Exercice2_SDZ

January 15, 2022

2. Résoudre $u_{tt} = c^2 u_{xx}$, $u(x,0) = \ln(1+x^2)$, $u_t(x,0) = 4+x$.

$$\phi(x) = \ln(1+x^2) \qquad \psi(x) = 4+x$$

On applique la formule générale

$$u(x,t) = \frac{1}{2}\Big(\phi(x+ct) + \phi(x-ct)\Big) + \frac{1}{2c}\int_{x-ct}^{x+ct} 4 + s ds$$

$$u(x,t) = \frac{1}{2}\Big(\ln(1+(x+ct)^2) + \ln(1+(x-ct)^2)\Big) + \frac{1}{2c}\Big(4s + \frac{s^2}{2}\Big)_{x-ct}^{x+ct}$$

$$u(x,t) = \frac{1}{2}\Big(\ln(1+(x+ct)^2) + \ln(1+(x-ct)^2)\Big) + \frac{1}{2c}\left(4x + 4ct + \frac{(x+ct)^2}{2} - 4x + 4ct - \frac{(x-ct)^2}{2}\right)$$

$$u(x,t) = \frac{1}{2}\Big(\ln(1+(x+ct)^2) + \ln(1+(x-ct)^2)\Big) + \frac{1}{2c}\left(8ct + 2xct\right)$$

$$\boxed{u(x,t) = \frac{1}{2}\Big(\ln(1+(x+ct)^2) + \ln(1+(x-ct)^2)\Big) + 4t + xt}$$