University of Lausanne
HEC Faculty

Quantitative Asset & Risk Management

Assignement 3

Group 19

Guillaume Pavé
guillaume.pave@unil.ch

Sebastien Gorgoni
sebastien.gorgoni@unil.ch

Nicolas Eon Duval
nicolas.eonduval@unil.ch

Liam Svoboda
liam.svoboda@unil.ch

Benoît Turbé
benoit.turbe@unil.ch

UNIL | Université de Lausanne
HEC Lausanne

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1 Introduction

This paper is an extension to the previous assignment on portfolio allocation, focusing this time on core concepts of risk management. Managing risk is a fast-growing sector in which financial institutions are increasingly interested in understanding concepts and dynamics behind an investor's biggest fear: losses. This report will describe various models and methods of risk assessment, which will then be applied to our three portfolios presented on the previous report, the Strategic Asset Allocation (SAA), the Tactical Asset Allocation (TAA) and the Real (Replication) Portfolio. Important concepts describing risks such as the Value at Risk (VaR) and the Expected Shortfall (ES) will be described and applied to practical portfolios. Finally, further analyses on the sources and consequences of risk will be made to give a global picture of risk in the past decade over a diversified range of seven assets. Code, data set and output are provided in the file and available on our Github **here**.

2 Ex-Ante Marginal Contribution to Risk

Having observed the evolution of our 7 indices in the out of sample period, we want to understand how the riskiness evolved across the period continuing off assignment 2, where we mainly focused on returns.

One way of doing this is by looking at the Marginal Contribution to Risk (MCR) progression. We will do this first only on the Strategic Asset Allocation (SAA) portfolio, which holds a static allocation across the period, and then on the Tactical Asset Allocation (TAA) portfolio, which is more nimble in that it can adapt its weights according to market situations. Finally, we look at the replication of the culmination of the SAA and the TAA, with the constraint of no investment grade, to see the total effect.

To compare the client's portfolio with ours adequately, we first define active weights. These are computed in equation (1) and allow us to truly see the additional effect of our portfolio management expertise:

$$\alpha_{\text{Active of Observed Portfolio}} = \alpha_{\text{Observed Portfolio}} - \alpha_{\text{Client Portfolio}} \tag{1}$$

Henceforth, for simplicity of notation, just consider α be the active weights.

As a reminder, the MCR is calculated as follows:

$$MCR = \frac{\partial \sigma_p}{\partial \alpha} = \frac{\Sigma \alpha}{\sigma_p} \tag{2}$$

Meaning that for each asset i, it can be computed in the following manner:

$$MCR_i = \frac{\partial \sigma_p}{\partial \alpha_i} = \frac{\sigma_{ip}}{\sigma_p} \tag{3}$$

Thereafter, we can compute the Risk Contribution (RC) of each asset to determine which asset brings the most risk to the portfolio. This is calculated using equation (4)

$$RC = \alpha' * MCR \tag{4}$$

Hence,

$$RC_i = \alpha_i * MCR_i \tag{5}$$

Results for our three portfolios are described in the following sections.

2.1 SAA Portfolio

Figure 1 – Ex-Ante MCR of SAA Portfolio

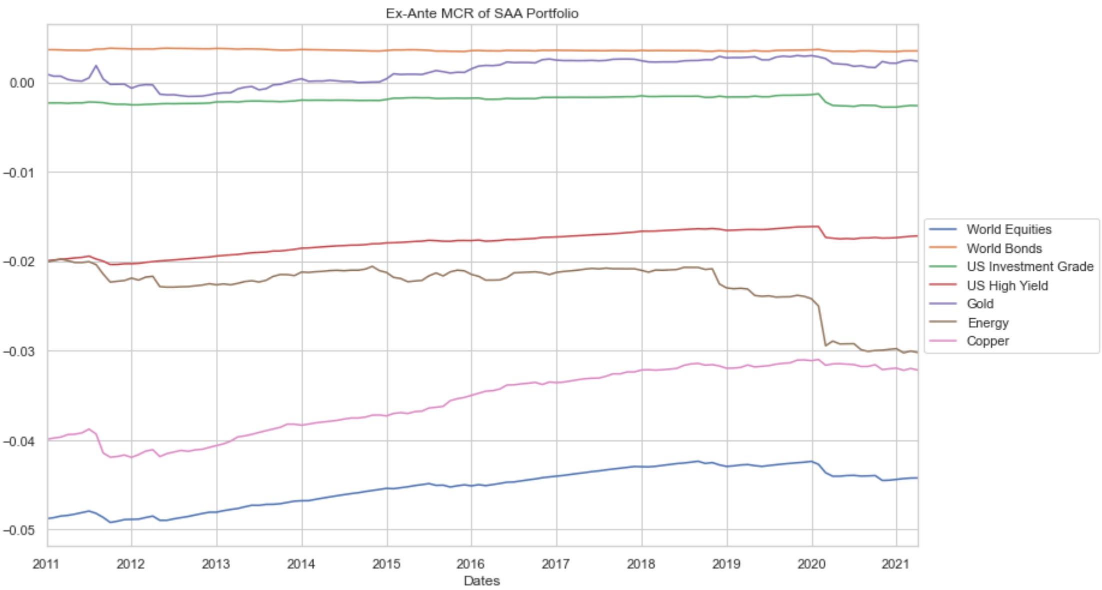


Table 1 – SAA Out-Sample MCR Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	-0.045314	0.003574	-0.002003	-0.017954	0.001200	-0.022658	-0.035602
std	0.002153	0.000097	0.000376	0.001239	0.001377	0.002734	0.003636
min	-0.049225	0.003421	-0.002798	-0.020390	-0.001596	-0.030263	-0.041947
max	-0.042379	0.003798	-0.001302	-0.016141	0.002980	-0.019772	-0.031003

For the more mathematically inclined, summary statics of the MCR are provided in table 1.

Given the static weights displayed in table 2, it is unsurprising that there are not many wild fluctuations between 2011 and 2021. Indeed, in ordinal terms, the assets stay the same as none of the lines cross.

We can, however, notice variations in terms of some of the values of the MCR_i. Most notably, we can distinguish two large shocks. The first one appeared in mid-2011, which seems to be linked to the Black Monday. The second one in early 2020, which we can, without doubt, be attributed to the COVID crisis. Looking at equation (3), one can interpret a decrease in MCR_i either as a decrease in σ_{ip} or as an increase in σ_p. In the two aforementioned shocks, the impact permeates all assets, albeit to a varying degree, meaning that we can probably attribute the variation to a change in σ_p.

Looking again at table 2, we see that the SAA is heavily weighted on World Bonds; we can hypothesize that increasing holdings in other assets would lower risk, thanks to the additional diversification they bring, dampening volatility. Hence, we observe mostly negative MCR_i for assets other than bonds. Gold, on the other hand, has a mostly positive MCR_i, probably due to its positive correlation with World Bonds, which in turn means a positive σ_{ip}, given the amount invested in World Bonds.

Table 2 – SAA Out-Sample Active Weights

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	-5.0e-01	0.3359	1.0e-02	9.161e-02	4.685-02	0.0	1.56e-02

Observing the RC from table 3, we notice that World Equities is the biggest source of risk, followed by Gold, the only two with a positive average RC.

Table 3 – SAA Out-Sample RC Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	0.022657	0.001200	-0.000020	-0.001645	0.000056	0.0	-0.000556
std	0.001077	0.000033	0.000004	0.000114	0.000065	0.0	0.000057
min	0.021190	0.001149	-0.000028	-0.001868	-0.000075	-0.0	-0.000655
max	0.024613	0.001276	-0.000013	-0.001479	0.000140	-0.0	-0.000484

