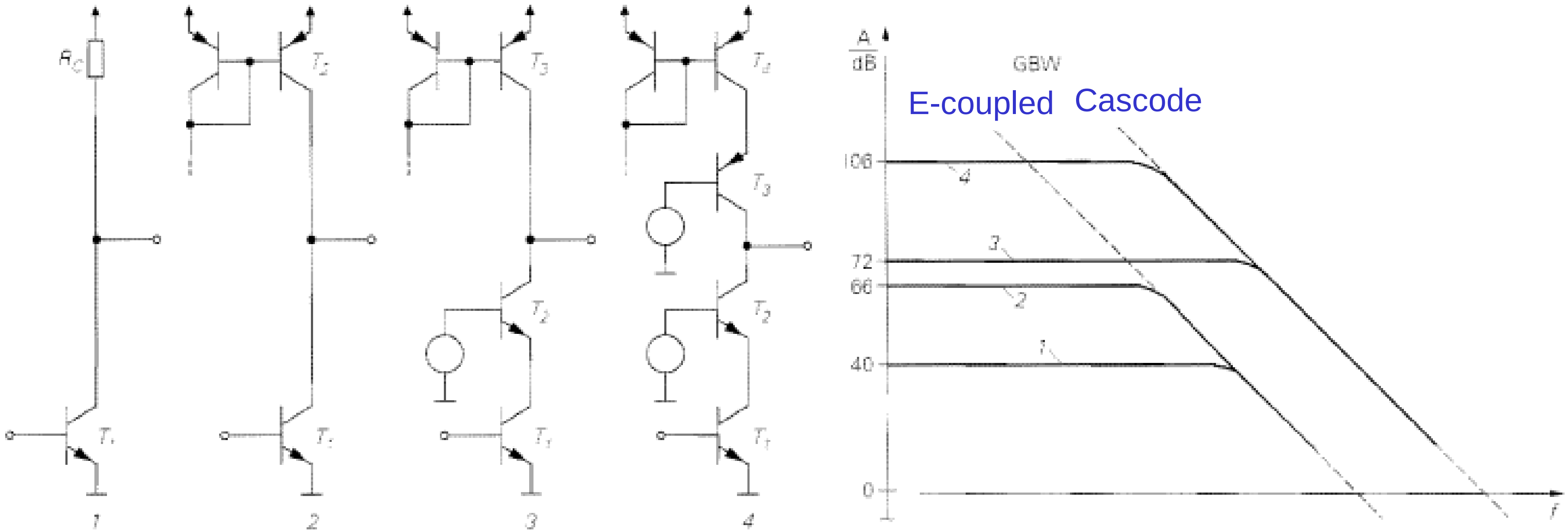
- Draw the schematic of a differential amplifier.
- Represent the differential amplifier by a control loop model, analogous to those for non inverting and inverting amplifiers.





Gain response comparison

The comparison shows the improvements obtained with active load and cascode circuits, of static gain and bandwidth.



Exercice (homework): Amplifier transfer functions

- For the 4 variants of the preceding slide, analytically estimate the passing band gain and cut-off frequency.

- Use the following parameters (all transistors identical):

  $r'_E = 25\Omega$, $r_{CE} = 100k\Omega$, $\beta = 100$, $R_C = 2.5k\Omega$, $C_M = 30pF$

- Simulate the 4 circuits using LTspice, and verify the gains and cut-off frequencies computed.

Amplifier transfer functions

- For the 4 variants of the preceding slide, analytically estimate the passing band gain and cut-off frequency.
- Use the following parameters (all transistors identical):

$$r'_E = 25\Omega,\ r_{CE} = 100k\Omega,\ \beta = 100,\ R_C = 2.5k\Omega,\ C_M = 30pF$$

$A_1 = -R_C/r'_E = -100 \rightarrow 40dB$ $\quad f_{c1} = 1/(2\pi\beta r'_E A_1 C_M) = 21.2kHz$

$A_2 = -r_{CE}/2r'_E = -2000 \rightarrow 66dB$ $\quad f_{c2} = 1/(2\pi\beta r'_E A_2 C_M) = 1.06kHz$

$A_3 = -r_{CE}/r'_E = -4000 \rightarrow 72dB$ $\quad f_{c3} = 1/(2\pi\beta r'_E 2C_M) = 1.06MHz$

$A_4 = -\beta r_{CE}/2r'_E = -200000 \rightarrow 106dB$ $\quad f_{c4} = 1/(2\pi\beta r'_E 2C_M) = 1.06MHz$

The corner frequency calculations only take into account input capacitance. The current mirrors have an output capacitance which at high gain (variant 4) will become the bandwidth limiting circuit.

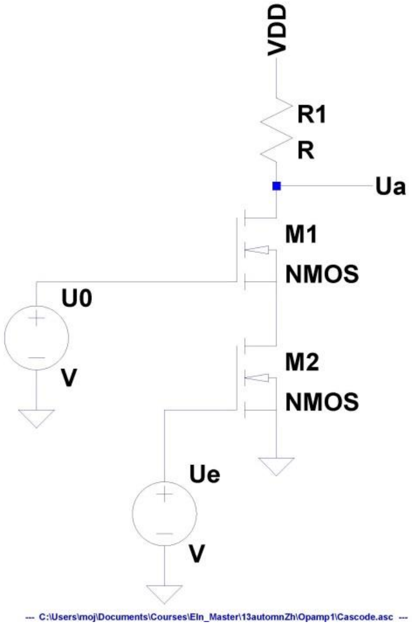
Exercice (homework): MOS-FET circuits

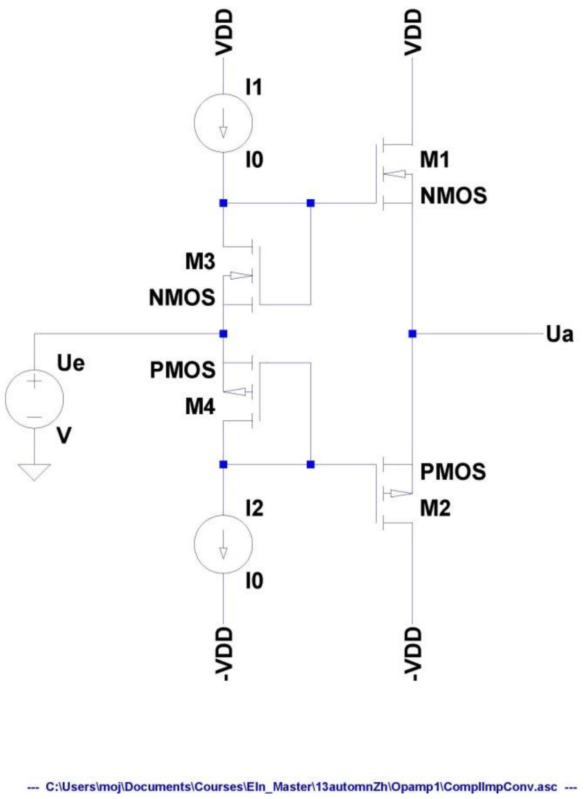
Redraw a cascode amplifier and a complementary impedance conversion stage schematic, each with MOS-FET instead of bipolar transistors.

