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A three-dimensional, iterative mapping procedure for the implementation of an ionosphere-magnetosphere anisotropic Ohm's law boundary condition in global magnetohydrodynamic simulations

Michael L. Goodman

Applied Research Corporation, Code 695 NASA Goddard Space Flight Center, Greenbelt, Maryland 20771, USA

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Abstract. The mathematical formulation of an iterative procedure for the numerical implementation of an ionosphere-magnetosphere (IM) anisotropic Ohm's law boundary condition is presented. The procedure may be used in global magnetohydrodynamic (MHD) simulations of the magnetosphere. The basic form of the boundary condition is well known, but a well-defined, simple, explicit method for implementing it in an MHD code has not been presented previously. The boundary condition relates the ionospheric electric field to the magnetic field-aligned current density driven through the ionosphere by the magnetospheric convection electric field, which is orthogonal to the magnetic field $\boldsymbol{B}$, and maps down into the ionosphere along equipotential magnetic field lines. The source of this electric field is the flow of the solar wind orthogonal to $\boldsymbol{B}$. The electric field and current density in the ionosphere are connected through an anisotropic conductivity tensor which involves the Hall, Pedersen, and parallel conductivities. Only the height-integrated Hall and Pedersen conductivities (conductances) appear in the final form of the boundary condition, and are assumed to be known functions of position on the spherical surface $R = R_1$ representing the boundary between the ionosphere and magnetosphere. The implementation presented consists of an iterative mapping of the electrostatic potential ψ, the gradient of which gives the electric field, and the field-aligned current density between the IM boundary at $R = R_1$ and the inner boundary of an MHD code which is taken to be at $R_2 > R_1$. Given the field-aligned current density on $R = R_2$, as computed by the MHD simulation, it is mapped down to $R = R_1$ where it is used to compute ψ by solving the equation that is the IM Ohm's law boundary condition. Then ψ is mapped out to $R = R_2$, where it is used to update the electric field and the component of velocity perpendicular to $\boldsymbol{B}$. The updated electric field and perpendicular velocity serve as new boundary conditions for the MHD simulation which is then used to compute a new field-aligned current density. This process is iterated at each time step. The required Hall and Pedersen conductances may be determined by any method of choice, and may be specified anew at each time step. In this sense the coupling between the ionosphere and magnetosphere may be taken into account in a self-consistent manner.

Correspondence to: M. L. Goodman

1 Introduction

This paper discusses the implementation for global magnetohydrodynamic (MHD) codes of an Ohm's law boundary condition imposed on a spherical surface $R = R_1$ representing the interface between the ionosphere and magnetosphere. The basic form of the boundary condition is well known (Wolf, 1975; Schindler and Birn, 1978), but an explicit method for implementing this boundary condition in an MHD code has not previously been presented in the literature. The implementation presented here involves an iterative mapping of the electrostatic potential and field-aligned currents between the ionosphere-magnetosphere (IM) boundary and the inner boundary of the MHD code which lies above the IM boundary.

The boundary condition relates the ionospheric electric field to the current density driven through the ionosphere by the magnetospheric convection electric field which is orthogonal to the magnetic field and maps down into the ionosphere along equipotential magnetic field lines. The source of the magnetospheric convection electric field is the flow of solar wind orthogonal to the magnetic field. The electric field and current density in the ionosphere are connected through an anisotropic conductivity tensor which includes the Hall, Pedersen, and parallel conductivities. These conductivities are assumed to be known functions of position on $R = R_1$, although they may be determined by any algorithm of choice. In circuit terminology, the ionosphere acts as a resistor which regulates currents in the magnetospheric current system. Despite its extremely small spatial extent compared with spatial scales in the magnetosphere, the ionosphere plays a major role in determining magnetospheric dynamics.

The current density is taken to consist of two parts. In spherical coordinates the first part is the radial current density which describes current flow into and out of the ionosphere. As shown in Sect. 2, the radial current density is equal to the current density parallel to the magnetic field times a geometric factor. The radial current density is assumed to be zero in the ionosphere which is a reasonable approximation since the bulk of the current flow in the ionosphere is perpendicular to the radial direction. The current flow is assumed to have a large radial component only in a boundary layer near the topside of the ionosphere. The second part of the current density is perpendicular to the radial direction and describes most of the current flow within the ionosphere. The magnetic field is taken to be a dipole field since, within about three Earth radii, the Earth's field is much larger than the external field generated by magnetospheric currents.

The boundary condition is discussed in terms of mapping the electrostatic potential and the current density parallel to the magnetic field between the IM boundary and an outer spherical surface which is taken to be the inner boundary for a global MHD code. The inner boundary for the MHD code is typically chosen several Earth radii above the ionosphere for numerical expediency. In order to resolve the MHD phenomena near the inner boundary the time step for the entire MHD code must scale as the inverse of the Alfvén speed near this boundary. As R decreases the Alfvén speed increases, mainly due to the increasing magnetic field strength. Eventually the increasing Alfvén speed corresponds to an unacceptably small time step in terms of obtaining numerical results in a reasonable time. This constraint suggests that the inner boundary be chosen as far out as possible. However, as the radius of the inner boundary increases the field lines that intersect this surface all map down into a spherical cap region of decreasing size about the north and south poles. Since one wants this region to include the auroral zone, the inner boundary cannot be chosen too far out. Empirically it is found that choosing the inner boundary to be at 3.5 Earth radii leads to a reasonable time step and corresponds to a spherical cap region with an arc radius of 32.31° measured from the poles, which includes most of the auroral region. Other choices of the position of the inner boundary, close to 3.5 Earth radii, may be used (Raeder *et al.*, 1995; Raeder, 1995).

Given the parallel current density on the outer surface, it is mapped down to $R = R_1$, where it is used to compute the electrostatic potential by solving the equation that is the Ohm's law boundary condition. The electrostatic potential is then mapped out to the outer boundary, where it is used to update the electric field and the component of velocity perpendicular to the magnetic field. The updated electric field and perpendicular velocity serve as new boundary conditions for the MHD code which is then used to compute a new parallel current density. This process is iterated with each time step.

The boundary condition which determines the electrostatic potential does not explicitly involve time although the current density which is the source term for the boundary condition changes with time as it is computed by the MHD code at each time step. Although the current density source term is time-dependent it changes on the ideal MHD time scale which is the Alfvén transit time for the magnetosphere. This time scale is much larger than the time scales that characterize the evolution of the ionosphere since the average Coulomb collision frequency, Alfvén speed, and inverse spatial scales are orders of magnitude larger in the ionosphere than in the magnetosphere. This large difference in time scales is what allows one to assume that the electric field in the ionosphere is electrostatic to a good approximation: the ionospheric electric field responds rapidly to changes in the magnetosphere.

Important parts of a realistic IM boundary condition which are not considered in this paper are the polar ion flux, cleft ion fountain, and the neutral wind. The polar ion flux and cleft ion fountain may serve as a major source of plasma for the magnetosphere (Chappell *et al.*, 1987), in particular for the magnetotail. The neutral wind is generated by temperature gradients due to solar heating, and by the viscous drag on neutral particles due to ion flows set up by the magnetospheric convection electric field which maps down into the ionosphere along equipotential magnetic field lines. The neutral wind, in turn, acts on the plasma through viscous drag, driving plasma across magnetic field lines, thereby modifying the local convection electric field which drives a current. This dynamo action of the neutral wind is an important source of current in addition to the field-aligned currents that originate in the magnetotail.

In Sect. 2, the Ohm's law boundary condition is presented and cast in the form of an equation for the electrostatic potential with the magnetic field-aligned current density as a source term, and with the conductivity tensor an assumed known function of the spherical angles θ and ϕ on $R = R_1$. At present, empirical maps of the conductivity are available from satellite and ground-based incoherent scatter radar measurements. In Sects. 3 and 4 prescriptions for mapping the electrostatic potential, and the magnetic field-aligned current density, between two concentric spherical surfaces are derived, respectively. In Sect. 5, the iterative procedure for implementing the boundary condition in an MHD code is given. In Sect. 6 the error introduced by assuming that the total magnetic field is a dipole field is briefly discussed. In Sect. 7 the advantages, limitations, and some possible extensions of the given form and implementation of the boundary condition are discussed. In particular, the questions of how to generate a ring current and a magnetic field-aligned electric field in a non-dissipative, one-fluid, isotropic pressure, global MHD simulation of the magnetosphere are briefly addressed by considering a generalized Ohm's law for the magnetosphere which includes Hall and electron pressure gradient effects.

