Quiz 1 A *family of sets over* $\mathbb{Z}$ is a set of sets of integers.
In other words a family of sets over $\mathbb{Z}$ is a subset of $\mathcal{P}(\ ints)$.
For example, $F_1 = \{\{1,2\},\{1,4,5\},\{6\}\}$ and $F_2 = \{\{1,2\},\{1,4,5\},\{2\}\}$ are both families of sets over $\mathbb{Z}$.

1. A *cover* for a set of integers S is a family F of sets over $\mathbb{Z}$ such that each element of S belongs to at least one set in F.
   For example F_1 is a cover for $\{1,6\}$, but F_2 is not. Express the predicate
   COVER(S,F) = "family F is a cover for set S"
   using only the predicate MEMBER, quantifiers, and logical connectives, where
   for any integer $e \in \mathbb{Z}$ and any subset of integers $S \subseteq \mathbb{Z}$,
   MEMBER(e,S) = "e is an element of S". **Solution**:

   $\forall e \in S, \exists f_i \in F,$ MEMBER(e,f$_i$)

2. A family of sets F over $\mathbb{Z}$ is *cover-free* if, for each set $S \in F$, the family of other sets in F is not a cover for S. For example, F_1 is cover-free, but F_2 is not. Express "F is cover-free" as a predicate using only the predicates = and MEMBER, quantifiers, and logical connectives. **Solution**:

   $\forall S \in F, \exists e \in S, \forall f_i \in F,\ (e \neq f_i) \vee (S = f_i)$

   Use the notation from the course slides and MIT book. Use parentheses to eliminate any possible ambiguities.