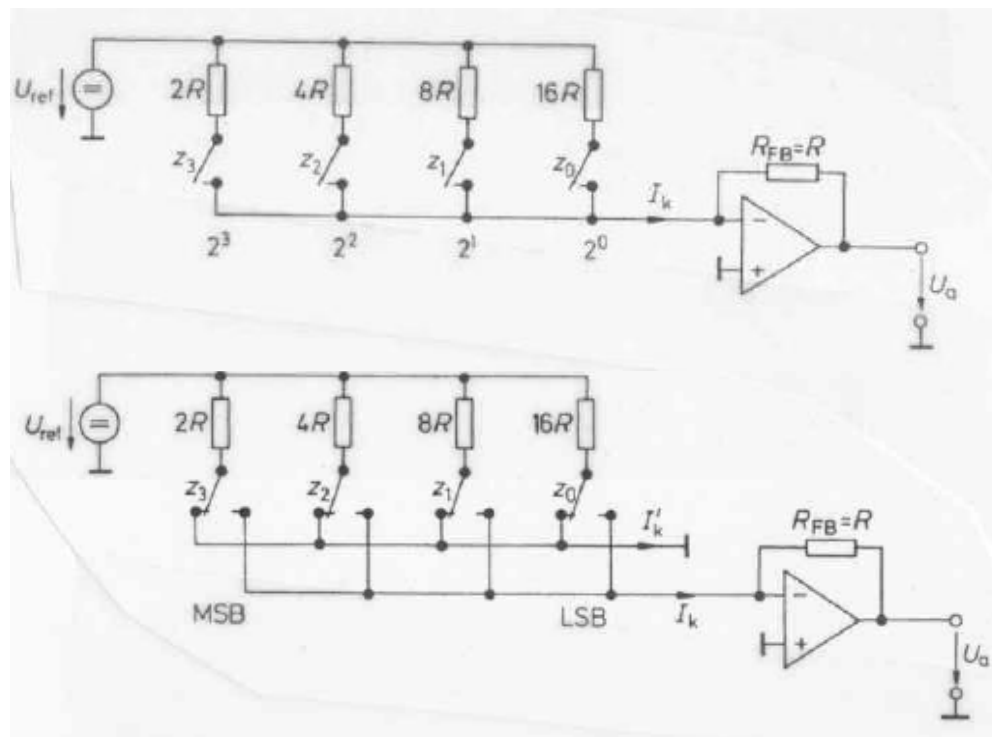


## Binary weighted structure

The two schematics below show binary weighted structures for DA conversion. The principal inconvenients of the upper structure are

- The load of  $U_{ref}$  is not constant, but depends on  $z$ .
- The parasitic capacitances of the switches are (dis)charged when switching.

Explain why the lower schematic solves these problems.



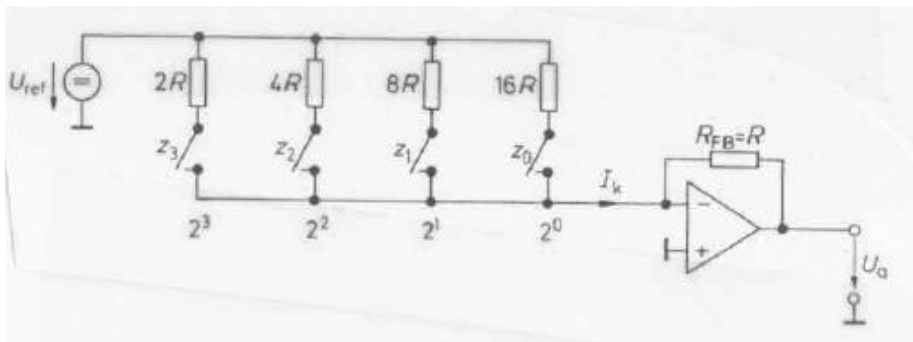
Exercice

## Binary weighted structure

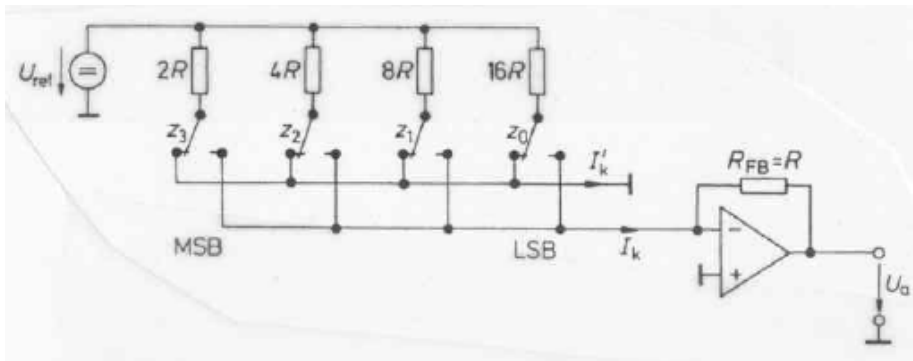
The two schematics below show binary weighted structures for DA conversion. The principal inconvenients of the upper structure are

- The load of  $U_{ref}$  is not constant, but depends on  $z$ .
- The parasitic capacitances of the switches are (dis)charged when switching.

Explain why the lower schematic solves these problems.



- The load of  $U_{ref}$  is always the same, independently of the switch positions, since current always flows into one of two collector rails.



- The connections of any switch are always at the same potential, so that no charging / discharging of stray capacitances occurs.

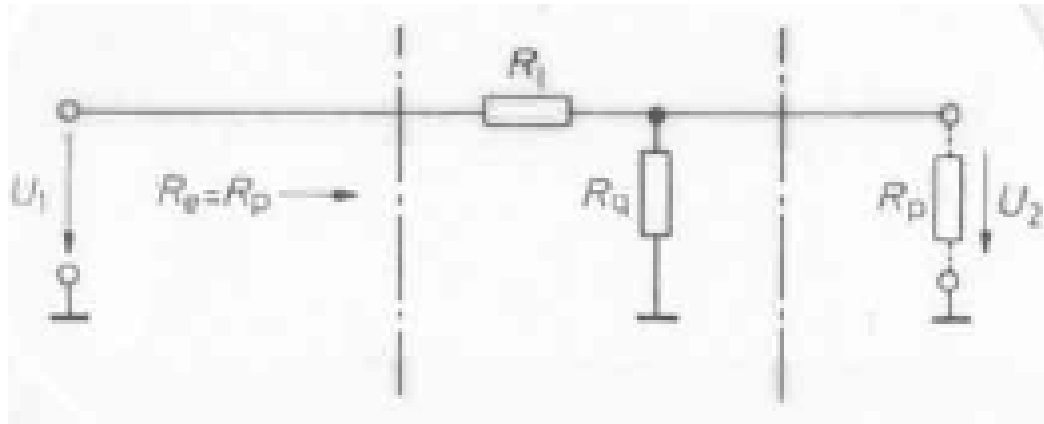
Exercice

## Generalized ladder network

Draw a schematic of a voltage output 4bit-DAC with an R/2R ladder network, using commutators between two output current rails.

