University of Lausanne
HEC Faculty

Quantitative Asset and Risk Management

Assignement 1

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Contents

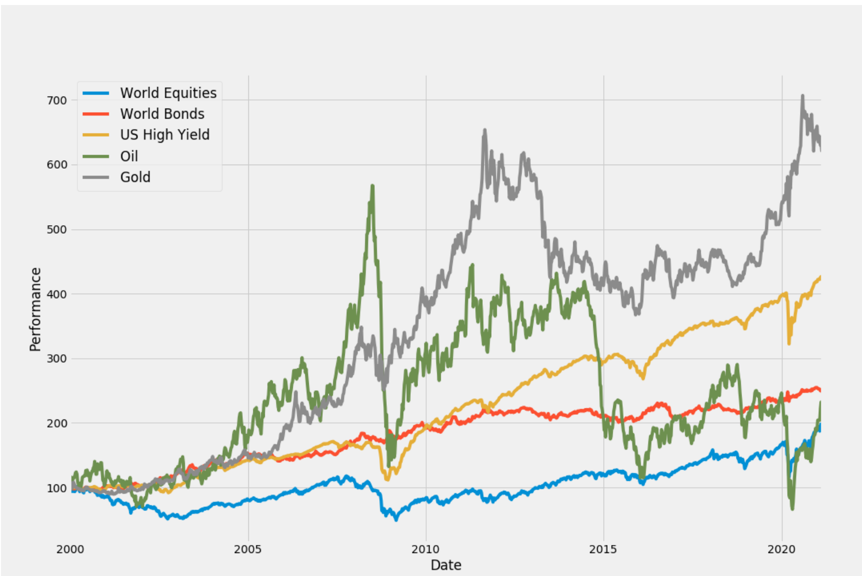
1 Introduction

This report aims to develop various types of optimal portfolios in the sense of Markovitz. The paper is separated into two parts. The first part will be about an optimal mean-variance portfolio allocation (i.e. EV allocation). The second part will be about an optimal mean-variance-skewness-kurtosis portfolio allocation (i.e. SK allocation). Our optimization methods will be based on the paper of E. Jondeau & M. Rockinger (2004) on *Optimal Portfolio Allocation Under Higher Moments*[1]. The rationale of using a model which considers higher central moments is that we no longer assume that asset returns are normally distributed but that they exhibit an asymmetric and/or fat-tailed distribution. We, therefore, incorporate the third and fourth central moments into the investors' utility function. Finally, we will compare the results of the EV and SK allocation. Both code and data set are available in our Github **here** [1].

2 Description of the Data Set

Our data set includes the prices of the World Equities index, World Bonds, US high yield bonds, oil and gold, from 1999 to 2021). We have in total 1,104 observations for each asset. The following graph depicts the cumulative returns of each asset class over our time frame.

Figure 1 – Entire Sample Cumulative Return



We notice on the one hand that both World Equities, World Bonds, US High Yield Bonds as well as Gold, have witnessed a general upward trend with different magnitudes. On the other

[1] https://papers.ssrn.com/sol3/papers.cfm?abstract_id=498322. Accessed: 21.03.21

hand, since the Great Financial Crisis in 2008, oil has been on a general downward trend, hitting a negative cumulative return in 2020 amid the COVID-19 pandemic, due to shrinkage of oil demand. Some assets such as the World Bonds are not particularly affected by economic or geopolitical downturns, in comparison. We can therefore notice that each asset class has its own particularity (i.e. different volatility, correlation to global markets, etc.). Our goal would be to account for such specificities when determining an optimal investment portfolio.

The data processing has been exclusively conducted on Python, using various libraries. The codes will be highlighted throughout this report, as guidance for the reader, and a full version will also be included in a separate file.

3 First Steps of Data Processing

Before starting the report, we will need to define some essential elements for our analysis. Let P_t be the stock price at time t. Then the one-period simple return from t-1 to t is:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

The cumulative return can be defined as the process of holding an asset for n period from t to $t+n$, which yield the n-period simple return:

$$r_t[n] = \frac{P_{t+n} - P_t}{P_t} \tag{2}$$

For this report, the original data is divided into two sub-parts:

1. **In-Sample:** Includes data from December 1999 to December 2017.
2. **Out-of-Sample:** Includes data from January 2018 to the end of the sample (February 2021).

The rationale of dividing our data is that the in-sample part is used to calibrate our asset allocations and the out-of-sample part allows us to how our hypothetical portfolio would have performed.

For this report, we used the following libraries on Python 3.8:

- Pandas (Version: 1.2.2)
- Scipy (Version: 1.6.1)

- Seaborn (Version: 0.11.1)
- Statsmodels (Version: 0.12.2)
- Matplotlib (Version: 3.3.4)
- Numpy (Version: 1.20.2)

```
## Import librairies
#####################
import os
import pandas as pd
import numpy as np
from scipy.stats import skew, kurtosis
from scipy.optimize import minimize
import matplotlib.pyplot as plt
from matplotlib import style
import seaborn as sns

style.use('fivethirtyeight') # librairies stylé

os.chdir("/Users/guillaume/MyProjects/PythonProjects/QARM/Assignements/A1")
print("Current working directory: {0}".format(os.getcwd()))

## Importation data
######################
df = pd.read_excel("Data_HEC_QAM_A1.xlsx", engine='openpyxl')

df = df.rename(columns={"Unnamed: 0":"date"})
df['date'] = pd.to_datetime(df['date'],format="%d.%m.%Y")

df.index = df['date']

## Condtion to create the two samples of data
#############################################
in_sample = df.loc[(df['date'] <= pd.to_datetime('2017-12-31'))].iloc[:,1:]
```

```
corr_insample = sns.heatmap(in_sample.corr(),annot=True) #Corrrelation
↪ matrix between assets (In-sample)
#corr_insample.set_title("Correlation matrix between assets (In-sample
↪ dataset)",pad=20,fontweight='bold')
plt.savefig("fig/corr_insample.png", bbox_inches = "tight")

out_sample = df.loc[(df['date'] > pd.to_datetime('2017-12-31'))].iloc[:,1:]
corr_outsample = sns.heatmap(out_sample.corr(),annot=True) #Correlation
↪ matrix between assets (Out-sample)
#corr_outsample.set_title("Correlation matrix between assets (Out-sample
↪ dataset)",pad=20,fontweight='bold')
plt.savefig("fig/corr_outsample.png", bbox_inches = "tight")

## Returns of "in-sample" dataset
#################################
num_lines_IS=np.size(in_sample,0)
simpleReturns_IS=((in_sample/in_sample.shift(1))-1).dropna()

## Returns of "out-of-samples" dataset
######################################
num_lines_OS=np.size(out_sample,0)
simpleReturns_OS=((out_sample/out_sample.shift(1))-1).dropna()

# cumulative returns of different assets (all dataset)
assets_cumul_return = (simpleReturns_OS+1).cumprod()
assets_cumul_return = assets_cumul_return*100
plt.rcParams["figure.figsize"] = (15,10)
assets_cumul_return.plot()
plt.legend(assets_cumul_return.columns,loc='lower left',fontsize='large')
plt.ylabel('Performance')
plt.xlabel('Date')
plt.savefig('fig/asset_cumul_return.png')
```

4 Mean-Variance Allocation

4.1 Question 1: In-sample Allocation

To perform a mean-variance optimisation for our portfolio allocations, we use a model developed by Jondeau & Rockinger (2004) which considers only the first two central moments (i.e. mean and standard deviation) and restrict ourselves to long-only positions. Under these conditions, we ignore the potential asymmetry of our assets' distributions. We will relax this assumption in the second part of our report when we will include the skewness and kurtosis into the optimization. This model will therefore share some strong similarities with the model developed by Markowitz (1952)[2], with the exception that short positions are not allowed. Thus the following equations will be required:

- An investor's utility function:

$$U(W_{t+1}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma} \tag{3}$$

  Where W_t denotes the wealth of the investor at time t. We consider a CRRA utility function since:

  1. There is a preference for mean returns and an aversion for standard deviation.
  2. It satisfies the requirement that the coefficient of relative risk aversion $\rho(W) = \gamma > 0$ is constant (i.e. independent of the initial wealth):

$$\rho(W) = -W\frac{U''(W)}{U'(W)} = -W\frac{-\gamma W^{-\gamma-1}}{W^{-\gamma}} = \gamma \tag{4}$$

- Its first and second-order derivatives:

$$U^{(1)}(W_{t+1}) = W_{t+1}^{-\gamma} \tag{5}$$

$$U^{(2)}(W_{t+1}) = -\gamma W_{t+1}^{-\gamma-1} \tag{6}$$

[2]https://www.jstor.org/stable/2975974?seq=1 Accessed: 21.03.21

- Using the second-order Taylor approximation around $\bar{W} = W_t(1+R_f)$, adapted from the model of Jondeau & Rockinger (2004), we first determine the utility of an investor:

$$U(W_{t+1}) = U(\bar{W}) + U^{(1)}(\bar{W})(W_{t+1} - \bar{W}) + \frac{1}{2}U^{(2)}(\bar{W})(W_{t+1} - \bar{W})^2 \tag{7}$$

Where R_f is the risk-free rate, which we use $R_f = 0.25\%$ due to low interest rates.