2.2 TAA Portfolio

Figure 2 – Ex-Ante MCR of TAA Portfolio

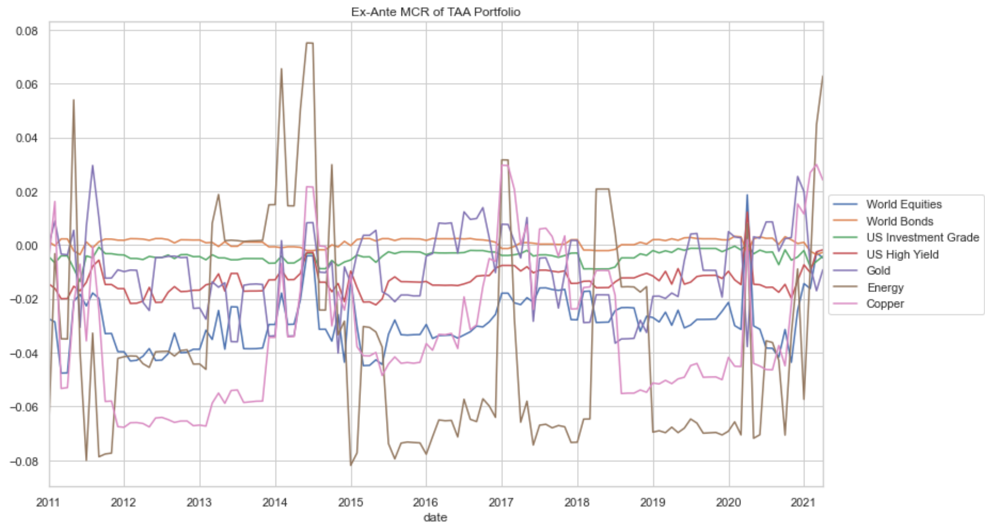


Table 4 – TAA Out-Sample MCR Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	-0.028845	0.000881	-0.004206	-0.013400	-0.009347	-0.036149	-0.032419
std	0.010325	0.001647	0.002203	0.004703	0.014733	0.039169	0.026715
min	-0.047619	-0.003561	-0.013618	-0.022253	-0.040206	-0.081911	-0.067752
max	0.018647	0.003230	0.000508	0.012229	0.029550	0.075084	0.029913

Considering the TAA portfolio, it becomes less clear than before since the weights change quite substantially throughout the period. Although we can see that World Bonds, US Investment Grade and US High Yield remain relatively flat, most assets are a bit all over the place. One asset that stands out as having a highly volatile MCR_i is the Energy index. Indeed, it reaches both the highest MCR_i of the entire period and then the lowest in less than a year. Although the price evolution of this index appears to be rather unexciting in the out-sample, safe for a

Table 6 – TAA Out-Sample RC Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	0.012576	-0.000349	-0.000073	-0.000320	0.003178	0.013570	0.009553
std	0.007724	0.000979	0.000749	0.002287	0.005185	0.012639	0.010014
min	0.001121	-0.001757	-0.001725	-0.004750	-0.001132	-0.001604	-0.004394
max	0.037661	0.004636	0.003180	0.008153	0.037767	0.039356	0.024654

minor blip around 2014-15, it appears that its in-sample performance made our strategy eager to short-sell it. A normal behaviour given that we use the in-sample covariance matrix. Indeed, observing table 5, we see that Energy was, on average, the most shorted asset. We also witness a similar behaviour with Copper. It is the second most shorted asset and also displays the highest standard deviation in weights meaning that our strategy saw fit to be heavily active on this asset.

Table 5 – TAA Out-Sample Active Weights Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	-0.414556	-0.505474	0.079582	0.040760	0.014987	-0.133709	-0.081590
std	0.199076	0.195614	0.207025	0.176489	0.267507	0.242623	0.288548
min	-0.863884	-1.500000	-0.363884	-0.363884	-1.000000	-0.500000	-0.500000
max	0.166667	-0.090744	0.666667	0.666667	0.500000	0.409256	0.500000

Observing the RC from table 6, we notice that Energy is the biggest source of risk, as we might have suspected from its erratic behaviour in figure 2.

2.3 Real Portfolio

Figure 3 – Ex-Ante MCR of Real Portfolio

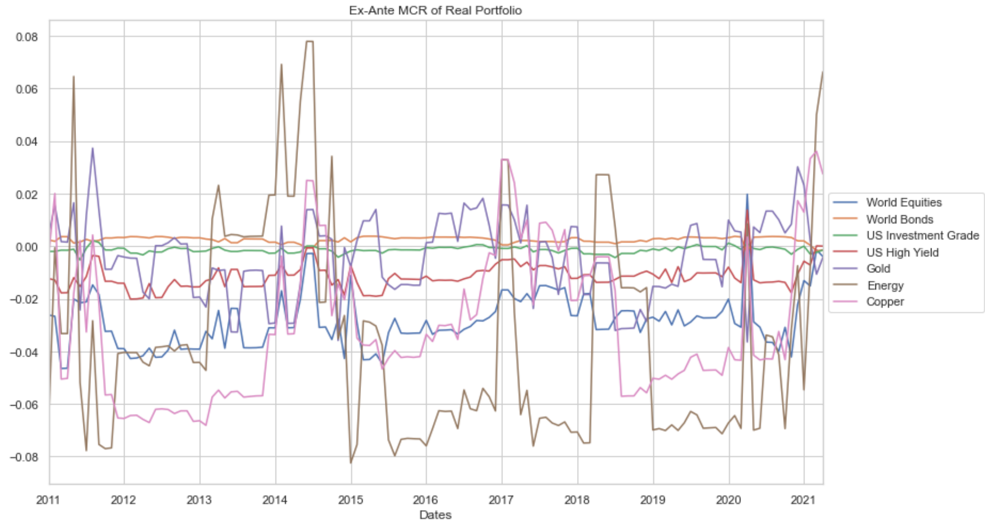


Table 7 – Real Portfolio Out-Sample MCR Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	-0.028408	0.002466	-0.001344	-0.011523	-0.003855	-0.034682	-0.029991
std	0.010387	0.001193	0.001167	0.004731	0.015157	0.040388	0.027783
min	-0.046526	-0.002614	-0.005281	-0.020113	-0.036453	-0.082502	-0.068166
max	0.019797	0.003811	0.002217	0.013793	0.037338	0.077955	0.036137

Looking at the portfolio as a whole, it is difficult to see much of a difference from the TAA at all. The reason being that the MCR_i in the SAA portfolio remain relatively stable due to the lack of change in the distribution of capital across assets, and the activity can be attributed to the more aggressive TAA portfolio.

By looking at the summary statistics of the active weights of the portfolio in table 8 and comparing it with table 5 and table 2, we can see that it is not simply an addition of the weights, due to the client's additional requirements.

Table 8 – Real Portfolio Out-Sample Active Weights Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	-0.417122	0.388267	5.905356e-20	0.162938	0.066820	-0.133180	-0.067723
std	0.199807	0.229755	9.413236e-19	0.198529	0.266989	0.242019	0.289444
min	-0.869692	-0.257329	-2.919137e-18	-0.315216	-0.915226	-0.499921	-0.493603
max	0.147089	1.015897	2.647148e-18	0.989387	0.575365	0.408654	0.515400

Observing the RC from table 9, we notice once again that Energy is the biggest source of risk as we might again have guessed from figure 3.

Table 9 – Real Portfolio Out-Sample RC Summary Statistics

	WE	WB	US IG	US HY	Gold	Energy	Copper
mean	0.012369	0.001081	-5.530542e-23	-0.001668	0.003000	0.013493	0.009229
std	0.007524	0.000766	1.744913e-21	0.002434	0.004609	0.012383	0.009450
min	0.000577	-0.000470	-8.154460e-21	-0.005830	-0.001267	-0.001614	-0.004309
max	0.036534	0.003132	3.683668e-21	0.013646	0.033363	0.037709	0.023595

3 Value-at-Risk & Expected Shortfall

3.1 SAA Portfolio

3.1.1 Variance Covariance Method

We will now go through the Value-at-Risk (VaR) and Expected-Shortfall (ES) of our SAA Portfolio, using the Variance Covariance Method. Before diving into our analysis, we will first define the Variance Covariance Method. The first step was to collect all the out-sample active weights determined on the previous reports. Then, we computed the total monthly returns from December 2010 (i.e. the first date of our in-sample) to the new observations of our out-sample on a holding based scenario, meaning that all historical portfolio returns have been calculated using the current portfolio's weights. As we are in an expanding setting, we did this procedure from the first date of our out-sample until its last date. As we would like to determine the VaR and ES of our portfolio, we defined the loss of the portfolio over the period

$[t, t + \Delta]$, as being the "negative returns":

$$L_{[t,t+\Delta]} = -r_{[t,t+\Delta]} \tag{6}$$

Thereafter, we computed the mean m and the standard deviation s of the historical holding-portfolio losses in each out-sample period. We assumed that the distribution of the losses is Gaussian. Thus, for a $N(m, s^2)$, distribution, we computed the VaR and ES at a θ level as follows:

