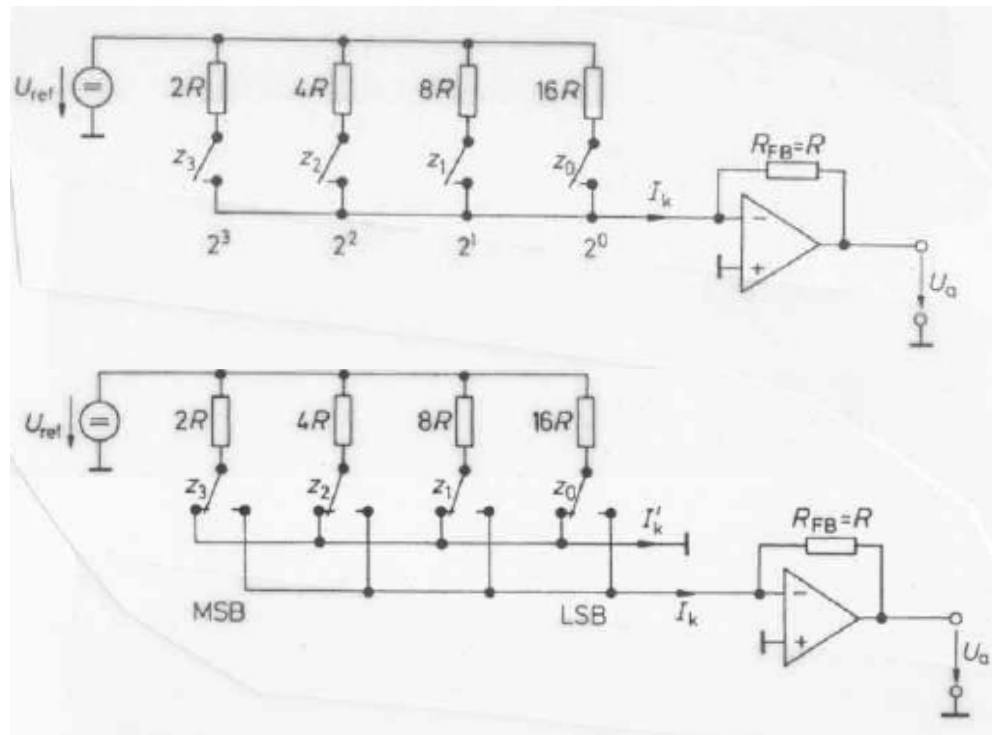


Binary weighted structure

The two schematics below show binary weighted structures for DA conversion. The principal inconvenients of the upper structure are

- The load of U_{ref} is not constant, but depends on z .
- The parasitic capacitances of the switches are (dis)charged when switching.

Explain why the lower schematic solves these problems.



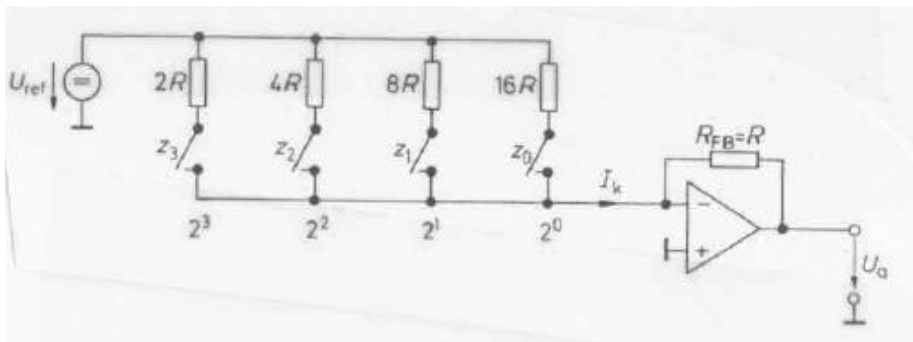
Exercice

Binary weighted structure

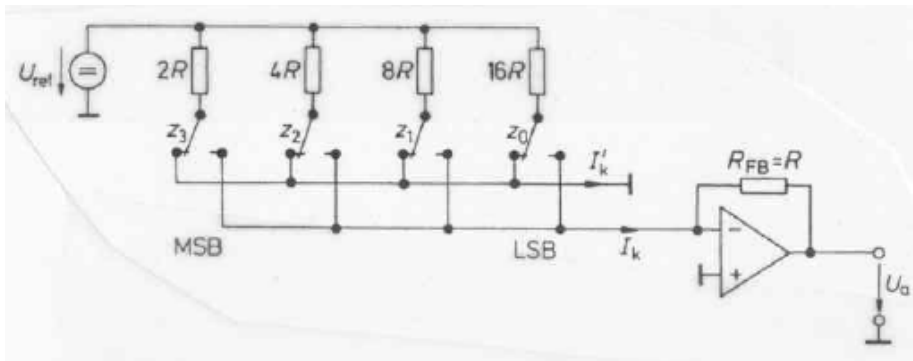
The two schematics below show binary weighted structures for DA conversion. The principal inconvenients of the upper structure are

- The load of U_{ref} is not constant, but depends on z .
- The parasitic capacitances of the switches are (dis)charged when switching.

Explain why the lower schematic solves these problems.



- The load of U_{ref} is always the same, independently of the switch positions, since current always flows into one of two collector rails.



- The connections of any switch are always at the same potential, so that no charging / discharging of stray capacitances occurs.

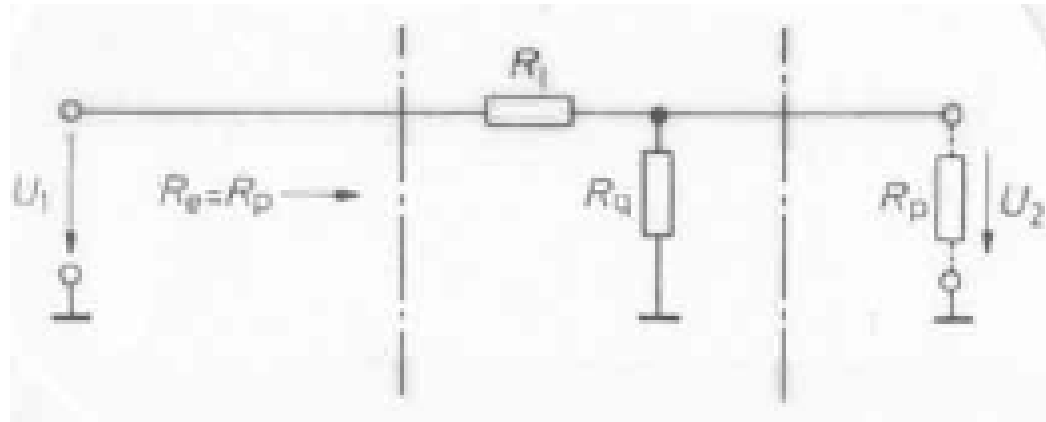
Exercice

Generalized ladder network

Draw a schematic of a voltage output 4bit-DAC with an R/2R ladder network, using commutators between two output current rails.