With the help of the circuit shown below, express  $R_l$  and  $R_p$  as functions of  $R_q$ , so that

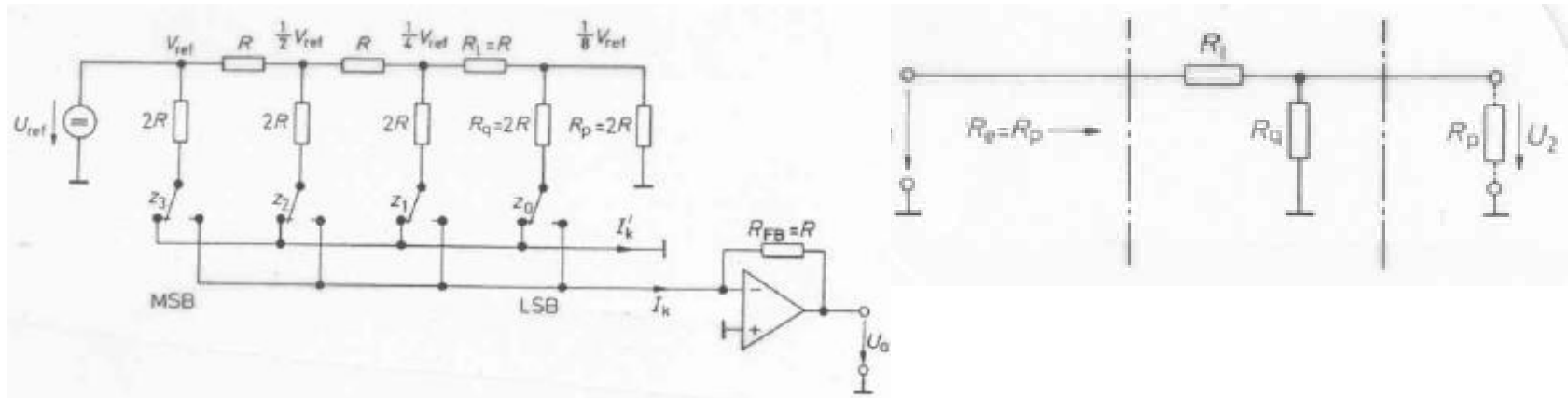
$$U_2/U_1 = \alpha$$



Exercice

# Generalized ladder network

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With the help of the circuit shown above, express  $R_l$  and  $R_p$  as functions of  $R_q$ , so that  $\alpha = U_2/U_1$

$$R_e = R_p = R_l + \frac{R_p R_q}{R_p + R_q} \Rightarrow R_l = \frac{R_p^2}{R_p + R_q}$$

$$\alpha = \frac{U_2}{U_1} = \frac{R_q}{R_p + R_q} \Rightarrow R_p = R_q \left( \frac{1}{\alpha} - 1 \right)$$

$$R_l = \alpha \left( \frac{1}{\alpha} - 1 \right)^2 R_q = \frac{(1-\alpha)^2}{\alpha} R_q$$

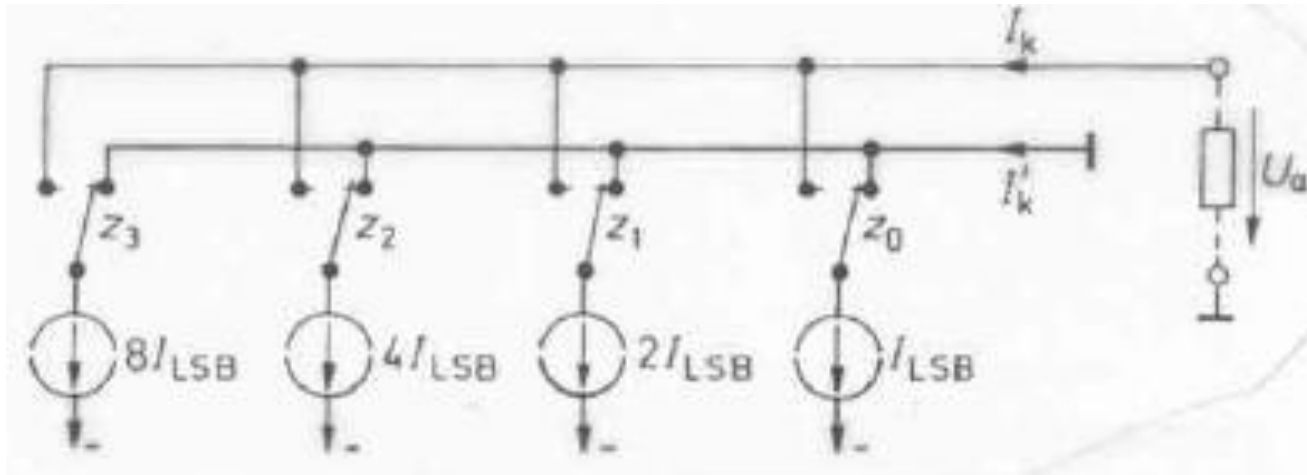
In particular, for  $\alpha = 0.5$ :

$$R_p = R_q = 2 R_l$$

Exercice

## Arrays of current sources

Instead of a reference voltage and a set of weighted resistors, a set of weighted current sources can be used for DA conversion as shown below.

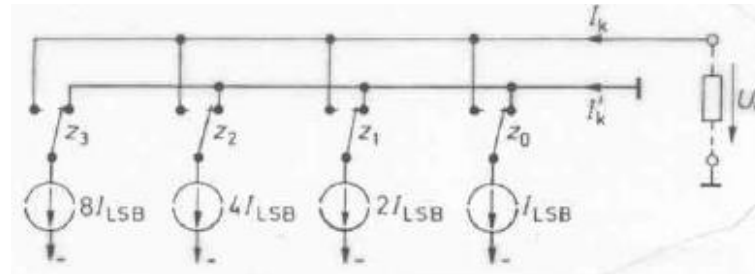


Propose an alternative circuit using a set of current sources of same value and a ladder network for DA conversion.

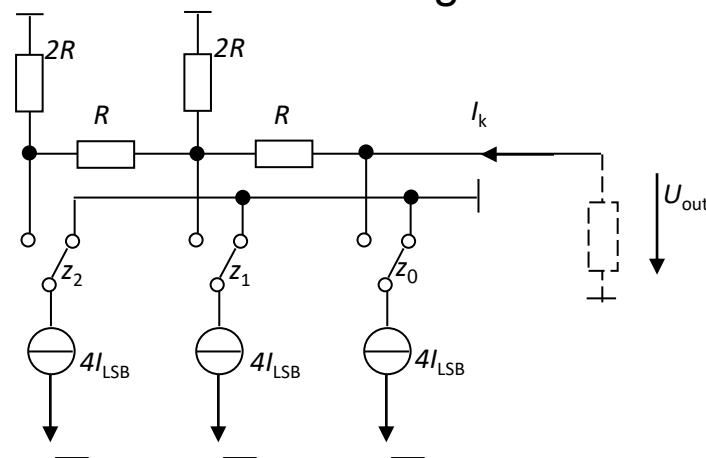
Exercise

## Arrays of current sources

Instead of a reference voltage and a set of weighted resistors, a set of weighted current sources can be used for D/A conversion as shown below.



