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Most Diversified Portfolio Construction & Views in Portfolio Optimization

Alternative Investments

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Abstract

The purpose of this paper[1] is to provide a most diversified portfolio using various asset classes. It presents the techniques used to construct portfolio as well as a thorough analysis of its results. Other investments techniques will be presented for comparison purposes. We included periods of market turbulence to analyze how our strategies behave and react during crises.

[1]Both code and data set are available in our **Github repository**.

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1 Introduction

Diversification in portfolio allocations has been one of the biggest criteria in investment decisions since its introduction by Harry Markowitz in the 20^{th} century with his paper on *Modern Portfolio Theory*[2]. One of the first type of diversification attempt was to create the popular 60% equities/40% bonds portfolio. Diversification allowed investors to reduce total risks of their portfolio by decreasing idiosyncratic risk an asset could carry, thus providing better risk-adjusted performances.

In this paper, we will create the most diversified portfolio using 3 different asset classes: equities, bonds and an alternative asset in commodities. To understand whether this portfolio allowed to minimize risks, we will also create 3 other types of allocations for comparison purposes: a maximum Sharpe ratio portfolio (with and without views), a minimum volatility portfolio and an equal-weights portfolio. We will present an in-depth performances and risk analysis of all portfolios. Furthermore, we were interested to do a comparison of the results by including our views concerning some assets.

We obtain that the maximum Sharpe ratio portfolio with views is the best portfolio in term of risk-adjusted performances with an annualized return of 4% and volatility of 8%. But in the case where we do not include those views, the results are clearly different with a negative Sharpe ratio. A simple equal-weight portfolio does not mitigate the potential risk as it generates an annualized STD of 12.5%, which is the highest among all the portfolios. On the contrary, the maximum diversification portfolio can provide one of the lowest volatility with 6% annualized, but also with low return with 1.5% annualized. On the extreme side, a minimum volatility can provide the lowest standard deviation of 4.9%, but also with one the lowest return of only 1.2%, providing some decent performances in term of risk-adjusted measures.

2 Data

To create our portfolios, we decided to have an exposure to 3 different indices in a distinct asset class:

- ***Stocks:*** SPX
- ***Bonds:*** Bloomberg (Barclays Capital) US Aggregate Bond Index
- ***Alternative assets:*** S&P Goldman Sachs Commodity Index (S&P GSCI)

In order to have an exposure to all indices, we decided to trade exchange-traded funds (ETF). We selected the following ETF based on the aforementioned indices:

- ***Stocks:*** SPDR S&P 500 ETF Trust (SPY)
- ***Bonds:*** iShares Core US Aggregate Bond ETF (AGG)
- ***Alternative assets:*** iShares S&P GSCI Commodity-Indexed Trust ETF (GSG)

We decided to collect the daily closing prices of all indices from 21-07-2006 until 01-12-2021,

[2]Markowitz (1952). https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1952.tb01525.x. Accessed on 18 December 2021.

as we are aiming to re-balance the portfolio daily. Several periods of market turbulence are included during this time period: the financial crisis of 2008, the greek government-debt crisis in 2010, the bear market of 2018, and finally the turbulence due to COVID-19 in 2020. As seen on table 1, we notice substantial differences in performances between the three assets since 2009. Indeed, equities have been on a bull rally since then, particularly after march/april 2020, when the COVID-19 pandemic started to spread around the world. Although the annualized returns were very high, risk was very high as shown on table 1, with a maximum drawdown of -56%. Nevertheless, it is clearly the best performing ETF from our selection based on these risk-adjusted measures. Indeed, our ETF on commodities has been performing terribly bad since the GFC of 2008 due mainly to poor economic conditions and low investments in this sector, thus obtaining a negative annualized return, while still being the most risky ETF. Indeed, commodities are not performing well during periods of strong turbulence like for instance during the financial crisis 2008 where our ETF experienced an important drawdown of -89%. On the other hand, the bond market did not performed well as shown by our ETF, mainly due to poor economic conditions globally. The returns on this investment has been particularly low, while still have very low risk, which is similar to keeping the capital in cash. Even during crises, the bond market is not very impacted, showing a very smooth but weak positive performance through time. We notice on figure 2 that there is a low negative correlation between bonds and commodities/equities, but a substantial positive correlation between commodities and equities. We collected the average 1-month LIBOR rate as a risk-free rate.

3 Methodology

From our raw daily prices, we decided to compute the daily simple returns. As we are aiming to determine the daily allocation of our portfolio, we lagged the returns by 1 day since they are computed using yesterday's closing price. As previously mentioned, we decided to implement 4 different investment strategies, the first one being at the center of our analysis:

1. Most Diversified Portfolio
2. Maximum Sharpe Ratio Portfolio (with and without views)
3. Minimum Volatility Portfolio
4. Equal-Weight Portfolio

The first model aims to seek the highest possible level of diversification by solving equation (1).

$$\max_{\alpha} D(\alpha) = \frac{\alpha'\sigma}{\sigma_p} \tag{1}$$

- Where $D(\alpha)$ is the diversification ratio and σ the vector of asset volatility $(\sigma_i(\alpha))$.