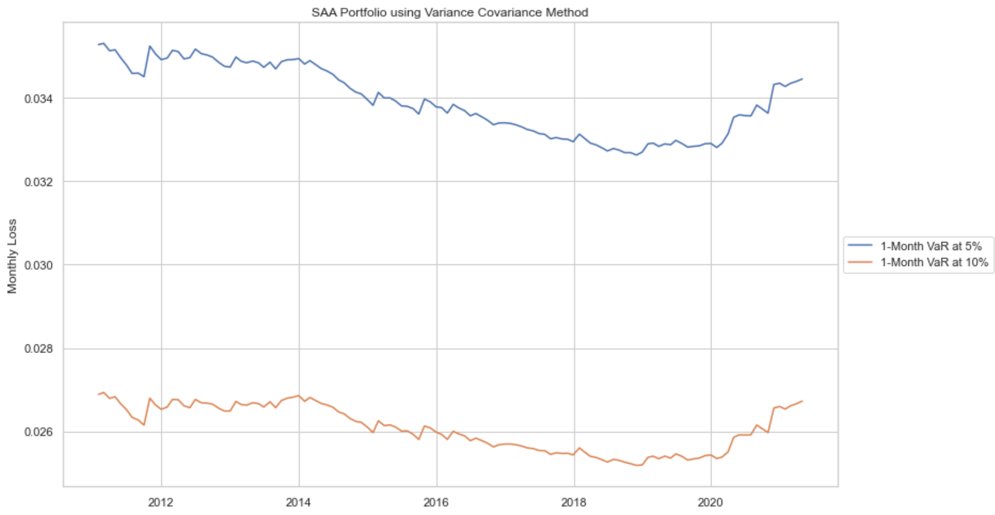
$$VaR_\theta = m + s * \Phi^{-1}(\theta) \tag{7}$$

$$ES_\theta = m + s * \frac{\varphi(\Phi^{-1}(\theta))}{1 - \theta} \tag{8}$$

Where φ is the PDF of the standard normal distribution and Φ^{-1} its inverse CDF. The ES is an alternative to the VaR, where it is more sensitive to the shape of the tail of our loss distribution[1].

By following the method above, we arrived at the following VaR curves for our two quantiles of 10% and 5%, over the out-of-sample period, using the active weights of our SAA portfolio:

Figure 4 – VaR of SAA Portfolio (Variance-Covariance Method)

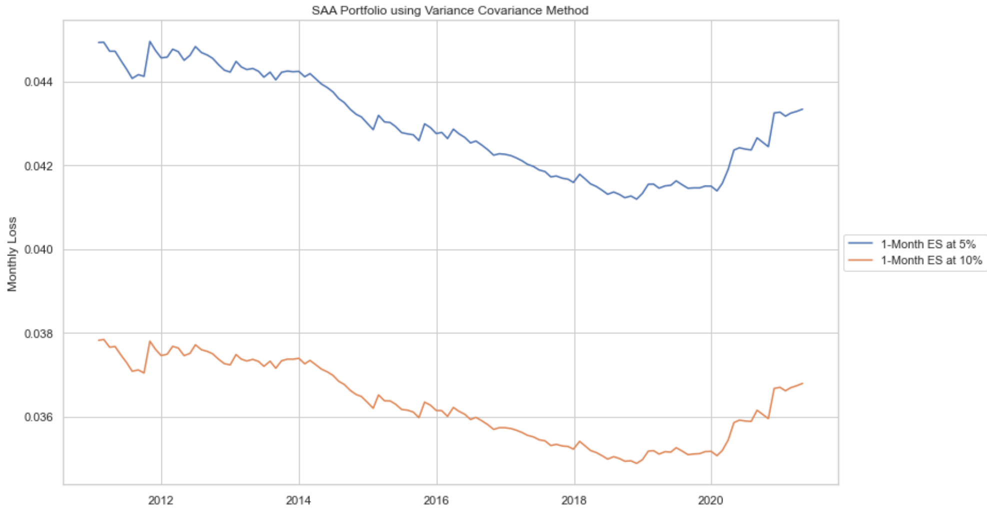


In figure 4, we can see a significant difference in value between both quantiles but similar in overall shape. Indeed, the SAA portfolio has endured a relatively large loss by the end of 2012, due to the European Debt Crisis, which was amplified by our large exposure in world bonds. However, we can see a downtrend in our monthly VaR for both quantiles. This downtrend was caused by strength in the performance of our SAA Portfolio as the world economy recovered from the crisis, showing positive momentum, which reduced the potential losses. Nevertheless,

[1] https://en.wikipedia.org/wiki/Expected_shortfall

these low-risk measures have been negatively affected by a severe shock in early 2020, as the World economy slows down due to COVID lockdowns. Similarly to VaR, figure 5 illustrates the ES of our SAA portfolio. Again, one can notice the steadiness of the portfolio post-GFC and the insufficient reaction of it to the COVID outbreak in the short term, leading to an increase in risk measures.

Figure 5 – ES of SAA Portfolio (Variance-Covariance Method)



Although the Variance-Covariance method is a fairly easy model to implement, the assumption of normality may seriously underestimate the tails of the distribution of returns, as returns/losses may not be Gaussian.

Table 10 – Monthly VaR/ES of SAA Portfolio using Variance-Covariance Method

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.033927	0.026068	0.042966	0.036309
std	0.000829	0.000540	0.001176	0.000920
min	0.032626	0.025184	0.041186	0.034882
25%	0.033126	0.025528	0.041829	0.035417
50%	0.033821	0.026036	0.042812	0.036183
75%	0.034782	0.026611	0.044190	0.037259
max	0.035303	0.026933	0.044951	0.037840

3.1.2 Historical Simulation

We then compute the VaR and ES based on historical simulation. To do so, we computed historical loss and then ranked them in descending order (i.e. from the greatest loss to the smallest). Finally, quantile at 90% and 95 % of VaR and ES have been computed as follows (ie. $\theta = 10\%$ and $\theta = 5\%$) :

$$VaR_\theta(L) = \tilde{L}_{[N(1-\theta)],N} \tag{9}$$

$$ES_\theta(L) = \frac{1}{[N(1-\theta)]} \cdot \sum_{i=1}^{[N(1-\theta)]} \tilde{L}_{i,N} \tag{10}$$

Therefore, VaR is the sample of loss above $N(1-\theta)$. Additionally, observing equation (10) one can notice that ES is the average amount of loss for the given percentage of cases (i.e. 5 and 10%).

Observing figures 6 and 7, one can notice that both measures behave similarly. Our SAA portfolio began in 2012 with values relatively high. These high values can be explained by high volatility in stock markets during the European debt crisis. Subsequently, both measures decreased steadily until 2020, illustrating a fairly safe strategy. However, a leap induced by the rise of the COVID Crisis appears in early 2020 in both metrics.

Figure 6 – VaR of SAA Portfolio (Historical Simulation)

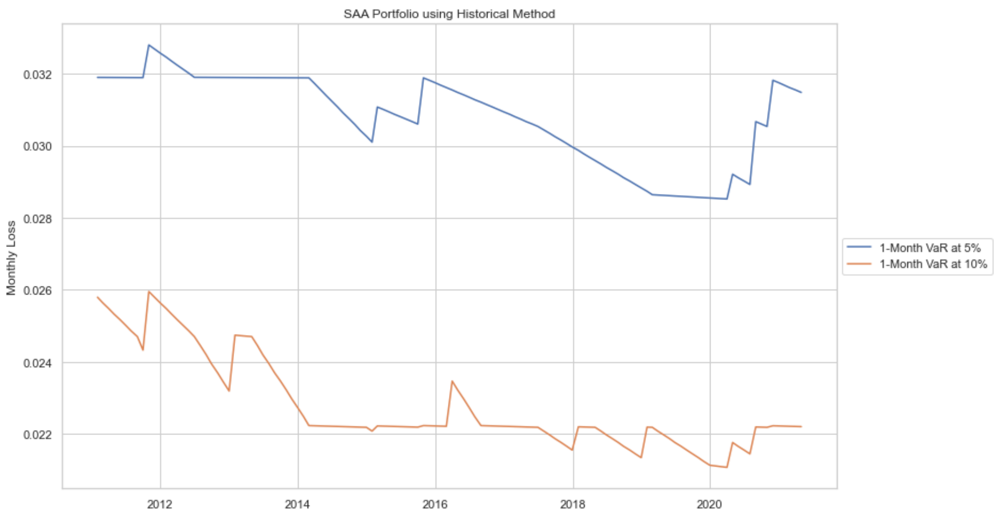
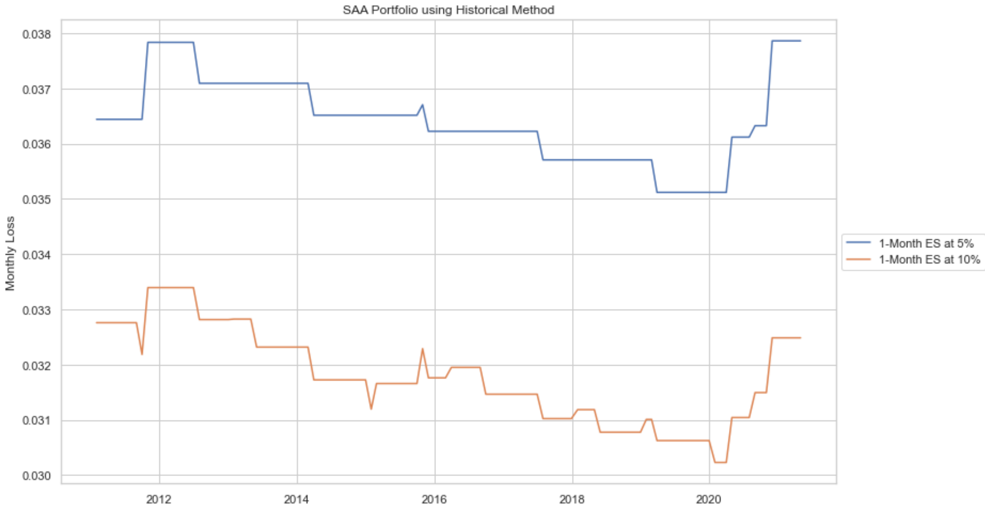