An Ohm's law IM boundary condition together with an iterative mapping procedure for the electrostatic potential and magnetic field-aligned currents is implemented in some form in existing global MHD simulations of the magnetosphere. Fedder and Lyon (1987) appear to be the first authors to implement such a boundary condition

However, they use a constant, scalar conductance in the Ohm's law, and therefore do not include the anisotropic effects due to the Hall and Pedersen conductances, nor take into account the differences in their magnitudes, and their variations over the IM interface. In addition, they do not present any detailed description of the form of the boundary condition, its mathematical classification (i.e. elliptic, parabolic, or hyperbolic) with reference to the boundary conditions necessary to uniquely determine its solution, or the mapping procedure that is used. The most recent work implementing an Ohm's law IM boundary condition in a global MHD simulation of the magnetosphere appears to be that of Raeder *et al.* (1995) and Raeder (1995), which includes the effects of Hall and Pedersen conductances. However, as in the case of Fedder and Lyon (1987), Raeder *et al.* (1995) and Raeder (1995) do not present any detailed description of the specific form of the boundary condition, of its mathematical classification and associated boundary conditions that ensure uniqueness, or of the mapping procedure. It is important that any work which implements an IM boundary condition in a global MHD simulation of the magnetosphere gives a detailed description and analysis of the form of the boundary condition and its method of implementation in order to allow for a judgement of its degree of validity, and to allow it to be compared with IM boundary conditions used by other authors. Although the present paper does not present the results of the implementation of such a boundary condition in a global MHD simulation of the magnetosphere, it does present a detailed description and analysis of the most general formulation of such a boundary condition, a discussion of its mathematical classification, and an explicit numerical prescription for building it into such a simulation. The general formulation of the boundary condition presented here may be used as a reference and a basis for its specific implementations using various forms for the Hall and Pedersen conductances which may be given as fixed maps or determined self-consistently with the magnetic field-aligned currents and the electrostatic potential according to some given algorithm.

CGS units are used unless explicitly indicated otherwise.

2 The Ohm's law boundary condition and the electrostatic potential

Let (R, θ, ϕ) be spherical coordinates with corresponding unit vectors $\hat{\boldsymbol{R}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}$. Let $\boldsymbol{E}, \boldsymbol{B}, \boldsymbol{j}$, and ψ be the electric field, dipole magnetic field, current density, and electrostatic potential, respectively. The magnetic field is given by

$$B_R = \frac{2a\cos\theta}{R^3}, \tag{1}$$

$$B_\theta = \frac{a\sin\theta}{R^3}, \tag{2}$$

$$B_\phi = 0, \tag{3}$$

where $a < 0$ for the Earth, and θ is measured from the north pole. Although $\boldsymbol{B}$ is independent of ϕ, the potential ψ is not independent of ϕ since the potential imposed on the field lines by the solar wind convection electric field, and the spatial distribution of solar radiation and field-aligned currents which determine the ionospheric conductivity are ϕ dependent.

2.1 Derivation of the equation for ψ

The divergence of the current density may be written as

$$\nabla\cdot\boldsymbol{j} = \frac{1}{R^2}\frac{\partial(R^2 j_R)}{\partial R} + \frac{1}{R\sin\theta}\left(\frac{\partial(j_\theta\sin\theta)}{\partial\theta} + \frac{\partial j_\phi}{\partial\phi}\right) \tag{4}$$

$$\equiv \frac{1}{R^2}\frac{\partial(R^2 j_R)}{\partial R} + \nabla_\perp\cdot\boldsymbol{j}_\perp, \tag{5}$$

where $\boldsymbol{j}_\perp = j_\theta\hat{\boldsymbol{\theta}} + j_\phi\hat{\boldsymbol{\phi}}$, and

$$\nabla_\perp = \frac{1}{R}\left(\left(\cot\theta + \frac{\partial}{\partial\theta}\right)\hat{\boldsymbol{\theta}} + \left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right)\hat{\boldsymbol{\phi}}\right).$$

Let the ionosphere lie in the range $r_1 < R < r_2$. It is assumed that j_R has a finite jump discontinuity at $R = r_2$ such that $j_R = 0$ for $R < r_2$ but $j_R \neq 0$ for $R > r_2$. $\boldsymbol{j}_\perp$ is assumed to be continuous across $R = r_2$. Current flows along magnetic field lines in the region $R > r_2$. It is the radial component of this current which enters or leaves the ionosphere. Inside the ionosphere where $R < r_2$ the current flows in the θ and ϕ directions. It is only in a thin boundary layer near $R = r_2$ where part of the current may be redirected in the radial direction and flow into or out of the ionosphere.

The integral of Eq. (5) over the thickness of the ionosphere is performed as follows. One has

$$\lim_{\varepsilon\to 0}\left(\int_{r_1}^{r_2-\varepsilon} + \int_{r_2-\varepsilon}^{r_2+\varepsilon}\right)\frac{1}{R^2}\frac{\partial(R^2 j_R)}{\partial R}\,dR$$

$$= \lim_{\varepsilon\to 0}\int_{r_2-\varepsilon}^{r_2+\varepsilon}\left(\frac{\partial j_R}{\partial R} + \frac{2j_R}{R}\right)dR \tag{6}$$

$$= \lim_{\varepsilon\to 0}\int_{r_2-\varepsilon}^{r_2+\varepsilon}\frac{\partial j_R}{\partial R}\,dR \tag{7}$$

$$\equiv j_R(r_2^+), \tag{8}$$

and

$$\lim_{\varepsilon\to 0}\left(\int_{r_1}^{r_2-\varepsilon} + \int_{r_2-\varepsilon}^{r_2+\varepsilon}\right)\nabla_\perp\cdot\boldsymbol{j}_\perp\,dR = \int_{r_1}^{r_2}\nabla_\perp\cdot\boldsymbol{j}_\perp\,dR \tag{9}$$

$$= -\int_{r_1}^{r_2}\nabla_\perp\cdot(\boldsymbol{\sigma}\cdot\nabla\psi)_\perp\,dR. \tag{10}$$

Here Ohm's law

$$\boldsymbol{j} = \boldsymbol{\sigma}\cdot\boldsymbol{E} = -\boldsymbol{\sigma}\cdot\nabla\psi \tag{11}$$

is used, where $\boldsymbol{\sigma}$ is the conductivity matrix. In reality, the convection electric field generated by the cross-magnetic-field flow of plasma moving with the neutral wind velocity $\boldsymbol{u}$ is important in determining $\boldsymbol{j}$, and may be included in

Eq. (11) by the replacement $\boldsymbol{E} \rightarrow \boldsymbol{E} + (\boldsymbol{u} \times \boldsymbol{B})/c$. Only the case $\boldsymbol{u} = \boldsymbol{0}$ is considered here.

Although the electric field may have a component along $\boldsymbol{B}$ in the ionosphere, the conductivity parallel to $\boldsymbol{B}$ is so large that ψ varies slowly along magnetic field lines. This, in addition to the fact that the θ coordinate of a magnetic field line does not vary significantly over the radial thickness of the ionosphere, implies that $\boldsymbol{E}$ varies slowly with R in the ionosphere. Next, since $r_1 = R_e(1 + \delta r_1/R_e)$, $r_2 = R_e(1 + \delta r_2/R_e)$, $R_e \sim 6.37 \times 10^3$ km (the radius of the Earth), $\delta r_1 \sim 60$ km, and $\delta r_2 \sim 400$ km one has $\delta r_1/R_e \ll 1$, and $\delta r_2/R_e \ll 1$, so that the factor R^{-1} in $\boldsymbol{\nabla}_\perp$ is almost constant between r_1 and r_2. It follows that

$$\int_{r_1}^{r_2} \boldsymbol{\nabla}_\perp \cdot (\sigma \cdot \boldsymbol{\nabla}\psi)_\perp \, dR \sim [\boldsymbol{\nabla}_\perp \cdot (\boldsymbol{\Sigma} \cdot \boldsymbol{\nabla}\psi)_\perp]_{R = R_e}, \tag{12}$$

where

$$\boldsymbol{\Sigma} \equiv \int_{r_1}^{r_2} \sigma \, dR \tag{13}$$

is the height-integrated conductivity matrix, called the conductance.

Since $\boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$ one has, setting $r_2^+ = R_e$ in Eq. (8)

$$j_R(R_e) = [\boldsymbol{\nabla}_\perp \cdot (\boldsymbol{\Sigma} \cdot \boldsymbol{\nabla}\psi)_\perp]_{R = R_e}. \tag{14}$$

This expression involves radial derivatives of ψ. However, since it is assumed that $\boldsymbol{B} \cdot \boldsymbol{\nabla}\psi = 0$ for $R \geq R_e$, it follows that

$$\frac{\partial \psi}{\partial R} = -\frac{\tan\theta}{2R}\frac{\partial \psi}{\partial \theta}. \tag{15}$$

Using Eq. (15) to eliminate the radial derivatives of ψ in Eq. (14) gives the following equation for ψ with $\boldsymbol{\Sigma}$ represented in spherical coordinates

$$\begin{aligned} j_R(R_e, \theta, \phi) = \frac{1}{R_e^2} \Bigg[& \left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)\frac{\partial^2 \psi}{\partial \theta^2} \\ & + \frac{1}{\sin\theta}\left(\Sigma_{\theta\phi} + \Sigma_{\phi\theta} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\frac{\partial^2 \psi}{\partial\phi\partial\theta} \\ & + \frac{\Sigma_{\phi\phi}}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \Bigg\{\frac{\partial}{\partial\theta}\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right) \\ & + \cot\theta\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right) \\ & + \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(\Sigma_{\phi\theta} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\Bigg\}\frac{\partial\psi}{\partial\theta} \\ & + \Bigg\{\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right) + \frac{1}{\sin^2\theta}\left(\frac{\partial\Sigma_{\phi\phi}}{\partial\phi}\right) \\ & + \frac{\Sigma_{\theta\phi}\cot\theta}{\sin\theta}\Bigg\}\frac{\partial\psi}{\partial\phi}\Bigg]_{R = R_e}. \end{aligned} \tag{16}$$

This is the basic equation for the electrostatic potential. Once the source term $j_R(R, \theta, \phi)$ and the conductance $\boldsymbol{\Sigma}(R, \theta, \phi)$ are known for $R = R_e$, Eq. (16) may be solved for $\psi(R_e, \theta, \phi)$ which determines $E_\theta(R_e, \theta, \phi)$ and $E_\phi(R_e, \theta, \phi)$. Then $E_R(R_e, \theta, \phi)$ is determined by Eq. (15). If $j_\parallel (\equiv \boldsymbol{j} \cdot \boldsymbol{B}/B)$ is given instead of j_R then j_R may be computed from

$$j_R = j_\parallel \frac{B_R}{B} \tag{17}$$

$$= -\frac{2j_\parallel \cos\theta}{(1 + 3\cos^2\theta)^{1/2}}. \tag{18}$$

As shown in Sect. 3, the magnetic field lines that cross the surface $R = 3.5\,R_e$ are all continuations of field lines that cross $R = R_e$ in the regions $\theta \leq 32.31°$ and $\pi - 32.31° \leq \theta \leq \pi$. In these regions, the dipole magnetic field is radial to within 17.54°, and the current density is radial to within 5%, as follows from Eq. (18).