Rewrite the gain and input capacitance equations for the cascode circuit with MOS-FETs.

MOS-FET circuits

Redraw a cascode amplifier and a complementary impedance conversion stage schematic, each with MOS-FET instead of bipolar transistors.





MOS-FET circuits

Rewrite the gain and input capacitance equations for the cascode circuit with MOS-FETs.

Gain: $A = -g_m R_D$ with drain resistance R_D and transconductance $gm = -2I_{DSS}/V_{GSB} \cdot (1-V_{GS}/V_{GSB})$

Input capacitance $C_{in} = C_{gs} + (1-A)C_{gd}$

With datasheet values C_{iss}, C_{rss}: $C_{in} = C_{iss} - C_{rss} + (1-A)C_{rss}$

Exercice:
Determination of I/O voltage ranges

- Look at the schematic of the OP741 amplifier (p.26) and determine the highest possible positive and negative input voltages.
- Do the same for the output.
- What will happen if an input pin is driven beyond the power supply voltages?

Determination of I/O voltage ranges

- Look at the schematic of the OP741 amplifier (p.26) and determine the highest possible positive and negative input voltages.
  Since T_5 is a Darlington transistor, $U_{BE5} = 1.2V$.
  $-14.4V < U_N, U_P < 13.4V$
- Do the same for the output.

  $-13.2V < U_a < 13.4V$
- What will happen if an input pin is driven beyond the power supply voltages?

  Driven beyond positive power supply, T1 or T2 will block: the device will therefore block.

  Driven beyond negative supply, T1 and T3 or T2 and T4 will go into reverse mode, leading to a short circuit.

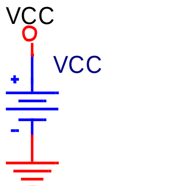
  Beyond power supply voltages, also input protection diodes switch on to prevent overvoltage damage.

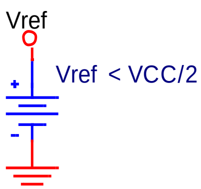
Exercice:
Single supply circuits

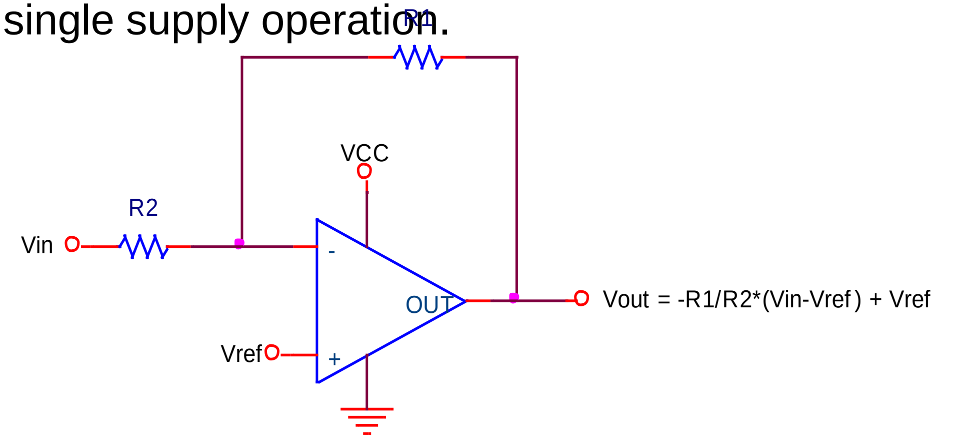
- Propose the standard operational amplifier circuits (inverting, non-inverting, differential amplifier) for single supply operation.

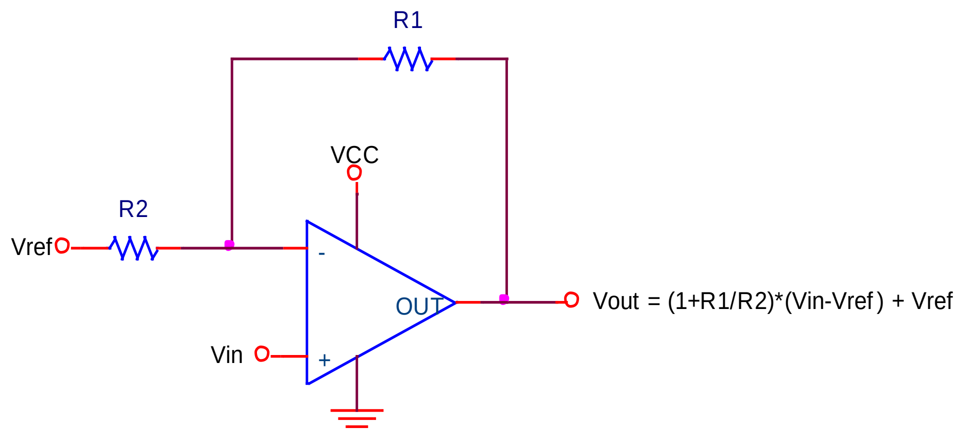
Single supply circuits (1)

- Propose the standard operational amplifier circuits (inverting, non-inverting, differential amplifier) for single supply operation.