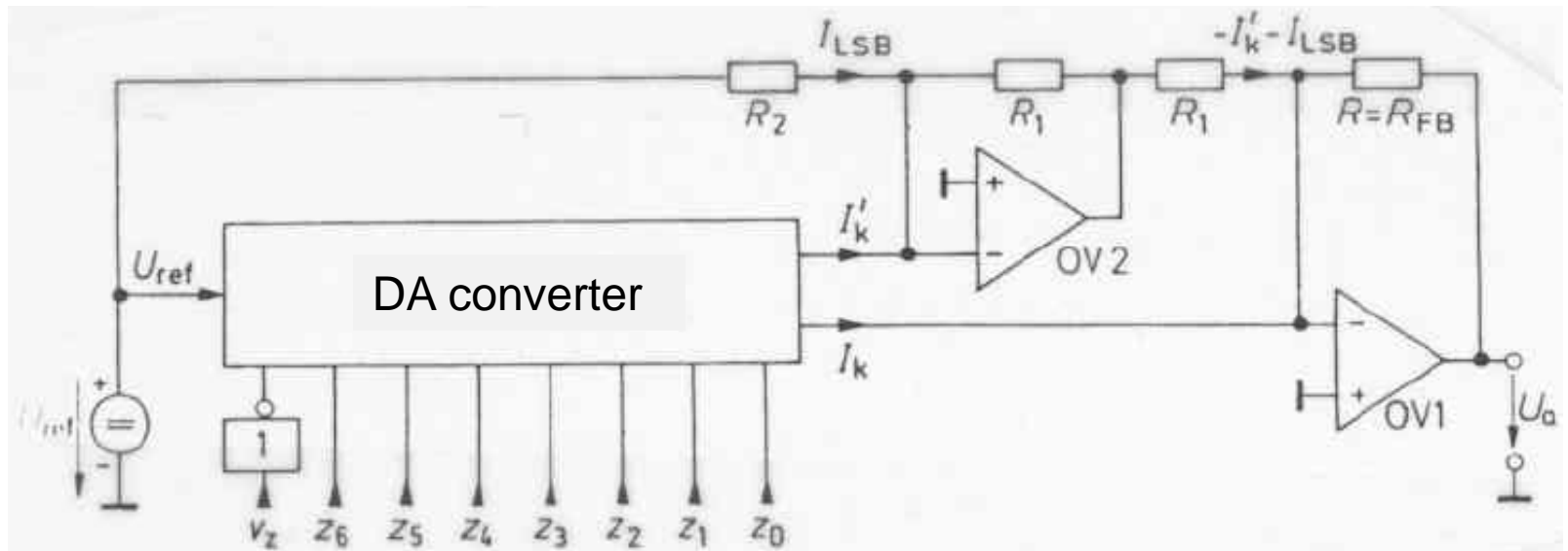
Propose an alternative circuit using a set of current sources of same value and a ladder network for D/A conversion. E.g. for three bits:



Exercice

## Bipolar output

For the circuit shown below, determine  $U_a$  as a function of  $Z$ , a signed binary number in 2's complement representation.  $I_k$  and  $I'_k$  are the two complementary current outputs of the DA converter.

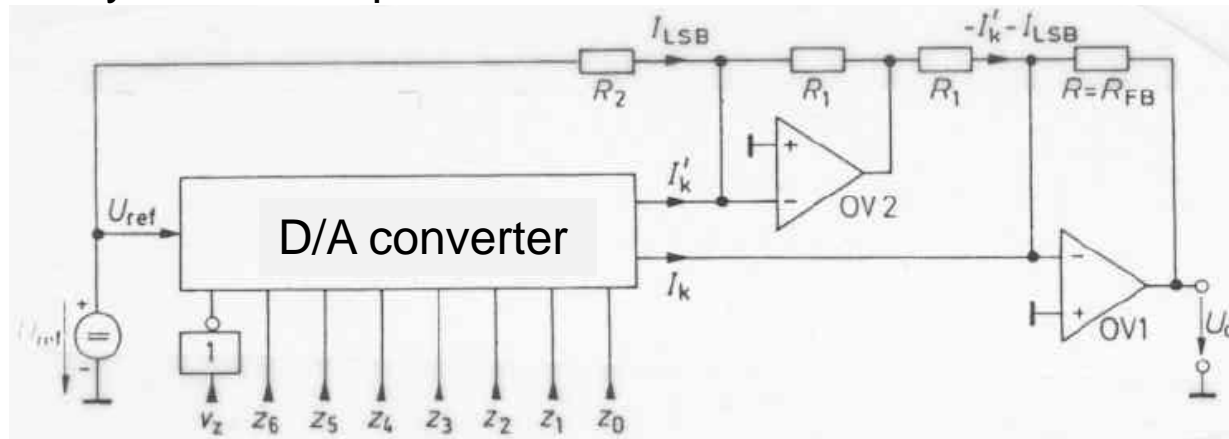


What is the range of  $U_a$ ?

Exercice

## Bipolar output

For the circuit shown below, determine  $U_a$  as a function of  $Z$ , a signed binary number in 2's complement representation.  $I_k$  and  $I'_k$  are the two complementary current outputs of the D/A converter.



$$-2^{n-1} < Z = [v_z, z_{n-1}, \dots, z_0] < 2^{n-1} - 1$$

$$I_k = \frac{U_{ref}}{R} \frac{Z + 2^{n-1}}{2^n}, \quad I'_k = \frac{U_{ref}}{R} \left( 1 - \frac{1}{2^n} \right) - I_k = \frac{U_{ref}}{R} \frac{2^{n-1} - Z - 1}{2^n}$$

