
1 Autres

1.1 Intégration par partie

$$\int_a^b u'v = uv \Big|_a^b - \int_a^b uv'$$

1.1.1 exemple

$$\begin{aligned}\int_0^1 x^2 \cdot \sin(n\pi x) dx &= \int_0^1 f dg = fg \Big|_0^1 - \int_0^1 g df \\ f &= x^2, dg = \sin(n\pi x) dx \\ df &= 2x \cdot dx, g = -\frac{\cos(n\pi x)}{n\pi} \\ &= -\frac{x^2 \cdot \cos(n\pi x)}{n\pi} \Big|_0^1 + \int_0^1 \frac{2x \cdot \cos(n\pi x)}{n\pi}\end{aligned}$$

1.2 Changement de variable

1.2.1 Méthode 1

Lorsque la dérivée $\varphi'(t)$ est présente

$$\int_a^b f(\varphi(t))\varphi'(t)dt = \int_{\varphi(a)}^{\varphi(b)} f(x)dx$$

1.2.2 Méthode 2

Si $\varphi'(t) = \varphi' = \text{constante}$

$$\int_a^b f(\varphi(t))dt = \frac{1}{\varphi'} \int_{\varphi(a)}^{\varphi(b)} f(x)dx$$

1.3 Solutions générales

$$\begin{array}{ll}X'' = -\beta^2 X & \longrightarrow X(x) = A \cos(\beta x) + B \sin(\beta x) \\X'' = \beta^2 X & \longrightarrow X(x) = A \cosh(\beta x) + B \sinh(\beta x) \\X'' = 0 & \longrightarrow X(x) = Ax + B\end{array}$$

1.4 Équation d'euler

$$e^{jx} = \cos(x) + j \sin(x)$$

1.5 Séparation en éléments simples

$$\begin{aligned} f(x) &= \frac{x(x+1)}{(x-1)(x-\textcolor{red}{0.25})(x-\textcolor{teal}{0.5})} \\ &\quad \downarrow \\ f(x) &= \frac{x(x+1)}{(x-\textcolor{teal}{1})(x-\textcolor{red}{0.25})(x-\textcolor{teal}{0.5})} \\ &\quad \downarrow \\ f(x) &= \frac{R_1}{(x-1)} + \frac{R_2}{(x-\textcolor{red}{0.25})} + \frac{R_3}{(x-\textcolor{teal}{0.5})} \end{aligned}$$

Attention ! Pas de $()^n$
dans le dénominateur.
Sinon résolution à la main

$$R_1 = \frac{\textcolor{teal}{1}(1+1)}{(1-\textcolor{red}{0.25})(1-\textcolor{teal}{0.5})}$$

$$R_2 = \frac{\textcolor{red}{0.25}(\textcolor{red}{0.25}+1)}{(\textcolor{red}{0.25}-1)(\textcolor{red}{0.25}-\textcolor{teal}{0.5})}$$

$$R_3 = \frac{\textcolor{teal}{0.5}(\textcolor{teal}{0.5}+1)}{(\textcolor{teal}{0.5}-1)(\textcolor{teal}{0.5}-\textcolor{red}{0.25})}$$