With the help of the circuit shown below, express R_l and R_p as functions of R_q , so that

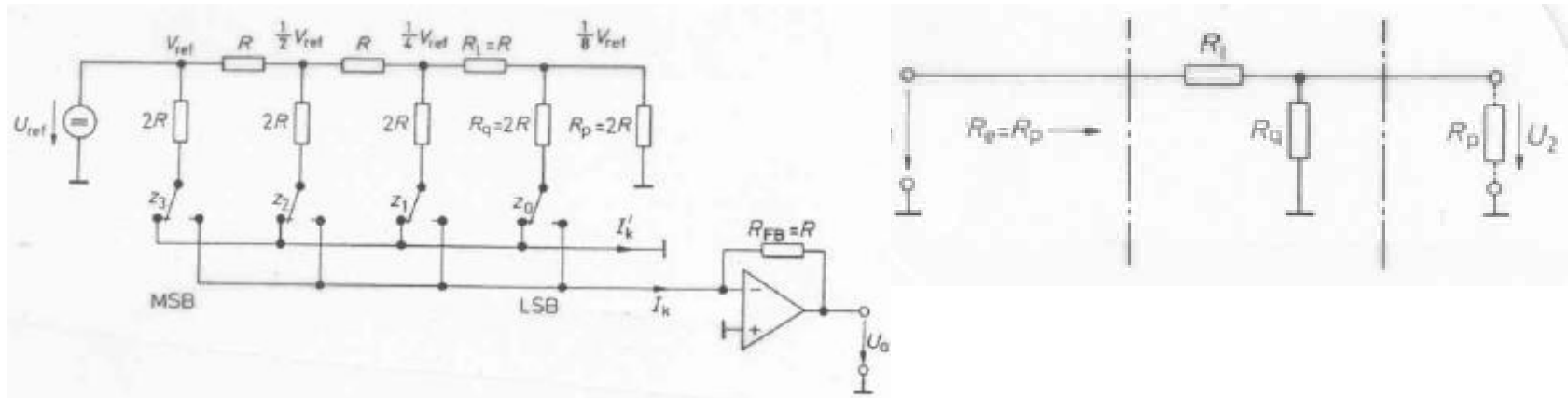
$$U_2/U_1 = \alpha$$



Exercice

Generalized ladder network

Draw a schematic of a voltage output 4bit-DAC with an R/2R ladder network, using commutators between two output current rails.



With the help of the circuit shown above, express R_l and R_p as functions of R_q , so that $\alpha = U_2/U_1$

$$R_e = R_p = R_l + \frac{R_p R_q}{R_p + R_q} \Rightarrow R_l = \frac{R_p^2}{R_p + R_q}$$

$$\alpha = \frac{U_2}{U_1} = \frac{R_q}{R_p + R_q} \Rightarrow R_p = R_q \left(\frac{1}{\alpha} - 1 \right)$$

$$R_l = \alpha \left(\frac{1}{\alpha} - 1 \right)^2 R_q = \frac{(1-\alpha)^2}{\alpha} R_q$$

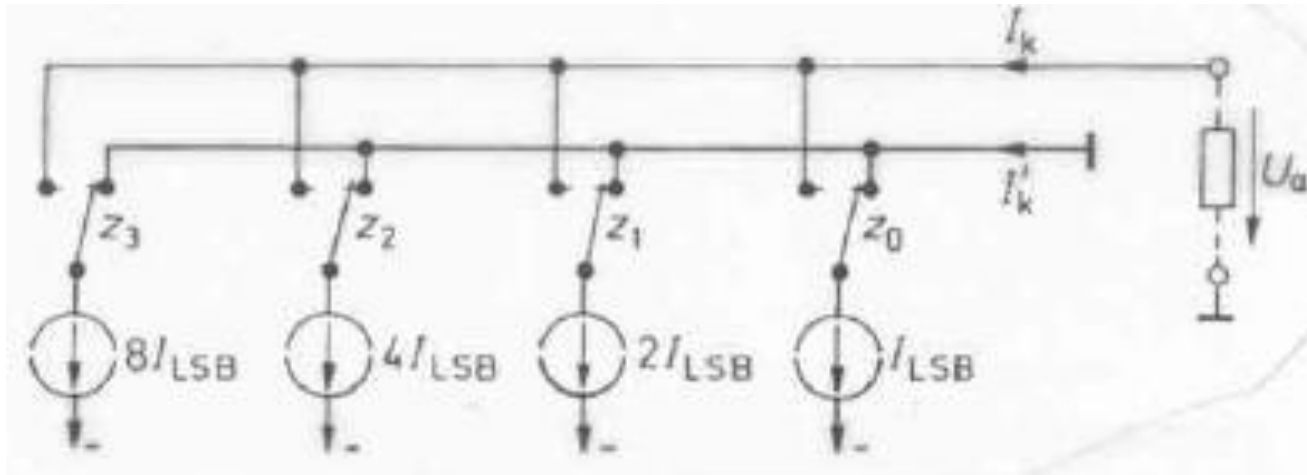
In particular, for $\alpha = 0.5$:

$$R_p = R_q = 2 R_l$$

Exercice

Arrays of current sources

Instead of a reference voltage and a set of weighted resistors, a set of weighted current sources can be used for DA conversion as shown below.

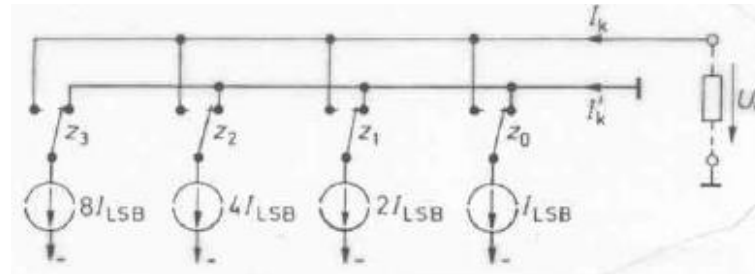


Propose an alternative circuit using a set of current sources of same value and a ladder network for DA conversion.