$$R_2 = \frac{R}{2^n} \Rightarrow I_{LSB} = \frac{U_{ref}}{2^n R}$$

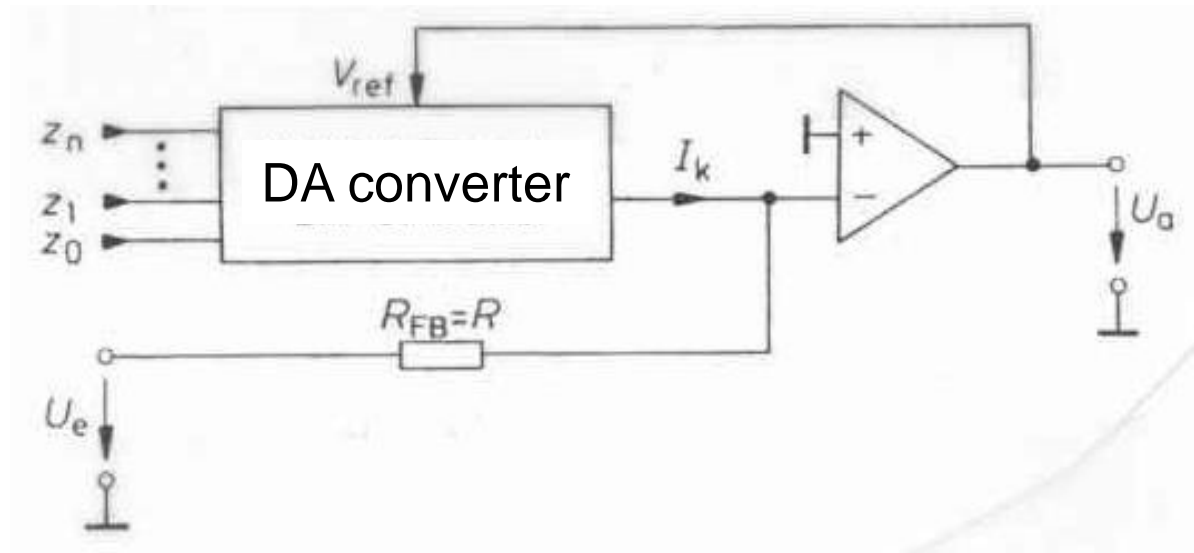
$$U_a = (I'_k + I_{LSB} - I_k)R = U_{ref} \frac{2^{n-1} - Z - 1 + 1 - Z - 2^{n-1}}{2^n} = -U_{ref} \frac{Z}{2^{n-1}}$$

Exercice



## Multiplication / division

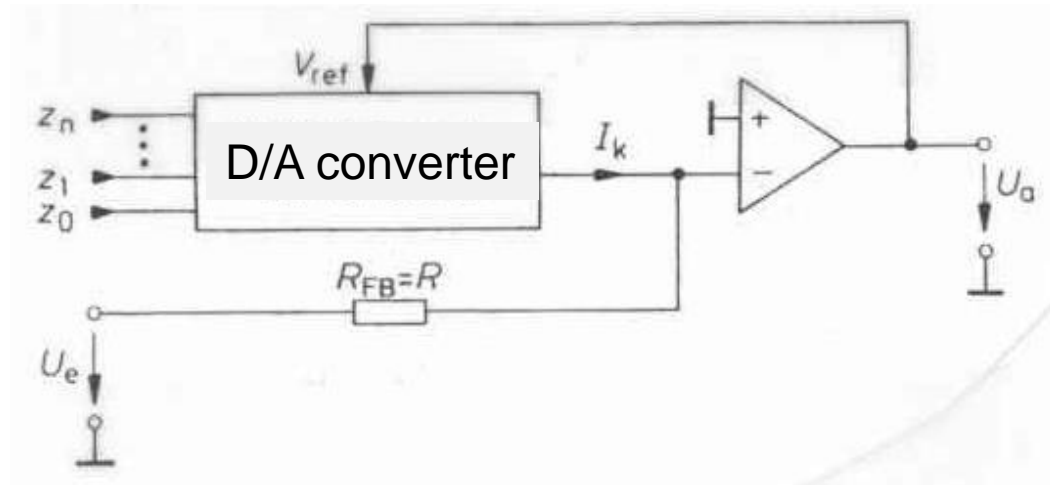
Show that in the circuit below, the output voltage  $U_a$  is proportional to  $U_e/Z$ .



Exercice

## Multiplication / division

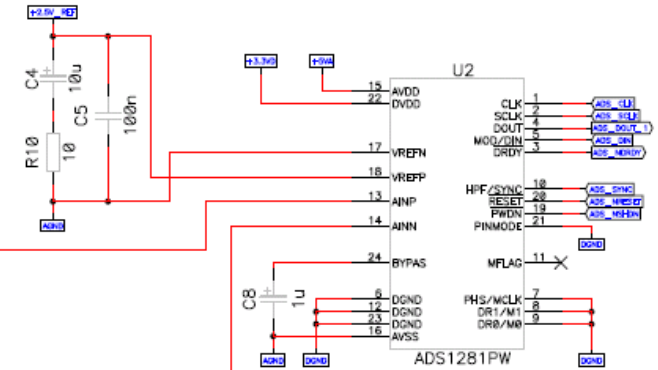
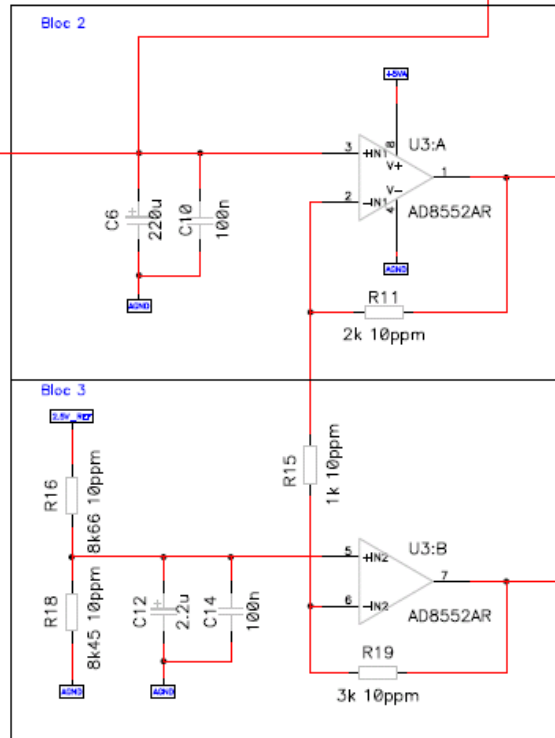
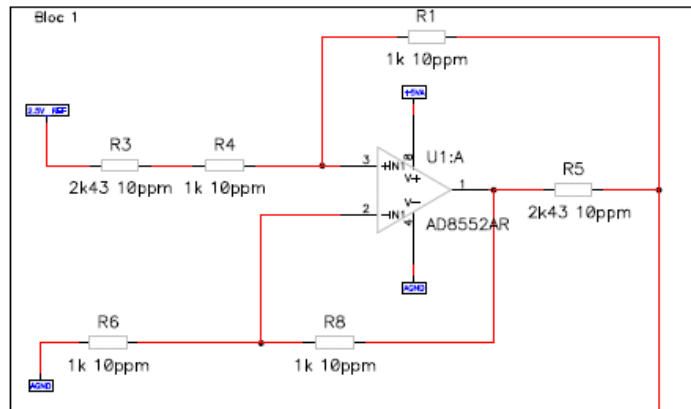
Show that in the circuit below, the output voltage  $U_a$  is proportional to  $U_e/Z$ .



$$I_k = \frac{V_{ref}}{R} \frac{Z}{2^n} = \frac{U_a}{R} \frac{Z}{2^n} = -\frac{U_e}{R_{FB}} = -\frac{U_e}{R} \Rightarrow U_a = -U_e \frac{2^n}{Z}$$

Exercice

# Pt1000 input circuit (1)



thermal balance by  
distributed temp. measurement Channel 1  
HAUTE ECOLE VALAISANNE