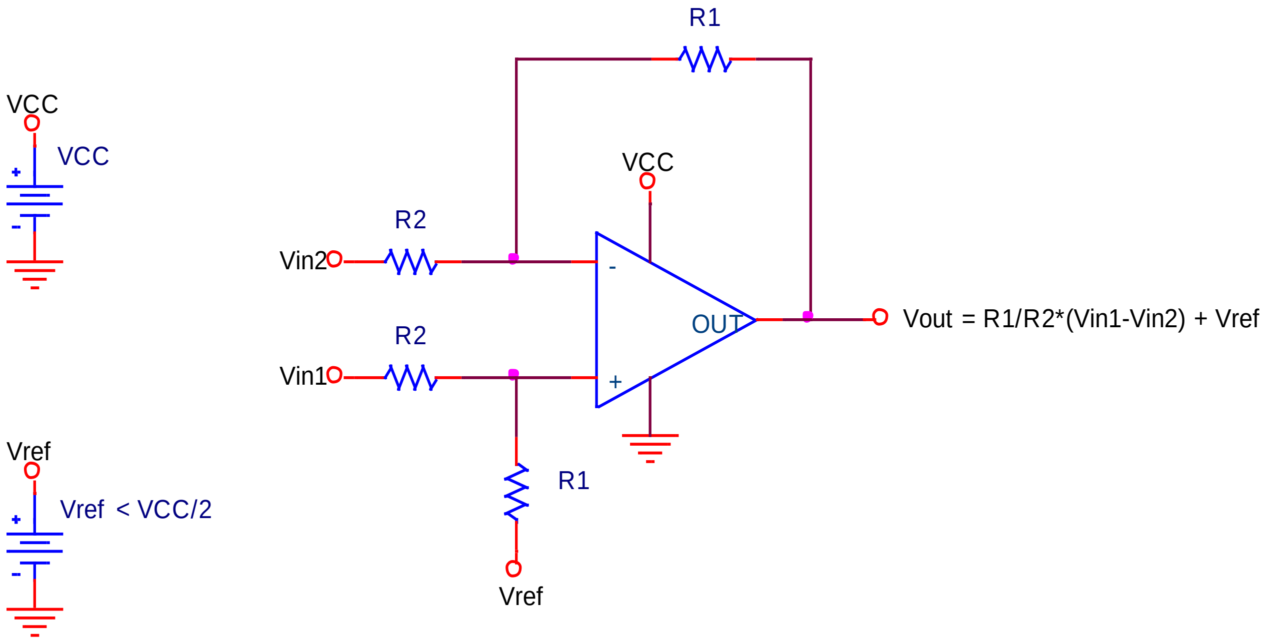






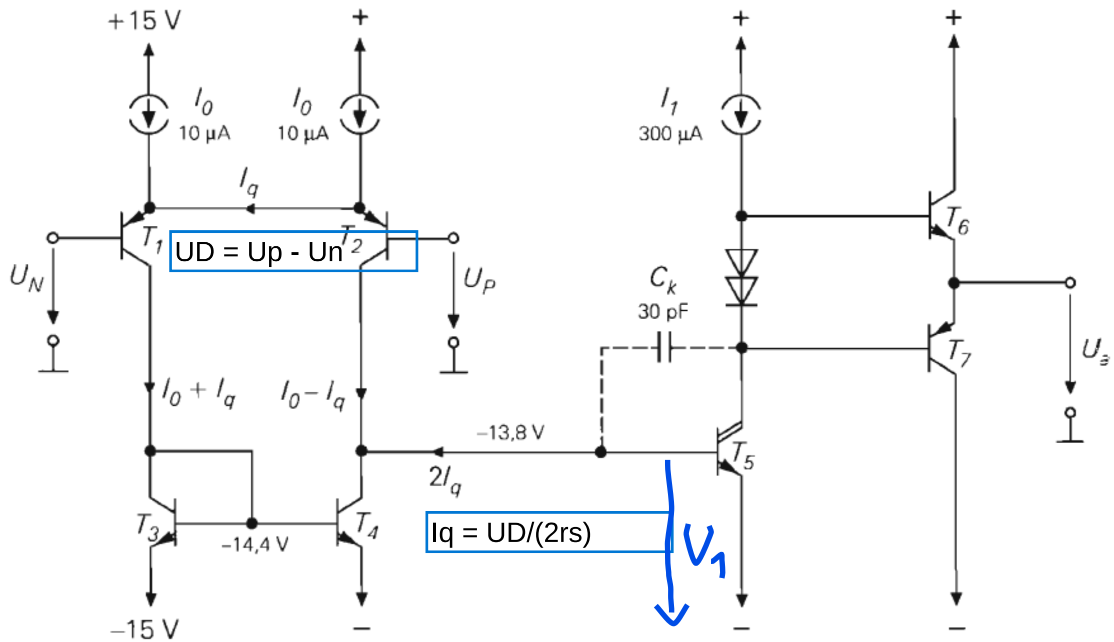


Single supply circuits (2)



Exercice: Analysis of opamp structure

Admit that the sources I_0 and I_1 need a minimum overhead voltage of 1.0V to operate. U_{BE} = 0.6V, U_T = 25mV for all transistors and diodes.
Total resistance connected to node of the basis of T_5 R_1 = 2MΩ, of the collector of T_5 R_2 = 100kΩ. Base currents are negligible. Transconductance of T_5: $1/r_{S5}$ = 5mA/V.



What is the admissible input voltage range of U_N, U_P ? What is the possible output voltage range of U_a ?
Determine the gain $A_0 = U_a/(U_P-U_N)$ of this opamp.
Admit that the capacitor C_k determines the dynamic behaviour of the circuit. What is the cut-off frequency of the open-loop transfer function ? What is the maximum slew rate ?
If the opamp is used as a linear inverting stage, which gain can be realised at 10kHz ?

Analysis of op-amp structure

What is the admissible input voltage range of U_N, U_P ? What is the possible output voltage range of U_a ?
Since T_5 is a Darlington transistor, $U_{BE5} = 1.2V$.
$-13.8V < U_N, U_P < 13.4V$ and $-13.2V < U_a < 13.4V$

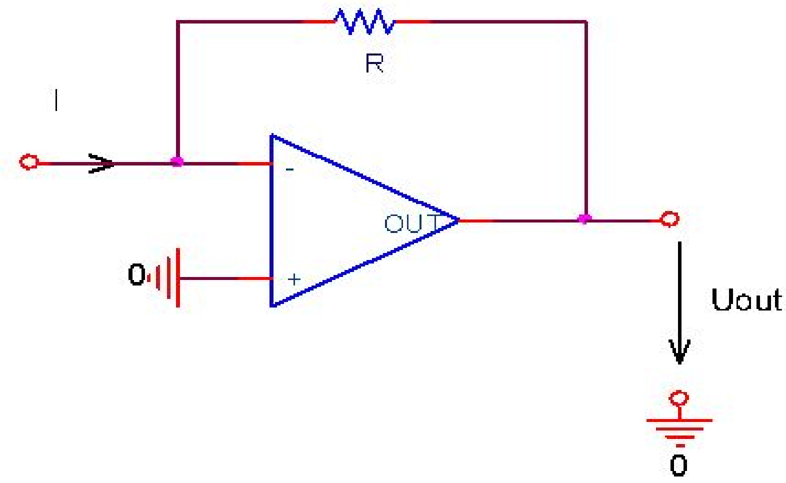
Determine the gain $A_0 = U_a/(U_P-U_N)$ of this op-amp ?
$r_S = U_T/I_0 = 2.5k\Omega$
$U_D = U_P - U_N$
$U_1 = -2R_1U_D/(2r_S) = -800\ U_D$
$U_a = –U_1R_2/r_{S5} = -U_1R_2/r_{s5} = 400'000\ U_D$
$A_0 = 400'000$