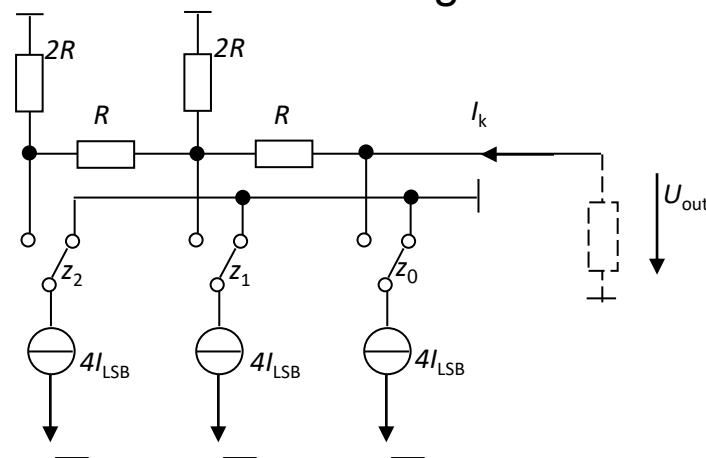
Exercise

Arrays of current sources

Instead of a reference voltage and a set of weighted resistors, a set of weighted current sources can be used for D/A conversion as shown below.



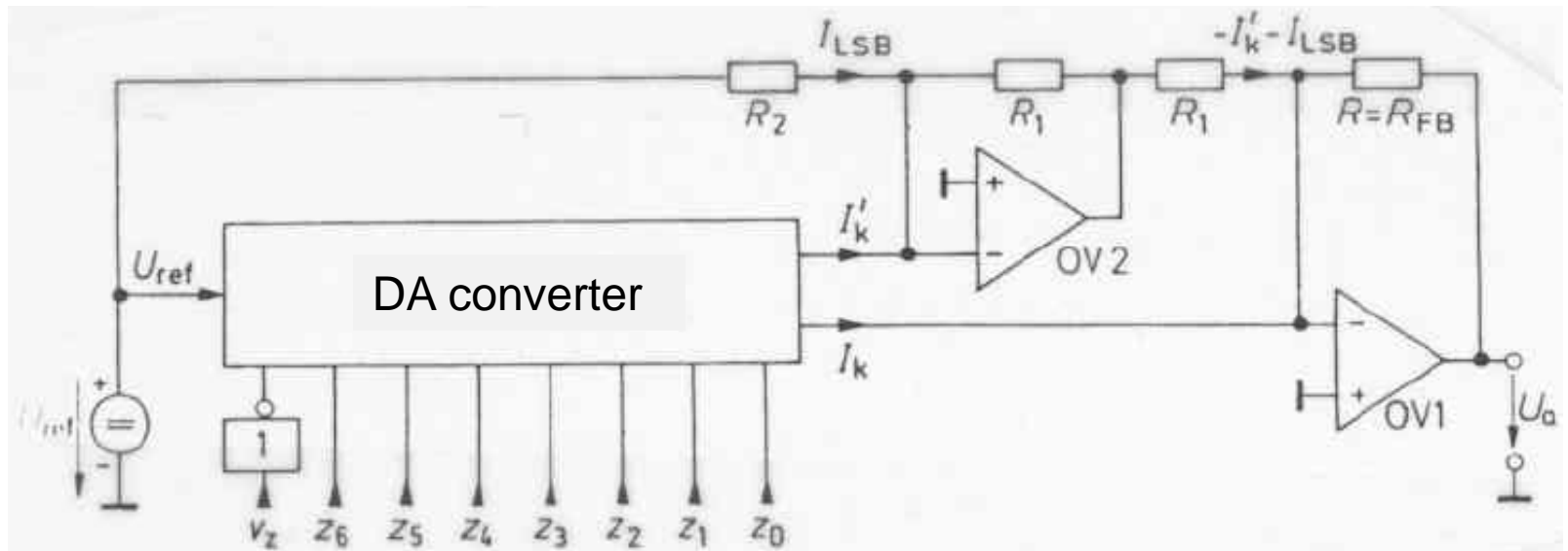
Propose an alternative circuit using a set of current sources of same value and a ladder network for D/A conversion. E.g. for three bits:



Exercice

Bipolar output

For the circuit shown below, determine U_a as a function of Z , a signed binary number in 2's complement representation. I_k and I'_k are the two complementary current outputs of the DA converter.

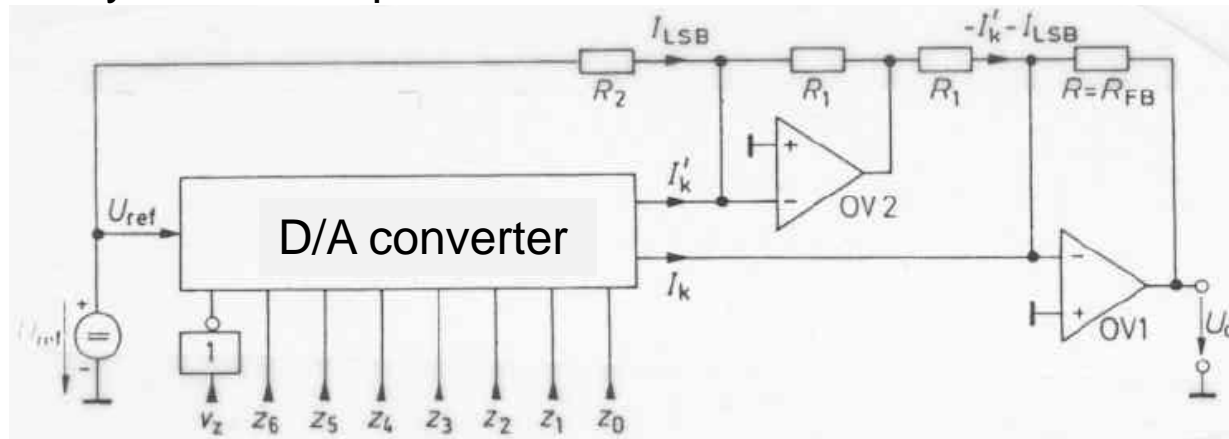


What is the range of U_a ?

Exercice

Bipolar output

For the circuit shown below, determine U_a as a function of Z , a signed binary number in 2's complement representation. I_k and I'_k are the two complementary current outputs of the D/A converter.



$$-2^{n-1} < Z = [v_z, z_{n-1}, \dots, z_0] < 2^{n-1} - 1$$

$$I_k = \frac{U_{ref}}{R} \frac{Z + 2^{n-1}}{2^n}, \quad I'_k = \frac{U_{ref}}{R} \left(1 - \frac{1}{2^n}\right) - I_k = \frac{U_{ref}}{R} \frac{2^{n-1} - Z - 1}{2^n}$$