The second method seeks to construct a portfolio with the highest possible average Sharpe Ratio (SR) across the period. To avoid over-exposure on commodities during period of crisis, we expressed our views by imposing a maximum allocation constraint of 30%. Additionally, we imposed a maximum allocation constraint in the optimization of 60% to equities and bonds respectively, as we had great conviction regarding the performances of the S&P500. Nevertheless, we exposed the results with and without views for comparison purposes. In mathematical

terms, we want to solve:

$$\max_{\alpha} SR = \frac{\bar{r}_p - r_f}{\sigma_p} \tag{2}$$

The third method aims to provide a portfolio with the lowest feasible volatility.

$$\min_{\alpha} \sigma_p = \alpha^T \Sigma \alpha \tag{3}$$

The final method seeks to construct a simple portfolio of equal-weights among all ETFs

$$\bar{r}_p = \frac{\sum_{i=1}^{N} r_i}{N} \tag{4}$$

For best practice, we decided to start the inception of the portfolio in 01-01-2007 in order to have enough prior data to estimate the covariance matrix and standard deviation. All portfolio weights have been determined using the prior day returns. Besides, we also computed the Sterling Ratio, which represents the annualized portfolio mean return subtracted by the risk-free rate and divided by the average largest portfolio draw down of the period. The Sterling and Burke Ratio are widely used by commodity trading advisers.

$$SterlingRatio_p = \frac{\bar{r}_p - r_f}{Largest\bar{D}rawdown} \tag{5}$$

$$BurkeRatio_p = \frac{\bar{r}_p - r_f}{\sqrt{AllDrawdowns^2}} \tag{6}$$

4 Results

After implementing the aforementioned strategies to our available ETFs, we obtained the performances shown on table 2.

4.1 Most Diversified Portfolio

As shown on table 3, the Most Diversified Portfolio has an annualized mean return of 1.476%, and an annualized volatility of 6%. In consequence, the corresponding Sharpe Ratio is then equal to 24.6%, by using the average 1-month LIBOR rate as a risk-free rate. We notice on table 3 that this strategy allocates 75%, 14% and 11% on average to the AGG, SPY and GSG respectively, as it aims to be the most diversified portfolio. The distribution of the return portfolio is very far from normality with a negative skewness of -1 and a kurtosis of 44.16. The majority of the returns are positive, according to the Hit Ratio. The most important decrease that the portfolio experienced through time is around -22.678%. In term of risk, this portfolio can mitigate the riskiness overall, as seen on figure 9, since the Value-at-Risk (VaR) and Expected Shortfall (ES) at 95% is relatively stable throughout time, except in 2008 and early 2020 due to the global financial crisis and the COVID19. Indeed, although idiosyncratic risk is minimized as we are aiming to provide the most diversified portfolio, the systematic risk is still significant due to financial crises.

4.2 Maximum Sharpe Ratio Portfolio

Concerning the Maximum Sharpe Ratio Portfolio when we do not incorporate any views in the optimization, we obtain the worst performing portfolio. Indeed, the annualized mean and volatility are respectively equal to -0.6% and 7%, as shown on table 2. The Sharpe ratio is therefore negative and the lowest, although the objective is to maximize it. This is due to an over-allocation of 100% in commodities during the crash of 2008, as seen on figure 6. Risk exposure is consequently significant and and the maximum drawdown as well, which reverberate the poor performances in the following years.

Consequently, we decided to incorporate our views as described in section 3. This allows us to avoid an over-exposure in commodities during the Great Financial Crisis. Additionally, as we had an over-exposure to bonds when no views were incorporated, we decided to increase the allocation to equities (consequently decreasing the weights to bonds) as we had great expectation about the US equity market. Therefore, with this portfolio, the annualized mean and volatility are respectively equal to 4% and 8%, as shown on table 2, thus obtaining the highest Sharpe Ratio which was the initial objective. We notice that this strategy allocates 59%, 39% and 2% on average to the AGG, SPY and GSG respectively, as it aims to maximize the Sharpe ratio, which explains a higher exposure to equities compared to the previous portfolio. The Maximum Drawdown observed during the period is equal to -31%, which is lower than the same portfolio without views as we have a lower exposure to commodities. Again, the probability distribution of the portfolio returns does not match the normal distribution characteristics since we obtain a negative skewness of -0.9 and a rather low kurtosis of 19 compared to the other portfolio. Similar to the previous portfolio, the risk can be volatile as seen on table 10, particularly in 2008 and 2020 due to extreme systematic risk related to financial crisis.