- Since the investor wants to maximize its expected utility function, we approximate it as:

$$\begin{aligned} E[U(W_{t+1})] &\approx U(\bar{W}) + U^{(1)}(\bar{W})E[W_{t+1} - \bar{W}] + \frac{1}{2}U^{(2)}(\bar{W})E[W_{t+1} - \bar{W}]^2 \\ &= U(\bar{W}) + U^{(1)}(\bar{W})E[W_{t+1} - \bar{W}] + \frac{1}{2}U^{(2)}(\bar{W})\sigma^2[W_{t+1}] \end{aligned} \tag{8}$$

We define the variance as:

$$\sigma^2[W_{t+1}] = E[(R_{p,t+1} - R_f)^2] = E[W_{t+1} - \bar{W}]^2 \tag{9}$$

With $W_{t+1} = (1 + R_{p,t+1})W_t$

- Replacing the equation (8) with elements determined above, we are left with:

$$E[U(W_{t+1})] \approx \frac{\bar{W}^{1-\gamma}}{1-\gamma} + \bar{W}^{1-\gamma}W_t E[(R_{p,t+1} - R_f)] - \frac{\gamma}{2!}\bar{W}^{-\gamma-1}W_t^2\sigma^2_{p,t+1} \tag{10}$$

Notice that maximizing a given positive function $+f(x)$ is equivalent of minimizing the same function when negative $-f(x)$.

Thus, we will minimize $-E[U(W_{t+1})]$ from equation (10), using the Sequential Least Squares Programming (SLSQP) iterative method as we have a non-linear optimization problem. It is performed with the following code:

```
def EV_criterion(weight,Lambda_RA,Returns_data):
    """

    this function computes the expected utility in the Markowitz case when
    investors have a preference for mean and variance
    Parameters
    ----------
   weight : list of floats
        weights in the investor's portfolio.
    Lambda_RA : int
        the risk aversion parameter.
    Returns_data : list of float
        the set of returns.
    Returns
    -------
    criterion : list of float
        optimal weights of assets in the portfolio.
    """
    portfolio_return=np.multiply(Returns_data,np.transpose(weight))
    portfolio_return=np.sum(portfolio_return,1)
    mean_ret=np.mean(portfolio_return,0)
    sd_ret=np.std(portfolio_return,0)
    skew_ret=skew(portfolio_return,0)
    kurt_ret=kurtosis(portfolio_return,0)
    W=1
    Wbar=1*(1+0.25/100)
    criterion=np.power(Wbar,1-Lambda_RA)/(1+Lambda_RA) +
    ↪ np.power(Wbar,-Lambda_RA)*W*mean_ret -
    ↪ Lambda_RA/2*np.power(Wbar,-1-Lambda_RA)*np.power(W,2)
    ↪ *np.power(sd_ret,2)
```

```
    #in order to maximize this formula in need to put negative sign =>
    ↪  because we use minimze -> -minimzeer = maximizer
    criterion=-criterion
    return criterion

## Function to run the two optimizers (EV and SK) in one time
#############################################################
def Optimizer(returnData):
    """


    Parameters
    ----------
    returnData : list of float
        DESCRIPTION.
    Returns
    -------
    res_SK : Object
        result of mean-variance-skewness-kurtosis optimizer
    res_EV : Object
        result of mean-variance optimizer
    """
    #starting points => 1% in each stock
    # weight for criterion :)
    x0 = np.array([0, 0, 0, 0, 0])+0.01

    # constraint for weight
    cons=({'type':'eq', 'fun': lambda x:sum(x)-1})
    Bounds= [(0 , 1) for i in range(0,5)] # boudns of weights -> 0 to 1

    Lambda_RA=3 #define teh risk aversion parameter

```

```
    res_SK = minimize(SK_criterion, x0, method='SLSQP', args=(Lambda_RA,np.array(returnData.iloc[:,0:5])), bounds=Bounds,constraints=cons,options={'disp': True})
    #res_SK.x: give the optimal weight
    res_EV = minimize(EV_criterion, x0, method='SLSQP', args=(Lambda_RA,np.array(returnData.iloc[:,0:5])), bounds=Bounds,constraints=cons,options={'disp': True})

    return (res_SK,res_EV)


############
## Part 1 ##
############

opt = Optimizer(simpleReturns_IS)
SK_w = opt[0].x
EV_w = opt[1].x

assets_cumul_return_IS = (simpleReturns_IS+1).cumprod()
assets_cumul_return_IS = assets_cumul_return_IS*100
plt.rcParams["figure.figsize"] = (15,10)
assets_cumul_return_IS.plot()
plt.legend(assets_cumul_return.columns,loc='upper left',fontsize='large')
plt.ylabel('Performance')
plt.xlabel('Date')
plt.savefig('fig/asset_cumul_return_IN.png')
```

The resulting asset allocations are the following:

Table 1 – Optimal Portfolio Allocation Under EV Allocation

	Weight	Avg. Ret.	STD
World Equities	0.000000	0.036289	0.169313
World Bonds	0.216421	0.046662	0.069291
US High Yield	0.344170	0.072764	0.072418
Oil	0.060850	0.113916	0.361528
Gold	0.378560	0.098734	0.173642

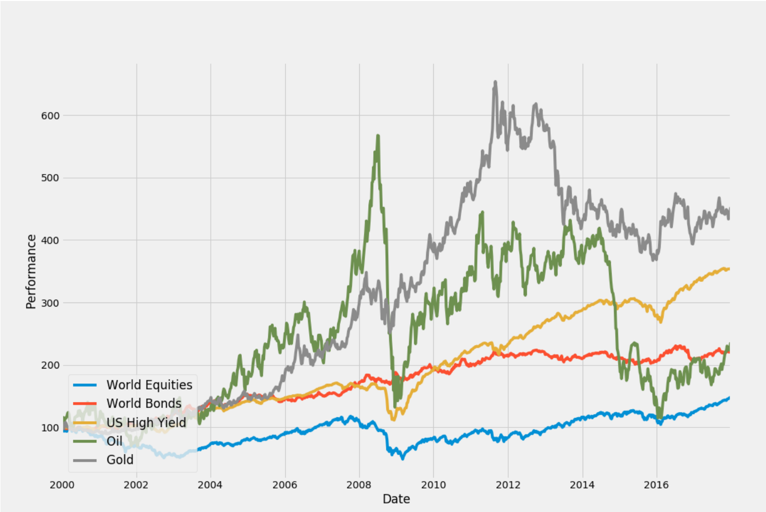
To anybody who has ever dealt with classical portfolio allocations, this will seem very peculiar.

- The most striking result is the complete absence of equities which would be quite uncommon in a real investment setting.
- The next problematic result which is closely linked to the previous point coupled with the low attribution to oil, is the lack of diversification which is generally something one wants to avoid.

Given the specification of our backwards-looking model, however, there are no real surprises. Observing Figure 2 we can see that:

- World Equities' downfall is that it has the lowest average returns out of all assets and a standard deviation almost as high as Gold's which has an average return almost 3 times larger.
- World Bonds had the lowest standard deviation but also a rather unimpressive performance, barely surpassing that of World Equities.
- US High Yield displayed a strong performance and relatively low standard deviation.
- Oil is penalized for its high volatility even if it has the highest average returns out of all the assets.
- Gold's strong performance during the in-sample period is reined in by relatively high volatility.