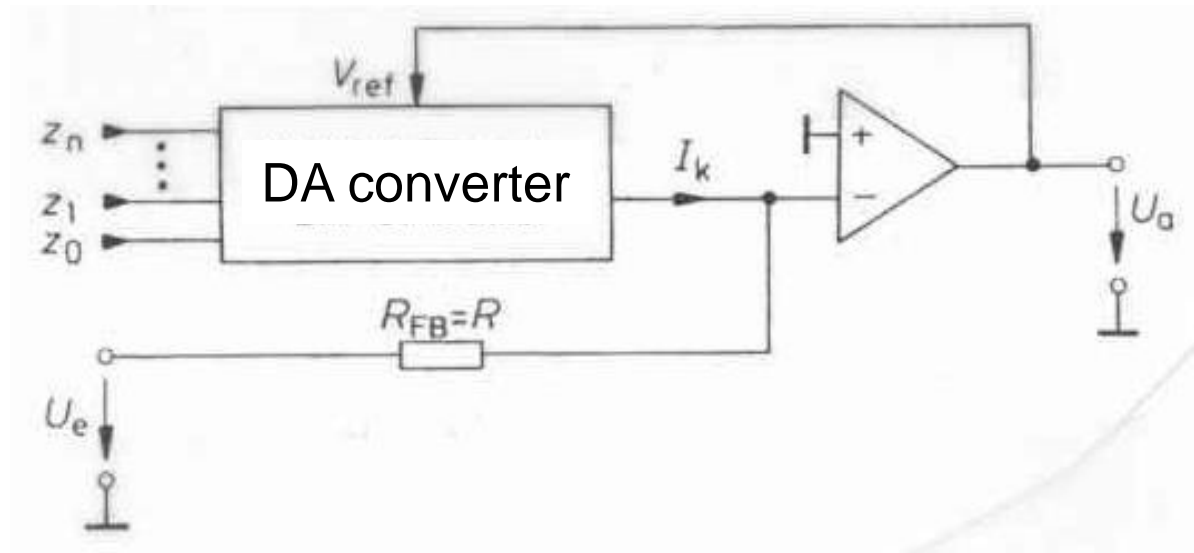
$$R_2 = \frac{R}{2^n} \Rightarrow I_{LSB} = \frac{U_{ref}}{2^n R}$$

$$U_a = (I'_k + I_{LSB} - I_k)R = U_{ref} \frac{2^{n-1} - Z - 1 + 1 - Z - 2^{n-1}}{2^n} = -U_{ref} \frac{Z}{2^{n-1}}$$

Exercice

Multiplication / division

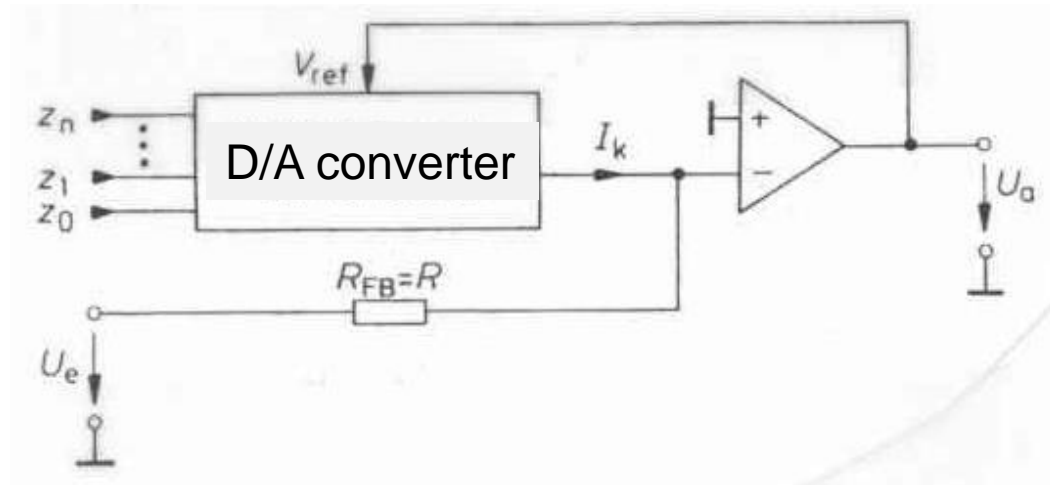
Show that in the circuit below, the output voltage U_a is proportional to U_e/Z .



Exercice

Multiplication / division

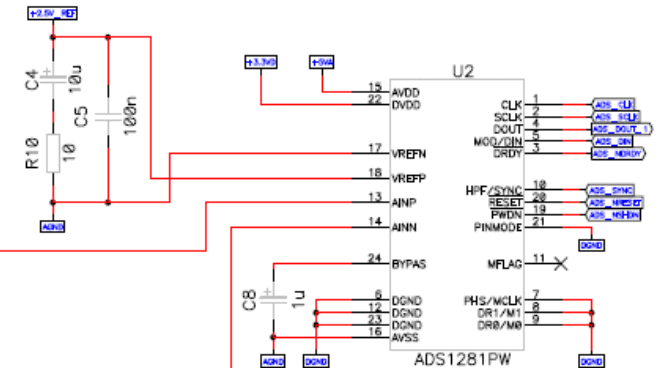
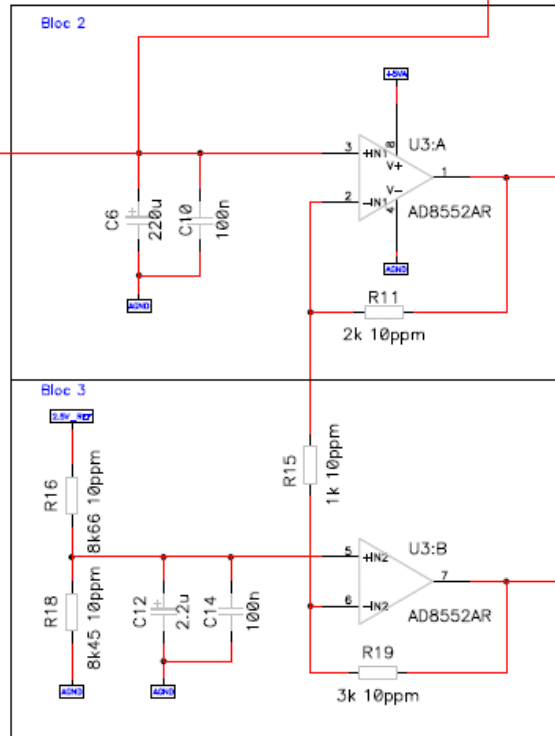
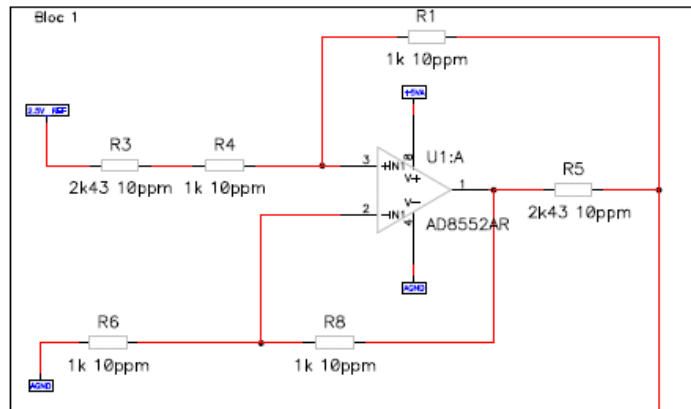
Show that in the circuit below, the output voltage U_a is proportional to U_e/Z .