4.3 Minimum Volatility Portfolio

In the minimum volatility, we obtain low an annualized return 1.3% and an annualized volatility of 4.9%, which the lowest among all other portfolios, as shown on table 2. Consequently, we obtain a decent Sharpe ratio of 0.26. Due to its objective of minimizing risk, we have a low max drawdown of -17%, and the returns are heavily left-skewed and with a high kurtosis, as shown on table. Its very low risk can be explained by its over-exposure to bonds, with approximately 90% in this asset class, as shown on table 3, which is explained by the low risk of it through the last years. Similar to the MDP portfolio, the risk is also controlled as seen on table 12, expect on 2008 and 2020 for the same reason.

4.4 Equal-Weights Portfolio

Finally, we obtained the following results for our Equally-Weighted Portfolio. As shown on table 2, the annualized mean return is equal to 2.65%, and the annualized volatility corresponds to 12.40%. We therefore obtain a poor Sharpe Ratio of 0.2. The Maximum Drawdown is equal to -44%, which is the highest of all portfolios. Indeed, by looking at figure 3, the performance is varying intensively, with a lot of drawdowns and up-downs through time. Although a simple equally-weighted portfolio can be efficient as it can avoids concentration in some positions, it does not monitor any performance measures (e.g. volatility), which can explain its high

riskiness. Compared to the previous portfolio, riskiness is uncontrolled as shown on table 13, since VaR and ES vary substantially throughout time, and is particularly high during the GFC and the start of COVID19 (March 2020).

4.5 Comparison of all Portfolios

Globally, the Maximum Sharpe Ratio with views Portfolio is the one providing the best annualized mean return compared to the two others but is also clearly quite volatile due to its exposure to equities and commodities. Nevertheless, we clearly see that incorporating our views can provide better performances compared to the same portfolio without views. The Equal-Weight portfolio is the most risky portfolio due to its over-exposure to commodities. As we previously showed, the performance of the EW Portfolio is varying a lot through time. Indeed, the MDP and MV Portfolio have a very smooth performance in comparison as shown on figure 3. It also explains why the maximum drawdown of the EW portfolio is the most important. With a very poor diversification, this portfolio is extremely sensible to ups and downs. We notice that the minimum volatility portfolio has similar performances as the most diversified portfolio. As they have similar Sharpe Ratio, the only trade-off is between returns and volatility, which can be a choice for investors with two different kind of risk aversion (i.e. lower and higher). The MDP portfolio can decrease significantly the idiosyncratic risk while still having some systematic risk. On the other hand, the MV portfolio aims to minimize the total risk of a portfolio. Furthermore, every portfolio returns have an empirical distribution that is not matching at all the characteristics of a normal distribution if we have a look on the higher moments.
However, if we would propose a specific portfolio technique to a client, we would decide between a MDP and SR with views portfolios. They would be suitable for either an investor risk-adverse or an investor seeking returns and is risk-neutral. The MDP provide some decent risk-adjusted performances while having good diversification and low risk. The SR portfolio with views can generate better risk-adjusted performances by having a higher exposure to equities.

5 Conclusion

To conclude, we were able to create and implement several portfolio construction strategies using diversified asset types. Therefore, we were able to build a comparative study between the final performances of our portfolios in order to give the best possible advice to the client in order to best satisfy his expectations. Since the Sharpe Ratio of the SR Portfolio with views is the highest among all portfolios, while having some non-marginal risk, this investment is perfectly adapted to a return-seeking and risk-neutral investor. A MDP portfolio is more suitable for risk-adverse investors still willing to obtain some decent raw returns, thanks to its diversification objective. An investor fully risk-adverse would potentially select the MV portfolio to minimize volatility.

We can also agree on the limitation of our asset space, as only 3 assets can limit our strategies to deliver decent performances. We believe that more than 3 ETFs and asset classes could allow us to grasp additional diversification effect, while generating better risk-adjusted performances.

6 Appendix

Table 1: Performances of all ETFs

	AGG	SPY	GSG
Annualized Return (%)	1.079	9.897	-3.340
Annualized STD (%)	5.161	20.257	23.752
Sharpe Ratio	0.209	0.489	-0.141
Sterling Ratio	-0.074	-0.175	0.037
Burke Ratio	0.000	0.000	-0.000
Max Drawdown (%)	-14.598	-56.474	-89.618
Hit Ratio (%)	53.901	55.313	52.543
Skewness	-2.832	-0.067	-0.484
Kurtosis	74.910	15.206	4.642
30d 95% VaR (%)	0.391	1.604	2.067
30d 95% ES (%)	0.519	2.053	2.650