DES	21.03.10	furrerd
REV	V1.0	
1/6	{Path} therm_bal_v_1.1.sch	

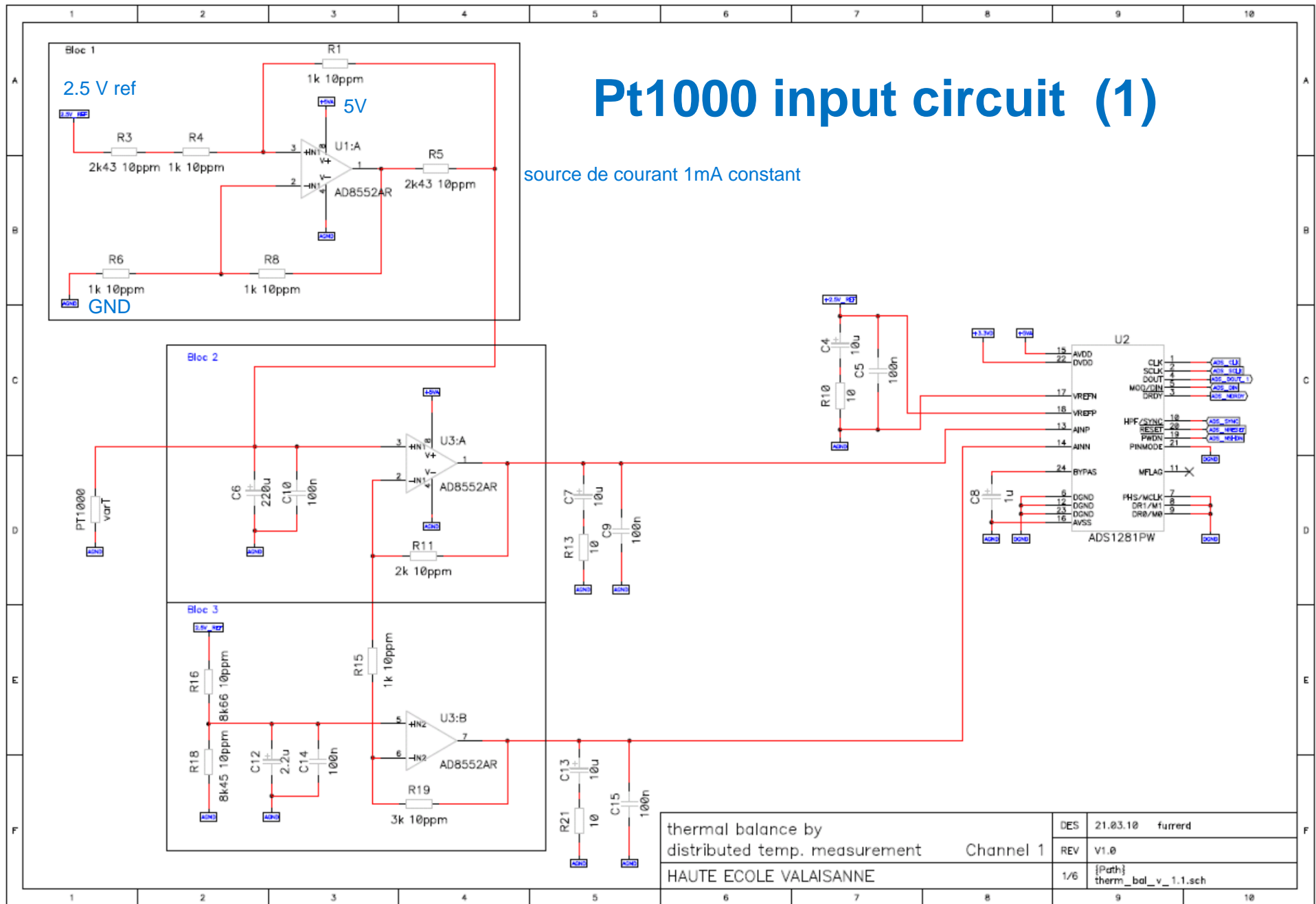
## Pt1000 input circuit (2)

The circuit on the slide above acquires a temperature measured by a Pt1000 resistor, through a 24 bits ADC (ADS1281), with the following characteristics :  $f_s = 10\text{Hz}$ , dynamic range of AINP and AINN :  $0 \dots V_{\text{ref}}$ , conversion of  $V_{\text{AINP}} - V_{\text{AINN}}$  within the range  $\pm V_{\text{ref}}$ .  $V_{\text{ref}} = 2.5\text{V}$  (net +2.5V\_REF).

- Describe in words the function of each of the 3 framed blocks.
- Why are resistors R16 and R18 not set to equal values ?
- Determine the temperature measurement range in  $^{\circ}\text{C}$  and the circuit sensitivity in  $\text{LSB}/^{\circ}\text{C}$ , knowing that the resistance of the Pt1000 sensor is  $1000\Omega$  at  $0^{\circ}\text{C}$ , increasing by  $0.4\%/^{\circ}\text{C}$ .
- Consider only voltage and current noises of the operational amplifiers AD8552:  $40\text{nV}/\text{rtHz}$  and  $2\text{fA}/\text{rtHz}$ , supposed to be uniform over frequency. What is the spectral density of the current noise in the Pt1000 sensor ? What is the spectral density of the voltage noise at inputs AINP and AINN of the A/D converter ? What is the signal-to-noise ratio between 0 and 5Hz after A/D conversion, taking into account quantization noise and noise in the operational amplifiers ?

# Pt1000 input circuit (1)

source de courant 1mA constant



Bloc 1:

$$U_i(-) = U_o \left( \frac{R_6}{R_6 + R_8} \right) = \frac{U_o}{2}$$

$$I_p = \frac{V_{ref} - U_i}{R_3 + R_4}$$

$$I_{out} = I_p + I_{R5}$$

$$= \frac{U_o - (U_i - R_1 I_p)}{R_5}$$

$$I_{out} = I_p + \frac{U_o - U_i + R_1 I_p}{R_5} = \frac{\frac{U_o}{2} + R_1 I_p}{R_5} + I_p$$

$$= \frac{U_o}{2R_5} + \frac{R_1 I_p}{R_5} + \frac{R_5 I_p}{R_5} = \frac{U_o}{2R_5} + \frac{(R_1 + R_5) I_p}{R_5} \rightarrow \frac{V_{ref} - U_i}{R_5} = \frac{U_o + 2V_{ref} - \frac{2U_o}{2}}{2R_5}$$
$$= \frac{V_{ref}}{R_5}$$

## Pt1000 input circuit (2)

The circuit on the slide above acquires a temperature measured by a Pt1000 resistor, through a 24 bits ADC (ADS1281), with the following characteristics :  $f_s = 10\text{Hz}$ , dynamic range of AINP and AINN :  $0 \dots V_{\text{ref}}$ , conversion of  $V_{\text{AINP}} - V_{\text{AINN}}$  within the range  $\pm V_{\text{ref}}$ .  $V_{\text{ref}} = 2.5\text{V}$  (net +2.5V\_REF).