$$I_k = \frac{V_{ref}}{R} \frac{Z}{2^n} = \frac{U_a}{R} \frac{Z}{2^n} = -\frac{U_e}{R_{FB}} = -\frac{U_e}{R} \Rightarrow U_a = -U_e \frac{2^n}{Z}$$

Exercice

Pt1000 input circuit (1)



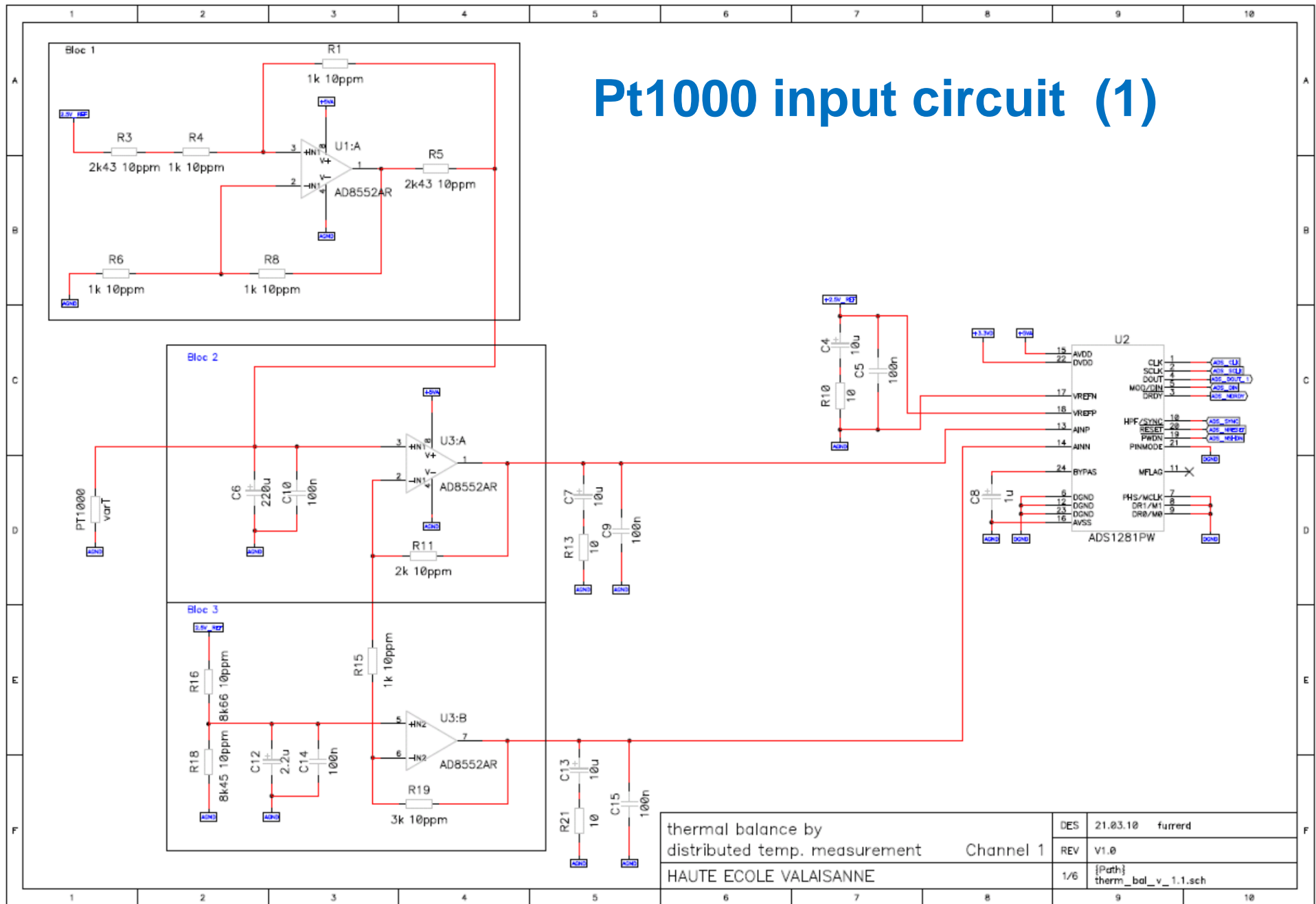
thermal balance by distributed temp. measurement	DES	21.03.10	furrerd
	REV	V1.0	
	1/6	{Path}	therm_bal_v_1.1.sch

Pt1000 input circuit (2)

The circuit on the slide above acquires a temperature measured by a Pt1000 resistor, through a 24 bits ADC (ADS1281), with the following characteristics : $f_s = 10\text{Hz}$, dynamic range of AINP and AINN : $0 \dots V_{\text{ref}}$, conversion of $V_{\text{AINP}} - V_{\text{AINN}}$ within the range $\pm V_{\text{ref}}$. $V_{\text{ref}} = 2.5\text{V}$ (net +2.5V_REF).

- Describe in words the function of each of the 3 framed blocks.
- Why are resistors R16 and R18 not set to equal values ?
- Determine the temperature measurement range in $^{\circ}\text{C}$ and the circuit sensitivity in $\text{LSB}/^{\circ}\text{C}$, knowing that the resistance of the Pt1000 sensor is 1000Ω at 0°C , increasing by $0.4\%/^{\circ}\text{C}$.
- Consider only voltage and current noises of the operational amplifiers AD8552: $40\text{nV}/\text{rtHz}$ and $2\text{fA}/\text{rtHz}$, supposed to be uniform over frequency. What is the spectral density of the current noise in the Pt1000 sensor ? What is the spectral density of the voltage noise at inputs AINP and AINN of the A/D converter ? What is the signal-to-noise ratio between 0 and 5Hz after A/D conversion, taking into account quantization noise and noise in the operational amplifiers ?

Pt1000 input circuit (1)



Pt1000 input circuit (2)

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➤ Describe in words the function of each of the 3 framed blocks.

$$U_i = U_o * R_6 / (R_6 + R_8) = U_o / 2$$

$$I_P = (V_{\text{ref}} - U_i) / (R_3 + R_4)$$

$$I_{\text{out}} = I_{R5} + I_P = (U_o - (U_i - R_1 I_P)) / R_5 + I_P = (U_o / 2 + R_1 I_P) / R_5 + I_P$$

$$I_{\text{out}} = U_o / (2 R_5) + I_P (R_1 + R_5) / R_5 = U_o / (2 R_5) + (V_{\text{ref}} - U_i) / R_5 = V_{\text{ref}} / R_5 = 1.03\text{mA}$$

Block 1 : constant current source

$$U_{iB} = V_{\text{ref}} * R_{18} / (R_{16} + R_{18}) = 1.235\text{V}$$

$$U_{iA} = U_{\text{PT1000}}$$

$$U_{\text{AINP}} = 3 U_{\text{PT1000}} - 2 U_{iB}$$

Block 2 : Range adaptation for input AINP of the ADC.

$$U_{\text{AINN}} = 4 U_{iB} - 3 U_{\text{PT1000}}$$

Block 3 : Generation of inverted signal for input AINN of the ADC.