Figure 7 – ES of SAA Portfolio (Historical Simulation)



The historical simulation may be a more suitable model than the covariance model, as it allows for non-normal returns since it is based on actual returns. Nevertheless, it is based on a strong underlying assumption that returns are *iid*, an assumption empirically disapproved.

Table 11 – Monthly VaR/ES of SAA Portfolio using Historical Simulation

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.030795	0.022784	0.036424	0.031799
std	0.001229	0.001289	0.000771	0.000841
min	0.028524	0.021077	0.035118	0.030229
25%	0.029944	0.022082	0.035706	0.031042
50%	0.031075	0.022216	0.036440	0.031724
75%	0.031890	0.023471	0.037093	0.032485
max	0.032799	0.025955	0.037861	0.033395

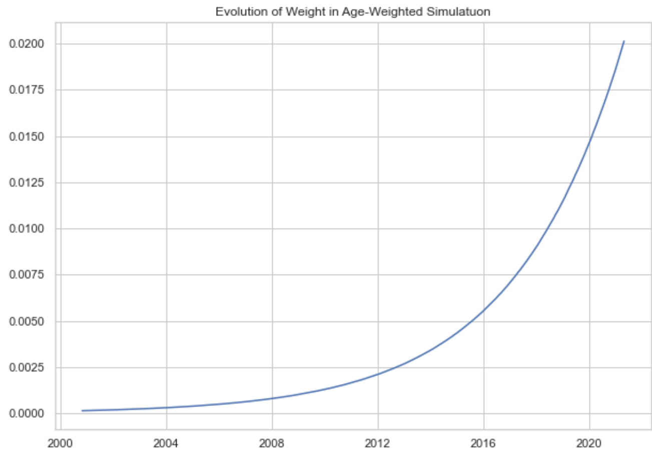
3.1.3 Age Weighted Simulation

We will now go through the Value-at-Risk (VaR) and Expected-Shortfall (ES) of our SAA Portfolio using the age-weighted simulation. Before diving into our analysis, let's first define the age-weighted simulation. One of the main issues with Historical Simulation is that past observations have the same weight as new ones (say, 1/N). This implies that risk estimates are unresponsive to major events (for instance, a market crash). A solution to reduce the effect of past observations is to use weights, increasing with the age of the observation:

$$w_i = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^N} \tag{11}$$

Where $\lambda \in [0, 1]$ is the memory parameter. In our analysis, we set a relatively low $\lambda = 0.98$, with which we can obtain, for example at a given date, the following evolution of weights:

Figure 8 – Evolution of Weight in Age-Weighted Simulation (Example)



The reason of selecting a $\lambda = 0.98$ is because we wanted a more concave weight evolution, thus allocating more weight to present data.

Once again, we computed the total monthly returns from December 2010 (i.e. the first date of our in-sample) until each new observations of our out-sample, on a holding based scenario, meaning that all historical portfolio returns have been calculated using the current portfolio's weights. As we are in an expanding setting, we did this procedure from the first date of our out-sample until March 2021. As we are looking to determine the VaR and ES of our portfolio, we defined the loss of the portfolio over the period $[t, t + \Delta]$, as being the "negative returns":

$$L_{[t,t+\Delta]} = -r_{[t,t+\Delta]} \tag{12}$$

Afterward, for each subsample of N past realization $L_{t-N+1}, ..., L_t$, we attribute weights w_i based on the age of the observations. Then we sorted the data by decreasing order $\tilde{L}_{1,N}, ..., \tilde{L}_{N,N}$, where $\tilde{L}_{1,N} \geq ... \geq \tilde{L}_{N,N}$, and we attributed the corresponding weights to the ordered sample as $\tilde{w}_1, ..., \tilde{w}_N$.Finally, we accumulated the weights $\tilde{w}_i$ until we reach a level of $1-\theta = 0.05$ (resp. $1 - \theta = 0.10$). If $\tilde{w}_\tau$ is such that $\sum_{i=1}^{\tau} \tilde{w}_i = 1 - \theta$, then:

- The VaR is the τ-th order statistic:

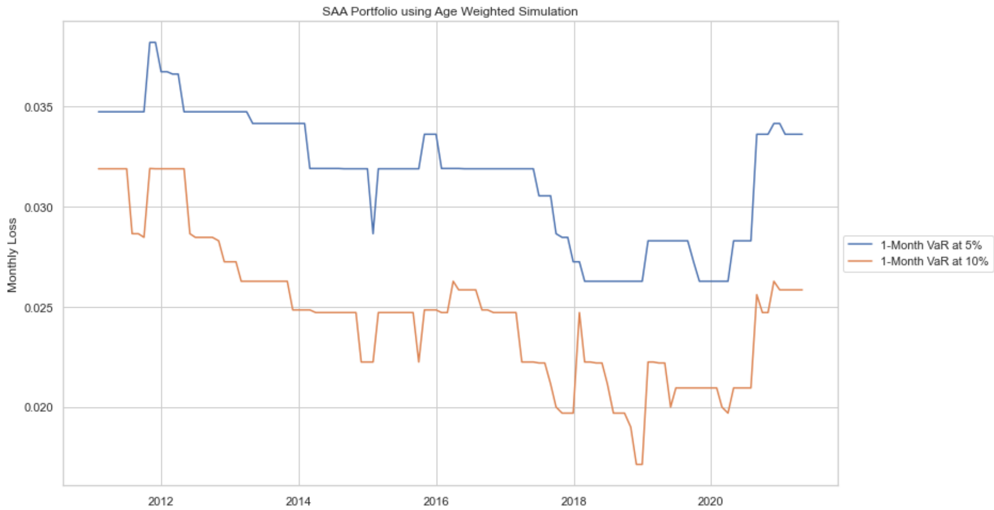
$$VaR_\theta(L) = \tilde{L}_{\tau,N} \tag{13}$$

- The ES is defined as the average of the realizations that are below this level:

$$ES_\theta(L) = \frac{1}{\tau}\sum_{i=1}^{\tau} \tilde{L}_{i,N} \tag{14}$$

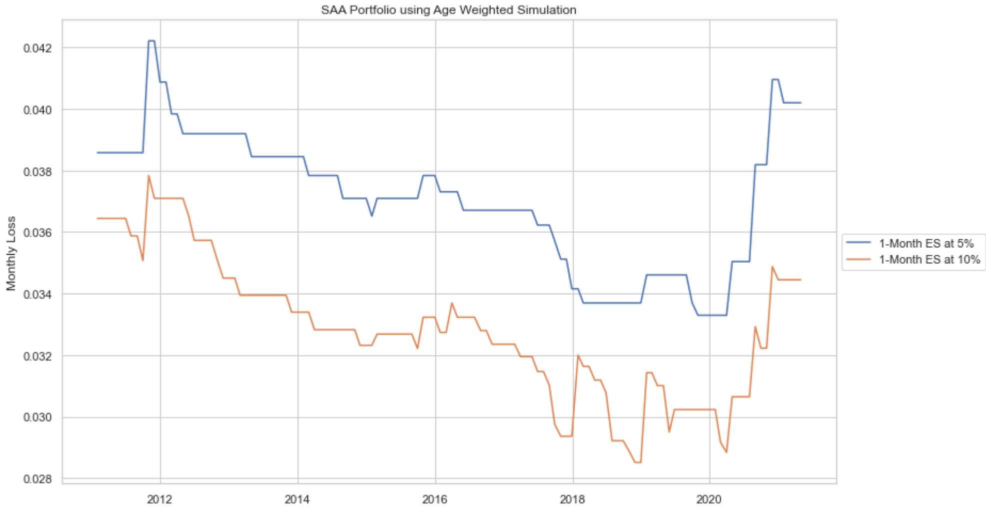
Therefore, we obtained the following evolution of the VaR:

Figure 9 – VaR of SAA Portfolio (Age Weighted Simulation)



At the beginning of the out-sample time horizon, we notice that there is a 5% probability of losing approximately 3.5% of the portfolio each month, which is still a low number compared to the portfolios presented in the next sections. The VaR of our SAA portfolio is at its peak during 2012, which can be explained by the Eurozone debt crisis, given the fact that World Bonds mainly determine this portfolio. Nevertheless, VaR decreased smoothly, reaching at its lowest a 5% probability of losing approximately 2.7%, although a small peak by the end of 2015 due to a rise in interest rates by the Fed. However, due to the COVID pandemic of 2020, all our asset classes were negatively impacted, which increased the VaR substantially at a quick pace. It ultimately stopped rising in mid-2020, thanks to both the implementation of government fiscal stimulus and the reopening of businesses, although it did not push the VaR down. By construction, we observe that the VaR at 90% level has a similar shape to the VaR at 95% level, but with lower annualized losses.

Figure 10 – ES of SAA Portfolio (Age Weighted Simulation)



Regarding the expected shortfall, since it is constructed similarly to the VaR, it is no surprise that they both have the same shape. However, since the ES is more sensitive to the shape of the tail of the loss distribution than the VaR, the expected monthly at a 5% probability is naturally higher.

As the age-weighted historical simulation is an improvement of the simple historical simulation, we notice that our risk estimates are clearly more responsive to major events as we allocate lower weights to past events.

Table 12 – Monthly VaR/ES of SAA Portfolio using Age-Weighted Simulation

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.031612	0.024704	0.037054	0.032850
std	0.003162	0.003551	0.002185	0.002237
min	0.026268	0.017123	0.033293	0.028511
25%	0.028288	0.022188	0.035037	0.031366
50%	0.031886	0.024706	0.037093	0.032795
75%	0.034147	0.026268	0.038577	0.034069
max	0.038196	0.031900	0.042211	0.037834

3.1.4 Summary

Overall, the long-term strategy and the main long position in World Bonds reflect the low risk of our SAA portfolio, represented by a low VaR and ES over time. Nevertheless, this portfolio is not spared either during periods of economic uncertainties, as seen during COVID, for example. Among all models to measure risk, the variance-covariance is the least effective since it relies on the assumption that returns are Gaussian. Hence not capturing the risk dynamic efficiently during periods of crisis. Relative to the latter model, the historical simulation could be a good alternative since it allows for non-normal returns as it is based on actual returns and can grasp more efficiently an accurate risk dynamic during more volatile periods. However, the latter model relies on a strong underlying assumption that the return process is iid, and thus attributing the same significance for all past observation. In this regards, the age-weighted historical simulation could be a better alternative since it reduces the effect of past observations, hence being more responsive to major events.

3.2 TAA Portfolio

3.2.1 Variance Covariance Method

Figures 11 and 12 illustrate the behaviour of both VaR and ES computed with the Variance Covariance method as described in section 3.1.1, applied to the TAA portfolio. As expected, it can be noted that TAA risk metrics are much more volatile than the ones observed for the SAA portfolio. Indeed, the TAA is driven by a short-term view, leading to a stronger response to market shift (i.e. unstable economies). In particular, we notice that the risk of this portfolio is at its peak during periods of high volatility (e.g. 2020) since it undertakes more aggressive positions.

Figure 11 – VaR of TAA Portfolio (Variance-Covariance Method)

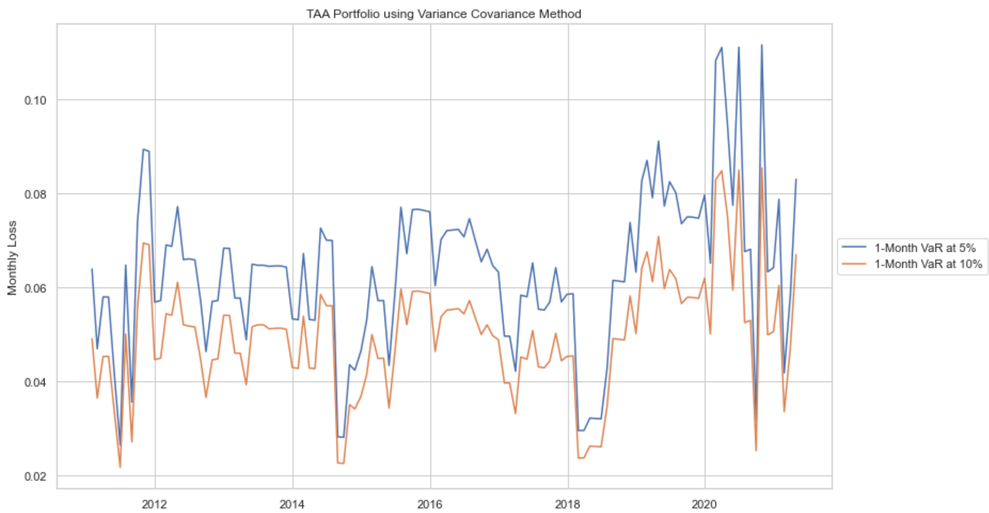


Figure 12 – ES of TAA Portfolio (Variance-Covariance Method)

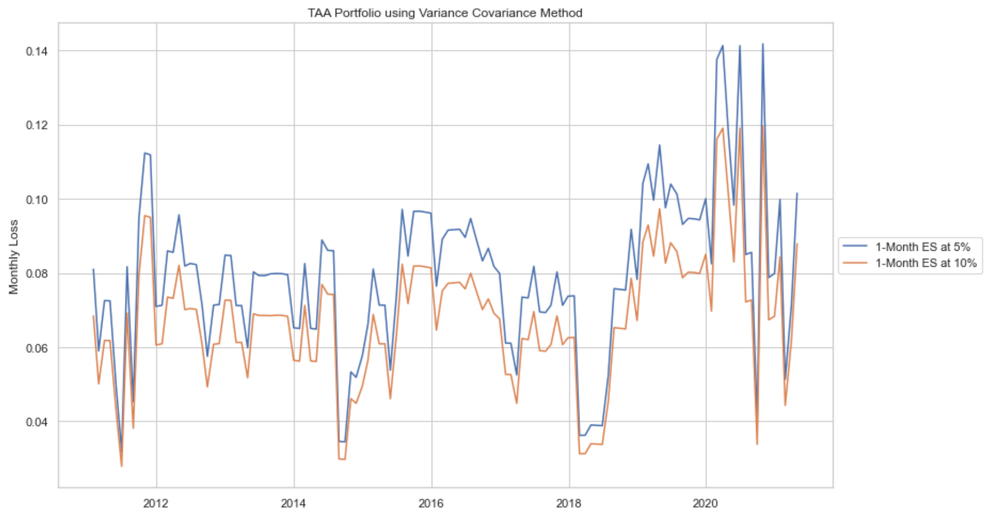


Table 13 – Monthly VaR/ES of TAA Portfolio using Variance-Covariance Method

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.063507	0.049690	0.079398	0.067695
std	0.016746	0.012642	0.021511	0.017999
min	0.026419	0.021666	0.031887	0.027860
25%	0.056860	0.044343	0.071144	0.060643
50%	0.064503	0.050191	0.079856	0.068543
75%	0.072846	0.056201	0.091759	0.077737
max	0.111614	0.085429	0.141731	0.119551

3.2.2 Historical Simulation

Similarly to section 3.1.2, equations (9) and (10) have been used to compute VaR and ES based on historical simulation. Quantile at 95% and 90% have been computed on our TAA positions. Figures 13 and 14 illustrate the results. Here, one can notice that both metrics behave on a larger scale compared to the values from the SAA portfolio. This illustrates the use of a sharper position in our short term view. In particular, the VaR and ES arise during periods of crisis since our TAA portfolio is in more aggressive positions during these periods.