➤ Describe in words the function of each of the 3 framed blocks.

$$U_i = U_o * R_6 / (R_6 + R_8) = U_o / 2$$

$$I_P = (V_{\text{ref}} - U_i) / (R_3 + R_4)$$

$$I_{\text{out}} = I_{R5} + I_P = (U_o - (U_i - R_1 I_P)) / R_5 + I_P = (U_o / 2 + R_1 I_P) / R_5 + I_P$$

$$I_{\text{out}} = U_o / (2 R_5) + I_P (R_1 + R_5) / R_5 = U_o / (2 R_5) + (V_{\text{ref}} - U_i) / R_5 = V_{\text{ref}} / R_5 = 1.03\text{mA}$$

Block 1 : constant current source

$$U_{iB} = V_{\text{ref}} * R_{18} / (R_{16} + R_{18}) = 1.235\text{V}$$

$$U_{iA} = U_{\text{PT1000}}$$

$$U_{\text{AINP}} = 3 U_{\text{PT1000}} - 2 U_{iB}$$

Block 2 : Range adaptation for input AINP of the ADC.

$$U_{\text{AINN}} = 4 U_{iB} - 3 U_{\text{PT1000}}$$

Block 3 : Generation of inverted signal for input AINN of the ADC.

## Pt1000 input circuit (3)

➤ Why are resistors R16 and R18 not set to equal values ?

This is to avoid saturation of inputs AINP and AINN close to full scale values.

➤ Determine the temperature measurement range in °C and the circuit sensitivity in LSB/°C, knowing that the resistance of the Pt1000 sensor is 1000Ω at 0°C, increasing by 0.4%/°C.

$$U_{\text{AINP}} = 0 \text{ for } U_{\text{Pt1000}} = 2 U_{\text{iB}} / 3 = 0.823\text{V} \Rightarrow R_{\text{Pt1000}} = 800\Omega, T_{\text{min}} = -50^\circ\text{C}$$

$$U_{\text{AINN}} = 0 \text{ for } U_{\text{Pt1000}} = 4 U_{\text{iB}} / 3 = 1.646\text{V} \Rightarrow R_{\text{Pt1000}} = 1600\Omega, T_{\text{max}} = 150^\circ\text{C}$$

Measurement range from -50°C to +150°C

$$U_{\text{AINNmax}} = U_{\text{AINPmax}} = 2U_{\text{iB}} = 2.470\text{V}$$

$$\text{Sensitivity} = 2^{24} * (U_{\text{AINNmax}} + U_{\text{AINPmax}}) / (2V_{\text{ref}}) / (T_{\text{max}} - T_{\text{min}}) = 82'879 \text{ LSB}/^\circ\text{C}$$



## Pt1000 input circuit (4)

➤ Consider only voltage and current noises of the operational amplifiers AD8552: 40nV/rHz and 2fA/rHz, supposed to be uniform over frequency. What is the spectral density of the current noise in the Pt1000 sensor ? What is the spectral density of the voltage noise at inputs AINP and AINN of the A/D converter ? What is the signal-to-noise ratio between 0 and 5Hz after A/D conversion, taking into account quantization noise and noise in the operational amplifiers ?

Spectral current noise density in the Pt1000 sensor:

$$I_{nPu} = (U_n - U_i) / (R_3 + R_4)$$

$$I_{noutu} = (2U_i - (U_i - U_n - R_1 I_{nPu})) / R_5 + I_{nPu} = 2U_n / R_5 = 32.9\text{pA/rHz}$$

$I_{nouti}$  is negligible, since at least one order of magnitude smaller than  $I_{noutu}$ .

$$I_{nout} = I_{noutu} = 32.9\text{pA/rHz in the Pt1000.}$$

The voltage noise of U3:A is amplified like  $U_{Pt1000}$ , the one of U3:B like  $U_{iB}$ .

Current noises are again negligible, given the level of resistors present.

Spectral density of noise contributed by U3:A

$$U_{nAINP} - U_{nAINN} = 6U_n = 240\text{nV/rHz}$$

Spectral density of noise contributed by U3:B

$$U_{nAINP} - U_{nAINN} = 6U_n = 240\text{nV/rHz}$$

## Pt1000 input circuit (5)

Spectral density of noise contributed by the current source (e.g. in centre of range,  $R_{Pt1000} = 1200\Omega$ )

$$U_{nAINP} - U_{nAINN} = 6R_{Pt1000}I_{nout} = 237\text{nV}/\text{rtHz}$$

Spectral density of total analog noise :

$$U_{nAINP} - U_{nAINN} = \text{sqrt}(240^2 + 240^2 + 237^2) [\text{nV}/\text{rtHz}] = 414\text{nV}/\text{rtHz}$$

Within the frequency band 0...5Hz :

$$(U_{nAINP} - U_{nAINN})_{\text{rms}} = 926\text{nV}$$

1LSB corresponds to  $2 \cdot 2.5\text{V}/2^{24} = 298\text{nV}$ , equivalent spectral density of quantization noise :  $U_{\text{LSB}}/\text{sqrt}(12) = 86\text{nV}/\text{rtHz}$

Spectral density of total noise :

$$U_{nAINP} - U_{nAINN} = \text{sqrt}(240^2 + 240^2 + 237^2 + 86^2) [\text{nV}/\text{rtHz}] = 423\text{nV}/\text{rtHz}$$

Within the frequency band 0...5Hz :

$$(U_{nAINP} - U_{nAINN})_{\text{rms}} = 945\text{nV}$$

$$\text{SNR} = 20 \log_{10} (V_{\text{ref}}/\text{sqrt}(2) / (U_{nAINP} - U_{nAINN})_{\text{rms}}) = 125\text{dB}$$

# Effect of clock jitter on conversion noise

We have a 20bit audio SAR ADC with a sampling clock rate of  $f_s = 48\text{ksps}$ . How much rms jitter of the ADC clock signal (suppose  $f_{\text{CLK}} = 24 \cdot f_s$ ) can be admitted, so that the conversion error remains below one LSB for sinusoidal full scale input ( $\pm 1\text{V}$ ) signals of frequency  $f = 4\text{kHz}$ ?