Table 2: Performances of all Portfolios

	MDP	SR (Without Views)	SR (With Views)	MV	EW
Annualized Return (%)	1.476	-0.063	4.004	1.291	2.655
Annualized STD (%)	6.003	7.220	8.269	4.882	12.471
Sharpe Ratio	0.246	-0.009	0.484	0.264	0.213
Sterling Ratio	-0.065	0.001	-0.126	-0.075	-0.060
Burke Ratio	0.000	-0.000	0.000	0.000	0.000
Max Drawdown (%)	-22.678	-42.631	-31.818	-17.326	-44.176
Hit Ratio (%)	53.129	54.301	55.286	53.688	53.635
Skewness	-1.005	-2.838	-0.920	-2.491	-0.698
Kurtosis	44.155	55.064	19.614	89.640	10.359
30d 95% VaR (%)	0.441	0.495	0.637	0.355	1.028
30d 95% ES (%)	0.581	0.639	0.833	0.475	1.316

Table 3: Average Asset Allocation

	MDP	SR (Without Views)	SR (With Views)	MV	EW
AGG	0.747	0.753	0.594	0.899	0.333
SPY	0.146	0.233	0.393	0.071	0.333
GSG	0.107	0.014	0.013	0.029	0.333

Figure 1: ETF Cumulative Returns

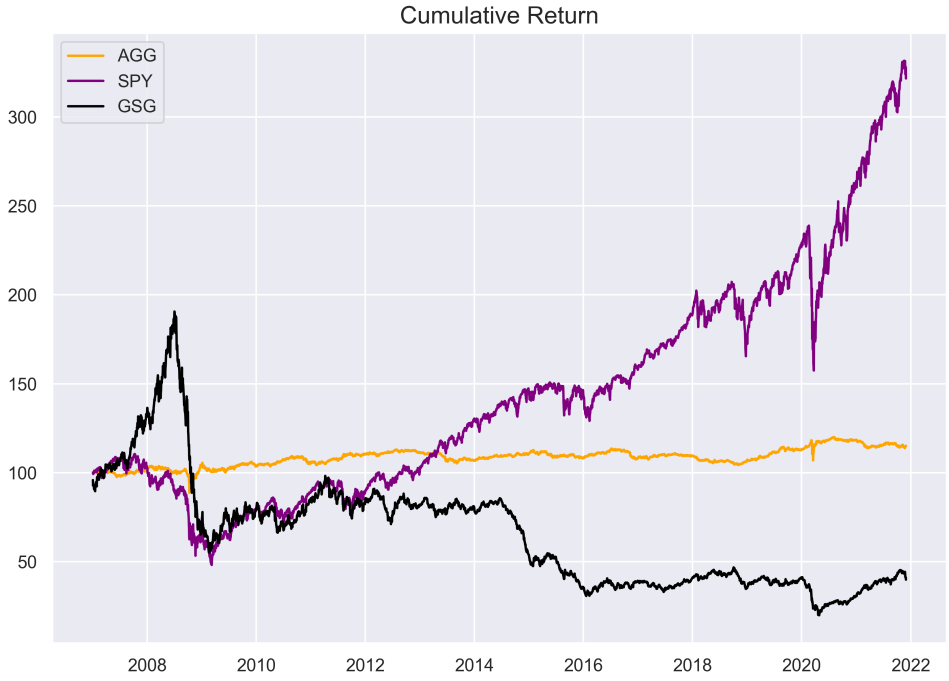

Figure 2: ETF Correlation Matrix

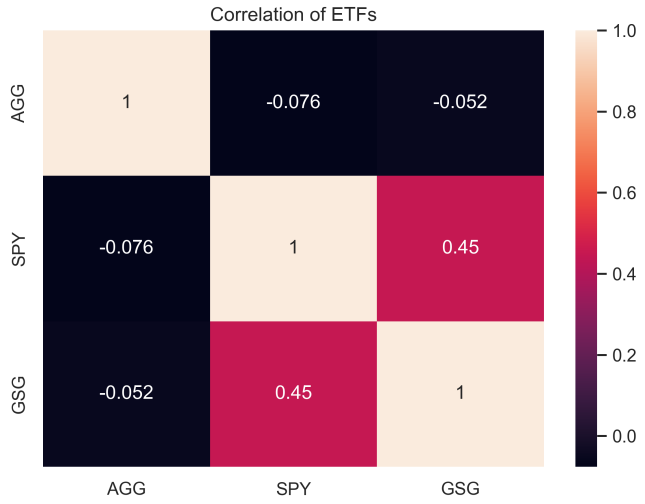

Figure 3: Portfolio Cumulative Returns

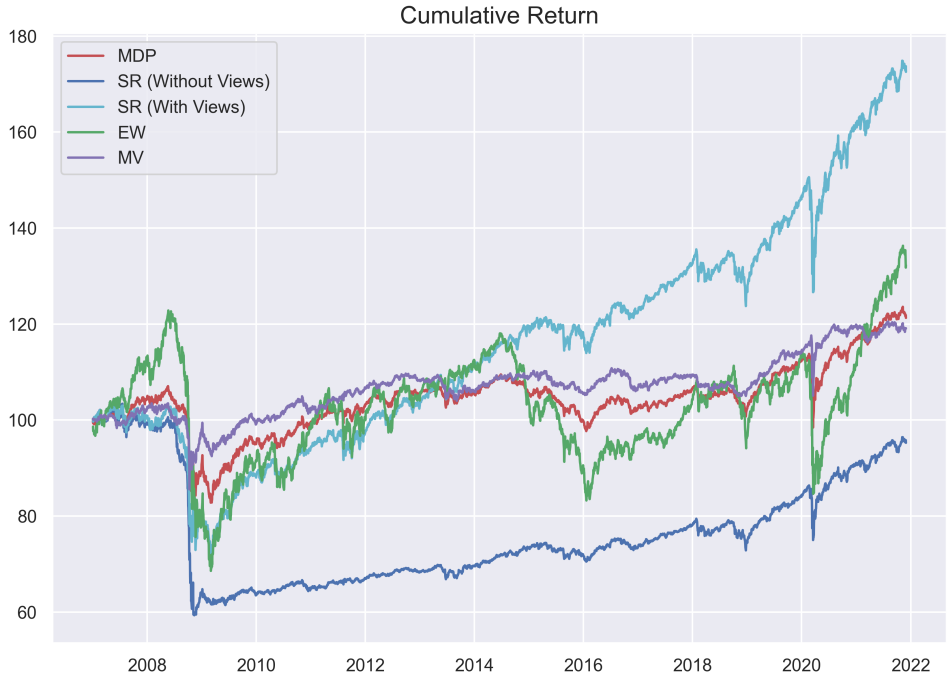

Figure 4: Portfolio & ETFs Cumulative Returns

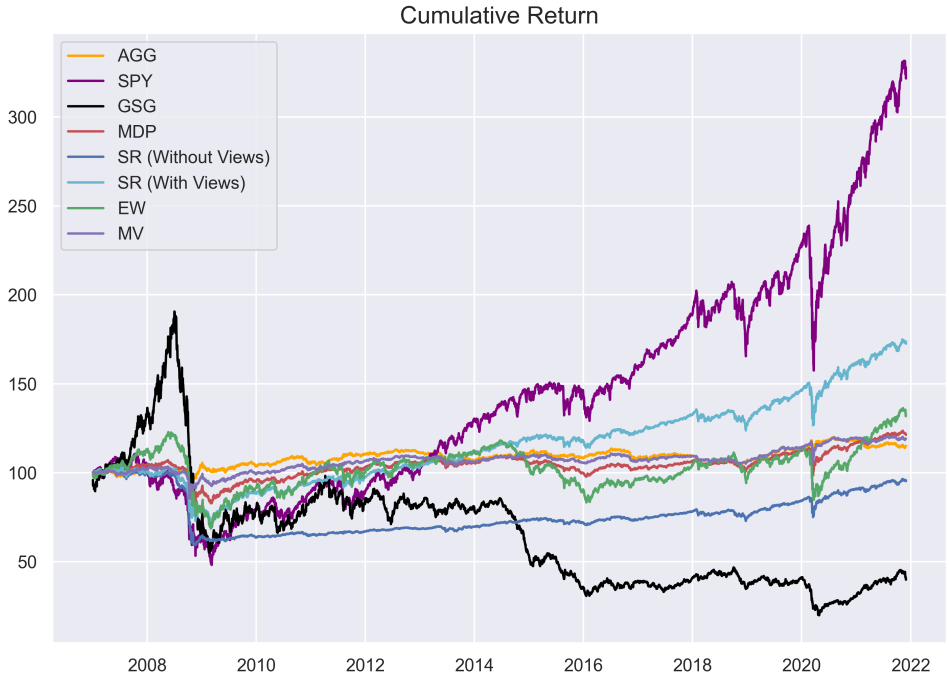

Figure 5: MDP Assets Allocation

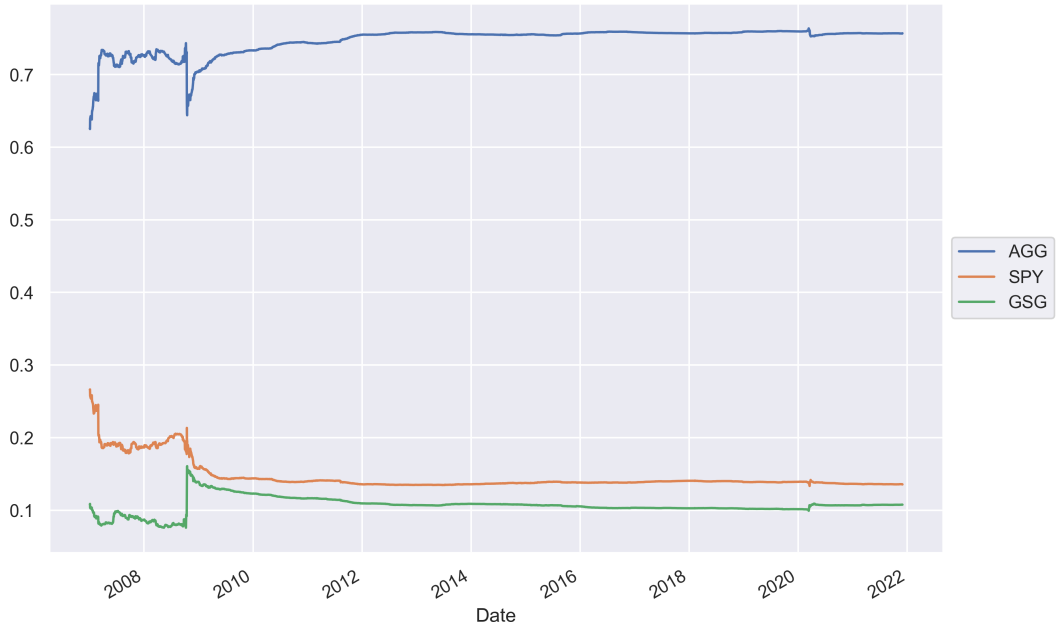

Figure 6: SR Assets Allocation (Without Views)

Figure 7: SR Assets Allocation (With Views)

