Exercice5_SDZ

November 26, 2021

[2]:
```
import numpy as np
import matplotlib.pyplot as plt
```

0.1 Exercice 5

Problème 5. Discretisez l'équation

$$-\mu u''(x) + u'(x) = 1$$

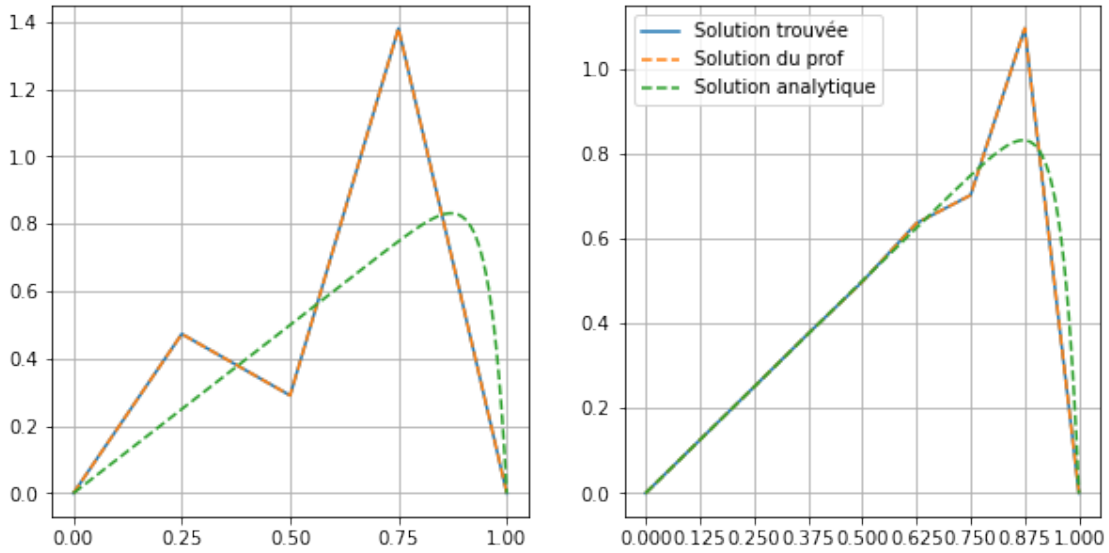
avec les conditions au bord Dirichlet homogènes $u(0) = u(1) = 0$ et le coefficient de diffusion $\mu = 0.04$ Utilisez les différences finies (stencil à 3 points), avec la discrétisation centrée pour le terme $u'(x)$ et avec la taille de maille constante:

- $h_a = 0.25$, et
- $h_b = 0.125$.

Solution: la solution analytique est $u(x) = x - \frac{1-e^{\frac{x}{\mu}}}{1-e^{\frac{1}{\mu}}}$. $\vec{u}_a = (0.4728, 0.2903, 1.3799)^T$ *et* $\vec{u}_b = (0.1250, 0.2499, 0.3755, 0.4977, 0.6356, 0.7018, 1.0945)^T$.

[7]:
```
mu = 0.04
ustart = 0
uend = 0

def analytic(x):
    return x - (1-np.exp(x/mu))/(1-np.exp(1/mu))

def th(x):
    match len(x)-2:
        case 3:
            return np.array([ustart, 0.4728, 0.2903, 1.3799, uend])
        case 7:
            return np.array([ustart, 0.1250, 0.2499, 0.3755, 0.4977, 0.6356, 0.
↪7018, 1.0945, uend])
    return np.zeros_like(x)
```

```
H = [0.25, 0.125]
plt.figure(figsize=(10, 5))
for i, h in enumerate(H):
    N = int(1/h) + 1
    x = np.linspace(0, 1, N)
    xos = np.linspace(0, 1, 1000)

    # Matrice A
    A = np.zeros([N-2, N-2])
    A += np.diagflat(np.ones(N-2)*2*mu)
    A += np.diagflat(np.ones(N-3)*(-mu+h/2), 1)
    A += np.diagflat(np.ones(N-3)*(-mu-h/2), -1)
    A = 1/h**2 * np.matrix(A)

    F = np.ones(N-2)
    u = np.zeros(N)
    F[0] += (mu + h/2)/h**2*ustart
    F[-1] += (mu-h/2)/h**2*uend
    u[1:-1] = A.I @ F
    u[0] = ustart
    u[-1] = uend

    plt.subplot(101 + len(H)*10 + i)
    plt.grid()
    plt.plot(x, u)
    plt.plot(x, th(x), '--')
    plt.plot(xos, analytic(xos), '--')
    plt.xticks(x)
plt.legend(['Solution trouvée', 'Solution du prof', 'Solution analytique'])
plt.show()
```



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