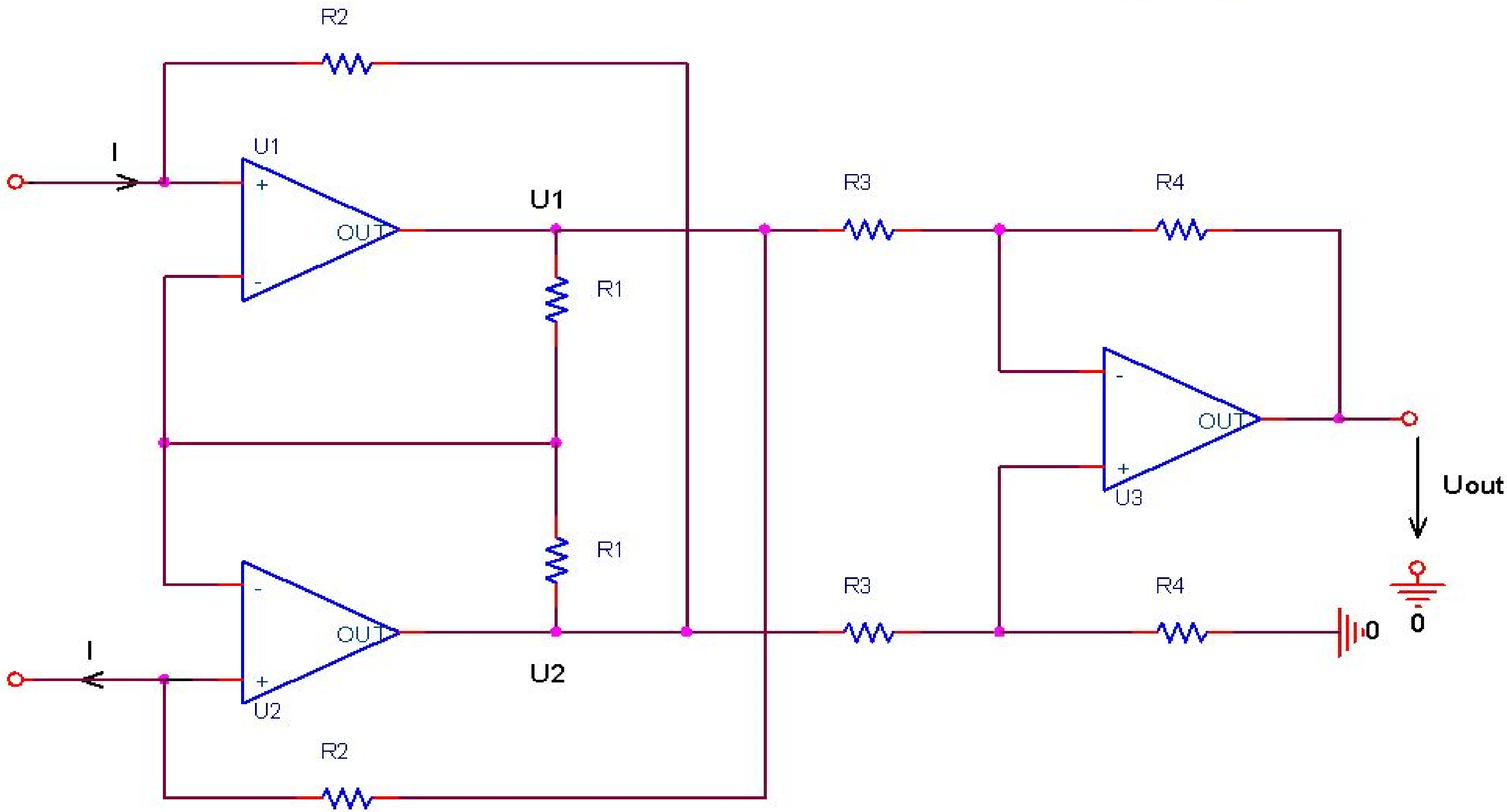
Admit that the capacitor C_k determines the dynamic behaviour of the circuit. What is the cut-off frequency of the open-loop transfer function ? What is the maximum slew rate ?
$f_0 \approx 1/(2\pi R_1C_k) \cdot r_{S5}/R_2 = 5.3Hz$
$S_R \approx I_1/C_k = 10V/\mu sec$

If the opamp is used as a linear inverting stage, which gain can be realised at 10kHz ?
$A_{max,10kHz} = 212$

Current to voltage conversion

Determine the transfer characteristic
of the following circuits:





Active current sensing amplifier

- Determine the transfer characteristic $U_{out} = f(I)$ of the two circuits:

  1st variant : $U_{out} = -R * I$

  2nd variant :
  $$U_2 = U_{in} - R_2 I$$
  $$U_1 = U_2 + 2(U_{in} - U_2) = U_{in} + R_2 I$$
  $$U_{out} = -2 \frac{R_4}{R_3} R_2 I$$

- How can the second circuit be made insensitive to common mode input voltage?

  The common mode sensitive stage is the differential stage.

  Therefore, the gain R_4/R_3 must be kept low, typically one.

Current amplifier

- Determine the transfer relations of the circuits in slides 11…20
- How does the output resistance of the circuit in slide 20 depend on R_N? Determine the transfer gain of the same circuit.
- Determine the transfer function describing the dynamic behaviour of the circuit in slide 18.

Current amplifier (1)

- Determine the transfer relations of the circuits in slides 11…20.

  In the following, we designate the transistor transconductance by $1/r_S$.

Slide 11:

$$S = \frac{I_{ak}}{U_D} = \frac{1}{r_S}$$

$$S_B = \frac{I_{ak}}{U_P} = \frac{1}{r_S + R_E}$$

$$A_B = \frac{U_a}{U_P} = \frac{R}{r_S + R_E}$$

Slide 12:

$$I_E = \frac{U_e}{r_S + R_E} = I_C$$

$$U_a = I_E\left(r_a \| R_C\right) = \frac{r_a \| R_C}{r_S + R_E} U_e \approx \frac{R_C}{R_E} U_e$$

Slide 13:

$$U_a = \frac{R_E}{r_S + R_E} U_e \approx U_e$$

Voltage follower:

$$2\frac{U_e - U_a}{r_S} - \frac{U_a}{R_E} = 0 \quad \Rightarrow \quad U_a = \frac{R_E}{R_E + r_S/2} U_e$$

Slide 14:

$$U_a = -I_E\left(r_a \| R_C\right) = -\frac{r_a \| R_C}{r_S + R_E} U_e \approx -\frac{R_C}{R_E} U_e$$

Current amplifier (2)

- Determine the transfer relations of the circuits in slides 16…25.

Slide 15:

$$I_q = \frac{U_{e1} - U_{e2}}{R_E + 2r_S}$$

$$U_{a1} = \frac{R_C \| r_a}{R_E + 2r_S}\left(U_{e1} - U_{e2}\right) = -U_{a2}$$

$$\frac{I_{a1}}{U_{e1} - U_{e2}} = \frac{1}{R_E + 2r_S}$$

Slide 17:

$$U_a = \frac{1}{C}\int I_C dt = \frac{1}{RC}\int U_e dt$$

$$U_a(s) = \frac{U_e(s)}{sRC}$$

Slide 18: see next exercice.

Slide 16:

$$I_1 = \frac{1}{R_G} U_2$$

$$I_2 = \frac{1}{R_G} U_1$$

Slide 19 lower circuit:

$$\text{Emitter}: \quad \frac{U_e - U_1}{r_S} + \frac{U_a - U_1}{R_N} - \frac{U_1}{R_1} = 0$$

$$\text{Collector}: \quad \frac{U_e - U_1}{r_S} + \frac{U_a - U_1}{R_N} - \frac{U_a}{r_a} = 0$$

$$\frac{U_a}{U_e} = \frac{1 + \dfrac{R_N}{2R_1}}{1 + \dfrac{1}{2r_a}\left(R_N + r_S\left(1 + \dfrac{R_N}{R_1}\right)\right) + \dfrac{r_S}{2R_1}} \cong 1 + \frac{R_N}{2R_1}$$

Taking frequency limitation into account : $\dfrac{U_a(s)}{U_e(s)} \cong \dfrac{1 + \dfrac{R_N}{2R_1}}{1 + sR_N C_a/2}$

Current amplifier (3)

- Determine the transfer relations of the circuits in slides 11…20.
- How does the output resistance of the circuit in slide 20 depend on R_N? Determine the transfer gain of the same circuit.

$$U_e = 0, \quad k_I > 1: \quad I_a = I_C - \frac{U_a}{R_N} = k_I I_E - \frac{U_a}{R_N} = -(k_I + 1)\frac{U_a}{R_N}$$

$$r_a = -\frac{U_a}{I_a} = \frac{R_N}{k_I + 1} = R_t$$

$$R_N = R_t(k_I + 1)$$

$$U_{a0} = U_e + R_N k_I I_E$$

$$I_E = \frac{U_e}{R_1(k_I + 1)} \quad \Rightarrow$$

$$U_{a0} = \left(1 + \frac{R_N k_I}{R_1(k_I + 1)}\right)U_e = \left(1 + \frac{R_t k_I}{R_1}\right)U_e$$

$$U_a = \frac{1}{2}U_{a0} = \frac{1}{2}\left(1 + \frac{R_t k_I}{R_1}\right)U_e$$

For unity gain, we need $R_1 = R_t k_I$.