Figure 2 – In-Sample Cumulative Return All Assets



4.2 Question 2: Out-of-Sample Portfolio Performances

Let $\mathbf{w_{EV}}$ be the optimal mean-variance allocation matrix (5 rows, 1 column) using the in-sample data set obtained in the previous question under a mean-variance allocation. Define $\mathbf{R_{i,t}^{OS}}$ as the weekly return of asset i at time t in the "out-of-sample" data set (i.e. from January 2018 to February 2021). Then, the weekly returns of our optimal allocation during the aforementioned time horizon is:

$$\mathbf{R_{EV,t}^{OS}} = \mathbf{R_{i,t}^{OS}} * \mathbf{w_{EV}} \tag{11}$$

We then computed the cumulative performance of this portfolio over the time horizon by indexing it at 100 in January 2018. We determined the following Python code:

```python
## function to  create the rolling performance for the out-sample dataset
######################################################################
def rollingperf(weight,returns,opt):
    return_test = np.multiply(returns,weight)
    sum_return = np.sum(return_test,1)
    perf = [100]

    for i in range(len(sum_return)):
        value = perf[i]*(1+sum_return.values[i])
        perf.append(value)

```

```
    df_perf = pd.DataFrame(perf,columns=["Performance"])
    df_perf.index = out_sample.index

    final_perf = sum_return.cumsum()
    if opt == "EV":
        df_perf.plot()
        plt.ylabel('Performance')
        plt.xlabel('Date')
        # plt.title(label="Cumul Perf. of Mean-Var portfolio by indexing at
        ↪  100 in january 2018",
        #    pad=20,
        #    fontweight='bold',
        #    color="black")
        plt.savefig('fig/EV_outsample.png', bbox_inches = "tight")
    else:
        df_perf.plot()
        plt.ylabel('Performance')
        plt.xlabel('Date')
        # plt.title(label="Cumul Perf. of Mean-Var-Skew-Kurt portfolio by
        ↪  indexing at 100 in january 2018",
        #    pad=20,
        #    fontweight='bold',
        #    color="black")
        plt.savefig('fig/SK_outsample.png', bbox_inches = "tight")
    return final_perf

# EV performance
################
#perf_EV_IS = rollingperf(simpleReturns_IS,EV_w,"EV") # Not necessary
perf_EV_OS = rollingperf(simpleReturns_OS,EV_w,"EV")

# cumulative returns of different assets ()
```

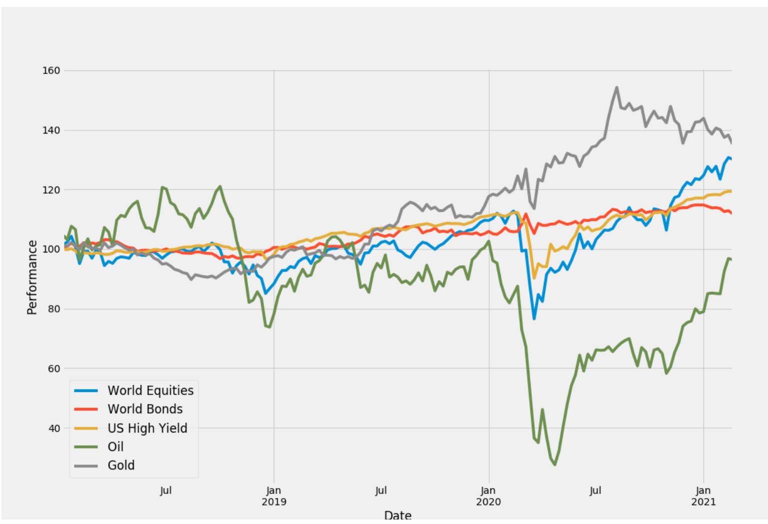
```
dfCumulRet = ((df.iloc[:,1:]/df.shift(1).iloc[:,1:] - 1).dropna() +
↪   1).cumprod()
dfCumulRet = dfCumulRet*100
dfCumulRet.plot()
plt.legend(dfCumulRet.columns,loc='best',fontsize='large')
plt.ylabel('Performance')
plt.xlabel('Date')
plt.savefig('fig/asset_cumul_return_outSample.png')
```

As a result, we obtained the following performances:

Figure 3 – Portfolio Performances Under EV Allocation

To better understand the performance of our portfolio, we also determined the cumulative performance from January 2018 to February 2020 of each asset classes available to us:

Figure 4 – Out-of-Sample Cumulative Return All Assets



From these results, we notice that our portfolio has witnessed a general upward movement, reaching a cumulative return of approximately +25% by February 2021. Nevertheless, it also faced medium to extreme jumps due mainly to economic uncertainty, geopolitical events and the global COVID-19 pandemic. We now will go through some of the main global episodes that impacted our portfolio. In 2018, our portfolio did not start in a good fashion. Indeed, in February 2018, markets plunged and volatility was extremely high as the Dow Jones Industrial Average plummeted by 1,500 basis points in a single day, amid concerns over rising inflation. Nevertheless, markets were able to partially recover at a quick pace, raising similarities with the Flash Crash of 2010[3]. Consequently, our portfolio witnessed a similar performance due to its exposure to oil price and bonds for example, although it doesn't have a stake in equities.

In 2018, after a few months of stability in all asset classes, the global markets plunged again at the end of the year, due to increasing tensions in the US-China trade dispute, which led to uncertainties regarding the tariffs on commodities, the rise in interest rates by the Federal Reserve as a result of tight monetary policies, as well as scepticism over global economic growth, partly caused by political tensions in the EU and the US (e.g. the rise of populism, Italy's budget proposal, Brexit). Such events negatively impacted global equities as the MSCI World Index fell by 7.1% by the end of 2018[4]. Although our portfolio doesn't have any exposure to equities, we were not immune since it also impacted the majority of the remaining assets. Indeed, the World Bonds prices plunged as volatility in global markets was increasing, which led to a rise in bonds yields and a decrease in their price. The same can be said about the US High-Yield bonds. Oil was the worst performing asset of our portfolio by the end of 2018. Indeed, as former president Trump brought back sanctions on Iran, Saudia Arabia increased their oil extraction due to the withdrawal of restrictive oil supply among OPEC members, and a lower-than-expected global demand led to a significant drop in oil prices, as Brent crude went down to −19.5% in 2018[5]. Nevertheless, the price of gold rose thanks to its use as a safe haven against market uncertainties and sluggish economic growth, thus partially mitigating losses from other assets[6].

After the sluggish performances by the end of 2018, our portfolio re-bounced strongly in 2019, reaching a cumulative return of approx. 12% by the end of the year, thanks to a booming global

[3] `https://www.cnbc.com/2018/02/05/why-the-stock-market-plunged-today.html` Accessed: 21.03.21

[4] `https://www.schroders.com/en/insights/economics/financial-markets-2018-the-year-in-review/` Accessed: 21.03.21

[5] `https://www.cnbc.com/2018/12/31/oil-prices-are-set-for-their-worst-year-since-2015.html` Accessed: 21.03.21

[6] `https://www.cnbc.com/2019/01/03/gold-markets-global-economy-asian-stock-markets-in-focus.html` Accessed: 21.03.21

market. Indeed, although the US-China trade war was still active, and the global economic growth was fairly weak, "2019 might just be the best year investors have ever had" according to Reuters on December 20, 2019[7]. This was mainly due to the Federal Reserve's actions of expansionary monetary policy by lowering interest rates, and the roaring US tech stocks which accounted significantly for the S&P500's gains of 2019. Consequently, all asset classes of our portfolio performed well during the year, raising the cumulative return up to 12%.

Thereafter, 2020 was a year defying global markets amid the COVID-19 pandemic. Initially in January and February, as China was the first country impacted by the virus, global investors were seeking safe-haven assets such as bonds to mitigate their risk. As lockdown and social-distancing measures were introduced in late February and the beginning of March to halt the spread of the virus, the majority of businesses were forced to halt their operations. This led to a huge drop in performances in all our asset classes, due to bearish sentiment in the global markets. The most impacted asset was oil. As we know, oil has a relatively high beta as demand for the commodity is highly related to economic activity[8]. The pandemic therefore caused a substantial price decline. In addition, rising tensions between Russia and Saudi Arabia, members of the OPEC organization, led to an oil price war as Saudi Arabia increased its supply substantially although demand was very low, exacerbating the price decrease[9]. Consequently, our portfolio underwent a steep decline.

