CSE 102 6-4-24

- Final Exam: Th 6/13 9:30 - 11:00 am

- SETs : Sun. 6/9 11:59 pm

  If response rate is $\geq 85\%$,

  I will add 0.5% to overall scores.

Theorem

at least $\binom{n}{2}$ edge probes must be performed (in worst case) to determine connectedness of a graph on n vertices.

Proof

consider an algorithm for this problem and run it against the following adversary.

Strategy: answer no to all edge probes, unless that answer would prove the graph is disconnected.

more Precisely:

maintain Two edge sets $A, B \subseteq E(K_n)$ where initially $A = E(K_n)$ and $B = \phi$.

Adversary performs following algorithm each time an edge is Probed

Probe(e)

1. if $A - e$ is connected
2. $\quad A = A - e$
3. $\quad$ answer No
4. else
5. $\quad B = B + e$
6. $\quad$ answer Yes

<u>note</u>: always

- $B \subseteq A$
- $A - B = \{$not yet probed edges$\}$
- A, B are both consistent with all the adversary's answers.

The following invariants are maintained over any seq. of edge probes

(a) A is always connected

(b) If A contains a cycle, then none of its edges belong to B.

<u>Proof</u>

deleting an edge from a cycle in A would leave A connected. Such an edge would never be added to B.

(c) B is acyclic. This follows from (b)

(d) If $A \neq B$, then B is disconnected

<u>Proof.</u>

Assume that B is connected. Then, being acyclic, B is a tree. Since $A \neq B$ and $B \subseteq A$ there exists $e \in A - B$. If e were added to B it would form a cycle with some other edges of B. (see treeness thm.) Since $B \subseteq A$, that cycle would also be in A. $\therefore$ A contains a cycle consisting of e together with other edges in B, contradicting (b). $\therefore$ B is disconn.

Suppos the algorithm halts after less than $\binom{n}{2}$ probes. Then at least one edge of K_n was not probed, hence $A-B \neq \emptyset$, $\therefore$ $A \neq B$.

(d) says B is disconnected, while (a) says A is connected. Both graphs are consistent with the seq. of answers given by adversary.

■

Ex. $n=4$. $\binom{n}{2} = \frac{4\cdot 3}{2} = 6$

Probe edges

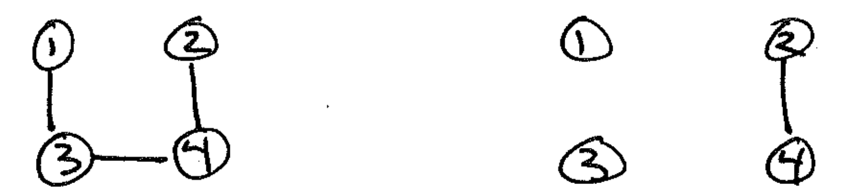
A B

$\{1,2\}$ no

$\{2,3\}$ no

$\{2,4\}$ yes

$\{1,4\}$ no



$\{3,4\}$ yes