However, it is important to take into account even small deviations of $\boldsymbol{B}$ from the radial direction for $R = R_e$. The reason is that such small deviations may correspond to large deviations in $\boldsymbol{B}$ in the magnetotail, and the effects of the IM boundary condition (i.e. of ionospheric processes) are propagated along field lines into the tail. Conversely, the effects of physical processes in the tail, such as reconnection and the redirection of the cross-tail current, are propagated along field lines into the ionosphere. In particular, collisional ionization due to precipitating electrons that flow from the magnetotail into the ionosphere determine, together with photoionization due to solar X-ray and ultraviolet radiation, the ionospheric electrical conductivity which in turn regulates the field-aligned currents. This global connection between the ionosphere and the tail region makes the IM boundary condition important in global magnetospheric modeling. One requirement for the validity of any model IM boundary condition is that the field-aligned currents in the auroral region do indeed map to the tail region.

2.2 The equation for ψ in terms of Hall and Pedersen conductances

In an (x, y, z) Cartesian coordinate system with the z axis pointing along $\boldsymbol{B}$, the conductance matrix has the well-known form

$$\boldsymbol{\Sigma}_{(x,y,z)} = \begin{pmatrix} \Sigma_p & -\Sigma_h & 0 \\ \Sigma_h & \Sigma_p & 0 \\ 0 & 0 & \Sigma_0 \end{pmatrix}, \tag{19}$$

where Σ_p, Σ_h and Σ_0 are the Pedersen, Hall, and parallel conductances, respectively. Transforming to spherical coordinates gives

$$\boldsymbol{\Sigma}_{(R,\theta,\phi)} = \begin{pmatrix} (\Sigma_0\cos^2\varepsilon + \Sigma_p\sin^2\varepsilon) & (\Sigma_p - \Sigma_0)\sin\varepsilon\cos\varepsilon & -\Sigma_h\sin\varepsilon \\ (\Sigma_p - \Sigma_0)\sin\varepsilon\cos\varepsilon & (\Sigma_0\sin^2\varepsilon + \Sigma_p\cos^2\varepsilon) & -\Sigma_h\cos\varepsilon \\ \Sigma_h\sin\varepsilon & \Sigma_h\cos\varepsilon & \Sigma_p \end{pmatrix}. \tag{20}$$

where ε is the angle between $\boldsymbol{B}$ and $\hat{\boldsymbol{R}}$, and

$$\cos\varepsilon = -\frac{2\cos\theta}{(1+3\cos^2\theta)^{1/2}}, \tag{21}$$

$$\sin\varepsilon = \frac{\sin\theta}{(1+3\cos^2\theta)^{1/2}}. \tag{22}$$

Equation (20) is valid for the northern and southern hemispheres (i.e. for $0 \le \theta \le \pi$).

Using Eq. (20) in Eq. (16) gives the following equation for ψ in terms of the Hall and Pedersen conductances which are assumed to be known functions of θ and ϕ for $R = R_e$

$$\begin{aligned} j_R(R_e,\theta,\phi) &= -\frac{2j_{\parallel}\cos\theta}{(1+3\cos^2\theta)^{1/2}} \\ &= \frac{1}{R_e^2}\Bigg[\Sigma_p\frac{\partial^2\psi}{\partial\theta^2} - \frac{\Sigma_h\tan\theta}{2(1+3\cos^2\theta)^{1/2}}\frac{\partial^2\psi}{\partial\phi\partial\theta} + \frac{\Sigma_p}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} \\ &\quad + \left\{\frac{\partial\Sigma_p}{\partial\theta} - \frac{(1+3\cos^2\theta)^{1/2}}{2\sin\theta\cos\theta}\frac{\partial\Sigma_h}{\partial\phi} + \Sigma_p\cot\theta\right\}\frac{\partial\psi}{\partial\theta} \\ &\quad + \left\{\frac{2\cot\theta}{(1+3\cos^2\theta)^{1/2}}\frac{\partial\Sigma_h}{\partial\theta} - \frac{2(1+3\cos^4\theta)\Sigma_h}{\sin^2\theta(1+3\cos^2\theta)^{3/2}}\right. \\ &\quad + \left.\frac{1}{\sin^2\theta}\frac{\partial\Sigma_p}{\partial\phi} + \frac{2\Sigma_h\cot^2\theta}{(1+3\cos^2\theta)^{1/2}}\right\}\frac{\partial\psi}{\partial\phi}\Bigg]_{R=R_e}. \end{aligned} \tag{23}$$

The coefficients in Eq. (23) are well-defined for $\theta \neq \pi/2$, and the equation has a well-defined limit as $\theta \to \pi/2$. The equator ($R = R_e, \theta = \pi/2, 0 \le \phi < 2\pi$) is the boundary of the northern and southern hemispheres in which Eq. (23) is to be solved. For $\theta \to \pi/2$, Eq. (23) reduces to

$$\left(\frac{\partial\psi}{\partial\theta}\right)_{(\theta=\pi/2)} = \frac{\Gamma}{\Sigma_h(\theta=\pi/2,\phi)}, \tag{24}$$

where Γ is an arbitrary constant. Choosing Γ provides a Neumann boundary condition for Eq. (23) on the equator. Since, as discussed in the following subsection, Eq. (23) is expected to be elliptic for realistic values of the Hall and Pedersen conductances in the Earth's ionosphere, a Neumann boundary condition is sufficient to uniquely determine the solution for ψ, to within an additive constant, in each hemisphere (Courant and Hilbert, 1962; John, 1982; Birkhoff and Lynch, 1984).

The conductance Σ_0 parallel to $\boldsymbol{B}$ does not, and cannot, appear in Eq. (23) since the electric field parallel to $\boldsymbol{B}$ is assumed to be zero. This may be seen directly from Eq. (14) which involves Σ_0 only as a coefficient of the electric field parallel to $\boldsymbol{B}$.

Equation (23) is linear if the conductances are given functions of position. In reality the conductances are functions of the electric field which regulates the field-aligned currents which in turn affect the degree of ionization and hence the conductances. Therefore, a proper treatment of the relationship between the conductances and electric field must result in a non-linear equation for ψ. If some prescription is used to update the conductances at each time step, then the equation for ψ remains linear, and the conductances are determined in an approximately self-consistent manner.

Almost all of the electron precipitation that contributes to the conductivity takes place within about 40° of the poles. Hence on the nightside the conductivity is essentially zero outside of these regions, and on the dayside the conductivity outside of these regions is entirely due to photoionization by solar radiation. This should be taken into account when deciding on what to use as the boundaries for Eq. (23) in the two hemispheres. On the nightside there are contributions to the conductivity due to the convection of solar radiation-generated plasma from the dayside to the nightside, and to the ionization created by Lyman-α radiation generated by the scattering of solar photons off neutral hydrogen near the polar regions. However, under most conditions ionization due to precipitating electrons is the dominant contribution to the conductivity on the nightside.

2.3 Classification of the equation for ψ and approximate expressions for the conductances

The numerical procedure for solving Eq. (23) for ψ depends upon the type of the equation, which is elliptic, parabolic, or hyperbolic if

$$1 - \alpha^2 f(\theta) \tag{25}$$

is negative, zero, or positive, respectively [see Chap. 2 of John (1982)]. Here

$$\alpha = \frac{\Sigma_p(\theta,\phi)}{\Sigma_h(\theta,\phi)}, \tag{26}$$

and

$$f(\theta) = \frac{16\cos^2\theta(1+3\cos^2\theta)}{\sin^4\theta}. \tag{27}$$

The mathematical type of Eq. (23) for the electrostatic potential is determined by the ratio of the Hall to Pedersen conductances, which in turn is in large measure (at least on the nightside) determined by the number flux and energy spectrum of the precipitating electrons. Therefore, any implementation of the boundary condition, Eq. (23), must take into account the possibility that the equation for the electrostatic potential may be elliptic, hyperbolic, or parabolic, and correspondingly, that the boundary conditions necessary to ensure a unique solution may be different for the different types. In particular, if the Hall and Pedersen conductances change during the course of the simulation, the type of the equation for the electrostatic potential and the corresponding boundary conditions necessary to ensure uniqueness may also change during the simulation. In addition, it is important to take the variable type of Eq. (23) into consideration when using it to model the ionosphere-magnetosphere coupling for planets other than Earth, since they may have precipitating particle spectra, solar irradiance fluxes, and other ionospheric properties which cause the Hall and Pedersen conductances to differ significantly from those of the Earth.

