A Course In Business Statistics

4th Edition

Chapter 13

Multiple Regression and Model Building

Chapter Goals

After completing this chapter, you should be able to:

- understand model building using multiple regression analysis
- apply multiple regression analysis to business decision-making situations
- analyze and interpret the computer output for a multiple regression model
- test the significance of the independent variables in a multiple regression model

Chapter Goals

(continued)

After completing this chapter, you should be able to:

- use variable transformations to model nonlinear relationships
- recognize potential problems in multiple regression analysis and take the steps to correct the problems.
- incorporate qualitative variables into the regression model by using dummy variables.

The Multiple Regression Model

Idea: Examine the linear relationship between
1 dependent (y) & 2 or more independent variables (x_i)

Population model:

Y-intercept

Population slopes

Random Error

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon$$

Estimated multiple regression model:

Estimated (or predicted) value of y

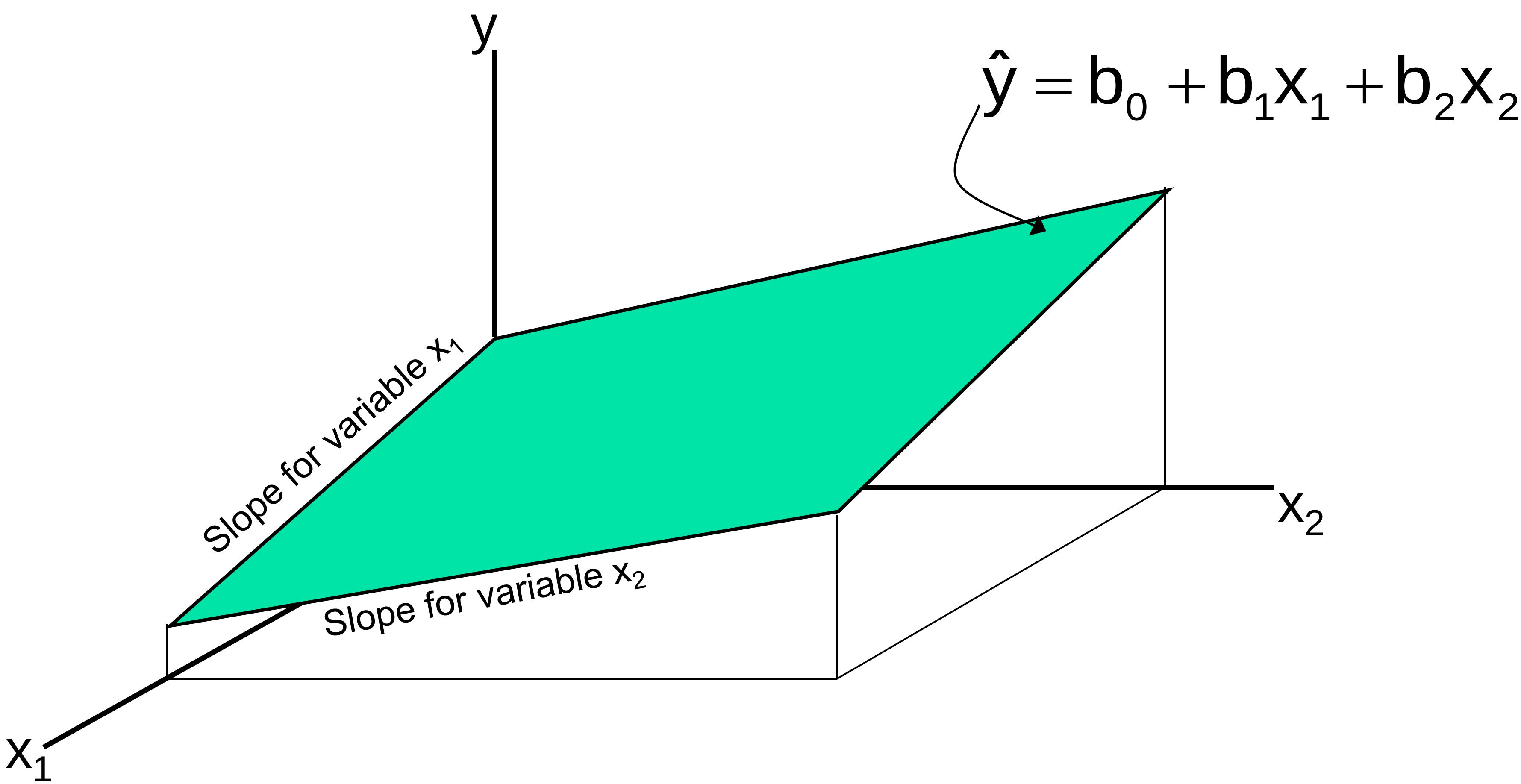
Estimated intercept

Estimated slope coefficients

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k$$

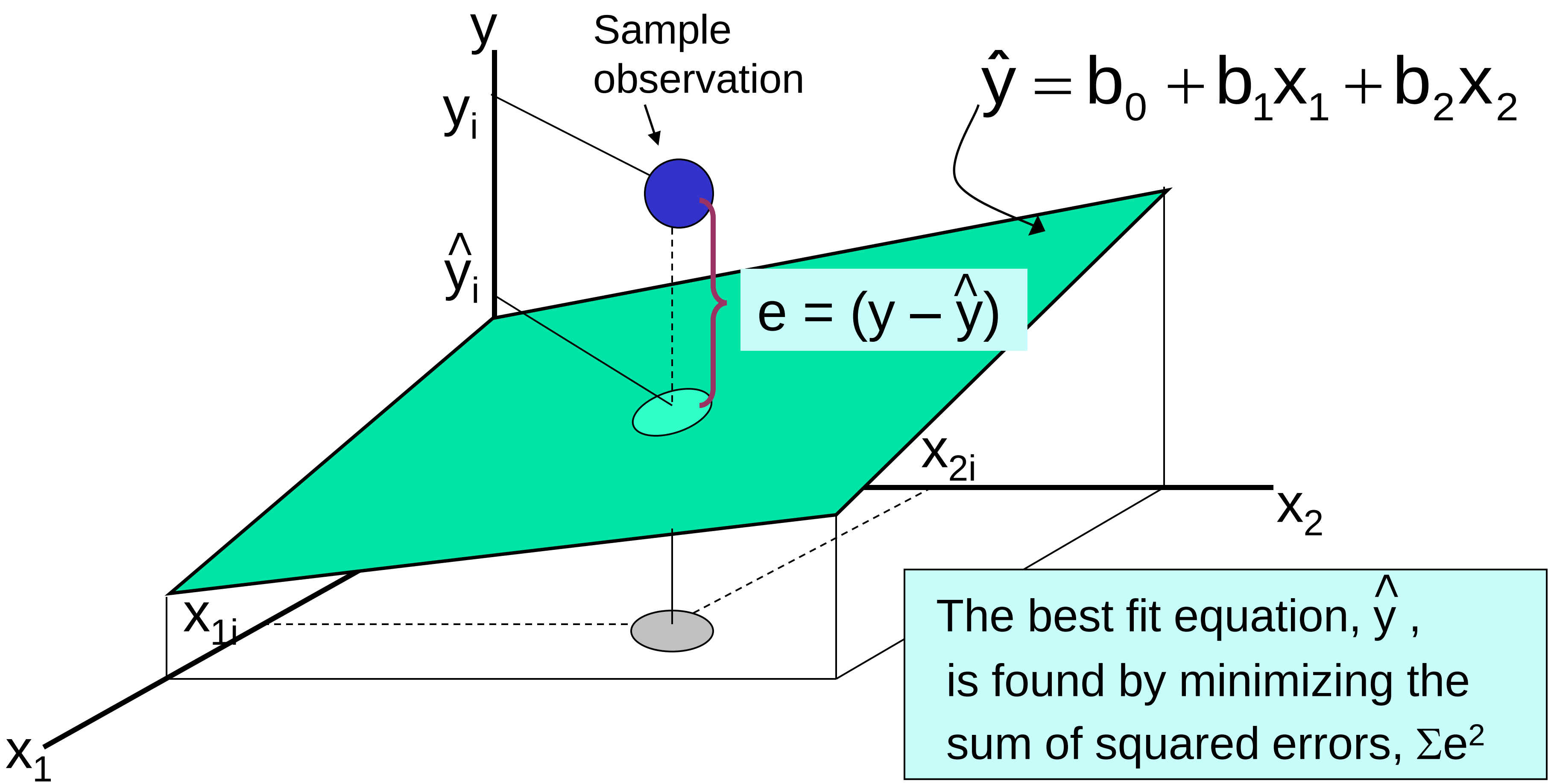
Multiple Regression Model

Two variable model



