CSE 102 4-25-24

<u>Ex.</u> $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

comp. $\frac{n}{\log n}$ to $n^{\log_2 2} = n^1$

observe: $\frac{n}{\log n} \neq O(n^{1-\varepsilon})$ for any $\varepsilon > 0$

why?

$$\frac{\frac{n}{\log n}}{n^{1-\varepsilon}} = \frac{\cancel{n} \cdot n^{\varepsilon}}{\cancel{n} \cdot \log n} \longrightarrow \infty \quad \text{as } n \to \infty$$

$\therefore \frac{n}{\log n} = \omega(n^{1-\varepsilon})$ $\therefore \frac{n}{\log n} \neq O(n^{1-\varepsilon})$

$\therefore$ not in case 1. $\therefore$ master thm does not apply.

Iteration Method

Ex. $$T(n) = \begin{cases} 1 & 1 \le n < 3 \\ 2T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + n & n \ge 3 \end{cases}$$

$$T(\text{☺}) = \text{☺} + 2T\left(\left\lfloor \frac{\text{☺}}{3} \right\rfloor\right) \qquad \underline{\underline{k}}$$

$$T(n) = n + 2T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) \qquad 1$$

$$= n + 2\left(\left\lfloor \frac{n}{3} \right\rfloor + 2T\left(\left\lfloor \frac{\lfloor n/3 \rfloor}{3} \right\rfloor\right)\right)$$

$$= n + 2\left\lfloor \frac{n}{3} \right\rfloor + 2^2\, T\left(\left\lfloor \frac{n}{3^2} \right\rfloor\right) \qquad 2$$

$$= n + 2\left\lfloor \frac{n}{3} \right\rfloor + 2^2\left(\left\lfloor \frac{n}{3^2} \right\rfloor + 2T\left(\left\lfloor \frac{\lfloor n/3^2 \rfloor}{3} \right\rfloor\right)\right)$$

$$= n + 2\left\lfloor \frac{n}{3} \right\rfloor + 2^2\left\lfloor \frac{n}{3^2} \right\rfloor + 2^3\, T\left(\left\lfloor \frac{n}{3^3} \right\rfloor\right) \qquad 3$$

$$\vdots$$

$$= \sum_{i=0}^{k-1} 2^i \left\lfloor \frac{n}{3^i} \right\rfloor + 2^k \cdot T\left(\left\lfloor \frac{n}{3^k} \right\rfloor\right)$$

recursion ends when

$$1 \le \left\lfloor \frac{n}{3^k} \right\rfloor < 3$$

$$\iff \quad 1 \le \frac{n}{3^k} < 3$$

$$\iff \quad 3^k \le n < 3^{k+1}$$

$$\iff \quad k \le \log_3(n) < k+1$$

$$\iff \quad \boxed{k = \lfloor \log_3(n) \rfloor}$$

$$\therefore \; T(n) = \sum_{i=0}^{k-1} 2^i \left\lfloor \frac{n}{3^i} \right\rfloor + 2^k,$$