In the following subsections, some of the existing models of Hall and Pedersen conductances which may be useful in implementing the IM boundary condition are presented and discussed. Detailed discussions of the origin and validity of these models are given in the references provided. The models presented are local in that the conductivity at a point is assumed to result from local ionization due to photoionization and electron-neutral collisions taking place at that point. However, especially at higher altitudes, nonlocal effects due to convective transport of ions may be important in determining the conductances. The effects of local turbulence, which alter the conductivity by increasing the Coulomb collision frequencies above their classical (i.e. near-Maxwellian) values, are also neglected.

2.3.1 Contribution to the conductances from precipitating electrons. On the nightside, where solar flux is negligible and the conductivity is due to field-aligned precipitating electrons, the type of Eq. (23) may be accurately estimated using a simple, approximate method for computing the Hall and Pedersen conductances given by Robinson *et al.* (1987), and used by Hardy *et al.* (1987) to construct global maps of the conductances from satellite measurements of the energy distribution of precipitating electrons in the northern auroral region ($\theta \leq 40^\circ$, $0 \leq \phi < 2\pi$). The maps generated by Hardy *et al.* (1987) are represented in analytic form and labeled by Kp. These maps may be used in Eq. (23) as a first approximation to including realistic conductances due to precipitating electrons in the IM boundary condition. Global maps of the conductances are also given by Spiro *et al.* (1982), using a smaller data set than Hardy *et al.* (1987).

According to Robinson *et al.* (1987) the Hall and Pedersen conductances may be accurately calculated from the relations

$$\Sigma_p = \frac{40E_0}{16 + E_0^2} I_E^{1/2}, \tag{28}$$

$$\frac{\Sigma_h}{\Sigma_p} = 0.45\, E_0^{0.85}, \tag{29}$$

and

$$E_0 = \frac{\int_{E_{min}}^{E_{max}} EF(E)\,dE}{\int_{E_{min}}^{E_{max}} F(E)\,dE}, \tag{30}$$

where E_0(keV), $F(E)$ (cm^{-2} s^{-1} keV^{-1}), I_E[ergs cm^{-2} s^{-1}], E_{min}, and E_{max} are the average electron energy, electron flux distribution function, electron energy flux, and minimum and maximum energies of the measured distribution function, respectively. The units of conductance are mhos. Given measurements of $F(E)$, I_E, E_{min}, and E_{max} along a satellite trajectory, the conductances may be computed. This method for determining the conductances is valid if $F(E)$ is close to being an isotropic (i.e. independent of pitch angle) Maxwellian for energies greater than 0.5 keV, which is the energy below which the contribution to the energy spectrum from low-energy secondary electrons, which do not contribute significantly to the conductances, becomes important (Banks *et al.*, 1974; Robinson *et al.*, 1987). Hence E_{min} should be set equal to 0.5 keV. $F(E)$ is isotropic if there is strong pitch angle diffusion in the source regions of the precipitating electrons. This appears to be the case, at least for plasma sheet precipitation associated with the diffuse aurora (Hones *et al.*, 1971; Meng *et al.*, 1979). $F(E)$ may be determined from radar measurements of the electron number density profile in the following manner. The measured electron density profile is combined with the electron continuity equation, which relates the electron density to the recombination and ionization rates, to determine the ionization rate. The ionization rate is then used to determine $F(E)$ through a deconvolution procedure involving successive fitting to a library of ionization rate profiles for monoenergetic electrons (Vondrak and Baron, 1976). In addition, once $F(E)$ is known, the magnetic field-aligned current density due to the precipitating electrons is given by the electron charge times the integral of $F(E)$ over E.

Using Eqs. (25), (26), and (29), Eq. (23) is elliptic if

$$E_0 < 2.56\,(f(\theta))^{0.5882}. \tag{31}$$

In the northern hemisphere $f(\theta)$ decreases with increasing θ, so that for $\theta < 32.31^\circ$ or 40°, Eq. (23) is elliptic in these regions if $E_0 < 91.85$ keV or 49.13 keV, respectively. As θ decreases from these values the corresponding values of the average energy below which Eq. (23) is elliptic increases from the upper bounds just given. For an isotropic Maxwellian energy distribution the average energy E_0 is twice the temperature (eV). The temperature of the precipitating electrons is expected to be similar to the electron temperature in the plasma sheet. Satellite measurements in the plasma sheet (Schield and Frank, 1970), and ionospheric electron energy deposition codes (Rees, 1963; Berger *et al.*, 1970; Banks *et al.*, 1974) combined with satellite measurements of auroral field-aligned currents and radar measurements of electron density (Robinson *et al.*, 1985) indicate that the temperature of precipitating electrons does not exceed about 4 keV, so the average energy is no more than about 8 keV, which is much less than the upper bounds on E_0 just derived. The same results hold for the corresponding auroral regions in the southern hemisphere. Hence, on the nightside, observational evidence suggests that Eq. (23) is elliptic for realistic models of the conductances.

The Hall and Pedersen conductances may also be determined from radar measurements of the electron density profile as follows. From the kinetic theory definitions of the Hall and Pedersen conductances (Chapman and Cowling, 1970), it follows that the conductances may be determined from a knowledge of the electron and total neutral particle number density profiles (Vickrey *et al.*, 1981). The neutral particle density profile may be obtained from thermospheric models (Banks and Kockarts, 1973). Here it is assumed that quasi-neutrality holds and the ions are singly charged, so that the electron and ion number densities are equal.

Global maps of the Hall and Pedersen conductances caused by precipitating electrons may be generated using empirical models such as the Rice Electron Precipitation

Model (Spiro *et al.*, 1982; Robinson *et al.*, 1987; Kamide *et al.*, 1989) and the AFGL Electron Precipitation Model (Hardy *et al.*, 1985, 1987; Robinson *et al.*, 1987). A brief description of these models, and other solar-terrestrial models, along with their application programs is given by Bilitza (1992).

2.3.2. Contribution to the conductances from solar radiation. On the dayside, electron precipitation and ionization due to solar radiation determine the Hall and Pedersen conductances. The contribution from electron precipitation is discussed in the preceding section. On the dayside, the contribution from the EUV and X-ray components of solar radiation may dominate the contribution from electrons. Numerous empirical fits to conductance calculations using radar measurements of electron density profiles and standard models of the thermosphere exist (e.g. Robinson and Vondrak, 1984; Moen and Brekke, 1993; Brekke and Moen, 1993) which attempt to isolate the contribution to the conductances from solar radiation. A recent fit (Moen and Brekke, 1993) gives

$$\Sigma_h = S_a^{0.53}(0.81\cos\chi + 0.54\cos^{1/2}\chi), \tag{32}$$

$$\Sigma_p = S_a^{0.49}(0.34\cos\chi + 0.93\cos^{1/2}\chi), \tag{33}$$

where χ is the solar zenith angle ($0 \le \chi \le \pi/2$) and S_a, in units of (10^{-22} W m^{-2} Hz^{-1}), is the solar radio flux index observed at 10.7 cm and adjusted to 1 AU. The units of conductance are mhos. The approximate variation of S_a is from 60 at solar minimum to 240 at solar maximum, which corresponds to a variation of about a factor of two in the conductances.

According to Eq. (25), Eq. (23) is elliptic in the region $\theta \le 40^\circ$ if $\Sigma_p/\Sigma_h > 1/f^{1/2}(40^\circ) = 0.081$. During magnetically quiet times solar radiation makes the dominant contribution to the conductances. Ignoring the difference in the variation of Eqs. (32) and (33) with S_a gives $\Sigma_p/\Sigma_h \ge 0.94$ for the solar conductances. Hence Eq. (23) is elliptic on the dayside in the region $\theta \le 40^\circ$ when electron precipitation is a minimum. For magnetically active times maps of $\Sigma_p^{total}/\Sigma_h^{total}$ should be computed for given models of the conductances to determine the regions on the dayside where Eq. (23) is elliptic, parabolic, or hyperbolic. The total conductances may in a first approximation be taken as the sums of the conductances due to solar radiation and electron precipitation.

3 Mapping of the electrostatic potential between concentric spherical surfaces

3.1 Derivation of the mapping function

Let $R = R_1$ and $R = R_2$ be concentric spherical surfaces with $R_2 > R_1$. As before $R = R_1$ represents the IM boundary and $R = R_2$ represents the inner boundary of an MHD simulation. Existing global MHD simulations of the magnetosphere typically set $R_2 \sim 3.5R_e$ (e.g. Fedder *et al.*, 1991). The dipole magnetic field lines are given by

$$R = L_0 \sin^2\theta, \tag{34}$$

$$L_0 = R\left(\theta = \frac{\pi}{2}\right), \tag{35}$$

$$\phi = \phi_0, \tag{36}$$

where the parameters L_0 and ϕ_0 satisfy $0 < L_0 < \infty$ and $0 \le \phi < 2\pi$.