Nevertheless, as countries began implementing economic recovery packages, especially the multi-billion stimulus package from the Federal Reserve, and strong expectations of markets' recovery, boosted by the quick development of a vaccine, markets rebounded at an incredible speed, particularly through US technology stocks and the huge increase of day-trading among retail investors. It was a matter of few weeks to regain pre-pandemic figures in our portfolio. From July to November 2020, global markets were stagnant, amid fears of the long process to recover from the pandemic. However, when Pfizer Inc. announced the high effectiveness of its COVID-19 vaccines on November 9 [10], markets regained confidence in a global economic recovery and all our asset classes witnessed an upward jump in their performances.

By the end of 2020 and beginning of 2021, although green investment was a hot topic, especially with the election of the president of the United States, Joe Biden, oil prices were increasing

[7] https://www.reuters.com/article/us-global-markets-2019-graphic-idUSKBN1YO266 Accessed: 21.03.21

[8] https://www.unigestion.com/fr/insight/a-deux-doigts-de-la-hausse-des-prix-du-petrole/ Accessed: 21.03.21

[9] https://www.ft.com/content/c9c3f8ac-64a4-11ea-a6cd-df28cc3c6a68 Accessed: 21.03.21

[10] https://www.pfizer.com/news/press-release/press-release-detail/pfizer-and-biontech-announce-vaccine-candidate-against Accessed: 25.03.21

steadily in line with its global demand.

When looking ahead for 2021, the main issue for investors is the latest rise in inflation due to the substantial consumption growth, caused by a colossal economic recovery package from governments. This rise in inflation is linked to the fact that oil is one of the main input for an economy and therefore a rise in its price will push the price of consumption goods upwards[11]. Although the Federal Reserve has pledged to keep its interest rate near zero, as the US is heading to one of its biggest economic growth in 40 years, investors are fearing that the central banks cannot maintain such a level of interest rate due to the increase in inflation[12]. According to a report from Unigestion [13], the main risk in 2021 is not a recession, but indeed inflation. Indeed, it believes that the impact on consumption growth by the fiscal stimulus has not been fully assimilated, thus creating expectations of a substantial consumption inflation wave. It indeed believes that the current inflation predictions by US economists are too conservative. Nevertheless, a rise in economic growth and asset price inflation is also expected, which can potentially generate upwards movements in our assets' performances.

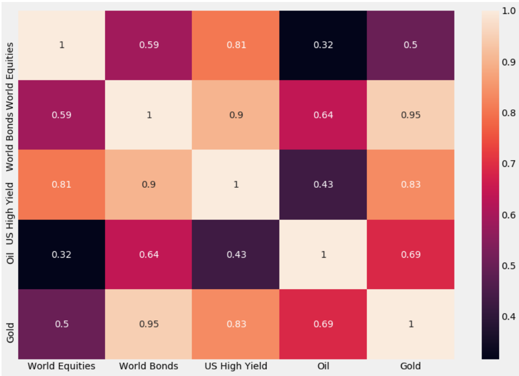
Figure 5 illustrates the correlation between asset classes, calculated on both In-sample and Out-Sample data. The highest noticeable correlations are 0.95 between Bond and Gold, 0.9 between Bond and High Yield and the 0.81 between High Yield and Gold. These strong correlations reinforce the hypothesis whereby these assets form a group of safe-haven investments. Within this group, the correlation of 0.95 between bond and gold can only be noted. These two asset classes have a relatively low correlation with equity (between 0.5 and 0.59) illustrating the fact that both Gold and bonds are used similarly as a hedge against equity surges. Overall, these tendencies seem to increase when comparing the In-sample and Out-sample data. For the Out-Sample Data specifically (see fig. 5b); one can only notice negative correlations across all asset classes with oil. This can mainly be explained by the consequent fall in oil in early 2020, while other asset classes continued to climb (see fig. 4).

[11] `https://www.investopedia.com/ask/answers/06/oilpricesinflation.asp` Accessed: 21.03.21

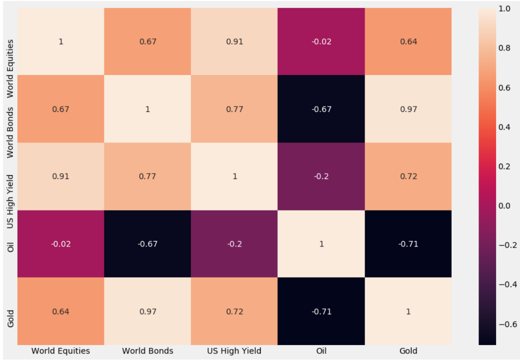
[12] `https://www.reuters.com/article/usa-fed/wrapup-6-fed-expects-growth-surge-inflation-jump-in-2021-but-no-rate-hike-idUSL1N2LF2JI/` Accessed: 21.03.21

[13] `https://www.unigestion.com/fr/insight/miviews-q1-2021-all-eyes-on-inflation/` Accessed: 21.03.21

Figure 5 – Correlation between Asset Class



(a) In-sample



(b) Out-sample

4.3 Question 3: Out-of-Sample and In-Sample Portfolio Characteristics

For this section, we computed the annualized return, the annualized volatility, the skewness as well as the kurtosis of our entire portfolio under the EV Allocation and all the financial assets available, both the in-sample data and the out-of-sample data. For the annualized return and annualized volatility, we multiplied the weekly return by 52 and the weekly standard-deviation by $\sqrt{52}$ given that there are 52 weeks in a year. Indeed, let r_i be the return in a week i, μ_W be the expected weekly return and μ_A be the annualized expected return, then:

$$\mu_A = E(r_1 + r_2 + ... + r_{52}) = 52\mu_W \tag{12}$$

Similarly, for the standard deviation, let $_A$ be the annualized volatility and σ_W be the weekly standard deviation. Then, with the assumption that the annualized variance is the sum of 52 weekly random variables independent and identically distributed, then we have:

$$(\sigma_A)^2 = Var(r_1 + r_2 + ... + r_{52}) = 52(\sigma_W)^2 \tag{13}$$

$$\sigma_A = \sqrt{52}\sigma_W \tag{14}$$

We implemented the following Python code:

```
 ## Function to compute Descriptive Statistics
#############################################
def Stat_descriptive(data,optimal_w,opt,sample):
    """

    Parameters
    ----------
    data : TYPE
        DESCRIPTION.
    optimal_w : TYPE
        DESCRIPTION.
    Returns
    -------
    output : Dataframe
        Dataframe of Descriptive Statistics

    """

    exp=np.mean(data,0)*52
    vol=np.std(data,0)*np.power(52,0.5)
    skew_ret=skew(data,0)
    kurt_ret=kurtosis(data,0)
    output=pd.DataFrame([optimal_w,exp,vol,skew_ret,kurt_ret],
    ↪   columns=["weight","AVg. Ret.","sigma", "Skew","Kurt"]);
    index = output.index
    columns = output.columns
    output = output.transpose()
    output.columns = columns
    output.index = [i for i in df.iloc[:,1:].columns]
    if opt == "EV":
        if sample == "IS":
            output.to_latex("table/IS_EV_stats_decript.tex")
```

```
        else:
            output.to_latex("table/OS_EV_stats_decript.tex")
    else:
        if sample == "IS":
            output.to_latex("table/IS_SK_stats_decript.tex")
        else:
            output.to_latex("table/OS_SK_stats_decript.tex")
    return output

## Descriptive Statistics with EV optimizer
###########################################
stat_EV_IS = Stat_descriptive(simpleReturns_IS,EV_w,"EV","IS")
stat_EV_OS = Stat_descriptive(simpleReturns_OS,EV_w,"EV","OS")

## Compute STD of optimal portfolio ##

#EV

port_return_IS_EV=np.multiply(simpleReturns_IS,np.transpose(EV_w))
port_return_IS_EV=np.sum(port_return_IS_EV,1)
exp_IS_EV=np.mean(port_return_IS_EV,0)*52
sd_re_IS_EV=np.std(port_return_IS_EV,0)*np.power(52,0.5)
skew_ret_IS_EV=skew(port_return_IS_EV,0)
kurt_ret_IS_EV=kurtosis(port_return_IS_EV,0)

port_return_OS_EV=np.multiply(simpleReturns_OS,np.transpose(EV_w))
port_return_OS_EV=np.sum(port_return_OS_EV,1)
exp_OS_EV=np.mean(port_return_OS_EV,0)*52
sd_re_OS_EV=np.std(port_return_OS_EV,0)*np.power(52,0.5)
skew_ret_OS_EV=skew(port_return_OS_EV,0)
kurt_ret_OS_EV=kurtosis(port_return_OS_EV,0)

