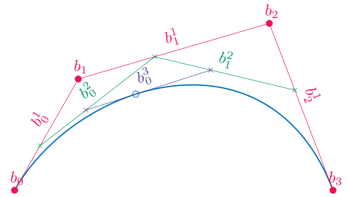
1 Courbes de Bézier



1.1 Triangle de Pascal

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \, 10 \quad 5 \quad 1 \end{array}$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$\binom{n}{k} = C_k^n = \frac{n!}{k!(n-k)!}$$

1.2 Polynômes de Bernstein

$$B_i^m(t) = \binom{m}{i} t^i (1-t)^{m-i}$$

n	$i=0$	$i=1$	$i=2$	$i=3$
0	$B_0^0(t)=1$			
1	$B_0^1(t)=1-t$	$B_1^1(t)=t$		
2	$B_0^2(t)=(1-t)^2$	$B_1^2(t)=2t(1-t)$	$B_2^2(t)=t^2$	
3	$B_0^3(t)=(1-t)^3$	$B_1^3(t)=3t(1-t)^2$	$B_2^3(t)=3t^2(1-t)$	$B_3^3(t)=t^3$

2 Résolution numérique

2.1 Doolittle

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$\begin{array}{c} 1 \\ 3 \\ 2 \qquad\qquad\qquad 5 \end{array} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{array}{c} \\ \\ 4 \end{array}$$

$$(1) \begin{cases} u_{11} &= a_{11} \\ u_{12} &= a_{12} \\ u_{13} &= a_{13} \end{cases} \qquad (2) \begin{cases} l_{21} &= \frac{a_{21}}{u_{11}} \\ l_{31} &= \frac{a_{31}}{u_{11}} \end{cases}$$

$$(3) \begin{cases} u_{22} &= a_{22} - l_{21}u_{12} \\ u_{23} &= a_{23} - l_{21}u_{13} \end{cases} \qquad (4) \left\{ l_{32} = \frac{a_{32} - l_{31}u_{12}}{u_{22}} \right.$$

$$(5) \left\{ u_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23} \right.$$

2.2 Cholesky

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} = \begin{pmatrix} \hat{l}_{11} & 0 & 0 \\ \hat{l}_{21} & \hat{l}_{22} & 0 \\ \hat{l}_{31} & \hat{l}_{32} & \tilde{l}_{33} \end{pmatrix} \cdot \begin{pmatrix} \hat{l}_{11} & \hat{l}_{21} & \hat{l}_{31} \\ 0 & \hat{l}_{22} & \hat{l}_{32} \\ 0 & 0 & \tilde{l}_{33} \end{pmatrix}$$

$$\hat{l}_{11} = \sqrt{a_{11}} \qquad \hat{l}_{21} = \frac{a_{12}}{\sqrt{a_{11}}}$$

$$\hat{l}_{31} = \frac{a_{13}}{\sqrt{a_{11}}} \qquad \hat{l}_{22} = \sqrt{a_{22} - \frac{a_{12}}{\sqrt{a_{11}}}}$$

$$\hat{l}_{32} = \frac{a_{23} - \frac{a_{13}a_{12}}{a_{11}}}{\sqrt{a_{22} - \frac{a_{12}}{a_{11}}}} \qquad \hat{l}_{33} = \sqrt{a_{33} - \hat{l}_{31}^2 - \hat{l}_{32}^2}$$

2.3 Permutations

Attention, si on utilise des permutations, alors

$$PA\vec{x} = P\vec{b}$$

3 Généralités

3.1 Matrices

3.1.1 Inverses

$$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{a} & -\frac{b}{ad} & \frac{be-cd}{adf} \\ 0 & \frac{1}{d} & -\frac{e}{fd} \\ 0 & 0 & \frac{1}{f} \end{pmatrix}$$

Même principe si on renverse

$$\left(M^T\right)^{-1} = \left(M^{-1}\right)^T$$

$$\begin{pmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{b}{ad} & \frac{1}{d} & 0 \\ \frac{be-cd}{adf} & -\frac{e}{fd} & \frac{1}{f} \end{pmatrix}$$