Low noise 75 Ohm driver

Propose a circuit based on a MAX436 and an OP37 amplifier capable of driving a 75 Ohm load with the noise floor of the output determined by the OP37 amplifier.

Determine the bandwidth of the driver and check its stability with 300pF capacitive load.
If an instability occurs, propose an appropriate circuit modification.

CC amplifier based high pass filter

- Determine the transfer function describing the dynamic behaviour of the circuit in slide 18. op amp2

  Neglect r_S with respect to R, then

$$\frac{U_{TP}(s)}{U_e(s)} = \frac{1}{1 + sC\,R^2/R_1 + s^2C^2R^2}$$

$$\frac{U_{BP}(s)}{U_e(s)} = \frac{sCR}{1 + sC\,R^2/R_1 + s^2C^2R^2}$$

$$f_0 = \frac{1}{2\pi RC}, \quad Q = \frac{R_1}{R}$$

Resonance frequency f_0 and quality factor Q can be adjusted independently.

Exercice
Noise performance data of OP37

Search for noise performance data in the OP37 datasheet.

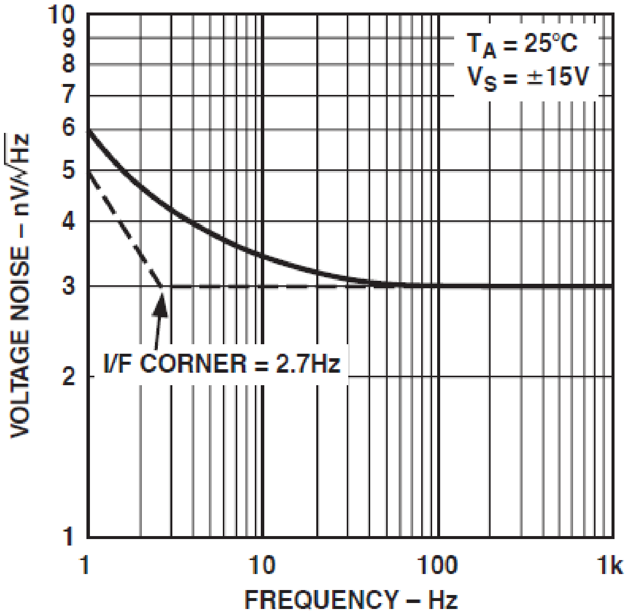
Determine the corner frequencies of voltage and current noise. At which input resistance will voltage and current white noise levels generate equivalent contributions at the output ?

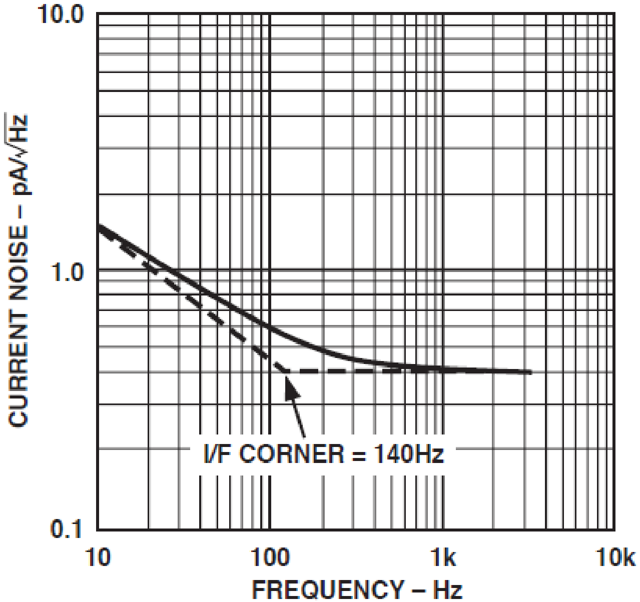
How high will the output noise then be ?

What will the noise corner frequency in this case become ?

Noise performance data of OP37

Search for noise performance data in the OP37 datasheet.





Determine the corner frequencies of voltage and current noise.

Voltage noise: 2.7Hz

Current noise: 140Hz

Noise performance data of OP37

At which input resistance will voltage and current white noise levels generate equivalent contributions at the output.

White noise levels of

Voltage noise: e_n = 3nV/rtHz Current noise: i_n = 0.4pA/rtHz

Source resistance for equivalent output noise contributions:

R_s = 3nV/0.4pA = 7.5kΩ

How high will the output noise then be?

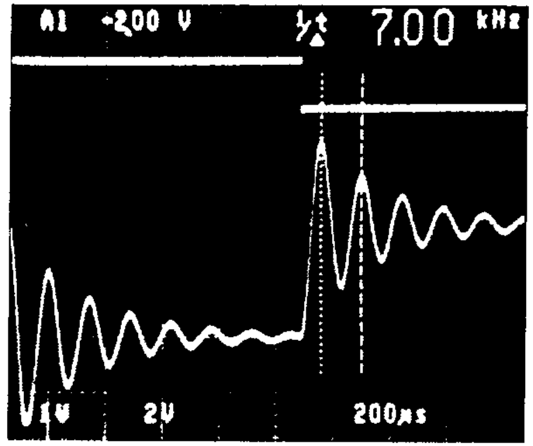
It will be sqrt(2) higher than the contribution of e_n or i_n alone.

What will the noise corner frequency in this case become?