```

```
sd_data_EV = {"Annualized return":[exp_IS_EV,exp_OS_EV], "Volatility":[sd_re_IS_EV,sd_re_OS_EV], "Skewness":[skew_ret_IS_EV,skew_ret_OS_EV], "Kurtosis":[kurt_ret_IS_EV,kurt_ret_OS_EV]}
sd_EV = pd.DataFrame(sd_data_EV,index=['Mean-Variance Portfolio','Skew-Kurtosis Portfolio'])
sd_EV.to_latex("table/comparison_EV.tex")
```

We obtained the following results:

Table 2 – Descriptive Table of the Portfolio Under EV Optimisation

	Return	Volatility	Skewness	Kurtosis
In-sample Portfolio	0.079450	0.086592	-0.384027	1.427645
Out-sample Portfolio	0.076996	0.093464	-1.860999	13.820387

Table 3 – In-Sample Descriptive Table of Assets Under EV Optimisation

	Weights	Avg. Ret.	STD	Skew.	Excess Kurt.
World Equities	0.000000	0.036289	0.169313	-0.851978	7.961114
World Bonds	0.216421	0.046662	0.069291	-0.028541	0.335057
US High Yield	0.344170	0.072764	0.072418	-1.895138	23.811137
Oil	0.060850	0.113916	0.361528	-0.299930	3.033220
Gold	0.378560	0.098734	0.173642	-0.074974	1.998239

Table 4 – Out-of-Sample Descriptive Table of Assets Under EV Optimisation

	Weights	Avg. Ret.	STD	Skew.	Excess Kurt.
World Equities	0.000000	0.105964	0.207377	-0.799844	6.203292
World Bonds	0.216421	0.037479	0.057553	0.035825	4.023223
US High Yield	0.344170	0.062039	0.105634	-1.903090	24.909059
Oil	0.060850	0.119505	0.507729	-0.009771	5.090989
Gold	0.378560	0.106354	0.141703	0.058696	3.917422

The three tables above depict the allocation, average annualized return, volatility, skewness and kurtosis, respectively for the portfolio and the assets of the In-Sample and the Out-of-Sample data with the EV Mean-Variance Optimization.

By comparing the tables with in-depth analysis, we can see that applying the initial in-sample allocation to the out-of-sample period is not optimal. There have been significant increases and decreases in annualized returns and volatility in our assets, between both samples. Starting with US High Yield, we can see that this asset had higher volatility and lower annual returns in the out-of-sample case, which should lead to a lower weight allocated to this asset. On the other hand, Gold had a higher average return and lower volatility in recent years, which should have given a higher allocation to this asset. The most striking difference between both samples is the performance of World Equities. These have performed far better in the out-of-sample case, by nearly tripling the annual return while having only a proportionally smaller increase in volatility. This risk/reward or Sharpe Ratio for World Equities increased dramatically between both samples, becoming a more interesting asset to invest in. Unfortunately, the out-of-sample portfolio could not benefit from this increase in performance of the World Equities, following the decision not to invest any of the money into the World Equities due to its poor performance in the in-sample. Meanwhile, Oil saw an increase in volatility while having a stable annualized return, and World Bonds saw a lower average return and volatility.

By taking into account these differences, World Equities and Gold should have had a higher allocation in the out-of-sample case, while World Bonds would have remained stable, and money allocated to US High Yield and Oil should have been lower had we been able to predict the future.

Before discussing skewness and kurtosis, we will briefly explain these measures. Skewness is the third central moment of a random variable and it describes the direction or the asymmetry of the distribution. Positive skewness means that the distribution is denser on the left side, with a long and thin tail running to the right. A negative skewness would mean the opposite. A neutral skewness would mean that the distribution is symmetrical like the Gaussian distribution for example. Kurtosis is the fourth central moment of a random variable and it describes how thick the tails are. We generally deal with excess kurtosis which uses the kusrtosis of a Normal distribution which is 3 as a benchmark. High excess kurtosis is known as leptokurtic, which means that the distribution is narrow around the mean and has flat and thin tails. Low kurtosis would be the opposite. Excess kurtosis of 0 shows that the distribution resembles a Gaussian distribution.

We know that our risk-averse investor (with a risk aversion coefficient of 3) would prefer positive skewness (preference for consistent smaller returns, than inconsistent higher returns) and a negative kurtosis (preference for a flatter curve). When looking at both tables, there aren't any striking differences between recent years and previous years. Coefficients have remained stable between both samples.

By comparing the in-sample and out-of-sample cases, on the one hand, one can conclude that major differences in annualized returns and volatility would have led to some large changes in allocations. On the other hand, skewness and kurtosis of our assets haven't seen significant differences between both samples, which would have impacted our allocation only in a meaningful way.

When looking at the portfolio performance, applying the same allocation to the out-sample period is also not efficient. Annualized return decreased when volatility increased, which in turn decreased the Sharpe Ratio (down from 0.919 to 0.824). The Sharpe Ratio describes the risk/reward trade-off of an allocation: the higher the ratio, the better, one gets more returns with the same amount of risk. Therefore, it would have been more reasonable to rebalance the allocation by using the comments made above regarding each asset to improve the Sharpe Ratio. Moreover, despite not taking into account skewness and kurtosis in the EV optimization, these moments have respectively decreased and increased.

5 Mean-Variance-Skewness-Kurtosis Optimal Allocation

5.1 Question 1: In-Sample Allocation

To perform a Mean-Variance-Skewness-Kurtosis optimisation for our portfolio allocations, we will again use a model by Jondeau & Rockinger (2004), this time including the first to fourth central moments. Indeed, we will also include the skewness and kurtosis into our optimal allocation, which assumes that the returns do not follow a normal distribution. Again we restrict ourselves to long-only positions. Thus the following equations will be required:

- An investor's utility function

$$U(W_{t+1}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma} \tag{15}$$

  Where W_t denotes the wealth of the investor at time t. We consider again a CRRA utility function since:

  1. There is a preference for mean returns and an aversion for standard deviation.

2. There is a **preference for skewness and aversion for kurtosis**.

3. It satisfies the requirement that the coefficient of relative risk aversion $\rho(W) = \gamma > 0$ is constant (i.e. independent of the initial wealth):

$$\rho(W) = -W\frac{U''(W)}{U'(W)} = -W\frac{-\gamma W^{-\gamma-1}}{W^{-\gamma}} = \gamma \tag{16}$$

- We consider the first to fourth order derivative of our utility function:

$$U^{(1)}(W_{t+1}) = W_{t+1}^{-\gamma} \tag{17}$$

$$U^{(2)}(W_{t+1}) = -\gamma W_{t+1}^{-\gamma-1} \tag{18}$$

$$U^{(3)}(W_{t+1}) = (\gamma+1)\gamma W_{t+1}^{-\gamma-2} \tag{19}$$

$$U^{(4)}(W_{t+1}) = -(\gamma+2)(\gamma+1)\gamma W_{t+1}^{-\gamma-3} \tag{20}$$

- Using the fourth-order Taylor approximation around $\bar{W} = W_t(1+R_f)$, we first determine the utility of the investor:

$$\begin{aligned} U(W_{t+1}) = U(\bar{W}) + U^{(1)}(\bar{W})(W_{t+1} - \bar{W}) + \frac{1}{2}U^{(2)}(\bar{W})(W_{t+1} - \bar{W})^2 \\ + \frac{1}{3!}U^{(3)}(\bar{W})(W_{t+1} - \bar{W})^3 + \frac{1}{4!}U^{(4)}(\bar{W})(W_{t+1} - \bar{W})^4 \end{aligned} \tag{21}$$

  Where R_f is the risk-free rate, which we decided to use $R_f = 0.25\%$ due to low interest rates.