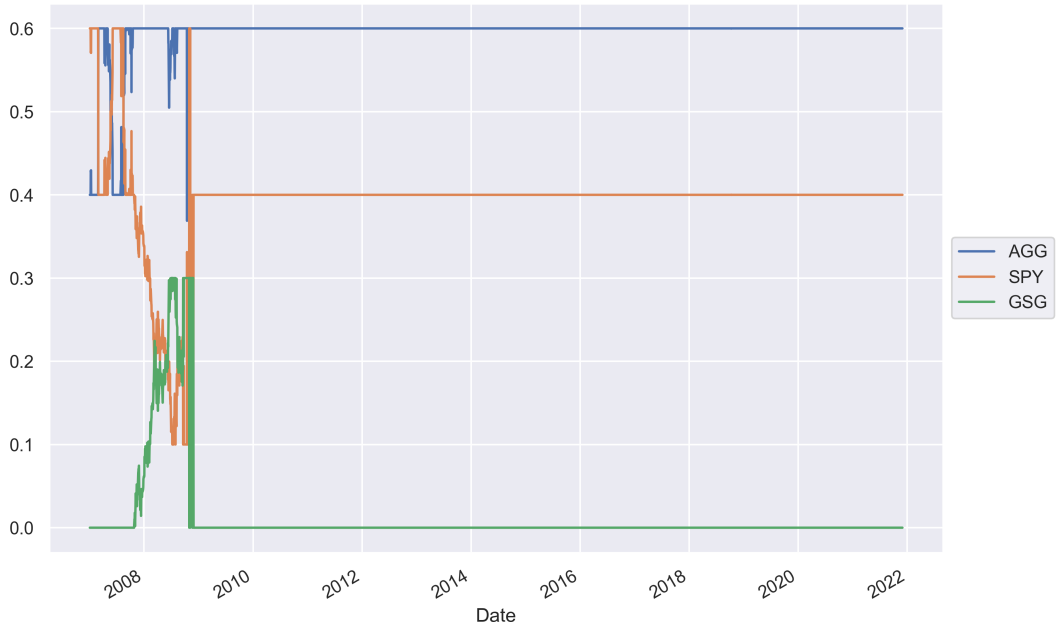

Figure 8: MV Assets Allocation

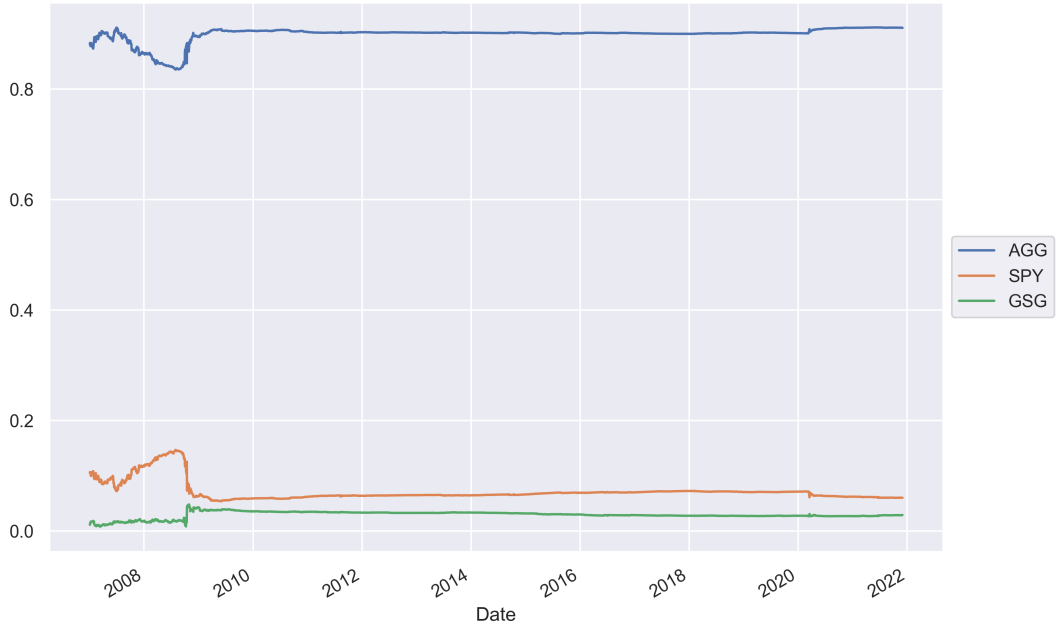

Figure 9: MDP Risk Analysis