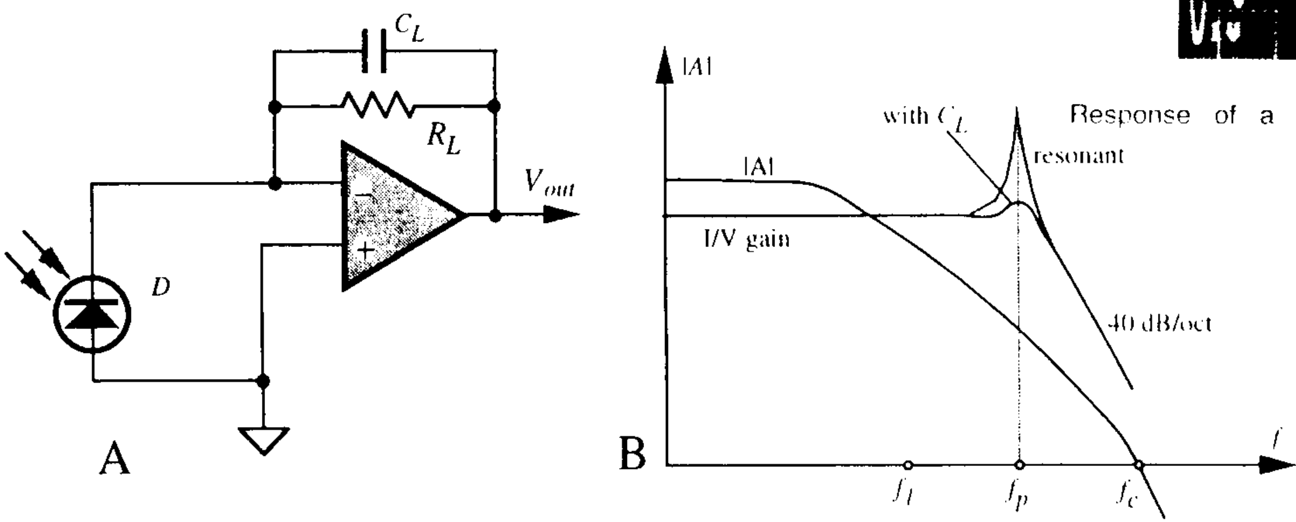
The higher of the voltage and current noise corner frequencies, i.e. 140Hz.

Exercice:
Noise floor of photodiode current to voltage converter

Determine the transfer function of the circuit below with and without the feedback capacitor C_L. Explain the improvement realized with C_L.



Response of a photodiode with uncompensated circuit.



Use of current-to-voltage converter (A) and the frequency characteristics (B).

Determine the output noise voltage noise density spectrum for an OP37 amplifier, BPW34 photodiode, R_L = 100kΩ, C_L such that Q = 1.

Measurement of photo-electric current

- Determine the transfer function of the circuit of slide 20 with and without the feedback capacitor C_L.

  We suppose that the photodiode is modeled by a light controlled current source I_{ph} in parallel with junction capacitance C_j, and the operational amplifier has a transfer function of

$$U_{out} / (U_{in+} - U_{in-}) = A_0/(1+s/\omega_0).$$

  To begin with, we determine the relation between I_{ph} and U_{out}, assuming an ideal operational amplifier. Without C_L, we have

$$U_{out}(s) = -R_L I_{ph}(s)$$

  and with C_L, $$U_{out}(s) = -\frac{R_L}{1+sR_LC_L} I_{ph}(s)$$

  Considering now the non-ideal behaviour of the operational amplifier, we can write a current balance equation for its negative input node, first without C_L,

$$I_{ph}(s) = \frac{U_{in-}(s) - U_{out}(s)}{R_L} + sC_j U_{in-}(s)$$

  Since $U_{in+} = 0$, the voltage $U_{in-}(s)$ can be replaced : $U_{in-}(s) = -U_{out}(1+s/\omega_0)/A_0$, and so

$$R_L I_{ph}(s) = -U_{out}(s)((1+sC_jR_L)(1+s/\omega_0)/A_0-1)$$

Measurement of photo-electric current

And finally $$\frac{U_{out}(s)}{R_L I_{ph}(s)} = -\frac{A_0}{A_0 + 1 + s\left(C_j R_L + \frac{1}{\omega_0}\right) + s^2 C_j R_L \frac{1}{\omega_0}} \cong -\frac{1}{1 + \frac{s}{A_0\omega_0} + s^2 \frac{C_j R_L}{A_0\omega_0}}$$

The approximation is valid if $A_0 >> 1$ and $C_j R_L << 1/\omega_0$. In our case, $A_0 \approx 1.5 \cdot 10^6$ $\omega_0 \approx 150$rad/sec and $C_j = 70$pF, $R_L = 100$kΩ.

This is a second order transfer function, with characteristic frequency

$$\omega_n = \sqrt{\frac{A_0\omega_0}{C_j R_L}}$$ and quality factor $$Q = \frac{1}{\frac{1}{\omega_0} + C_j R_L}\sqrt{\frac{C_j R_L (A_0 + 1)}{\omega_0}} \cong \sqrt{A_0 \omega_0 C_j R_L}$$

With the numeric values of the OP37 and BPW34 (see above), we have a very weakly damped resonance with $Q \approx 40$ at $f_n = \omega_n / 2\pi \approx 900$kHz.

We now add C_L to the circuit, and get

$$I_{ph}(s) = \left(\mathrm{U}_{\text{in-}}(s) - \mathrm{U}_{\text{out}}(s)\right)\left(\frac{1}{R_L} + sC_L\right) + sC_j \mathrm{U}_{\text{in-}}(s)$$

and with the same substitutions and assumptions as before

$$\frac{U_{out}(s)}{R_L I_{ph}(s)} \cong -\frac{1}{1 + s\left(C_L R_L + \frac{1}{A_0\omega_0}\right) + s^2 \frac{(C_j + C_L)R_L}{A_0\omega_0}}$$

Measurement of photo-electric current

This is again a second order transfer function, with reduced characteristic frequency

$$\omega_n' = \sqrt{\frac{A_0\omega_0}{(C_j + C_L)R_L}} \cong \omega_n \quad \text{and quality factor} \quad Q' \cong \frac{1}{\omega_n'\left(C_L R_L + \dfrac{1}{A_0\omega_0}\right)} \cong \frac{Q}{1 + A_0\omega_0 C_L R_L}$$

where we assumed $C_j >> C_L$ which must be assured by the selection of C_L.

- Explain the improvement realized with C_L.

Without C_L, Q may be high ($Q \approx 40$ in our case).

By selecting an appropriate value of C_L, $Q' = 1$ can be realized. With our numeric values: $C_L \approx 1.7$pF. ω'_n does practically not change with respect to ω_n.

Measurement of photo-electric current

- Determine the output noise voltage noise density spectrum for an OP37 amplifier, BPW34 photodiode, R_L = 100kΩ, C_L such that Q = 1.

  BPW34: C_j = 70pF
  OP37: $\omega_0 = 2\pi \cdot 30$Hz, $A_0 = 1.5 \cdot 10^6$

- For R_L = 100kΩ, Q = 1, we need C_L = 1.7pF (very small C_L is sufficient!)

- The photodetector bandwidth is approximately 900kHz.

- The noise sources are the diode i_{nD} = sqrt(NEP/R_L) = 630pA/rtHz, the operational amplifier i_n = 0.4pA/rtHz above 140Hz, e_n = 3nV/rtHz above 2.7Hz, and the resistor e_L = 41nV/rtHz

- The diode noise is clearly the most important at the amplifier output, $e_{out} = i_{nD} * R_L$ = 63µV/rtHz

- A lower corner frequency (1/f increase in spectral density of noise power) is not given in the BPW34 datasheet, but certainly exists.