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Largest error at zero crossing of sine oscillation, since slope is maximum there:

$$\left. \frac{du}{dt} \right|_{t=0} = \hat{U} \omega$$

Amplitude error resulting from sampling instant uncertainty  $\Delta t_A$ :

$$\Delta u = \hat{U} 2\pi f \Delta t_A$$

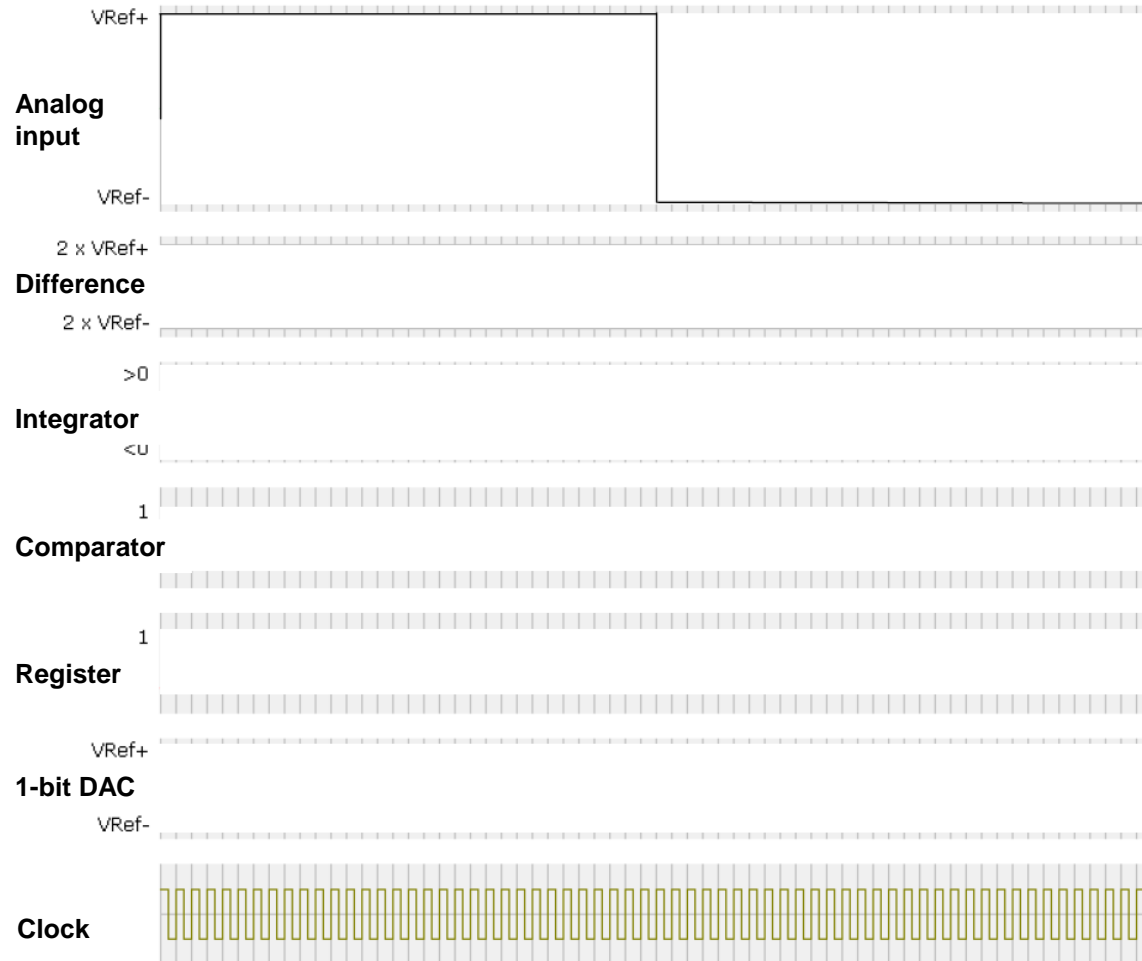
The amplitude of the sine signal is half of the input range  $U_{\text{max}}$ , so :

$$\hat{U} \leq \frac{U_{\text{FS}}}{2}, \quad U_{\text{LSB}} = \frac{U_{\text{FS}}}{2^n} \quad \Rightarrow \quad \Delta t_{A, \text{pk-pk}} \leq \frac{1}{2^n \pi f}, \quad \Delta t_{A, \text{rms}} \leq \frac{1}{6.6 \cdot 2^n \pi f}$$

With given numeric values,  $\Delta t_{A, \text{rms}} < 11.5\text{psec}$ .

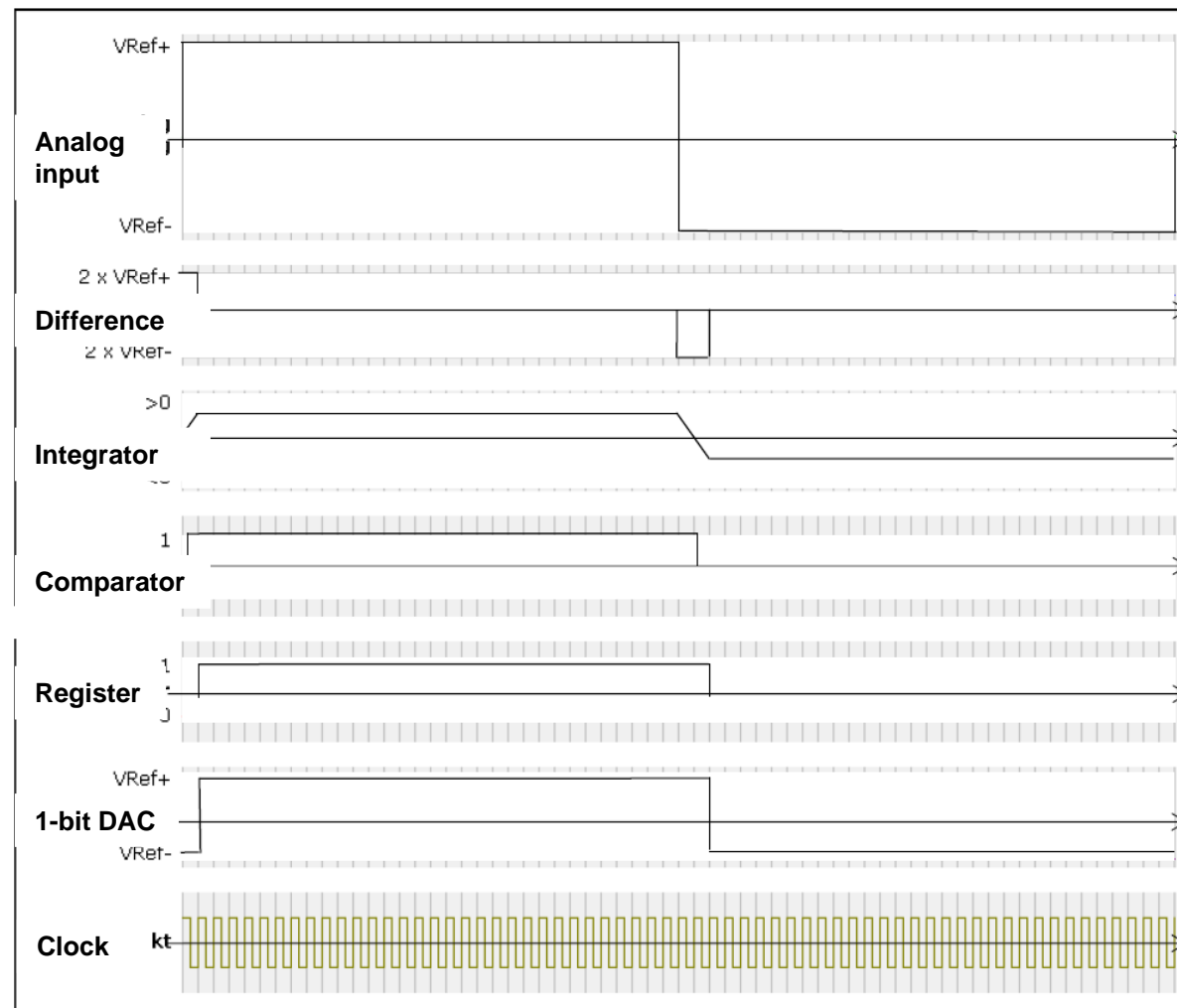
# Signal forms

Trace the forms of signals in an analog first order modulator for a rectangular input signal:



Retrace the bitstream after decimation by 8.