Figure 13 – VaR of TAA Portfolio (Historical Simulation)

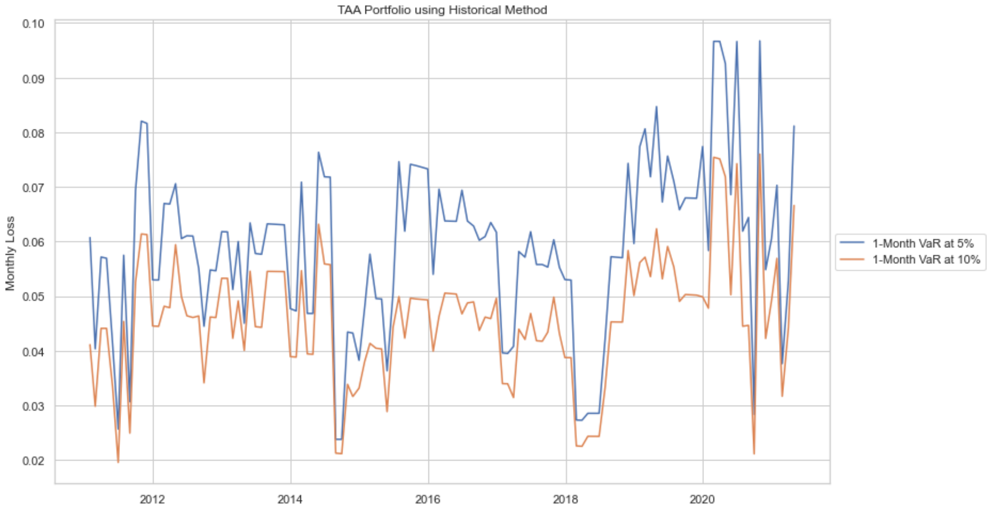


Figure 14 – ES of TAA Portfolio (Historical Simulation)

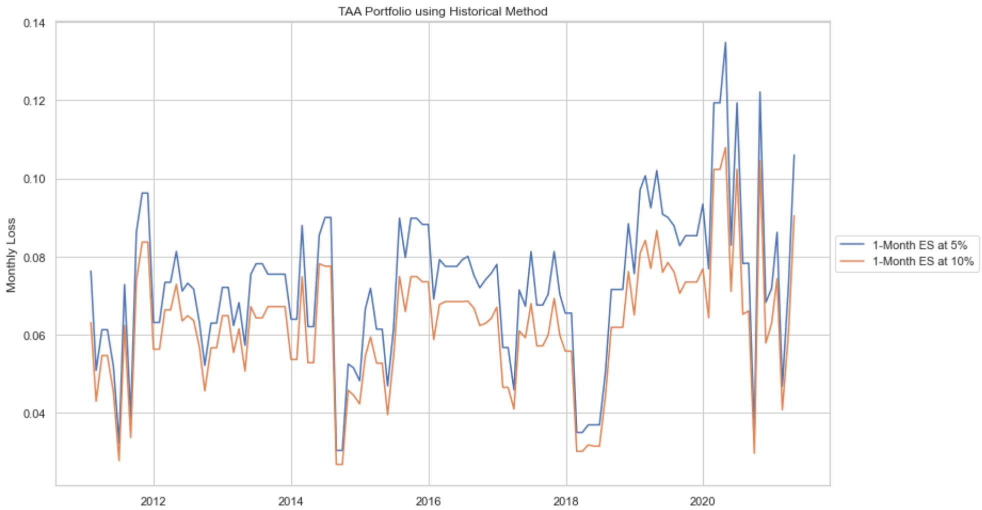


Table 14 – Monthly VaR/ES of TAA Portfolio using Historical Simulation

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.059003	0.045804	0.072725	0.062544
std	0.015642	0.011441	0.019308	0.016168
min	0.023806	0.019573	0.030355	0.026774
25%	0.051110	0.040281	0.062795	0.054629
50%	0.060406	0.046139	0.073042	0.063892
75%	0.068136	0.050502	0.083522	0.073045
max	0.096732	0.076009	0.134829	0.107939

3.2.3 Age Weighted Simulation

After applying the method described in the previous sections, we obtained the following evolution of the VaR and ES at a 90% and 95% level:

Figure 15 – VaR of TAA Portfolio (Age Weighted Simulation)

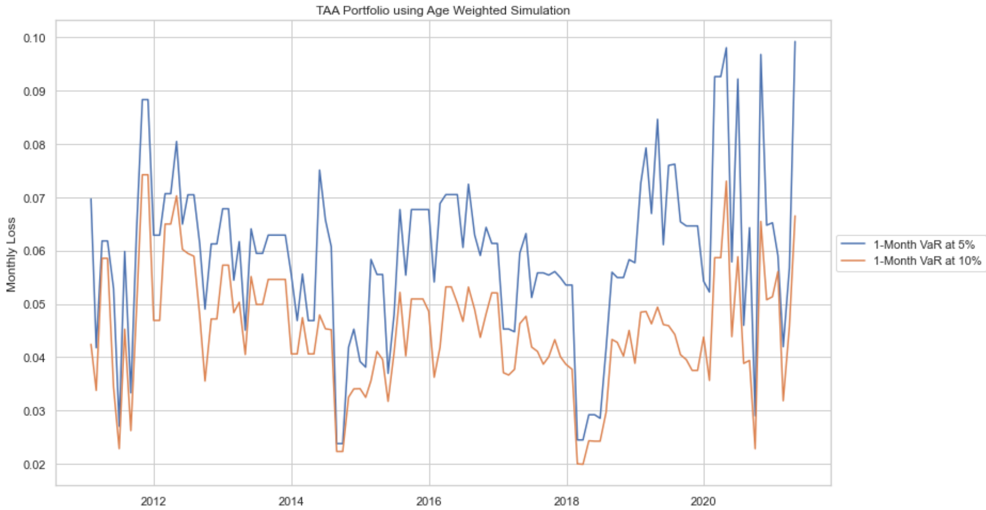
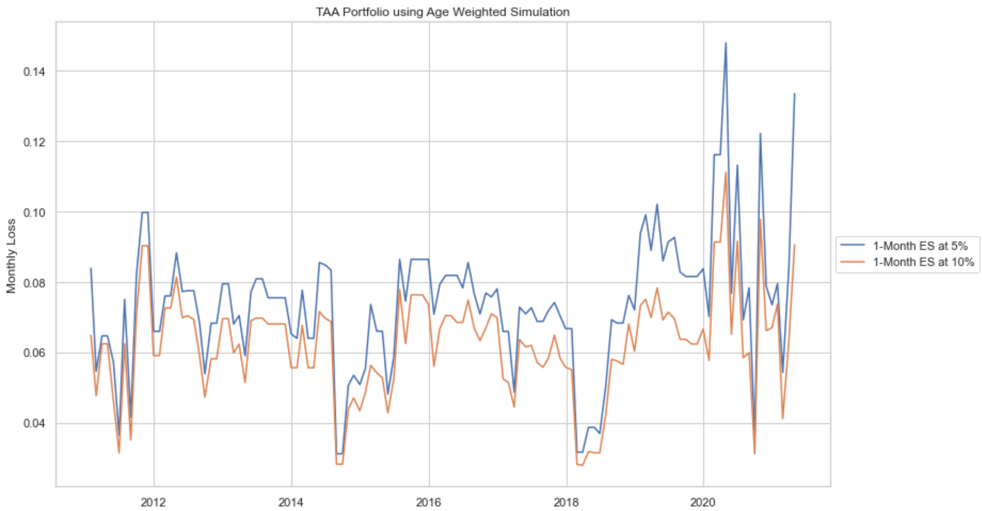


Figure 16 – ES of TAA Portfolio (Age Weighted Simulation)



Compared to the SAA Portfolio, we notice that variation of the TAA component's VaR is more substantial due to the short-term bets undertaken when using the value and momentum factor. Furthermore, we notice that the VaR and ES are higher in periods of crisis, reaching, for example, at their highest a monthly loss of approximately 12% (resp. 14%) with a 5% probability during the COVID pandemic. Clearly, we notice that taking long/short positions over a short amount of time does increase the risk of our portfolio. Additionally, taking dramatic positions during periods of high volatility (i.e. high level of VIX index) does aggravate the volatility of our position(as demonstrated in the previous report). Nevertheless, we were able to mitigate some of the risks with the use of an ex-ante risk budget of only 2% on the value and momentum factors distinctly. Hence, the riskiness of the TAA portfolio is fairly high, particularly during periods of crisis (e.g. European Debt crisis, COVID Pandemic), which might be a concern for risk-averse investors.