Pt1000 input circuit (3)

➤ Why are resistors R16 and R18 not set to equal values ?
This is to avoid saturation of inputs AINP and AINN close to full scale values.

➤ Determine the temperature measurement range in °C and the circuit sensitivity in LSB/°C, knowing that the resistance of the Pt1000 sensor is 1000Ω at 0°C, increasing by 0.4%/°C.

$$U_{\text{AINP}} = 0 \text{ for } U_{\text{Pt1000}} = 2 U_{\text{iB}} / 3 = 0.823\text{V} \Rightarrow R_{\text{Pt1000}} = 800\Omega, T_{\text{min}} = -50^\circ\text{C}$$

$$U_{\text{AINN}} = 0 \text{ for } U_{\text{Pt1000}} = 4 U_{\text{iB}} / 3 = 1.646\text{V} \Rightarrow R_{\text{Pt1000}} = 1600\Omega, T_{\text{max}} = 150^\circ\text{C}$$

Measurement range from -50°C to +150°C

$$U_{\text{AINNmax}} = U_{\text{AINPmax}} = 2U_{\text{iB}} = 2.470\text{V}$$

$$\text{Sensitivity} = 2^{24} * (U_{\text{AINNmax}} + U_{\text{AINPmax}}) / (2V_{\text{ref}}) / (T_{\text{max}} - T_{\text{min}}) = 82'879 \text{ LSB}/^\circ\text{C}$$

Pt1000 input circuit (4)

➤ Consider only voltage and current noises of the operational amplifiers AD8552: 40nV/rHz and 2fA/rHz, supposed to be uniform over frequency. What is the spectral density of the current noise in the Pt1000 sensor ? What is the spectral density of the voltage noise at inputs AINP and AINN of the A/D converter ? What is the signal-to-noise ratio between 0 and 5Hz after A/D conversion, taking into account quantization noise and noise in the operational amplifiers ?

Spectral current noise density in the Pt1000 sensor:

$$I_{nPu} = (U_n - U_i) / (R_3 + R_4)$$

$$I_{noutu} = (2U_i - (U_i - U_n - R_1 I_{nPu})) / R_5 + I_{nPu} = 2U_n / R_5 = 32.9\text{pA/rHz}$$

I_{nouti} is negligible, since at least one order of magnitude smaller than I_{noutu} .

$$I_{nout} = I_{noutu} = 32.9\text{pA/rHz in the Pt1000.}$$

The voltage noise of U3:A is amplified like U_{Pt1000} , the one of U3:B like U_{iB} .

Current noises are again negligible, given the level of resistors present.

Spectral density of noise contributed by U3:A

$$U_{nAINP} - U_{nAINN} = 6U_n = 240\text{nV/rHz}$$

Spectral density of noise contributed by U3:B

$$U_{nAINP} - U_{nAINN} = 6U_n = 240\text{nV/rHz}$$

Pt1000 input circuit (5)

Spectral density of noise contributed by the current source (e.g. in centre of range, $R_{Pt1000} = 1200\Omega$)

$$U_{nAINP} - U_{nAINN} = 6R_{Pt1000}I_{nout} = 237\text{nV}/\text{rtHz}$$

Spectral density of total analog noise :

$$U_{nAINP} - U_{nAINN} = \sqrt{240^2 + 240^2 + 237^2} [\text{nV}/\text{rtHz}] = 414\text{nV}/\text{rtHz}$$

Within the frequency band 0...5Hz :

$$(U_{nAINP} - U_{nAINN})_{\text{rms}} = 926\text{nV}$$

1LSB corresponds to $2 \cdot 2.5\text{V}/2^{24} = 298\text{nV}$, equivalent spectral density of quantization noise : $U_{\text{LSB}}/\sqrt{12} = 86\text{nV}/\text{rtHz}$

Spectral density of total noise :

$$U_{nAINP} - U_{nAINN} = \sqrt{240^2 + 240^2 + 237^2 + 86^2} [\text{nV}/\text{rtHz}] = 423\text{nV}/\text{rtHz}$$

Within the frequency band 0...5Hz :

$$(U_{nAINP} - U_{nAINN})_{\text{rms}} = 945\text{nV}$$

$$\text{SNR} = 20 \log_{10} (V_{\text{ref}}/\sqrt{2} / (U_{nAINP} - U_{nAINN})_{\text{rms}}) = 125\text{dB}$$

Effect of clock jitter on conversion noise

We have a 20bit audio SAR ADC with a sampling clock rate of $f_s = 48\text{ksps}$. How much rms jitter of the ADC clock signal (suppose $f_{\text{CLK}} = 24 \cdot f_s$) can be admitted, so that the conversion error remains below one LSB for sinusoidal full scale input ($\pm 1\text{V}$) signals of frequency $f = 4\text{kHz}$?

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Largest error at zero crossing of sine oscillation, since slope is maximum there:

$$\left. \frac{du}{dt} \right|_{t=0} = \hat{U} \omega$$

Amplitude error resulting from sampling instant uncertainty Δt_A :