# Signal forms

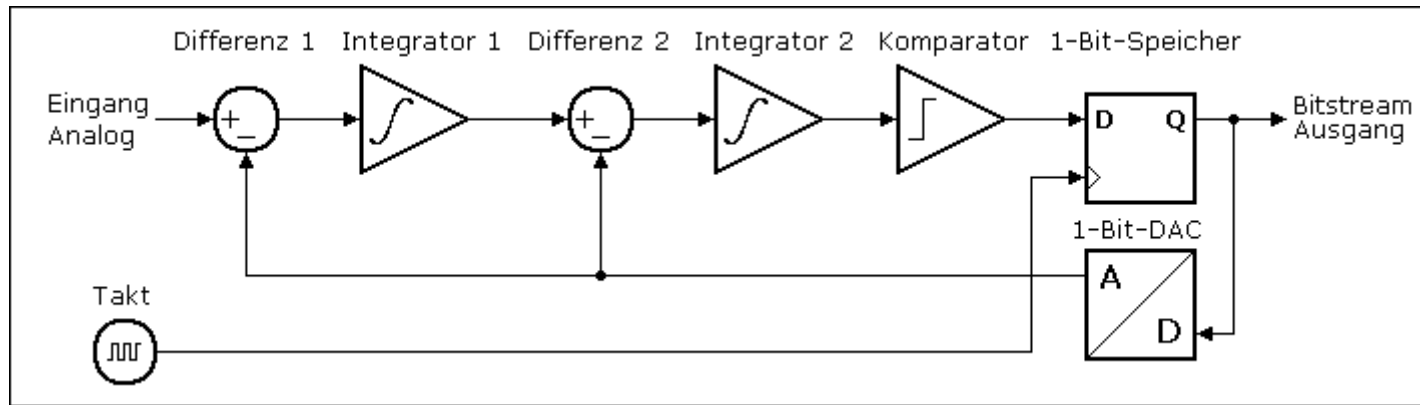


Bitstream after decimation by 8:



Solution of exercises

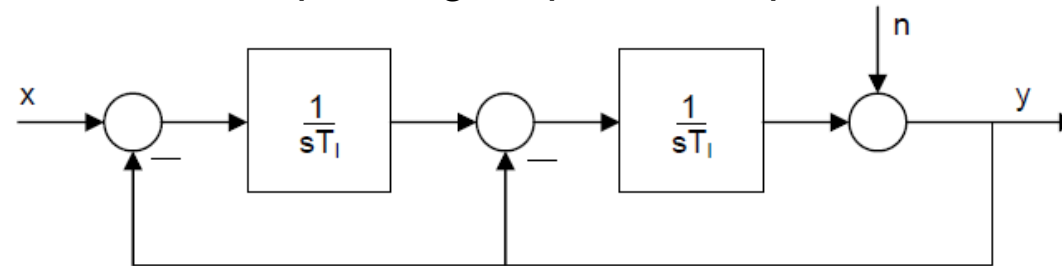
# Second order modulator transfer functions



- Determine the equivalent continuous transfer functions of a second order delta-sigma modulator, for the signal and for the noise.
- Represent the corresponding amplitude responses in a Bode diagram.

## Second order modulator transfer functions

- Determine the equivalent continuous transfer functions of a second order delta-sigma modulator, for the signal and for the noise.
- Represent the corresponding amplitude responses in a Bode diagram.



$$y = n + \frac{1}{sT_I} \left( -y + \frac{1}{sT_I} (x - y) \right)$$

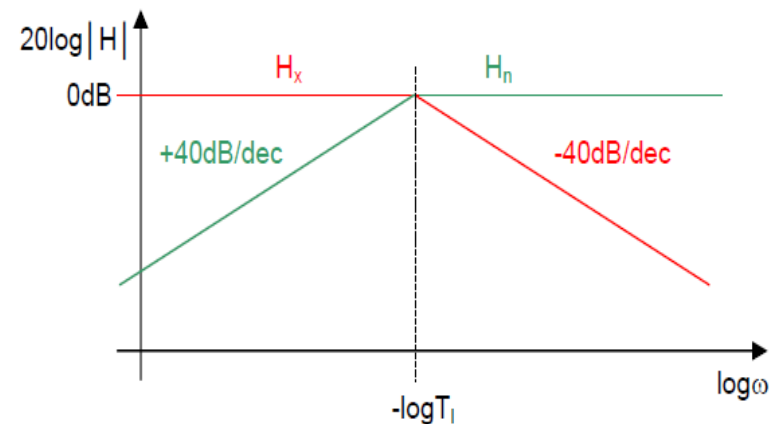
$$y(s^2T_I^2 + sT_I + 1) = ns^2T_I^2 + x$$

Signal transfer:

$$H_x = \frac{y}{x} = \frac{1}{1 + sT_I + s^2T_I^2}$$

Noise transfer:

$$H_n = \frac{y}{n} = \frac{s^2T_I^2}{1 + sT_I + s^2T_I^2}$$





# Signal to noise and oversampling ratios

- What is the maximum signal-to-noise ratio of a 16bit ADC? Suppose a sinusoidal input signal, and that the quantisation noise dominates all other noises. The noise is considered as white noise.
- Which resolution, expressed as number of bits (ENOB), can have an ADC with a second order delta-sigma modulator, operating at an oversampling ratio of 128?
- Consider a delta-sigma ADC with ENOB = 16bits and an input signal bandwidth of 1kHz. The oversampling ratio is 128. Which order is required for an analog anti-aliasing filter?

## Signal to noise and oversampling ratios (2)

- Consider a delta-sigma ADC with 16bit resolution and an input signal bandwidth of 1kHz. The oversampling ratio is 128. Which order is required for an analog anti-aliasing filter?

A maximum attenuation of  $-SNR = -98\text{dB} \cong 80'000$  is required at  $f > 128 f_{c, \text{Antialias}}$ .

E.g. for a Butterworth filter, the order  $n$  is given by

$$H_{\text{Antialias}}^2 = \frac{1}{1 + \left( \frac{f}{f_{c, \text{Antialias}}} \right)^{2n}} \Rightarrow n = \frac{\log\left( \frac{1}{H_{\text{Antialias}}^2} - 1 \right)}{2 \log\left( \frac{f}{f_{c, \text{Antialias}}} \right)} \approx 2.3 \Rightarrow n = 3$$

# Second order high pass filters

- Propose circuits for second order switched capacitor high-pass filters.

## Second order high pass filters

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