Let $(R, \theta, \phi) = (R_2, \theta_2, \phi_2)$. This point lies on the field line defined by $L_0 = R_2/\sin^2\theta_2$ and $\phi_0 = \phi_2$. Then the point

$$R = R_1,$$

$$\theta = \arcsin\left(\frac{R_1 \sin^2\theta_2}{R_2}\right)^{1/2},$$

$$\phi = \phi_2, \tag{37}$$

lies on the same field line.

Hence if $\psi(R_1, \theta, \phi)$ is known, and if ψ is constant along magnetic field lines, then $\psi_{R=R_2}$, is given by the mapping function

$$\psi(R_2, \theta, \phi) = \psi\left(R_1, \arcsin\left(\frac{R_1 \sin^2\theta}{R_2}\right)^{1/2}, \phi\right). \tag{38}$$

If $R_2 = 3.5R_e$ and $R_1 = R_e$ then all field line segments that intersect the hemisphere ($R = R_2, \theta \le \pi/2$) map down to the northern hemispheric region ($R = R_1, \theta \le \arcsin 3.5^{-1/2} = 32.31^\circ$), and all field line segments that intersect the hemisphere ($R = R_2, \pi/2 \le \theta \le \pi$) map down to the southern hemispheric region ($R = R_1$, $\pi - 32.31^\circ \le \theta \le \pi$). These regions around the poles cover most of the auroral regions. Each field line segment that intersects $R = R_2$ also intersects $R = R_1$, although the converse is not true.

3.2 Velocity and electric field update at $R = R_2$

Let $\boldsymbol{V}_\perp$ be the MHD fluid velocity perpendicular to $\boldsymbol{B}$. Since $\psi(R_2, \theta_2, \phi_2)$ is known from Eq. (38), one immediately has

$$E_\theta(R_2, \theta_2, \phi_2) = -\frac{1}{R_2}\frac{\partial\psi(R_2, \theta_2, \phi_2)}{\partial\theta_2}, \tag{39}$$

and

$$E_\phi(R_2, \theta_2, \phi_2) = -\frac{1}{R_2 \sin\theta_2}\frac{\partial\psi(R_2, \theta_2, \phi_2)}{\partial\phi_2}. \tag{40}$$

Fixing θ_2, ϕ_2, and R_1, and differentiating $\psi(R_2, \theta_2, \phi_2)$ with respect to R_2 gives

$$E_R(R_2, \theta_2, \phi_2) = \frac{\tan\theta_1}{2R_2}\frac{\partial\psi(R_1, \theta_1, \phi_1)}{\partial\theta_1} \tag{41}$$

$$= -\frac{R_1 \tan\theta_1}{2R_2} E_\theta(R_1, \theta_1, \phi_1), \tag{42}$$

where (θ_1, ϕ_1) is given by Eq. (37) with $(\theta, \phi) = (\theta_1, \phi_1)$. If $R_1 = R_2$ then Eq. (42) reduces to Eq. (15), as required.

Next

$$V_\perp(R_2,\theta_2,\phi_2) = \frac{c(\boldsymbol{E}(R_2,\theta_2,\phi_2)\times\boldsymbol{B})}{B^2}, \tag{43}$$

which, together with Eqs. (39)–(42), determine the updates to $\boldsymbol{E}$ and $\boldsymbol{V}_\perp$ on $R = R_2$. Here c is the speed of light and the Ohm's law for a perfectly conducting ion fluid is assumed, which is the Ohm's law used in existing global MHD simulations of the magnetosphere, with the exception of a recently implemented ad hoc resistive term added to this Ohm's law to allow for magnetic reconnection (Raeder, 1995).

4 Mapping of the field-aligned current density between concentric spherical surfaces

Again let $R_2 > R_1$. Suppose that $j_\parallel(R_2,\theta,\phi)$ is known. One wants to compute $j_\parallel(R_1,\theta,\phi)$ which is the source term in Eq. (23) for ψ derived in Sect. 2.

Consider an infinitesimal magnetic flux tube which intersects $R = R_2$ and $R = R_1$ at the area elements $R_2^2\,d\Omega_2 = R_2^2 \sin\theta_2\,d\theta_2\,d\phi_2$ and $R_1^2 d\Omega_1 = R_1^2 \sin\theta_1\,d\theta_1\,d\phi_1$, respectively. Since (R_1,θ_1,ϕ_1) and (R_2,θ_2,ϕ_2) are on the same field line, it follows from Eq. (37) that

$$\sin^2\theta_1 = \left(\frac{R_1}{R_2}\right)\sin^2\theta_2, \tag{44}$$

and

$$\phi_1 = \phi_2, \tag{45}$$

which, for fixed R_1 and R_2 implies that

$$d\Omega_1 = \frac{R_1\cos\theta_2}{R_2\cos\theta_1(\theta_2)}\,d\Omega_2, \tag{46}$$

and

$$\theta_1(\theta_2) = \arcsin\left(\frac{R_1\sin^2\theta_2}{R_2}\right)^{1/2}. \tag{47}$$

Let the angle between $\hat{\boldsymbol{R}}(R_i,\theta_i,\phi_i)$ and the magnetic flux tube be ε_i $(i = 1,2)$. Then $0 \le \varepsilon_i \le \pi/2$ and

$$\cos\varepsilon_i = \left.\frac{|B_R|}{B}\right|_{(R_i,\theta_i,\phi_i)} = \frac{2|\cos\theta_i|}{(1+3\cos^2\theta_i)^{1/2}}. \tag{48}$$

Conservation of charge requires that

$$j_\parallel(R_1,\theta_1,\phi_1)R_1^2\,d\Omega_1\cos\varepsilon_1 = j_\parallel(R_2,\theta_2,\phi_2)R_2^2\,d\Omega_2\cos\varepsilon_2. \tag{49}$$

Combining Eqs. (46), (48), and (49) gives the required mapping function

$$j_\parallel(R_1,\theta_1,\phi_1) = \left(\frac{R_2}{R_1}\right)^3\left(\frac{1+3\cos^2\theta_1(\theta_2)}{1+3\cos^2\theta_2}\right)^{1/2} \times j_\parallel(R_2,\theta_2,\phi_2), \tag{50}$$

with $\theta_1(\theta_2)$ given by Eq. (47).

5 Iterative procedure for implementing the IM boundary condition

The following iterative procedure may be used for implementing the IM boundary condition in an MHD simulation. Let $j_\parallel$ (or j_R)(R_2,θ_2,ϕ_2) be given. Compute $j_\parallel(R_1,\theta_1,\phi_1)$ using Eq. (50). Compute $\psi(R_1,\theta_1,\phi_1)$ using Eq. (23) for given maps of $\Sigma_h(R_1,\theta_1,\phi_1)$ and $\Sigma_p(R_1,\theta_1,\phi_1)$. Compute $\psi(R_2,\theta_2,\phi_2)$ using Eq. (38). Compute $\boldsymbol{E}(R_2,\theta_2,\phi_2)$ and $\boldsymbol{V}_\perp(R_2,\theta_2,\phi_2)$ using Eqs. (39)–(43), and use them as new boundary conditions in the MHD simulation to compute a new value for $j_\parallel(R_2,\theta_2,\phi_2)$. Iterate this procedure as required.

The above procedure does not give any prescription for updating $\boldsymbol{V}_\parallel$, the velocity parallel to $\boldsymbol{B}$ at $R = R_2$. Then, for given initial conditions, subsequent updates to $\boldsymbol{E}$ and $\boldsymbol{V}_\perp$ at each time step are either sufficient to determine $\boldsymbol{V}_\parallel$ or $\boldsymbol{V}_\parallel(R_2,\theta,\phi,t)$ must be specified. Since $\boldsymbol{V}_\parallel$ has a radial component, it partly determines the particle flux into and out of the ionosphere. In addition, the boundary condition on $\boldsymbol{V}_\parallel(R = R_2)$ influences the charging of the ionosphere in the following manner. This boundary condition partly determines the MHD solution for $\boldsymbol{B}$. Although $\boldsymbol{B}(R = R_2)$ is a dipole field, it is not curl-free since its derivative normal to the surface $R = R_2$ is not that of a dipole field (otherwise $\boldsymbol{j}\,(R = R_2) \equiv 0$). Then $\boldsymbol{V}\times\boldsymbol{B}(R = R_2) \neq 0$, and determines $\boldsymbol{j}\,(R = R_2)$, which in turn determines the electrostatic potential $\psi(R = R_1)$ on the ionospheric surface. From $\psi(R = R_1)$ the ionospheric electric field $\boldsymbol{E}(R = R_1)$ is computed. Calculating the divergence of $\boldsymbol{E}(R = R_1)$ then yields the charge density on the ionospheric surface.