- As the investor wants to maximize its expected utility function, we approximate it as:

$$
\begin{aligned}
E[U(W_{t+1})] &\approx U(\bar{W}) + U^{(1)}(\bar{W})E[W_{t+1} - \bar{W}] + \frac{1}{2}U^{(2)}(\bar{W})E[W_{t+1} - \bar{W}]^2 \\
&+ \frac{1}{3!}U^{(3)}(\bar{W})E[W_{t+1} - \bar{W}]^3 + \frac{1}{4!}U^{(4)}(\bar{W})E[W_{t+1} - \bar{W}]^4 \\
&= U(\bar{W}) + U^{(1)}(\bar{W})E[W_{t+1} - \bar{W}] + \frac{1}{2}U^{(2)}(\bar{W})\sigma^2[W_{t+1}] \\
&+ \frac{1}{3!}U^{(3)}(\bar{W})s^3[W_{t+1}] + \frac{1}{4!}U^{(4)}(\bar{W})k^4[W_{t+1}]
\end{aligned}
\tag{22}
$$

We define the variance, skewness and kurtosis as[14]:

$$\sigma^2[W_{t+1}] = E[(R_{p,t+1} - R_f)^2] = E[W_{t+1} - \bar{W}]^2 \tag{23}$$

$$s^3[W_{t+1}] = E[(R_{p,t+1} - R_f)^3] = E[W_{t+1} - \bar{W}]^3 \tag{24}$$

$$\kappa^4[W_{t+1}] = E[(R_{p,t+1} - R_f)^4] = E[W_{t+1} - \bar{W}]^4 \tag{25}$$

Where $W_{t+1} = (1 + R_{p,t+1})W_t$.

Notice that the definition of the skewness and kurtosis differ from their statistical definitions as the standardized central higher moments:

$$S[X] = E\left[\left(\frac{X - \mu}{\sigma}\right)^3\right] \tag{26}$$

$$K[X] = E\left[\left(\frac{X - \mu}{\sigma}\right)^4\right] \tag{27}$$

Where X is a random variable (e.g. asset returns), μ is the sample mean and σ the sample standard deviation. When we consider a sample of n values, where m_i is the sample i^{th} central moment, then the empirical skewness and kurtosis:

$$\hat{S} = \frac{m_3}{m_2^{\frac{3}{2}}} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{\frac{3}{2}}} \tag{28}$$

[14] We decided to determine the mean, variance, skewness and excess kurtosis using our Python libraries instead of computing them manually to increase their precision.

$$\hat{K} = \frac{m_4}{m_2^2} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^2} \tag{29}$$

- Replacing the equation (22) by elements determined above, we are left with:

$$\begin{aligned} E[U(W_{t+1})] \approx \frac{\bar{W}^{1-\gamma}}{1-\gamma} + \bar{W}^{1-\gamma} W_t E[(R_{p,t+1} - R_f)] - \frac{\gamma}{2!}\bar{W}^{-\gamma-1} W_t^2 \sigma_{p,t+1}^2 \\ + \frac{(\gamma+1)\gamma}{3!}\bar{W}^{-2-\gamma} W_t^3 s_{p,t+1}^3 - \frac{(\gamma+2)(\gamma+1)\gamma}{4!}\bar{W}^{-\gamma-3} W_t^4 \kappa_{p,t+1}^4 \end{aligned} \tag{30}$$

Notice that maximizing a given positive function $+f(x)$ is equivalent of minimizing the same function when negative $-f(x)$.

Thus, we will minimize $-E[U(W_{t+1})]$ from equation (22), using the Sequential Least-Squares Programming (SLSQP) iterative method as we have a non-linear optimization problem.

```
#Optimization function
def SK_criterion(weight,Lambda_RA,Returns_data):
    """


    this function computes the expected utility in the Markowitz case when
    investors have a preference for skewness and kurtosis
    Parameters
    ----------
    weight : list of float
        weights in the investor's portfolio.
    Lambda_RA : int
        the risk aversion parameter.
    Returns_data : list of float
        the set of returns.
    Returns
    -------
    criterion : list of float
        optimal weights of assets in the portfolio.
    """
```

```
    portfolio_return=np.multiply(Returns_data,np.transpose(weight))
    portfolio_return=np.sum(portfolio_return,1)
    mean_ret=np.mean(portfolio_return,0)
    sd_ret=np.std(portfolio_return,0)
    skew_ret=skew(portfolio_return,0)
    kurt_ret=kurtosis(portfolio_return,0)
    W=1
    Wbar=1*(1+0.25/100)
    # use CRRA function + les dérivées => create A,B,C D => permet de créer
    ↪   la taylor expansion

    criterion=np.power(Wbar,1-Lambda_RA)/(1+Lambda_RA)+
    ↪   np.power(Wbar,-Lambda_RA)*W*mean_ret-
    ↪   Lambda_RA/2*np.power(Wbar,-1-Lambda_RA)*
    ↪   np.power(W,2)*np.power(sd_ret,2)+
    ↪   Lambda_RA*(Lambda_RA+1)/(6)*np.power(Wbar,-2-Lambda_RA)*
    ↪   np.power(W,3)*skew_ret-Lambda_RA*(Lambda_RA+1)*
    ↪   (Lambda_RA+2)/(24)*np.power(Wbar,-3-Lambda_RA)*
    ↪   np.power(W,4)*kurt_ret
    criterion=-criterion
    return criterion



def Optimizer(returnData):
    """
    Parameters
    ----------
    returnData : Float
        DESCRIPTION.
    Returns
    -------
    res_SK : Object
        result of mean-variance-skewness-kurtosis optimizer
```

```
    res_EV : Object
        result of mean-variance optimizer
    """
    # starting points => 1% in each stock
    # initialize weight
    x0 = np.array([0, 0, 0, 0, 0]) + 0.01

    # constraint for weight
    cons = ({'type': 'eq', 'fun': lambda x: sum(x) - 1})
    Bounds = [(0, 1) for i in range(0, 5)]  # boudns of weights -> 0 to 1

    Lambda_RA = 3  # define the risk aversion parameter
 #Calculate the optimal weight for the Mean-Variance-Skewness-Kurtosis
 ↪  Optimisation
    res_SK = minimize(SK_criterion, x0, method='SLSQP', args=(Lambda_RA,
     ↪  np.array(returnData.iloc[:, 0:5])), bounds=Bounds, constraints=cons,
     ↪  options={'disp': True})

 #Calculate the optimal weight for the Mean-Variance Optimisation
    res_EV = minimize(EV_criterion, x0, method='SLSQP', args=(Lambda_RA,
     ↪  np.array(returnData.iloc[:, 0:5])), bounds=Bounds, constraints=cons,
     ↪  options={'disp': True})

    return (res_SK, res_EV)

opt = Optimizer(simpleReturns_IS)
SK_w = opt[0].x
EV_w = opt[1].x
```

The resulting asset allocations are the following:

Table 5 – Optimal Portfolio Allocation Under SK Optimisation

	Weight	Avg. Ret.	STD	Skew.	Excess Kurt.
World Equities	0.0000000	0.036289	0.169313	-0.851978	7.961114
World Bonds	0.9916035	0.046662	0.069291	-0.028541	0.335057
US High Yield	0.0000000	0.072764	0.072418	-1.895138	23.811137
Oil	0.0083964	0.113916	0.361528	-0.299930	3.033220
Gold	0.0000000	0.098734	0.173642	-0.074974	1.998239

If the asset allocations under the EV optimisation yielded slightly unrealistic results, KS optimisation is completely preposterous.

- Yet again, World Equities are omitted entirely but now even Gold and High Yield bonds are shunned.
- Although we do have a stake in Oil, it accounts for less than a percent.
- Hence, diversification, one of the pillars of a asset management, is virtually non-existent.

This goes to show that the model is very limited in terms of building a reliable portfolio since such an allocation would be ill-advised in a real investment strategy to say the least.