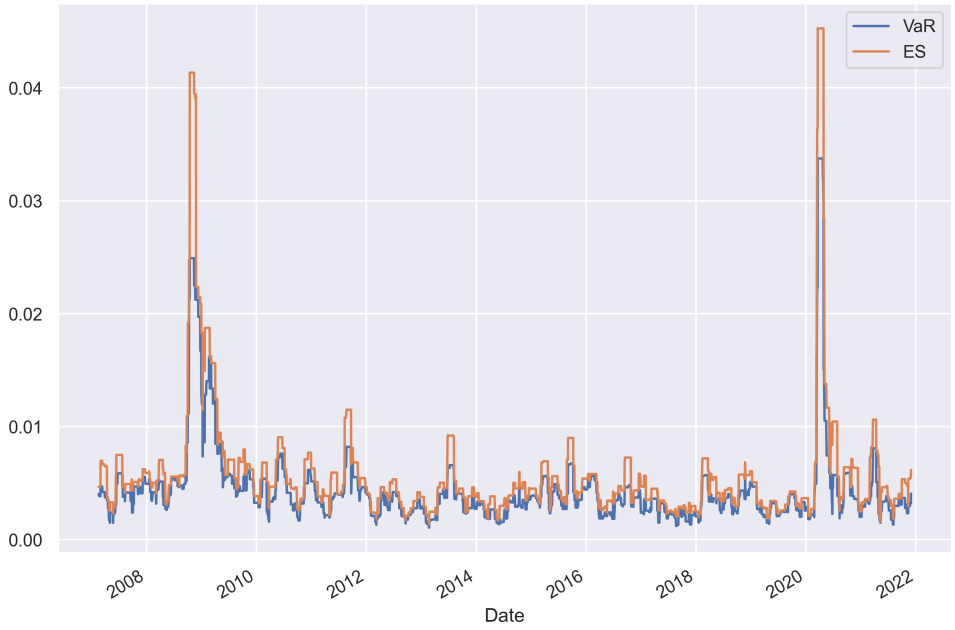

Figure 10: SR Risk Analysis (Without Views)

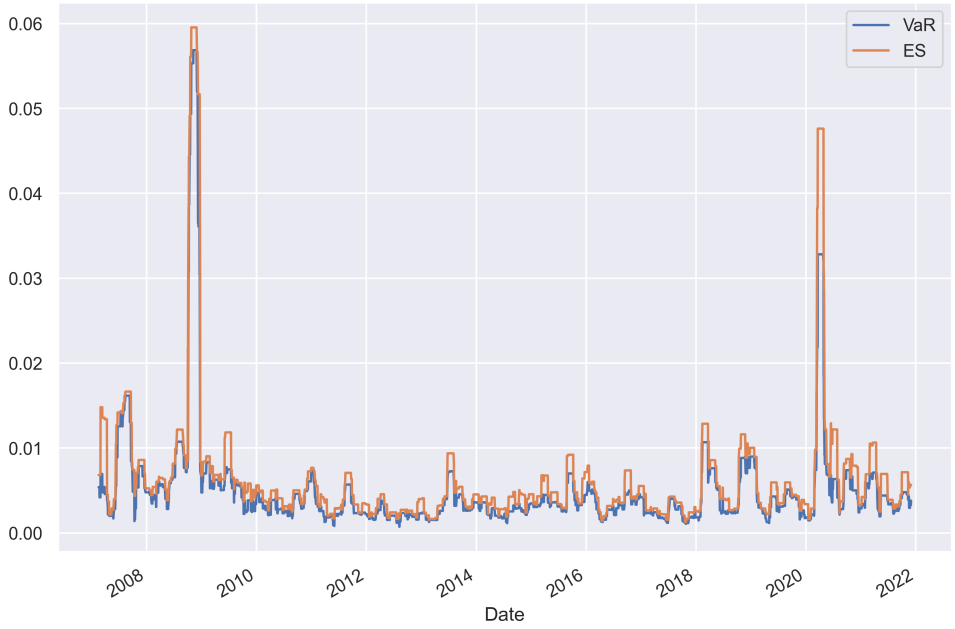

Figure 11: SR Risk Analysis (With Views)