$$\Delta u = \hat{U} 2\pi f \Delta t_A$$

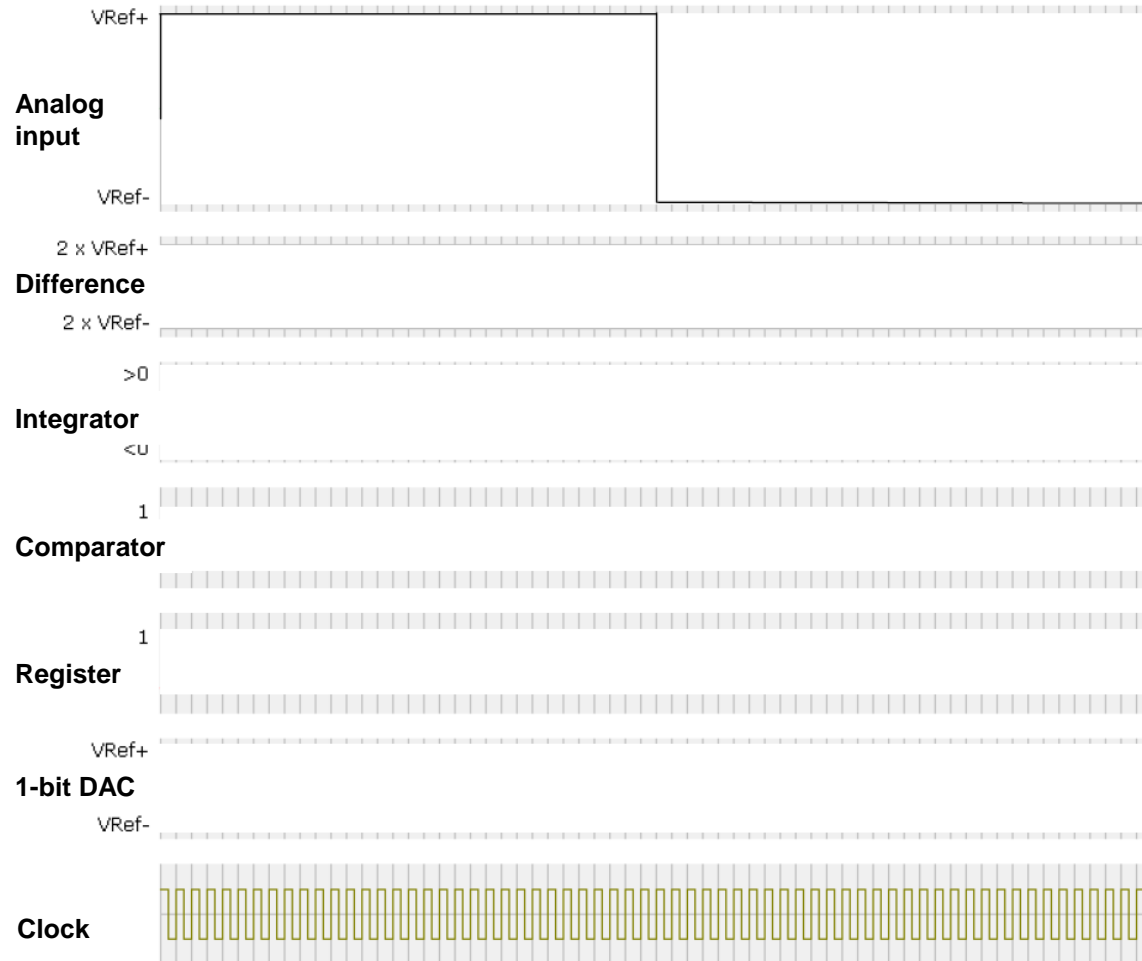
The amplitude of the sine signal is half of the input range U_{max} , so :

$$\hat{U} \leq \frac{U_{\text{FS}}}{2}, \quad U_{\text{LSB}} = \frac{U_{\text{FS}}}{2^n} \quad \Rightarrow \quad \Delta t_{A, \text{pk-pk}} \leq \frac{1}{2^n \pi f}, \quad \Delta t_{A, \text{rms}} \leq \frac{1}{6.6 \cdot 2^n \pi f}$$

With given numeric values, $\Delta t_{A, \text{rms}} < 11.5\text{psec}$.

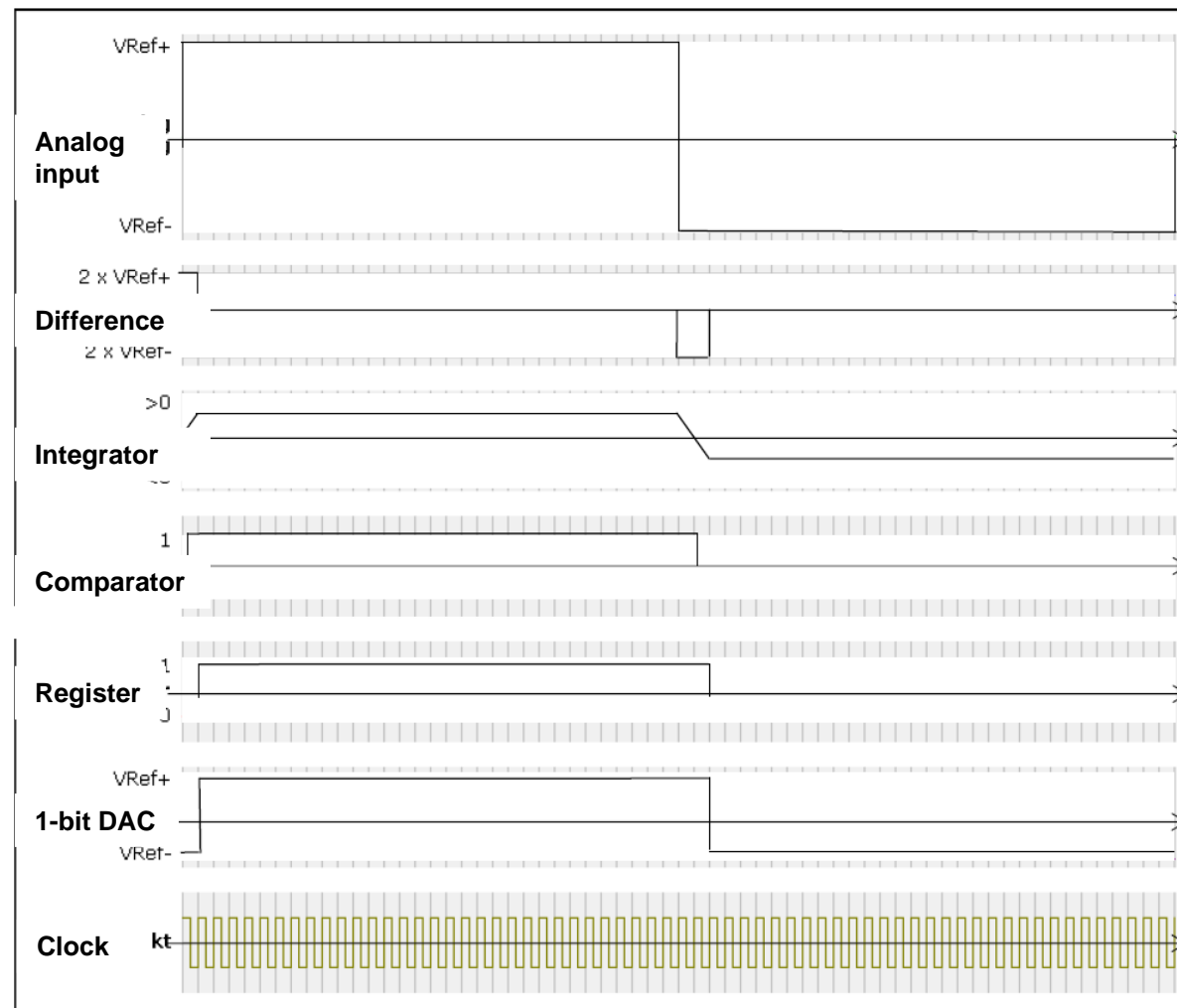
Signal forms

Trace the forms of signals in an analog first order modulator for a rectangular input signal:



Retrace the bitstream after decimation by 8.

