

1 Équations aux différences

EDO vs Equation aux différences

$$\frac{dy(t)}{dt} = ay(t) \rightarrow y(t+1) = (a+1)y(t)$$

1.0.1 Exemple équation aux différences

$$\begin{aligned} y(k+n) + a_{n-1}y(k+n-1) + \dots + a_0y(k) &= 0 \\ \lambda^{k+n} + a_{n-1}\lambda^{k+n-1} + \dots + a_0\lambda^k &= 0 \\ \lambda^n + a_{n-1}\lambda^{n-1} + \dots + a_0\lambda &= 0 \end{aligned}$$

1.0.2 Exemple EDO

$$\frac{dy}{dt} = ay \rightarrow y(t) = Ce^{at}$$

$$y(0) = Ce^{a \cdot 0} = Ce^0 = C$$

$$y(t) = y(0)e^{at}$$

$$\frac{d^2y}{dt^2} + \omega^2y = 0$$

$$my'' = -ky$$

$$\lambda^2 + \frac{k}{m} = 0 \rightarrow \lambda = \pm j\sqrt{\frac{k}{m}} = \pm j\omega$$

$$y(t) = C_1e^{j\omega t} + C_2e^{-j\omega t} = A\sin(\omega t) + B\cos(\omega t)$$