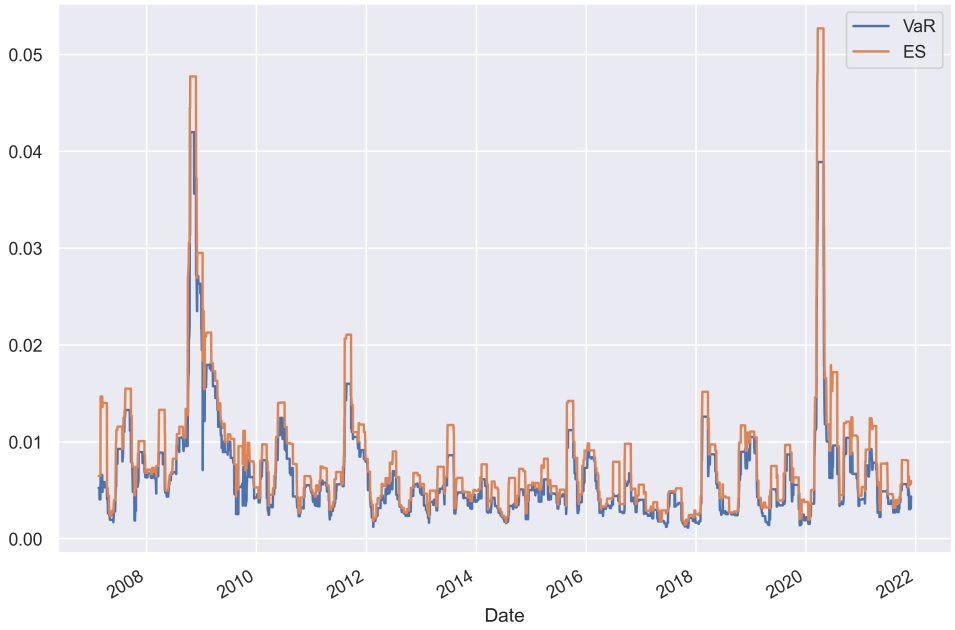

Figure 12: MV Risk Analysis

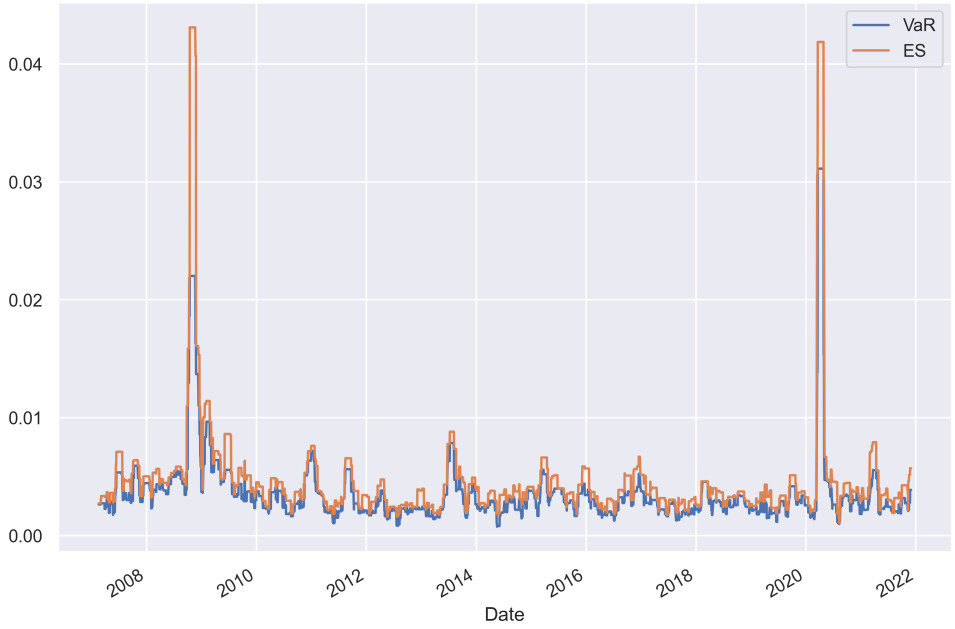

Figure 13: EW Risk Analysis