Stabilization of capacitive load

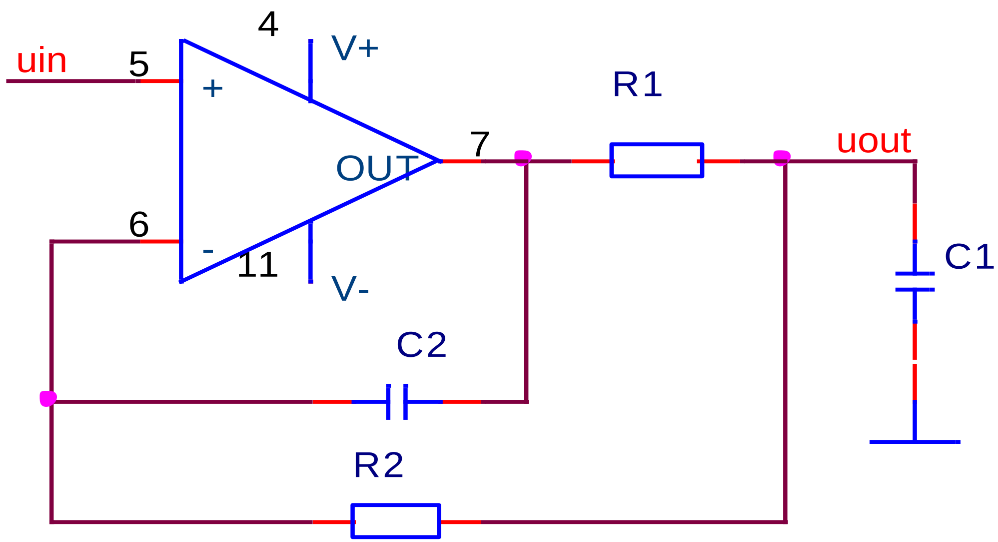
The following circuit represents a voltage follower driving a capacitive load.

Determine its transfer function without and with R1, R2, C2. Use a first order model for the operational amplifier.

What is the use of R2 ?

Suppose an OP37 operational amplifier, and C1= 100nF.

Propose values for R1, R2 and C2, so as to make sure that the amplifier has Q = 1. What bandwidth can be achieved?



Stabilization of capacitive load (1)

$$u_{out}=u_{amp,out}\frac{\frac{1}{sC_1}}{\frac{R_1\left(R_2+\frac{1}{sC_2}\right)}{R_1+R_2+\frac{1}{sC_2}}+\frac{1}{sC_1}}=u_{amp,out}\frac{1+sC_2\left(R_1+R_2\right)}{sC_1R_1\left(1+sC_2R_2\right)+1+sC_2\left(R_1+R_2\right)}$$

$$u_{in}=\frac{R_2u_{amp,out}+\frac{1}{sC_2}u_{out}}{R_2+\frac{1}{sC_2}}=\frac{sC_2R_2u_{amp,out}+u_{out}}{1+sC_2R_2}\quad\Rightarrow\quad u_{amp,out}=\frac{u_{in}\left(1+sC_2R_2\right)-u_{out}}{sC_2R_2}$$

$$\begin{aligned}
u_{out}&=\frac{u_{in}\left(1+sC_2R_2\right)-u_{out}}{sC_2R_2}\,\frac{1+sC_2\left(R_1+R_2\right)}{sC_1R_1\left(1+sC_2R_2\right)+sC_2\left(R_1+R_2\right)+1}\\
&=\frac{\left(1+sC_2R_2\right)\left(1+sC_2\left(R_1+R_2\right)\right)}{sC_2R_2\left(\left(1+sC_1R_1\right)\left(1+sC_2R_2\right)+sC_2R_1\right)+1+sC_2\left(R_1+R_2\right)}u_{in}\\
&=\frac{\left(1+sC_2R_2\right)\left(1+sC_2\left(R_1+R_2\right)\right)}{\left(1+sC_2R_2\right)sC_2R_1+sC_2R_2\left(\left(1+sC_1R_1\right)\left(1+sC_2R_2\right)\right)+1+sC_2R_2}u_{in}\\
&=\frac{1+sC_2\left(R_1+R_2\right)}{sC_2R_1+sC_2R_2\left(1+sC_1R_1\right)+1}u_{in}\\
&=\frac{1+sC_2\left(R_1+R_2\right)}{1+sC_2\left(R_1+R_2\right)+s^2C_2C_1R_2R_1}u_{in}
\end{aligned}$$

Stabilization of capacitive load (2)

In normalized form:

$$\frac{u_{out}}{u_{in}} = \frac{1 + sC_2\left(R_1 + R_2\right)}{1 + a_1 \frac{s}{2\pi f_g} + b_1 \frac{s^2}{\left(2\pi f_g\right)^2}}$$

$$\frac{b_1}{\left(2\pi f_g\right)^2} = C_2 C_1 R_2 R_1 \,, \quad \frac{a_1}{2\pi f_g} = C_2\left(R_1 + R_2\right)$$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{b_1}{C_1 a_1 2\pi f_g} = 256\Omega \,, \quad \text{we choose e.g. } R_1 = 270\Omega \quad \Rightarrow \quad R_2 = 4.94k\Omega \,, C_2 = 1.97nF$$

With stationary voltages, R_2 feeds back U_{out} to U_{in}. Thus, no static voltage output error appears.

Using a passive RC low-pass in front of the circuit, the zero of the transfer function u_{out}/u_{in} can be compensated, and an ideal low-pass behaviour is achieved.

The calculation above is done with an ideal op-amp.

Stabilization of capacitive load (1)

$$u_{out} = u_{amp,out}\frac{\frac{1}{sC_1}}{\frac{R_1\left(R_2+\frac{1}{sC_2}\right)}{R_1+R_2+\frac{1}{sC_2}}+\frac{1}{sC_1}} = u_{amp,out}\frac{1+sC_2(R_1+R_2)}{sC_1R_1(1+sC_2R_2)+1+sC_2(R_1+R_2)}$$

$$u_{in} = \frac{R_2u_{amp,out}+\frac{1}{sC_2}u_{out}}{R_2+\frac{1}{sC_2}} = \frac{sC_2R_2u_{amp,out}+u_{out}}{1+sC_2R_2} \quad \Rightarrow \quad u_{amp,out} = \frac{u_{in}(1+sC_2R_2)-u_{out}}{sC_2R_2}$$

$$u_{out} = \frac{u_{in}(1+sC_2R_2)-u_{out}}{sC_2R_2}\frac{1+sC_2(R_1+R_2)}{sC_1R_1(1+sC_2R_2)+sC_2(R_1+R_2)+1}$$

$$= \frac{(1+sC_2R_2)(1+sC_2(R_1+R_2))}{sC_2R_2((1+sC_1R_1)(1+sC_2R_2)+sC_2R_1)+1+sC_2(R_1+R_2)}u_{in}$$

$$= \frac{(1+sC_2R_2)(1+sC_2(R_1+R_2))}{(1+sC_2R_2)sC_2R_1+sC_2R_2((1+sC_1R_1)(1+sC_2R_2))+1+sC_2R_2}u_{in}$$

$$= \frac{1+sC_2(R_1+R_2)}{sC_2R_1+sC_2R_2(1+sC_1R_1)+1}u_{in}$$

$$= \frac{1+sC_2(R_1+R_2)}{1+sC_2(R_1+R_2)+s^2C_2C_1R_2R_1}u_{in}$$

Stabilization of inductive load (1)

We suppose an ideal op-amp. The shunt voltage drop on R_s is called U_s.