Multiple Regression Model

Two variable model



The best fit equation, $\hat{y}$, is found by minimizing the sum of squared errors, Σe^2

Multiple Regression Assumptions

Errors (residuals) from the regression model:

$$e = (y - \hat{y})$$

- The errors are normally distributed
- The mean of the errors is zero
- Errors have a constant variance
- The model errors are independent

Model Specification

- Decide what you want to do and select the dependent variable
- Determine the potential independent variables for your model
- Gather sample data (observations) for all variables

The Correlation Matrix

- Correlation between the dependent variable and selected independent variables can be found using Excel:
  - Tools / Data Analysis… / Correlation

- Can check for statistical significance of correlation with a t test

Example

- A distributor of frozen desert pies wants to evaluate factors thought to influence demand
  - Dependent variable: Pie sales (units per week)
  - Independent variables: Price (in $)
    Advertising ($100's)
- Data is collected for 15 weeks

Pie Sales Model

Week	Pie Sales	Price ($)	Advertising ($100s)
1	350	5.50	3.3
2	460	7.50	3.3
3	350	8.00	3.0
4	430	8.00	4.5
5	350	6.80	3.0
6	380	7.50	4.0
7	430	4.50	3.0
8	470	6.40	3.7
9	450	7.00	3.5
10	490	5.00	4.0
11	340	7.20	3.5
12	300	7.90	3.2
13	440	5.90	4.0
14	450	5.00	3.5
15	300	7.00	2.7