If $V_R(R = R_2)$ is constrained to be zero, this essentially means that there is no ion flow, and hence no mass flow, through the surface $R = R_2$. In this case, the local mass flow into the ionosphere is zero and the current flow into the ionosphere is carried entirely by the electrons. The constraint $V_R(R = R_2) = 0$ together with Eq. (43) then determine $\boldsymbol{V}_\parallel(R_2,\theta,\phi,t)$. Specifically, $\boldsymbol{V}_\parallel(R = R_2)$ is determined as follows. Since $\boldsymbol{V}_\perp$ is known from Eq. (43) one has $\boldsymbol{V}_\perp = \alpha\hat{\boldsymbol{R}} + \beta\hat{\theta} + \gamma\hat{\phi}$ where α, β, γ are known functions of R_2, θ_2, ϕ_2. Since $B_\phi = V_R(R = R_2) = 0$ one has $\boldsymbol{V}_\parallel = (V_\theta B_\theta/B)\hat{\boldsymbol{B}}$. It remains to determine V_θ. Since $V_\theta \equiv (\boldsymbol{V}_\perp + \boldsymbol{V}_\parallel)\cdot\hat{\boldsymbol{\theta}} = \beta + V_\theta(B_\theta/B)^2$, it follows that $V_\theta = \beta(B/B_R)^2$, which expresses V_θ, and hence $\boldsymbol{V}_\parallel$, in terms of known quantities.

An alternative, but more complex, approach to determining $\boldsymbol{V}_\parallel$ is not to constrain $V_R(R = R_2)$ but instead to require that the mass density be constant in a neighborhood of $R = R_2$. Then the continuity equation implies that $\boldsymbol{V}$ is divergence-free on $R = R_2$. Since $\boldsymbol{V}_\perp$ is known from Eq. (43), the divergence-free condition on $\boldsymbol{V}$ becomes a first order partial differential equation for $\boldsymbol{V}_\parallel(R = R_2,\theta,\phi)$ involving only derivatives with respect to R and θ. Hence, in order to solve this equation, information about the variation of $\boldsymbol{V}$ normal to the surface $R = R_2$, as well as in the surface with respect to variations in θ, must be specified in a suitable boundary condition.

To within the accuracy of observations the net current flow into the ionosphere is zero, and this is guaranteed to be the case for the MHD solution to the extent to which the divergence-free condition on the current density is satisfied numerically.

Since the electric field on $R = R_2$ is electrostatic, it follows from Faraday's law that the magnetic field on this surface is independent of time and hence remains a dipole field throughout the simulation.

6 Deviation of *B* from a dipole field

Some remarks on the fact that the actual magnetic field is not a dipole field are in order. Let $\boldsymbol{B}_d$ and $\boldsymbol{B}_{mhd}$ be the Earth's dipole field and the magnetic field due to the plasma as computed by the MHD simulation, respectively. Since $\boldsymbol{B}_d$ is curl free, the current density $\boldsymbol{j}$ is entirely due to $\boldsymbol{B}_{mhd}$. Throughout this paper it is assumed that $\boldsymbol{B} = \boldsymbol{B}_d$ wherever $\boldsymbol{B}$ is used in a calculation. This assumption introduces errors of order

$$\left.\frac{|\boldsymbol{B}_{mhd}|}{|\boldsymbol{B}_d|}\right|_{R \sim 3.5R_e} \ll 1 \tag{51}$$

and

$$\left.\frac{|\boldsymbol{B}_d - \boldsymbol{B}_{IGRF}|}{|\boldsymbol{B}_d|}\right|_{R \sim 3.5R_e} \ll 1 \tag{52}$$

in the MHD momentum equation and Faraday's law. Here $\boldsymbol{B}_{IGRF}$ is the International Geomagnetic Reference Field for the internal field of the Earth (Langel, 1987), and the inequality in Eq. (51) follows from the assumption that $|\boldsymbol{B}_{mhd}|$ is on the order of the magnitude of the total field generated by the actual magnetospheric current system.

It is only important to take $\boldsymbol{B}_{mhd}$ into account in the computation of $\boldsymbol{j}$. This is done to within numerical accuracy in the MHD simulation.

7 Discussion

Any realistic formulation and implementation of an IM boundary condition in a global MHD simulation must include a prescription for updating the conductances in response to the time-varying state of the magnetosphere and to the seasonal and solar cycle variation of the solar radiation flux into the ionosphere. On the nightside, collisional ionization due to incoming electrons associated with upward-directed field-aligned currents dominates the determination of the conductivities. On the dayside, photoionization by solar radiation dominates the determination of the conductivities. A model for the conductances which at least depends on these two main sources of ionization must be used to calculate the conductances in a self-consistent manner.

The main advantage of the IM boundary condition model presented here is its generality in allowing for an arbitrary variation of the conductances over the spherical surface representing the IM boundary. Any algorithm may be used to calculate the conductances. In particular a time-dependent algorithm that has as inputs quantities calculated by an MHD simulation at its inner boundary may be used to determine the conductances self-consistently with the state of the magnetosphere as represented by the output of the simulation.

The obvious limitation of the IM boundary condition model presented here is its relatively simple treatment of the IM coupling. The actual coupling is complex (Kamide and Baumjohann, 1933), and involves kinetic effects which cannot be modeled by a surface boundary condition and which lie outside the scope of MHD. In particular, the model does not allow for an electric field parallel to $\boldsymbol{B}$, though such fields are believed to be important in particle acceleration, and hence current generation, in the auroral region.

However, the IM boundary condition model presented here should be useful as a step towards taking IM coupling into account in a self-consistent and realistic manner when the model is implemented in a global MHD simulation. The model can probably be extended to include the effects of polar and neutral winds without much difficulty, and these would be important extensions.

Existing global MHD simulations of the magnetosphere assume that the ions are perfectly conducting (i.e. $\boldsymbol{E} + (\boldsymbol{V} \times \boldsymbol{B})/c = 0$) which implies that the parallel electric field $E_\parallel \equiv \boldsymbol{E} \cdot \boldsymbol{B}/B = 0$. In reality, parallel electric fields exist and are known to be important in determining $j_\parallel$ and hence the ionospheric conductivities (Stern, 1983; Kamide and Baumjohann, 1993; Burke *et al.*, 1994). These fields are detected above the ionosphere, at an altitude ~ 5000 km, and accelerate ions up into the magnetosphere, and electrons down into the ionosphere (Kelly, 1989). It is unlikely that a realistic mechanism for generating $E_\parallel$ can be built into a surface boundary condition such as the one presented here. In that case, the only way of generating a parallel electric field in a non-dissipative, one-fluid, isotropic pressure MHD model is to use an Ohm's law in the magnetosphere given by

$$\boldsymbol{E} + \frac{\boldsymbol{V} \times \boldsymbol{B}}{c} = \frac{1}{en_e}\left(\frac{\boldsymbol{j} \times \boldsymbol{B}}{c} - \nabla p_e\right), \tag{53}$$

which is the Ohm's law that follows from the two-fluid MHD Euler equations, neglecting gravity and electron inertia. Here p_e, n_e, and e are the electron pressure, electron number density, and magnitude of the electron charge, respectively. It is emphasized that Eq. (53) is the most general Ohm's law for a two-fluid, non-dissipative MHD model neglecting gravity and electron inertia. If the electrons and ions have adiabatic or constant temperature ideal gas equations of state then p_e may be expressed as a function of ρ or p, respectively, where ρ is the MHD mass density (the ion mass density) and p is the MHD pressure (the sum of the ion and electron pressures). When the electron pressure is expressed in terms of ρ or p the corresponding MHD model with the Ohm's law given by Eq. (53) formally becomes a one-fluid model since the electron pressure is then a derived quantity.

The Ohm's law given by Eq. (53) allows for a parallel electric field given by

$$E_\parallel = -\frac{\boldsymbol{B} \cdot \nabla p_e}{en_e B}. \tag{54}$$

Hence, the standard ideal MHD Ohm's law, or even this Ohm's law modified by adding the Hall term, does not allow for a parallel electric field, but within the context of the general two-fluid, non-dissipative MHD model neglecting gravity and electron inertia, which contains the electron pressure gradient term in Ohm's law, there is a parallel electric field, given by Eq. (54), due to a variation of the electron pressure along the magnetic field. The electric field given by Eq. (54) can drive a flow of the ions H^+ and He^+ from the high-latitude ionosphere, in the altitude range $\sim$ 600–3000 km, into the magnetosphere (Kelly, 1989). This ion flow may be subsonic or supersonic. However, whether or not, within the context of MHD, $E_\parallel$ given by Eq. (54) may generate realistic field-aligned currents in the magnetosphere above the high-latitude ionosphere, and thereby contribute to electron precipitation into the ionosphere can only be determined by using Eq. (53) in a global MHD simulation. This has not yet been done, but is worth exploring as an MHD mechanism for accelerating particles along the magnetic field in the magnetosphere.

For the quiet-time ring current, the large-scale electric field $\boldsymbol{E} \sim \boldsymbol{0}$, in which case there is no cross-magnetic field ion flow, and hence no ion contribution to the ring current, if the ions are assumed to be perfectly conducting. Since the actual ring current is carried mainly by westward-drifting positive ions (mostly protons) it follows that the ion fluid in the quiet-time ring current is not perfectly conducting. The standard argument for neglecting the Hall term in Eq. (53) is that

$$\left|\frac{1}{ZeN_i}\left(\frac{\boldsymbol{J}\times\boldsymbol{B}}{c}\right)\right| \ll \left|\frac{\boldsymbol{U}\times\boldsymbol{B}}{c}\right|, \tag{55}$$

which is equivalent to $|J_\perp^{total}| \ll |J_\perp^{ions}|$. However, in the quiet-time ring current the ions carry most of the current (Lui and Hamilton, 1992), so that the Hall term must be retained in Ohm's law to properly describe the current flow. In the quiet-time ring current the electrons are cold compared with the ions since $T_e \approx 0.14\ T_i$(Lui and Hamilton, 1992), where T_e and T_i are the electron and ion temperatures, respectively. Hence, electron pressure makes a relatively small, though significant, contribution to the total plasma pressure and so should be included in Ohm's law in order to properly describe the quiet-time ring current.