The reason for these results are as follows:

- World Equities remain off the table due to their poor Sharpe Ratio. We would probably allocate even less money to this assets if it were possible because of the high kurtosis
- World Bonds' receives almost all the attention despite unimpressive average returns since they had by far the lowest Excess kurtosis and Volatility and also the largest skewness
- US High Yield had such high kurtosis that it had no place in the portfolio any more.
- Oil is propped up from second-worst to second-best due to it having the highest return and reasonable higher central moments.
- Gold falls out of favor and is no longer the leading asset class in our portfolio. Even if it would seem that it would be better that Oil given that it bests it on volatility, skewness and kurtosis its lower average returns are enough to tip the scale in favor of Oil.

Under this allocation, we believe there is some irregularity regarding the weights on Oil and Gold. Indeed, under the EV allocation, gold was the favorite asset due to its relatively high return and modest volatilty. Nevertheless, under the SK allocation, although Gold has a higher Sharpe ratio, as well as higher skewness and lower kurtosis than Oil in the in-sample, it is not selected under the SK allocation. We believe this irregularity could be produced in the optimization on Python.

5.2 Question 2: Out-of-Sample Portfolio Performances

Let $\mathbf{w_{SK}}$ be the optimal mean-variance-skewness-kurtosis allocation matrix (5 rows, 1 column) using the "in-sample" data set. Define $\mathbf{R_{i,t}^{OS}}$ as the weekly returns of the five assets i at time t in the "out-of-sample" data set (i.e. from January 2018 to February 2021). Then, the weekly returns of our optimal allocation during the aforementioned time horizon is:

$$\mathbf{R_{SK,t}^{OS}} = \mathbf{R_{i,t}^{OS}} * \mathbf{w_{SK}} \tag{31}$$

We then computed the cumulative performance of this portfolio over the time horizon by indexing it at 100 in January 2018. We implemented the following code:

```
## function to  create the rolling performance for out-sample dataset
######################################################################
def rollingperf(weight,returns,opt):
    return_test = np.multiply(returns,weight)
    sum_return = np.sum(return_test,1)
    perf = [100]

    for i in range(len(sum_return)):
        value = perf[i]*(1+sum_return.values[i])
        perf.append(value)

    df_perf = pd.DataFrame(perf,columns=["Performance"])
    df_perf.index = out_sample.index

    final_perf = sum_return.cumsum()
    if opt == "EV":
        df_perf.plot()
        plt.ylabel('Performance')
        plt.xlabel('Date')
```

```
        # plt.title(label="Cumul Perf. of Mean-Var portfolio by indexing at
        ↪ 100 in january 2018",
        #   pad=20,
        #   fontweight='bold',
        #   color="black")
        plt.savefig('fig/EV_outsample.png', bbox_inches = "tight")
    else:
        df_perf.plot()
        plt.ylabel('Performance')
        plt.xlabel('Date')
        # plt.title(label="Cumul Perf. of Mean-Var-Skew-Kurt portfolio by
        ↪ indexing at 100 in january 2018",
        #   pad=20,
        #   fontweight='bold',
        #   color="black")
        plt.savefig('fig/SK_outsample.png', bbox_inches = "tight")
    return final_perf

# SK performance
###############
#perf_SK_IS = rollingperf(simpleReturns_IS,SK_w,"SK") # Not necessary
perf_SK_OS = rollingperf(simpleReturns_OS,SK_w,"SK")
```

As a result, we obtained the following performance:

Figure 6 – Portfolio Performances Under SK Allocation

From this result, we notice that our portfolio has witnessed a general upward movement, reaching a cumulative return of approximately +12% on February 2021. Nevertheless, it also faced jumps and drift due mainly to economic uncertainties, geopolitical events and the global pandemic of COVID-19, but at a different amplitude compared to the mean-variance portfolio. As the latter is almost entirely determined by the World Bond, we will go into details through the main events that impacted the performances for this asset only. In February 2018, as global equities plunged and volatility was abnormally high, World Bonds performed poorly alongside the hike of US Treasury yields due to concerns about a rising inflation [15]. During Q4 of 2018, the Federal Reserve was hawkish, seeking to normalize its monetary policy by gradually increasing rates following the financial crisis in 2008 [16]. Consequently, a rise in interest rate pushed the price of bonds downward, as illustrated by the negative cumulative performance in our portfolio. Ultimately, as the market was facing turmoil during this period, in 2019 the Federal Reserve decided to cut the interest rates in favour of investors. Therefore, as mentioned in the first part, all asset classes, including World Bonds, were performed tremendously well.

Nevertheless, as mentioned in the first section, during the early days of the COVID-19 pandemic, investors were seeking safer investments amid concerns over global growth: equities were down, but government bonds were up due to lower yields [17]. Therefore, this scenario explains the significant upward spike in our portfolio's performances. However, when markets froze by the end of March 2020, as countries were enforcing lock-downs, bonds became extremely volatile and illiquid, amid fears of waves of defaults. Nevertheless, thanks to a quick reaction by Central Banks by implementing monetary policies and various fiscal stimuli, it allowed investors to quickly regain confidence in the credit market. Consequently, the World Bonds were able to regain pre-pandemic figures, but still raising uncertainties about its volatility and liquidity [18]. Although our portfolio under the SK allocation was under-diversified, it wasn't hit as badly as our previous EV portfolio, as we didn't have exposure to oil and US high yields. As explained in the first part, when Pfizer Inc. announced the high effectiveness of its COVID-19 vaccines in November 2020, World Bonds witnessed an upward jump, which is reflected in an increase in our portfolio's performances.

Since the beginning of 2021, as the fear of a higher-than-expected inflation is growing among in-

[15] `https://www.schroders.com/en/insights/economics/monthly-markets-review---february-2018/` Accessed 26.03.21

[16] `https://www.cnbc.com/2018/09/26/fed-hikes-rates-by-a-quarter-point.html` Accessed: 21.03.21

[17] `https://www.schroders.com/en/insights/economics/monthly-markets-review---february-2020/` Accessed: 25.03.21

[18] `https://www.juliusbaer.com/it/insights/markets-explained/what-the-covid-19-shock-teaches-bond-investors/` Accessed: 25.03.21

vestors, and the expectation of the Federal Reserve to enter into an interest rate hike, although they pledged against it, bond prices are starting to decreases, which impacts our portfolio negatively[19]. As we are too early to determine the consequences of higher inflation and higher economic growth, only time will let us know the outlook of our portfolio's performance.

5.3 Question 3: Out-of-Sample and In-Sample Portfolio Characteristics

For this section, we computed the annualized return, the annualized volatility, the skewness as well as the kurtosis of our entire portfolio under the SK Allocation and all the financial assets available, both the in-sample data and the out-of-sample data. For the annualized return and annualized volatility, we applied the same method as explained in section 4.3. We implemented the following Python code:

```python
 ## Function to compute Descriptive Statistics
###############################################
def Stat_descriptive(data,optimal_w,opt,sample):
    """


    Parameters
    ----------
    data : TYPE
        DESCRIPTION.
    optimal_w : TYPE
        DESCRIPTION.
    Returns
    -------
    output : Dataframe
        Dataframe of Descriptive Statistics


    """


    exp=np.mean(data,0)*52
    vol=np.std(data,0)*np.power(52,0.5)
```