Signal forms

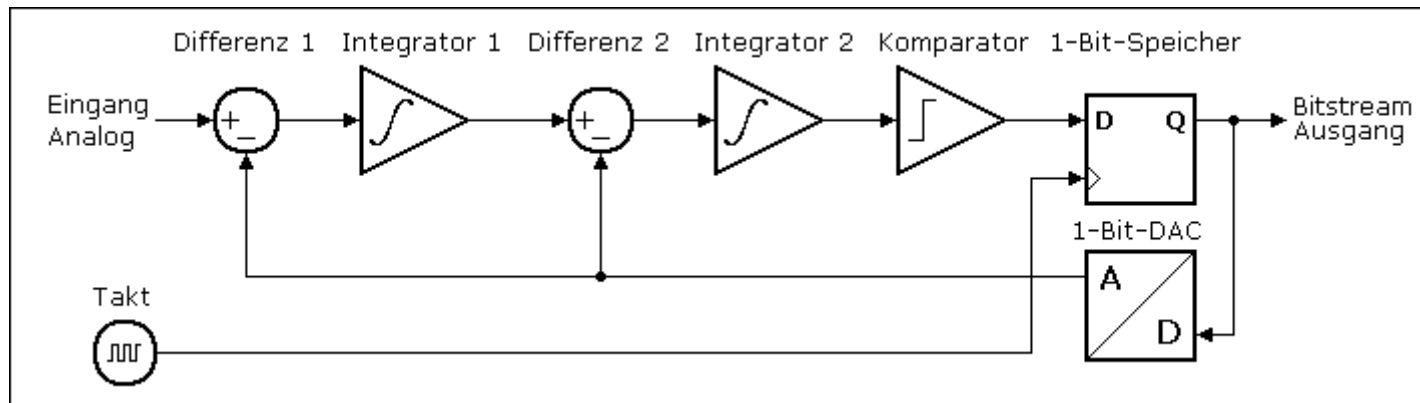


Bitstream after decimation by 8:



Solution of exercises

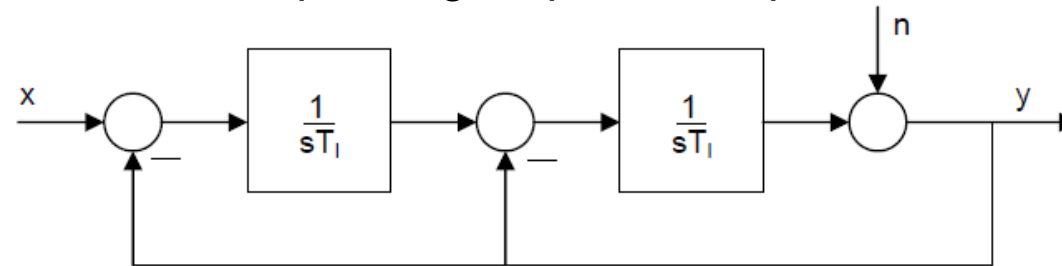
Second order modulator transfer functions



- Determine the equivalent continuous transfer functions of a second order delta-sigma modulator, for the signal and for the noise.
- Represent the corresponding amplitude responses in a Bode diagram.

Second order modulator transfer functions

- Determine the equivalent continuous transfer functions of a second order delta-sigma modulator, for the signal and for the noise.
- Represent the corresponding amplitude responses in a Bode diagram.



$$y = n + \frac{1}{sT_I} \left(-y + \frac{1}{sT_I} (x - y) \right)$$

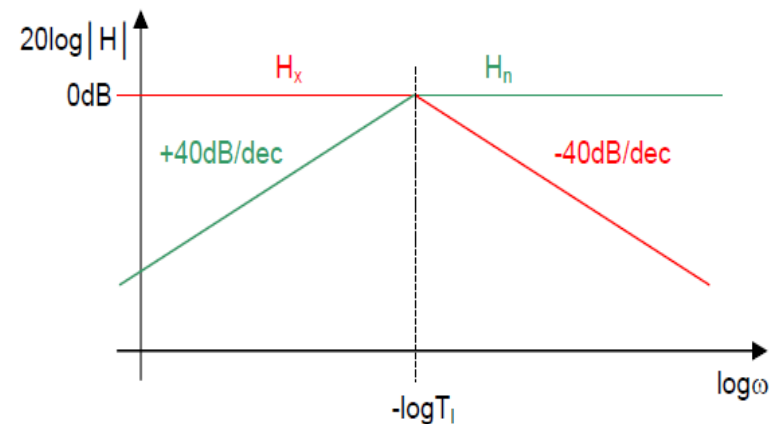
$$y(s^2T_I^2 + sT_I + 1) = ns^2T_I^2 + x$$

Signal transfer:

$$H_x = \frac{y}{x} = \frac{1}{1 + sT_I + s^2T_I^2}$$

Noise transfer:

$$H_n = \frac{y}{n} = \frac{s^2T_I^2}{1 + sT_I + s^2T_I^2}$$



Signal to noise and oversampling ratios

- What is the maximum signal-to-noise ratio of a 16bit ADC? Suppose a sinusoidal input signal, and that the quantisation noise dominates all other noises. The noise is considered as white noise.
- Which resolution, expressed as number of bits (ENOB), can have an ADC with a second order delta-sigma modulator, operating at an oversampling ratio of 128?
- Consider a delta-sigma ADC with ENOB = 16bits and an input signal bandwidth of 1kHz. The oversampling ratio is 128. Which order is required for an analog anti-aliasing filter?

Signal to noise and oversampling ratios (2)

- Consider a delta-sigma ADC with 16bit resolution and an input signal bandwidth of 1kHz. The oversampling ratio is 128. Which order is required for an analog anti-aliasing filter?

A maximum attenuation of $-SNR = -98\text{dB} \cong 80'000$ is required at $f > 128 f_{c, \text{Antialias}}$.

E.g. for a Butterworth filter, the order n is given by

$$H_{\text{Antialias}}^2 = \frac{1}{1 + \left(\frac{f}{f_{c, \text{Antialias}}} \right)^{2n}} \Rightarrow n = \frac{\log\left(\frac{1}{H_{\text{Antialias}}^2} - 1 \right)}{2 \log\left(\frac{f}{f_{c, \text{Antialias}}} \right)} \approx 2.3 \Rightarrow n = 3$$

Second order high pass filters

- Propose circuits for second order switched capacitor high-pass filters.

Second order high pass filters

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