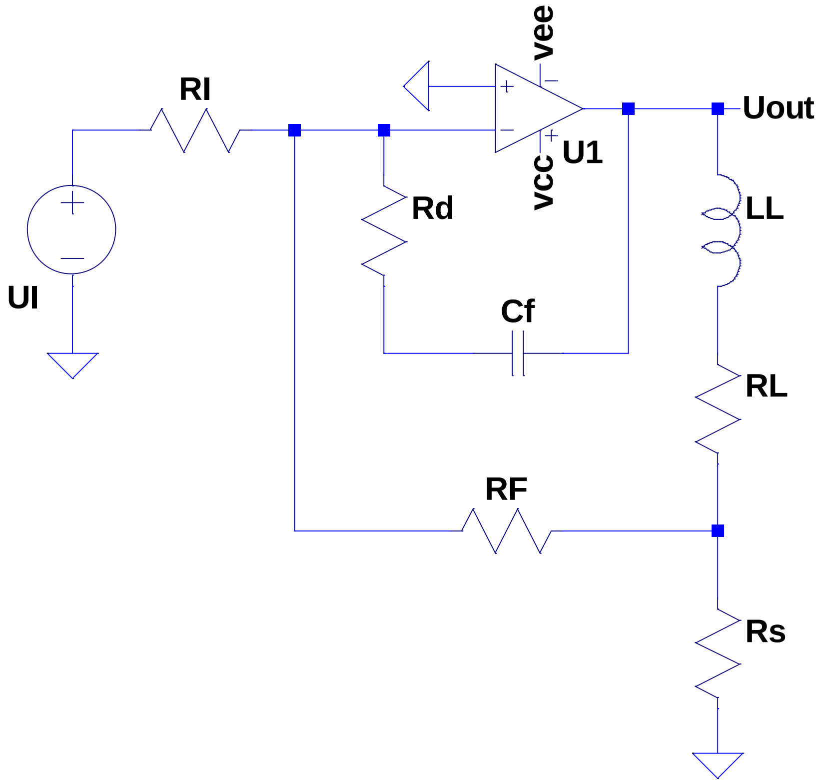
First consider the circuit without the correction $R_d - C_f$:

We suppose that the current through R_F is negligibly small compared to the current through R_s, causing voltage drop U_s. The circuit works as transimpedance amplifier, from U_I to I_L.

$$\frac{U_s}{R_F} = -\frac{U_I}{R_I}, \qquad \frac{U_{out}}{U_s} = 1 + \frac{R_L + sL_L}{R_s}$$

$$U_{out} = -R_F \frac{R_s + R_L + sL_L}{R_s R_I} U_I$$

$$I_L = \frac{U_s}{R_s} = -\frac{R_F}{R_I R_s} U_I$$



Stabilization of inductive load (2)

To take the open-loop gain frequency response of the op-amp into account, we have to introduce the negative input pin potential U_N:

$$U_{out} = -\frac{A_0}{1+\frac{s}{\omega_0}} U_N, \qquad \frac{U_I - U_N}{R_I} = \frac{U_N - U_s}{R_F} \Leftrightarrow U_N = \frac{U_I R_F + U_s R_I}{R_F + R_I}$$

$$U_{out} = -\frac{A_0}{1+\frac{s}{\omega_0}} \frac{U_I R_F + U_{out}\frac{R_s R_I}{R_s + R_L + sL_L}}{R_F + R_I}$$

$$U_{out}\left(\left(1+\frac{s}{\omega_0}\right)(R_F + R_I)(R_s + R_L + sL_L) + A_0 R_s R_I\right) = -A_0 U_I R_F (R_s + R_L + sL_L)$$

$$\frac{U_{out}}{U_I} \cong -\frac{\frac{R_F(R_L + sL_L)}{R_s R_I}}{1 + s\frac{(R_F + R_I)(R_s + R_L + \omega_0 L_L)}{\omega_0 A_0 R_s R_I} + s^2 \frac{L_L(R_F + R_I)}{\omega_0 A_0 R_s R_I}} = -\frac{\frac{R_F(R_L + sL_L)}{R_s R_I}}{1 + \frac{s}{Q\omega_n} + \frac{s^2}{{\omega_n}^2}}$$

assuming $R_s \ll R_L$ and $R_L \ll \omega_0 L_L$:

Stabilization of inductive load (3)

$$\omega_n = \sqrt{\frac{\omega_0 A_0 R_s R_I}{L_L(R_F + R_I)}}, Q = \frac{\omega_n}{\omega_0} = \sqrt{\frac{A_0 R_s R_I}{\omega_0 L_L(R_F + R_I)}}$$

Since A_0 is very large, the quality factor Q will also become very high, making the circuit marginally stable and prone to transient oscillations.

The same computation with the additional branch R_d – C_f gives

$$\frac{U_I - U_N}{R_I} = \frac{U_N - U_s}{R_F} + \frac{U_N - U_{out}}{R_d + \frac{1}{sC_f}} \Leftrightarrow U_N = \frac{(U_I R_F + U_s R_I)\left(R_d + \frac{1}{sC_f}\right) + U_{out} R_F R_I}{R_F R_I + (R_F + R_I)\left(R_d + \frac{1}{sC_f}\right)}$$

$$U_{out} \cong -\frac{A_0}{1 + \frac{s}{\omega_0}} \frac{\left(U_I R_F + U_{out}\frac{R_s R_I}{sL_L}\right)(sC_f R_d + 1) + U_{out} sC_f R_F R_I}{(R_F R_I + R_I R_d + R_d R_F)sC_f + R_F + R_I}$$

$$U_{out}\left(1 + \frac{s}{\omega_0}\right)\left((R_F R_I + R_I R_d + R_d R_F)sC_f + R_F + R_I\right) + U_{out} A_0 R_I\left(\frac{R_s}{sL_L}(sC_f R_d + 1) + sC_f R_F\right) = -A_0 U_I R_F(sC_f R_d + 1)$$

Stabilization of inductive load (4)

The circuit has in principle third order dynamics, but the approximation in the modelisation process yielded a second order model, easier to handle:

$$\frac{U_{out}}{U_I} \cong -\frac{A_0 \frac{R_F}{R_F + R_I}\left(sC_f R_d + 1\right)}{1 + s\frac{R_F R_I}{R_F + R_I} C_f A_0 + s^2 \frac{R_F R_I + R_I R_d + R_d R_F}{R_F + R_I}\frac{C_f}{\omega_0}}$$

$$\omega_n = \sqrt{\frac{(R_F + R_I)\omega_0}{(R_F R_I + R_I R_d + R_d R_F) C_f}},$$

$$Q = \frac{R_F + R_I}{R_F R_I \omega_n C_f A_0} = \frac{1}{R_F R_I A_0}\sqrt{\frac{(R_F + R_I)(R_F R_I + R_I R_d + R_d R_F)}{\omega_0 C_f}}$$

With a suitable choice of R_d and C_f, a quality factor of 1 can be reached.

The additional feedback impedance R_d should be about 10 times smaller than the inductive load impedance feedback $\cong R_F/R_s * \omega L_L$ at the cross-over frequency ω_c, to guarantee good damping.

For the given numeric values, $R_F = 1k\Omega$, $R_s = 1.2\Omega$, $\omega_c = 2\pi * 1kHz$, $L_L = 159mH$, $R_d \cong 82k\Omega$.