Table 15 – Monthly VaR/ES of TAA Portfolio using Age-Weighted Simulation

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.059254	0.045086	0.073502	0.062107
std	0.015486	0.011245	0.019175	0.014881
min	0.023824	0.019948	0.031172	0.027881
25%	0.053375	0.038815	0.065921	0.055583
50%	0.060903	0.045296	0.075229	0.063412
75%	0.067673	0.051031	0.081810	0.069716
max	0.099160	0.074196	0.147801	0.111134

3.2.4 Summary

Overall, all models have been able to conclude that our TAA portfolio is far riskier than the SAA portfolio. This can be explained by the long/short positions in short term, and the portfolio is more aggressive during periods of high volatility. Once again, the variance-covariance model is the least effective as it assumes that returns are normally distributed and do not effectively respond to major events (e.g. financial crisis). In the case of the TAA portfolio, the historical simulation and the age-weight simulation yield comparable results since they share many assumptions (e.g. both allow for non-normal returns). Nevertheless, we believe that the age-weighted historical simulation is more appropriate since it reduces the effect of past observations and thus being more responsive to major recent events.

3.3 Real Portfolio

3.3.1 Variance Covariance Method

Accordingly to previous sections, the Variance Covariance method has been used to calculate risk measures to the Real Portfolio. These results are depicted in figures 17 and 18. The VaR varies between 2% and 11%, a range close to the one of the TAA portfolio although much higher than the one of the SAA computed with a similar method (between 3.2% and 3.5%).

These results indicate that the inclusion of short-term views strongly contributes to the risk of the real portfolio, compared to the client's benchmark, as we are working with active weights. Nevertheless, as we include the SAA component in the real portfolio, we notice a slight decrease in term of VaR and ES, indicating that long-term views can potentially mitigate some part of the portfolio's risk induced by our short-term bets.

Figure 17 – VaR of Real Portfolio (Variance Covariance Method)

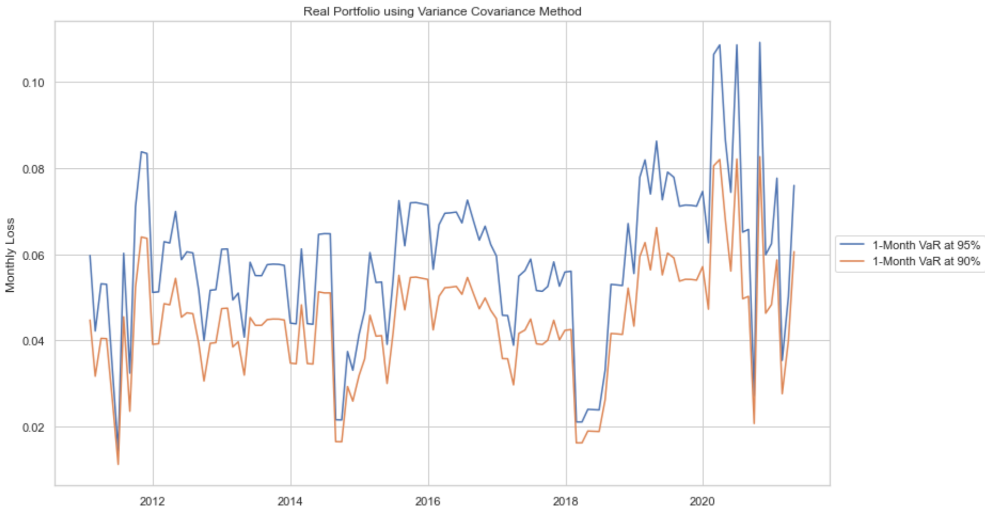


Figure 18 – ES of Real Portfolio (Variance Covariance Method)

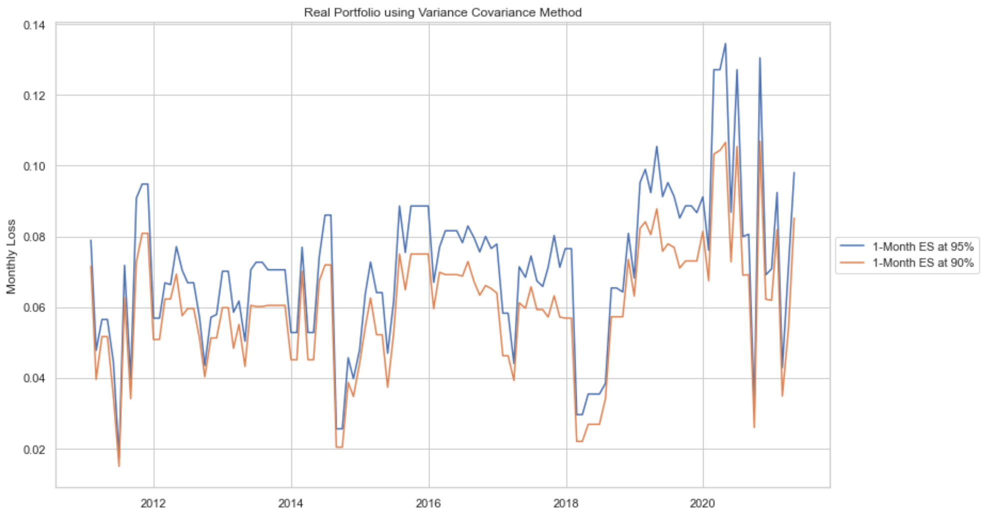


Table 16 – Monthly VaR/ES of Real Portfolio using Variance-Covariance Method

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.058125	0.044538	0.073753	0.062244
std	0.017819	0.013423	0.022913	0.019159
min	0.014152	0.011050	0.017720	0.015093
25%	0.051060	0.039106	0.064710	0.054658
50%	0.058431	0.044929	0.073879	0.062489
75%	0.069619	0.052321	0.089399	0.074746
max	0.109430	0.082845	0.140008	0.117489

3.3.2 Historical Simulation

Finally, consistent with section 3.1.2, both VaR and ES based on Real portfolio historical measures have been computed. Figures 19 and 20 illustrate these results. The risk measures of the Real portfolio tend to oscillate around their mean. However, it can be seen that both metrics undergo a sharp upwards rise in early 2020 due to the pandemic. This jump can be explained by the radical positions of the TAA portfolio in times of high volatility.

Figure 19 – VaR of Real Portfolio (Historical Simulation)

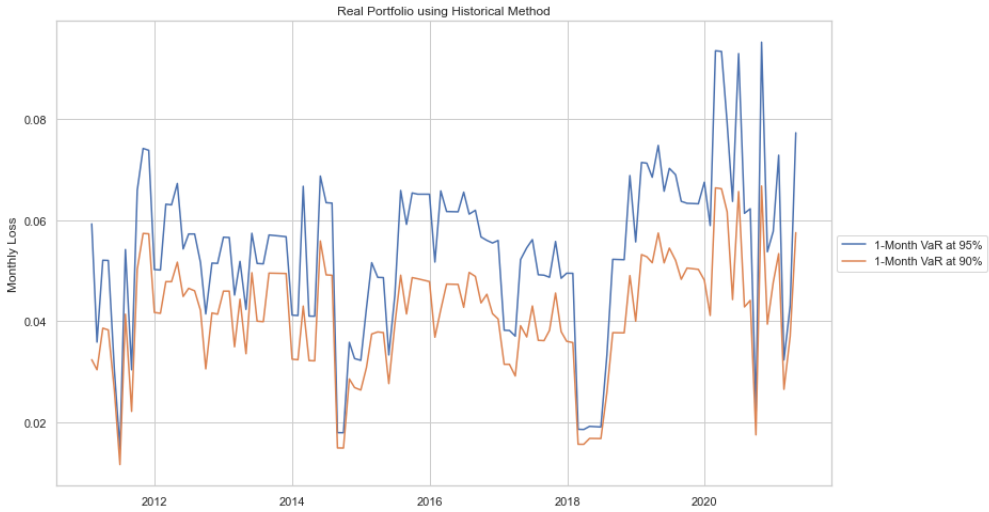


Figure 20 – ES of Real Portfolio (Historical Simulation)