Existing global MHD simulations of the magnetosphere (Brecht *et al.*, 1982; Ogino, 1986; Fedder and Lyon, 1987; Watanabe and Sato, 1990; Walker *et al.*, 1993; Ogino *et al.*, 1994; Tanaka, 1994; Raeder *et al.*, 1995; Raeder, 1995) assume that the ions are perfectly conducting, or allow for the addition of an ad hoc resistive term in Ohm's law for purposes of numerical stability, and for modeling overall resistive effects in the magnetosphere, especially in regions of enhanced current density where reconnection is believed to occur. However, since resistivity does not play a significant role in maintaining the quiet-time ring current, and since as just pointed out the ion fluid in the quiet-time ring current is not perfectly conducting (i.e. the electric field in the frame of the ions is not zero) it follows that these simulations cannot produce a quiet-time (i.e. $\boldsymbol{E} \sim \boldsymbol{0}$) ring current. In fact, there is no mention of a ring current in the literature presenting the results of these simulations, which strongly suggests that either no ring current is generated in these simulations or that a ring current may be generated but it is unphysical. However, if the pressure is isotropic, but Ohm's law given by Eq. (53) is used, then the Hall and electron pressure gradient terms on the right-hand side of Eq. (53) allow for a cross-magnetic field ion flow even when $\boldsymbol{E} = \boldsymbol{0}$. This suggests that using Eq. (53) as Ohm's law in the magnetosphere may produce a ring current in a global MHD simulation which assumes an isotropic pressure, although anisotropic pressure effects are known to be significant, but not dominant, in the quiet- and storm-time ring current (Lui *et al.*, 1987; Lui and Hamilton, 1992). The curvature current density, which is eastward and is caused by an anisotropic pressure, is rarely more than 25% of the total current density, although it may be comparable to the total current density in the inner region of the ring current ($L < 4$). Hence, as a first approximation to modeling the ring current using a single-fluid MHD model, one may assume an isotropic pressure and neglect electron pressure effects, but must include the Hall term in Ohm's law. From the analysis of Lui *et al.* (1987) and Lui and Hamilton (1992), it appears that anisotropic pressure effects are comparable to or dominate electron pressure effects in determining the total ring current density, that both effects make significant contributions to the total current density, but that these contributions are usually small compared to the contribution of the ion pressure gradient. The possibility of generating a ring current in a single-fluid, global MHD simulation of the magnetosphere by including the Hall term and, of lesser importance, the electron pressure gradient term, in Ohm's law should be explored further.

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References

Banks, P. M., and G. Kockarts, *Aeronomy. Part A.* Academic Press, New York, 1973.

Banks, P. M., C. R. Chappell, and A. F. Nagy, A new model for the interaction of auroral electrons with the atmosphere: spectral degradation, backscatter, optical emission and ionization. *J. Geophys. Res.*, **79**, 1459, 1974.

Berger, M. J., S. M. Seltzer, and K. Maeda, Energy deposition by auroral electrons in the atmosphere. *J. Atmos. Terr. Phys.*, **32**, 1015, 1970.

Bilitza, D., Solar-terrestrial models and application software. *Planet. Space Sci.*, **40**, 541, 1992.

Birkhoff, G., and R. Lynch, *Numerical Solution of Elliptic Problems.* SIAM, Philadelphia, 1984.

Brecht, S. H., J. G. Lyon, J. A. Fedder, and K. Hain, A time-dependent three-dimensional simulation of the Earth's magnetosphere: reconnection events, *J. Geophys. Res.*, **87**, 6098, 1982.

Brekke, A., and J. Moen, Observations of high-latitude ionospheric conductances – a review, *J. Atmos. Terr. Phys.*, **55**, 1493, 1993.

Burke, W. J., J. S. Machuzak, N. C. Maynard, E. M. Basinska, G. M. Erickson, R. A. Hoffman, J. A. Slavin, and W. B. Hanson, Auroral ionospheric signatures of the plasma sheet boundary layer in the evening sector, *J. Geophys. Res.*, **99,** 2489, 1994.

Chapman, S., and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases*, Cambridge University Press, New York, 1970.

Chappell, C. R., T. E. Moore, and J. A. Waite, Jr., The ionosphere as a fully adequate source of plasma for the Earth's magnetosphere, *J. Geophys. Res.*, **92,** 5896, 1987.

Courant, R., and D. Hilbert, *Methods of Mathematical Physics, vol. 2*, Interscience, New York, 1962.

Fedder, J. A., and J. G. Lyon, The solar wind-magnetoshpere-ionosphere current-voltage relationship, *Geophys. Res. Lett.*, **14,** 880, 1987.

Fedder, J. A., C. M. Mobarry, and J. G. Lyon, Reconnection voltage as a function of IMF clock angle, *Geophys. Res. Lett.*, **18,** 1047, 1991.

Hardy, D. A., M. S. Gussenhoven, and E. Holeman, A statistical model of the auroral electron precipitation, *J. Geophys. Res.*, **90,** 4229, 1985.

Hardy, D. A., M. S. Gussenhoven, and R. Raistrick, Statistical and functional representations of the pattern of auroral energy flux, number flux, and conductivity, *J. Geophys. Res.*, **92,** 12275, 1987.

Hones, E. W., Jr., J. R. Asbridge, S. J. Bame, and S. Singer, Energy spectra and angular distributions of particles in the plasma sheet and their comparison with rocket measurement over the auroral zone, *J. Geophys. Res.*, **76,** 63, 1971.

Kamide, Y., and W. Baumjohann, *Magnetosphere-ionosphere Coupling*, Springer, Berlin, Heidelberg, New York, 1993.

Kamide, Y., Y. Ishihara, T. L. Killeen, J. D. Craven, L. A. Frank, and R. A. Heelis, Combining electric field and aurora observations from DE-1 and -2 with ground magnetometer records to estimate ionospheric electromagnetic quantities, *J. Geophys. Res.*, **94,** 6723, 1989.

Kelly, M. C., *The Earth's Ionosphere: Plasma Physics and Electrodynamics*, Academic Press, New York, 1989.

John, F., *Partial Differential Equations*, Springer, Berlin, Heidelberg, New York, 1982.

Langel, R. A., The main geomagnetic field, in *Geomagnetism*, Ed. J. A. Jacobs, Academic Press, New York, 1987.

Lui, A. T. Y., and D. C. Hamilton, Radial profiles of quiet-time magnetospheric parameters, *J. Geophys. Res.*, **97,** 19325, 1992.

Lui, A. T. Y., R. W. McEntire, and S. M. Krimigis, Evolution of the ring current during two geomagnetic storms, *J. Geophys. Res.*, **92,** 7459, 1987.

Meng, C.-I., B. Mauk, and C. E. McIlwain, Electron precipitation of evening diffuse aurora and its conjugate electron fluxes near the magnetospheric equator, *J. Geophys. Res.*, **84,** 2545, 1979.

Moen, J., and A. Brekke, The solar flux influence on quiet-time conductances in the auroral ionosphere, *Geophys. Res. Lett.*, **20,** 971, 1993.

Ogino, T., A three-dimensional MHD simulation of the interaction of the solar wind with the Earth's magnetosphere: the generation of field-aligned currents, *J. Geophys. Res.*, **91,** 6791, 1986.

Ogino, T., R. J. Walker, and M. Ashour-Abdalla, A global magnetohydrodynamic simulation of the response of the magnetosphere to a northward turning of the interplanetary magnetic field, *J. Geophys. Res.*, **99,** 11027, 1994.

Raeder, J., Global MHD simulations of the dynamics of the magnetosphere: weak and strong solar wind forcing, in *Substorms 2: Proc. Second Internat. Conf. on Substorms*, 1995.

Raeder, J., R. J. Walker and M. Ashour-Abdalla, The structure of the distant geomagnetic tail during long periods of northward IMF, *Geophys. Res. Lett.*, **22,** 349, 1995.

Rees, M. H., Auroral ionization and excitation by incident energetic electrons, *Planet. Space Sci.*, **11,** 1209, 1963.

Robinson, R. M., and R. R. Vondrak, Measurements of E-region ionization and conductivity produced by solar illumination at high latitudes, *J. Geophys. Res.*, **89,** 3951, 1984.

Robinson, R. M., R. R. Vondrak, and T. A. Potemra, Auroral zone conductivities within the field-aligned current sheets, *J. Geophys. Res.*, **90,** 9688, 1985.

Robinson, R. M., R. R. Vondrak, K. Miller, T. Dabbs, and D. Hardy, On calculating ionospheric conductances from the flux and energy of precipitating electrons, *J. Geophys. Res.*, **92,** 2565, 1987.

Schield, M. A., and L. A. Frank, Electron observations between the inner edge of the plasma sheet and the plasmasphere, *J. Geophys. Res.*, **75,** 5401, 1970.

Schindler, K., and J. Birn, Magnetospheric physics, *Phys. Rep.*, **47,** No. 2, 109, 1978.

