

Exercice1_SDZ

January 15, 2022

1. Résoudre $u_{tt} = c^2 u_{xx}$, $u(x, 0) = e^x$, $u_t(x, 0) = \sin(x)$.

On utilise la fonction générale

$$u(x, t) = \frac{1}{2}(\phi(x+ct) + \phi(x-ct)) + \frac{1}{2c} \int_{x-ct}^{x+ct} \sin(s) ds = \frac{1}{2}(e^{x+ct} + e^{x-ct}) + \frac{1}{2c}(-\cos(x+ct) + \cos(x-ct))$$

On peut simplifier un peu les expressions

$$\frac{1}{2}(e^{x+ct} + e^{x-ct}) + \underbrace{\frac{1}{2c}(-\cos(x+ct) + \cos(x-ct))}_{\frac{1}{2}(\sin(x)\sin(ct))} = e^x \underbrace{\frac{1}{2}(e^{ct} + e^{-ct})}_{\cosh(ct)} + \frac{1}{2}\sin(x)\sin(ct)$$