The initial equation given by the goodman paper is as follows:

$$j_R = \frac{1}{R_e^2}\Bigg[\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)\frac{\partial^2\psi}{\partial\theta^2}$$

$$+ \frac{1}{\sin\theta}\left(\Sigma_{\theta\phi} + \Sigma_{\phi\theta} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\frac{\partial^2\psi}{\partial\phi\partial\theta}$$

$$+ \frac{\Sigma_{\phi\phi}}{\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \Bigg\{\frac{\partial}{\partial\theta}\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)$$

$$+ \cot\theta\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)$$

$$+ \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(\Sigma_{\phi\theta} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\Bigg\}\frac{\partial\psi}{\partial\theta}$$

$$+ \left\{\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial\Sigma_{\phi\phi}}{\partial\phi} + \frac{\Sigma_{\theta\phi}\cot\theta}{\sin\theta}\right\}\frac{\partial\psi}{\partial\phi}\Bigg]_{R=R_e}$$

By moving the R_E^2 term to the left side and then multiplying both sides by $\sin^2\theta$, we arrive at:

$$\sin^2\theta\cdot R_e^2 j_R = \Bigg[\sin^2\theta\cdot\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)\frac{\partial^2\psi}{\partial\theta^2}$$

$$+ \sin\theta\left(\Sigma_{\theta\phi} + \Sigma_{\phi\theta} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\frac{\partial^2\psi}{\partial\phi\partial\theta}$$

$$+ \Sigma_{\phi\phi}\frac{\partial^2\psi}{\partial\phi^2} + \Bigg\{\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)$$

$$+ \sin\theta\cos\theta\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)$$

$$+ \sin\theta\cdot\frac{\partial}{\partial\phi}\left(\Sigma_{\phi\theta} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\Bigg\}\frac{\partial\psi}{\partial\theta}$$

$$+ \left\{\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right) + \frac{\partial\Sigma_{\phi\phi}}{\partial\phi} + \Sigma_{\theta\phi}\cdot\cos\theta\right\}\frac{\partial\psi}{\partial\phi}\Bigg]_{R=R_e}$$

As given by the comment on the paper by O.Amm, the conductance tensor is:

$$\Sigma_{(R,\theta,\phi)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Sigma_0\Sigma_P}{C} & \frac{\Sigma_0\Sigma_H(-\cos\varepsilon)}{C} \\ 0 & \frac{\Sigma_0\Sigma_H\cos\varepsilon}{C} & \Sigma_P + \frac{\Sigma_H^2\sin^2\varepsilon}{C} \end{pmatrix}$$

From this, it is clear that $\Sigma_{\theta\phi} = -\Sigma_{\phi\theta}$ and $\Sigma_{\theta R} = \Sigma_{\phi R} = 0$. We can therefore, make subsitutions $\Sigma_{\psi\theta} = -\Sigma_{\theta\psi}$ on lines 2 and 6:

$$
\begin{aligned}
\sin^2\theta \cdot R_e^2 j_R = \Bigg[& \sin^2\theta \cdot \left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right)\frac{\partial^2\psi}{\partial\theta^2} \\
& + \sin\theta\left(\frac{\Sigma_{\phi R}\tan\theta}{2}\right)\frac{\partial^2\psi}{\partial\phi\partial\theta} \\
& + \Sigma_{\phi\phi}\frac{\partial^2\psi}{\partial\phi^2} + \Bigg\{\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right) \\
& + \sin\theta\cos\theta\left(\Sigma_{\theta\theta} - \frac{\Sigma_{\theta R}\tan\theta}{2}\right) \\
& + \sin\theta\cdot\frac{\partial}{\partial\phi}\left(-\Sigma_{\theta\phi} - \frac{\Sigma_{\phi R}\tan\theta}{2}\right)\Bigg\}\frac{\partial\psi}{\partial\theta} \\
& + \left\{\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right) + \frac{\partial\Sigma_{\phi\phi}}{\partial\phi} + \Sigma_{\theta\phi}\cdot\cos\theta\right\}\frac{\partial\psi}{\partial\phi}\Bigg]_{R=R_e}
\end{aligned}
$$

Additionally, we can remove all terms with a $\Sigma_{\psi R}$ or $\Sigma_{\theta R}$:

$$
\begin{aligned}
\sin^2\theta \cdot R_e^2 j_R = \Bigg[& \sin^2\theta\cdot\Sigma_{\theta\theta}\frac{\partial^2\psi}{\partial\theta^2} \\
& + \Sigma_{\phi\phi}\frac{\partial^2\psi}{\partial\phi^2} \\
& + \left\{\sin^2\theta\cdot\frac{\partial\Sigma_{\theta\theta}}{\partial\theta} + \sin\theta\cos\theta\cdot\Sigma_{\theta\theta} - \sin\theta\cdot\frac{\partial\Sigma_{\theta\phi}}{\partial\phi}\right\}\frac{\partial\psi}{\partial\theta} \\
& + \left\{\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right) + \frac{\partial\Sigma_{\phi\phi}}{\partial\phi} + \Sigma_{\theta\phi}\cdot\cos\theta\right\}\frac{\partial\psi}{\partial\phi}\Bigg]_{R=R_e}
\end{aligned}
$$

The $\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right)$ term in the first ϕ derivative coefficient can be expanded with the quotient rule, giving:

$$
\begin{aligned}
\sin^2\theta\cdot\frac{\partial}{\partial\theta}\left(\frac{\Sigma_{\theta\phi}}{\sin\theta}\right) &= \sin^2\theta\cdot\frac{\sin\theta\partial_\theta\Sigma_{\theta\phi} - \Sigma_{\theta\phi}\cos\theta}{\sin^2\theta} \\
&= \sin\theta\cdot\frac{\partial\Sigma_{\theta\phi}}{\partial\theta} - \Sigma_{\theta\phi}\cdot\cos\theta
\end{aligned}
$$

This cancels out the other term of $\Sigma_{\theta\phi}\cdot\cos\theta$ in the same coefficient. Separating out the coefficients, we arrive at the following expression for the original equation:

$$
\begin{aligned}
\sin^2\theta R_E^2 j_r &= \kappa_{\theta\theta}\frac{\partial^2\psi}{\partial\theta^2} + \kappa_{\psi\psi}\frac{\partial^2\phi}{\partial\psi^2} + \kappa_\theta\frac{\partial\psi}{\partial\theta} + \kappa_\phi\frac{\partial\psi}{\partial\phi} \\
\kappa_{\theta\theta} &= \sin^2\theta\cdot\Sigma_{\theta\theta} \\
\kappa_{\psi\psi} &= \Sigma_{\phi\phi} \\
\kappa_\theta &= \sin^2\theta\cdot\frac{\partial\Sigma_{\theta\theta}}{\partial\theta} + \sin\theta\cos\theta\cdot\Sigma_{\theta\theta} - \sin\theta\cdot\frac{\partial\Sigma_{\theta\phi}}{\partial\phi} \\
\kappa_\phi &= \sin\theta\cdot\frac{\partial\Sigma_{\theta\phi}}{\partial\theta} + \frac{\partial\Sigma_{\phi\phi}}{\partial\phi}
\end{aligned}
$$