Spiro, R. W., P. H. Reiff, and L. J. Maher, Jr., Precipitating electron energy flux and auroral zone conductances – an empirical model, *J. Geophys. Res.*, **87,** 8215, 1982.

Stern, D. P., Electric currents and voltage drops along auroral field lines, *Space Sci. Rev.*, **34,** 317–1983.

Tanaka, T., Finite-volume TVD scheme on an unstructural grid system for three-dimensional MHD simulation of inhomogeneous systems including strong back-ground potential fields, *J. Comp. Phys.*, **111,** 381, 1994.

Vickrey, J. F., R. R. Vondrak, and S. J. Matthews, The diurnal and latitudinal variation of auroral zone ionospheric conductivity, *J. Geophys. Res.*, **86,** 65, 1981.

Vondrak, R. R., and M. J. Baron, Radar measurements of the latitudinal variation of auroral ionization, *Radio Sci.*, **11,** 939, 1976.

Walker, R. J., T. Ogino, J. Raeder, and M. Ashour-Abdalla, A global magnetohydrodynamic simulation of the magnetosphere when the interplanetary magnetic field is southward: the onset of magnetotail reconnection, *J. Geophys. Res.*, **98,** 17235, 1993.

Watanabe, K., and T. Sato, Global simulation of the solar wind-magnetosphere interaction: the importance of its numerical validity, *J. Geophys. Res.*, **95,** 75, 1990.

Wolf, R. A., Ionosphere-magnetosphere coupling, *Space Sci. Rev.*, **17,** 537, 1975.

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Letter to the Editor

Comment on "A three-dimensional, iterative mapping procedure for the implementation of an ionosphere-magnetosphere anisotropic Ohm's law boundary condition in global magnetohydrodynamic simulations" by Michael L. Goodman

O. Amm

Finnish Meteorological Institute, Department of Geophysics, P.O. Box 503, FIN-00101 Helsinki, Finland

in Ann. Geophysicae, 13, pp. 843–853, 1995

In a recent issue, Goodman (1995) presented an approach to implement the ionospheric Ohm's law in its general, anisotropic form as a boundary condition for magnetospheric MHD simulations. This approach consists of three steps for each time step of the MHD simulation: 1. Mapping the field-aligned current density $j_{\parallel}$ given by the MHD code at its boundary next to the earth down to the ionosphere; 2. There from calculating the ionospheric electric potential ψ by Ohm's law, assuming that the height integrated Hall (Σ_H) and Pedersen (Σ_P) conductances are given; 3. Mapping ψ back to the earthward boundary of the MHD simulation area where it is used as a boundary condition for the next MHD code step. The mapping formulas are given assuming that the earth's magnetic field is a dipole field. While this approach is clearly valuable in general, an inconsistency appeared in the second, "ionospheric" step. Goodman (1995) correctly assumes that if the ionosphere lies in a radial range $r_1 < R < r_2$, the radial currents j_R must be zero for $R < r_2$, i.e., inside the ionosphere, and then jump to some possibly nonzero value for $R > r_2$, i.e., in the region of field-aligned currents. However, the conductance tensor $\Sigma_{(R,\theta,\phi)}$ he uses (his equation (20)) clearly does not provide $j_R = 0$ inside the ionosphere. That tensor is the correct transformation of the Cartesian coordinate form of Ohm's law [equation (19) in Goodman (1995)] to spherical coordinates, but both tensors do not take into account the secondary polarisation effects that occur inside the ionosphere if the magnetic field lines are not perpendicular to it: The primary currents in the ionosphere will be perpendicular to the tilted magnetic field lines, i.e., have a radial component. Since the Hall and Pedersen currents cannot continue over the boundaries of the ionosphere at $R = r_1$ and $R = r_2$, they build up space charges there which cause a secondary electric field that subsequently causes secondary Hall and Pedersen currents. This process will continue until a stationary situation is reached, i.e., no further space charges are produced, synonymous with $j_R = 0$. Hence, this condition is provided only by the secondary process. It can easily be shown that the characteristic time for reaching the stationary situation is of the order 10^{-6} s, i.e., practically instantaneous in the context of MHD code time steps.

Including the polarisation effects leads to an effective conductance tensor (to be applied with the primary electric field) (e.g., Rishbeth and Garriott, 1969)

$$\Sigma_{(R,\theta,\phi)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\Sigma_0 \Sigma_P}{C} & \dfrac{\Sigma_0 \Sigma_H(-\cos\varepsilon)}{C} \\ 0 & \dfrac{\Sigma_0 \Sigma_H \cos\varepsilon}{C} & \Sigma_P + \dfrac{\Sigma_H^2 \sin^2\varepsilon}{C} \end{pmatrix} \tag{1}$$

where Σ_0 is the conductance parallel to the magnetic field, ε the angle between the magnetic field lines and the normal on the ionosphere, and $C = \Sigma_0 \cos^2\varepsilon + \Sigma_P \sin^2\varepsilon$. Some important features of the above tensor are: 1. $j_R = 0$ is fulfilled (upper row zero). Whereas j_R is zero inside the ionosphere, the field-aligned currents above the ionosphere are in general nonvanishing, since they are given of the horizontal ionospheric currents as they result from tensor (1); 2. The primary E_R has no influence since the final value of E_R is fixed by the condition $j_R = 0$ (first column zero); 3. For $\varepsilon = 0$, eq. (1) and eq. (20) of Goodman (1995) are identical; 4. For $\varepsilon = 90°$ (i.e., at the equator in a dipole field earth), $\Sigma_{\phi\phi}$ results into the wellknown Cowling conductance $\Sigma_C = \Sigma_P + \Sigma_H^2/\Sigma_P$, in contrast to eq. (20) of Goodman (1995).

Therefore, tensor (1) instead of Goodman's tensor (20) should be inserted into his equation (16). The central coupling equation (23) of Goodman (1995) that connects j_R above the ionosphere and the ionospheric conductances with the horizontal electric field, is then rewritten (without inserting the special dipole case, i.e., $\varepsilon = \varepsilon(\theta)$) as

$$
\begin{aligned}
j_R(R_E, \theta, \phi) = \frac{1}{R_E^2} \Bigg[& \frac{\partial^2 \psi}{\partial \theta^2} \frac{\Sigma_0 \Sigma_P}{C} \\
& + \frac{\partial^2 \psi}{\partial \phi^2} \left\{ \frac{1}{\sin^2 \theta} \left(\Sigma_P + \frac{\Sigma_H^2 \sin^2 \varepsilon}{C} \right) \right\} \\
& + \frac{\partial \psi}{\partial \theta} \left\{ \frac{\partial}{\partial \theta} \left(\frac{\Sigma_0 \Sigma_P}{C} \right) + \cot \theta \frac{\Sigma_0 \Sigma_P}{C} \right. \\
& \qquad \left. + \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \left(\frac{\Sigma_0 \Sigma_H \cos \varepsilon}{C} \right) \right\} \qquad (2) \\
& + \frac{\partial \psi}{\partial \phi} \left\{ \frac{\partial}{\partial \theta} \left(\frac{\Sigma_0 \Sigma_H (-\cos \varepsilon)}{C \sin \theta} \right) \right. \\
& \qquad + \frac{1}{\sin^2 \theta} \frac{\partial}{\partial \phi} \left(\Sigma_P + \frac{\Sigma_H^2 \sin^2 \varepsilon}{C} \right) \\
& \qquad \left. + \frac{\Sigma_0 \Sigma_H (-\cos \varepsilon) \cot \theta}{C \sin \theta} \right\} \Bigg]_{R = R_E} .
\end{aligned}
$$

As discussed in Goodman (1995), his equation (23) can be elliptic, parabolic, or hyperbolic in terms of solving for ψ, depending on the values of θ and of the ratio $\Sigma_P(\theta, \phi)/\Sigma_H(\theta, \phi)$, resulting also in different types of boundary conditions required. The physical reason for that behaviour remains unclear. In contrast, eq. (2) above is elliptic throughout. Hence, the discussion in section 2.3 of Goodman's paper is unneccessary.

References

Goodman, M. L., A three-dimensional, iterative mapping procedure for the implementation of an ionosphere-magnetosphere anisotropic Ohm's law boundary condition in global magnetohydrodynamic simulations. *Ann. Geophysicae*, **13,** 843, 1995.

Rishbeth, H., and Garriot, O. K., *Introduction to ionospheric physics*, International Geophysics Series Vol. 14, edited by J. van Mieghem, Academic Press, New York, 1969.

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Letter to the Editor

Reply to Olaf Amm's comment on "A three-dimensional, iterative mapping procedure for the implementation of an ionosphere-magnetosphere anisotropic Ohm's law boundary condition in global magnetohydrodynamic simulations" by Michael L. Goodman

M. L. Goodman

Nasa Goddard Space Flight Center, Greenbelt MD 20771, USA

The conductance tensor used by Amm is valid when the space charge polarization effect mentioned by Amm is important. However, it is unlikely that this effect is important in the ionosphere on MHD time scales since the high conductivity along magnetic field lines causes any net space charge to be neutralized on time scales characteristic of the displacement current which are much shorter than MHD time scales. The upper and lower boundaries of the ionosphere are connected by highly conducting magnetic field lines. Quasineutrality will be maintained, so net space charge effects will be small.