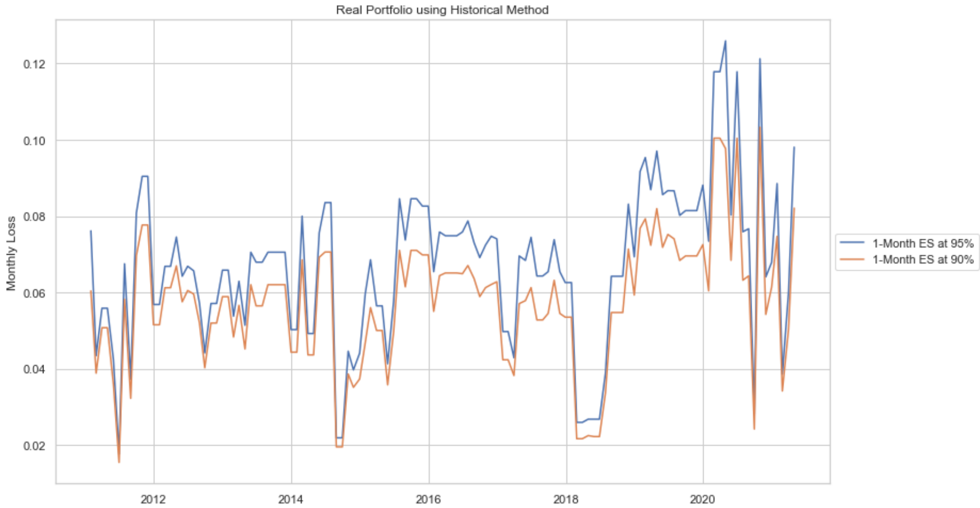


Table 17 – Monthly VaR/ES of Real Portfolio using Historical Simulation

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.054001	0.041078	0.066901	0.057328
std	0.016121	0.011441	0.020496	0.017012
min	0.014723	0.011313	0.017556	0.015543
25%	0.047723	0.035828	0.056265	0.049900
50%	0.055778	0.041868	0.068002	0.058855
75%	0.065550	0.048247	0.079906	0.068490
max	0.093990	0.066279	0.121819	0.102748

3.3.3 Age Weighted Simulation

Finally, we determine the VaR and ES of the real portfolio's active weights using the age-weighted simulation.

Figure 21 – VaR of Real Portfolio (Age Weighted Simulation)

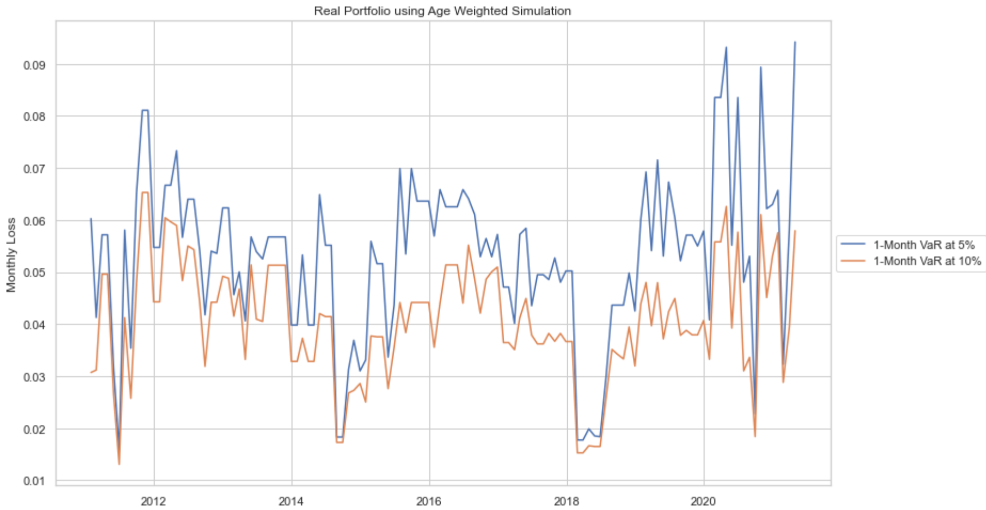
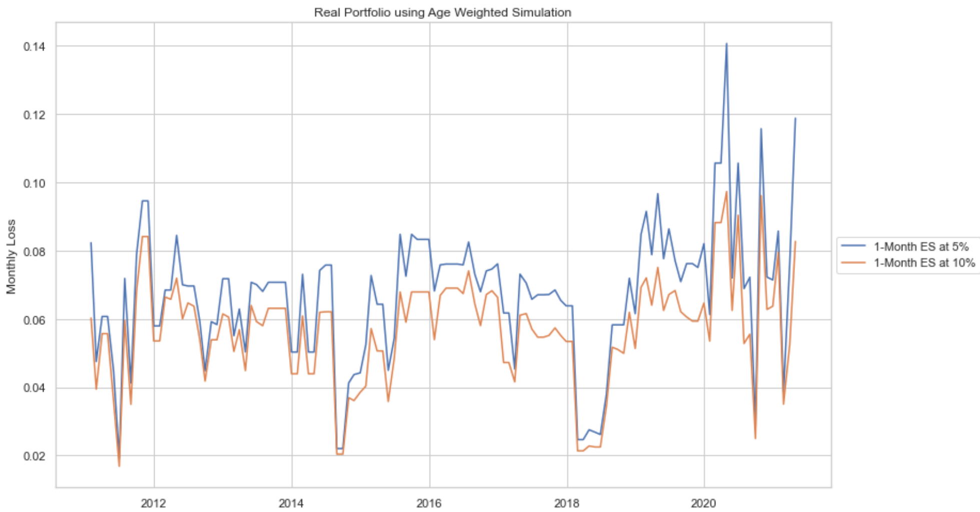


Figure 22 – ES of Real Portfolio (Age Weighted Simulation)



Similarly to the historical simulation, the VaR and ES show significant variability, particularly during periods of crisis: the monthly loss at a 5% probability is substantial during these periods. The risk of this portfolio mainly emerges from the TAA portfolio since we set short-term bets. Once again, when compared to the other methods of measure risk, the age-weighted simulation is more advanced in terms of modelling since it does not assume that returns are normal and allocates more significance into present observations. Consequently, this model is able to capture more accurately the risk of our real portfolio.

Table 18 – Monthly VaR/ES of Real Portfolio using Age-Weighted Simulation

	VaR 95%	VaR 90%	ES 95%	ES 90%
count	124.000000	124.000000	124.000000	124.000000
mean	0.052986	0.040626	0.067033	0.056746
std	0.015747	0.011289	0.019940	0.015759
min	0.015747	0.013068	0.019224	0.016838
25%	0.043658	0.034099	0.058265	0.050585
50%	0.054744	0.040839	0.070031	0.059290
75%	0.062395	0.048647	0.076068	0.065889
max	0.094218	0.065300	0.140662	0.097304

3.3.4 Summary

Overall, all models have been able to conclude that, because of the TAA component (i.e. long/short position in short-term and being more aggressive during periods of high volatility), our Real portfolio can be significantly risky, particularly during periods of crisis. Nevertheless, this portfolio is slightly less risky compared to the TAA portfolio standalone, thanks to the addition of the SAA portfolio. Indeed, the latter's addition mitigates some risks by including long-term views and a major position in World bond, which can be considered a low-risk security. Again, the variance-covariance model is the least effective model as it assumes that returns are normally distributed. However, we notice that both historical simulation and variance-covariance yield "sticky" VaR and ES relative to the age-weighted simulation since they attribute the same significance to all past observation. In this logic, the age-weighted simulation seems to be, once again, the most appropriate model to measure the risk of our portfolio.

4 Conclusion

This report analyses different risk sources of our previously constructed SAA, TAA and Real portfolio. In our case, this analysis is critical as our TAA allocation undertook aggressive positions. Therefore, avoiding an analysis of the risk carried by our portfolios could lead to subsequent losses and destroy our client's wealth. Hence, we computed risk measures such as the Value at Risk (VaR) and Expected Shortfall (ES).

We showed that, by construction, the ES is consistently more significant than the VaR. This

is to be expected as ES is the mean of the values superior to VaR. Hence, ES is more sensitive to the shape of the tail of loss distribution than VaR. Thus there is no wrong or right risk measures, it will be all down to the investors and their incentives.
For each graph illustrating our results, we decided to change the threshold (5% and 10%) of VaR and ES. This allowed us to observe the sensibility of the VaR and ES with a different threshold. Overall, we observed similar trends over both VaR and ES for each portfolio and similar method.

We found that across all risk measures and methods, all our portfolios have been affected by two significant events, namely the European debt crisis of 2012 and the COVID Crisis in early 2020.

Finally, we showed that TAA allocation is the riskiest strategy when compared to SAA and Real portfolio. Indeed, the risk carried by our Real portfolio is primarily induced by position coming from our TAA portfolio. This is intuitive as the TAA portfolio is based on short term views.

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