[19] https://blog.en.erste-am.com/yield-opportunity-in-the-bond-market/ Accessed: 25.03.21

```
    skew_ret=skew(data,0)
    kurt_ret=kurtosis(data,0)
    output=pd.DataFrame([optimal_w,exp,vol,skew_ret,kurt_ret],
     columns=["weight","AVg. Ret.","sigma", "Skew","Kurt"]);
    index = output.index
    columns = output.columns
    output = output.transpose()
    output.columns = columns
    output.index = [i for i in df.iloc[:,1:].columns]
    if opt == "EV":
        if sample == "IS":
            output.to_latex("table/IS_EV_stats_decript.tex")
        else:
            output.to_latex("table/OS_EV_stats_decript.tex")
    else:
        if sample == "IS":
            output.to_latex("table/IS_SK_stats_decript.tex")
        else:
            output.to_latex("table/OS_SK_stats_decript.tex")
    return output

## Descriptive Statistics with SK optimizer
###########################################
stat_SK_IS = Stat_descriptive(simpleReturns_IS,SK_w,"SK","IS")
stat_SK_OS = Stat_descriptive(simpleReturns_OS,SK_w,"SK","OS")

## Compute STD of optimal portfolio ##


#SK


port_return_IS_SK=np.multiply(simpleReturns_IS,np.transpose(SK_w))
port_return_IS_SK=np.sum(port_return_IS_SK,1)
exp_IS_SK=np.mean(port_return_IS_SK,0)*52
```

```
sd_re_IS_SK=np.std(port_return_IS_SK,0)*np.power(52,0.5)
skew_ret_IS_SK=skew(port_return_IS_SK,0)
kurt_ret_IS_SK=kurtosis(port_return_IS_SK,0)

port_return_OS_SK=np.multiply(simpleReturns_OS,np.transpose(SK_w))
port_return_OS_SK=np.sum(port_return_OS_SK,1)
exp_OS_SK=np.mean(port_return_OS_SK,0)*52
sd_re_OS_SK=np.std(port_return_OS_SK,0)*np.power(52,0.5)
skew_ret_OS_SK=skew(port_return_OS_SK,0)
kurt_ret_OS_SK=kurtosis(port_return_OS_SK,0)


sd_data_SK = {"Annualized return":[exp_IS_SK,exp_OS_SK],
↪  "Volatility":[sd_re_IS_SK,sd_re_OS_SK],
↪  "Skewness":[skew_ret_IS_SK,skew_ret_OS_SK],
↪  "Kurtosis":[kurt_ret_IS_SK,kurt_ret_OS_SK]}
sd_OS = pd.DataFrame(sd_data_SK,index=['In-sample Portfolio','Out-sample
↪  Portfolio'])
sd_OS.to_latex("table/comparison_SK.tex")
```

Consequently, we obtained the following results:

Table 6 – Descriptive Table of Portfolio Under SK Allocation

	Return	Volatility	Skewness	Kurtosis
In-sample Portfolio	0.047226	0.068826	-0.040973	0.316911
Out-sample Portfolio	0.038168	0.057320	-0.173101	4.464439

Table 7 – In-Sample Descriptive Table of Assets Under SK Allocation

	Weight	Avg. Ret.	STD	Skew.	Excess Kurt.
World Equities	0.000000	0.036289	0.169313	-0.851978	7.961114
World Bonds	0.991603	0.046662	0.069291	-0.028541	0.335057
US High Yield	0.000000	0.072764	0.072418	-1.895138	23.811137
Oil	0.008396	0.113916	0.361528	-0.299930	3.033220
Gold	0.000000	0.098734	0.173642	-0.074974	1.998239

Table 8 – Out-of-Sample Descriptive Table of Assets Under SK Allocation

	Weight	Avg. Ret.	STD	Skew.	Excess Kurt.
World Equities	0.000000	0.105964	0.207377	-0.799844	6.203292
World Bonds	0.991603	0.037479	0.057553	0.035825	4.023223
US High Yield	0.000000	0.062039	0.105634	-1.903090	24.909059
Oil	0.008396	0.119505	0.507729	-0.009771	5.090989
Gold	0.000000	0.106354	0.141703	0.058696	3.917422

We now have the portfolio, in-sample and out-of-sample tables as in part 4.3, this time optimized for skewness and kurtosis as well. The concepts of skewness and kurtosis have been described in part 4.3. As stated earlier, our investor with a risk aversion coefficient of 3 (slightly risk averse) prefers higher skewness and mean returns and lower kurtosis and volatility. Therefore, they will invest almost all their wealth into World Bonds, which has the highest skewness and lowest kurtosis and volatility although experiencing a relatively low average return.

Comparing the in-sample to the out-of-sample performance under the optimized the allocation, one can see that the allocation is no longer optimal in the out-of-sample case. Indeed, applying the initial allocation decreases skewness and increases the kurtosis. The slightly risk-averse investor would have liked the opposite. Moreover, the Sharpe Ratio of this allocation has also decreased, going from 0.686 in the in-sample portfolio to 0.666, which makes the allocation less optimal.

The reason is that, Gold now has the highest skewness and the lowest kurtosis making it more desirable again. An increase in skewness and a decrease in kurtosis for Gold means that the asset has become safer with a higher density around lower returns with a thicker tail. It is of no surprise to see an increase in the skewness of Gold, which is often considered as a safe asset in times of turbulence, which was the case in the out-of-sample case, which included a small recession in the end of 2019 and the Covid-19 crash in early 2020.

On the other hand, World Bonds have seen a small increase in skewness and a large increase in kurtosis. The large increase in kurtosis, which is feared by risk-averse investors, would have led to a much lower allocation in the out-of-sample case. Furthermore, the same reasoning regarding skewness as in the Gold case for volatility and safety can be applied to World Bonds, which are also considered as a safe asset in turbulent times.

Furthermore, other assets such as World Equities, US High Yield and Oil have seen small moves in skewness in kurtosis, not significant enough to drastically change their allocation.

Finally, one can clearly see that slightly risk-averse investors get what they desire when optimizing their portfolio for skewness and kurtosis. The volatility remains low, which comes at the cost of a low average annualized return.

6 Comparison Between the EV & SK Allocation

Under the EV allocation, our portfolio was composed of all our assets available except equities. When including the skewness and kurtosis into our optimization, the portfolio would be built from only World Bonds and Oil, as risk-averse investors are weary of negative skewness and positive kurtosis. Consequently, our SK portfolio would be able to withstand a strong economic crisis, such as the beginning of COVID-19 as the losses were smaller compared to our EV portfolio. Additionally, the SK portfolio witnessed lower volatility in comparison, which would be appropriate for a strong risk-averse investor. Nevertheless, this portfolio also has some drawbacks. Indeed, by emphasizing the importance of lower volatility, our SK portfolio could not grasp the global market upside potential, as it only produced a cumulative return of approximately +12% by February 2021, compared to +25% under the EV portfolio. Alternatively, the Sharpe ratio of the SK portfolio (0.67) was also lower than its the EV portfolio (0.82), when considering a risk-free rate of approximately 0. Additionally, as the SK allocation includes mainly one asset, it is not hedged against specific adverse events for this asset class: its poor diversification makes it less desirable if such criterion is essential for an investor. To conclude this section, we cannot conclude which portfolio is the more favourable. It is all down to the investor's preferences:

- If the investor is only concerned about the returns and volatility of its portfolio, while benefiting from the diversification effect and ignoring its skewness and kurtosis, the EV is more suitable.
- If the investor is also concerned about skewness and kurtosis, an SK allocation would be more suitable, although it would have generated lower returns in the Out-of-Sample time frame.

7 Conclusion

In the description section, we have acknowledged the asymmetric distribution of returns over 5 different assets classes: namely World Equities, Worlds Bonds, Us high Yields Bonds, Oil and Gold. The first part of the report only takes into account the two first moments (i.e. Mean and Standard Deviation) and therefore assumes a Normal distribution of returns. Additionally, long positions only have been added as a constraint. Given these assumptions, the minimization of variance on In-sample data resulted in a portfolio excluding equities and allowing only a small allocation to oil. When comparing the in-sample and out-sample results of this allocation we notice that this allocation is no longer optimal namely US high yield, Gold and world Equities would have their weights modified if we could have known how their returns would behave out of the sample.

We then moved to the second part of our report consisting of an allocation including the four central moments, allowing us to relax the normal distribution of returns hypothesis. The Mean-Variance-Skewness-Kurtosis optimisation resulted in an allocation of 99% in world Bonds, less than 1% in Oil and no investment in the remaining assets classes. These results are coherent with the preference of our investor having a risk aversion coefficient set to 3 and therefore preferring assets with higher skewness and lower kurtosis. However, using this result on the Out-of-Sample data resulted in a sub-optimal performance as the asset having the highest out-of-sample skewness turned out to be Gold.

Overall, one can notice that using backwards-looking optimization for a static portfolio allocation does not allow us to build efficient portfolios. Indeed, in the case of EV allocation, we have shown that using allocation computed on the In-Sample data did not lead to the minimum variance portfolio Out-of-Sample data. Likewise, we showed that there existed a portfolio having higher skewness and lower kurtosis out-sample than the one using the allocations calculated with In-sample data.

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