Multiple regression model:

$$\widehat{\text{Sales}} = b_0 + b_1\,(\text{Price}) + b_2\,(\text{Advertising})$$

Correlation matrix:

	Pie Sales	Price	Advertising
Pie Sales	1		
Price	-0.44327	1	
Advertising	0.55632	0.03044	1

Interpretation of Estimated Coefficients

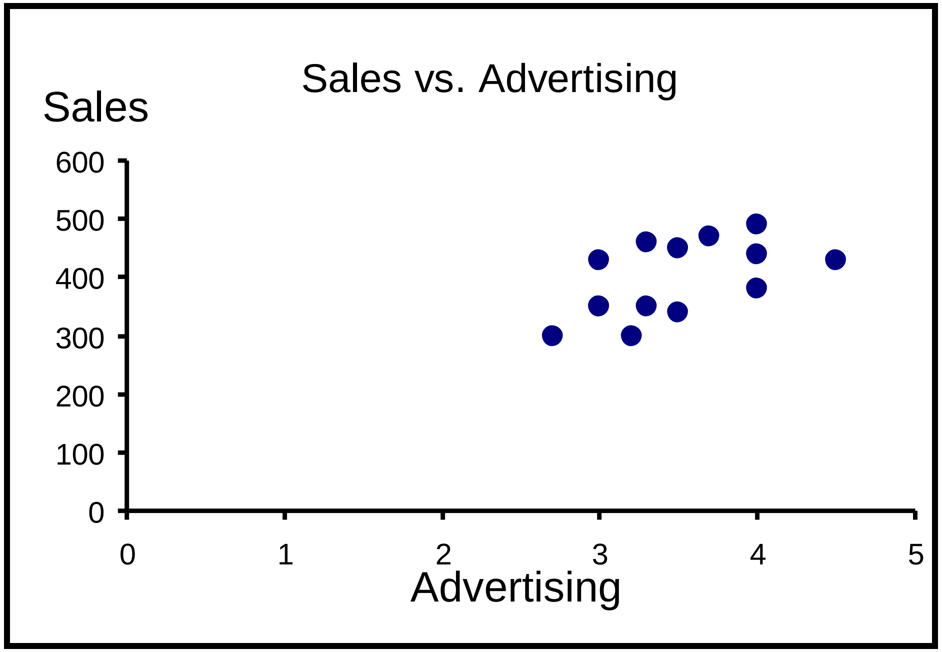
- Slope (b_i)
  - Estimates that the average value of y changes by b_i units for each 1 unit increase in X_i holding all other variables constant
  - Example: if b_1 = -20, then sales (y) is expected to decrease by an estimated 20 pies per week for each $1 increase in selling price (x_1), net of the effects of changes due to advertising (x_2)
- y-intercept (b_0)
  - The estimated average value of y when all x_i = 0 (assuming all x_i = 0 is within the range of observed values)

Pie Sales Correlation Matrix

	Pie Sales	Price	Advertising
Pie Sales	1		
Price	-0.44327	1	
Advertising	0.55632	0.03044	1

- Price vs. Sales : r = -0.44327
  - There is a negative association between price and sales
- Advertising vs. Sales : r = 0.55632
  - There is a positive association between advertising and sales

Scatter Diagrams

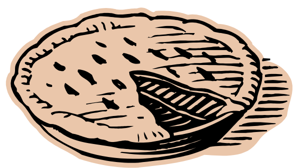


Estimating a Multiple Linear Regression Equation

- Computer software is generally used to generate the coefficients and measures of goodness of fit for multiple regression

- Excel:
  - Tools / Data Analysis... / Regression
- PHStat:
  - PHStat / Regression / Multiple Regression…

Multiple Regression Output



Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

$$Sales = 306.526 - 24.975(Price) + 74.131(Advertising)$$

ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
Total	14	56493.333			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	306.52619	114.25389	2.68285	0.01993	57.58835	555.46404
Price	-24.97509	10.83213	-2.30565	0.03979	-48.57626	-1.37392
Advertising	74.13096	25.96732	2.85478	0.01449	17.55303	130.70888

The Multiple Regression Equation

$$\widehat{\text{Sales}} = 306.526 - 24.975(\text{Price}) + 74.131(\text{Advertising})$$

where
Sales is in number of pies per week
Price is in $
Advertising is in $100's.

b_1 = -24.975: sales will decrease, on average, by 24.975 pies per week for each $1 increase in selling price, net of the effects of changes due to advertising

b_2 = 74.131: sales will increase, on average, by 74.131 pies per week for each $100 increase in advertising, net of the effects of changes due to price

Using The Model to Make Predictions

Predict sales for a week in which the selling price is $5.50 and advertising is $350:

$$\widehat{\text{Sales}} = 306.526 - 24.975(\text{Price}) + 74.131(\text{Advertising})$$

$$= 306.526 - 24.975(5.50) + 74.131(3.5)$$

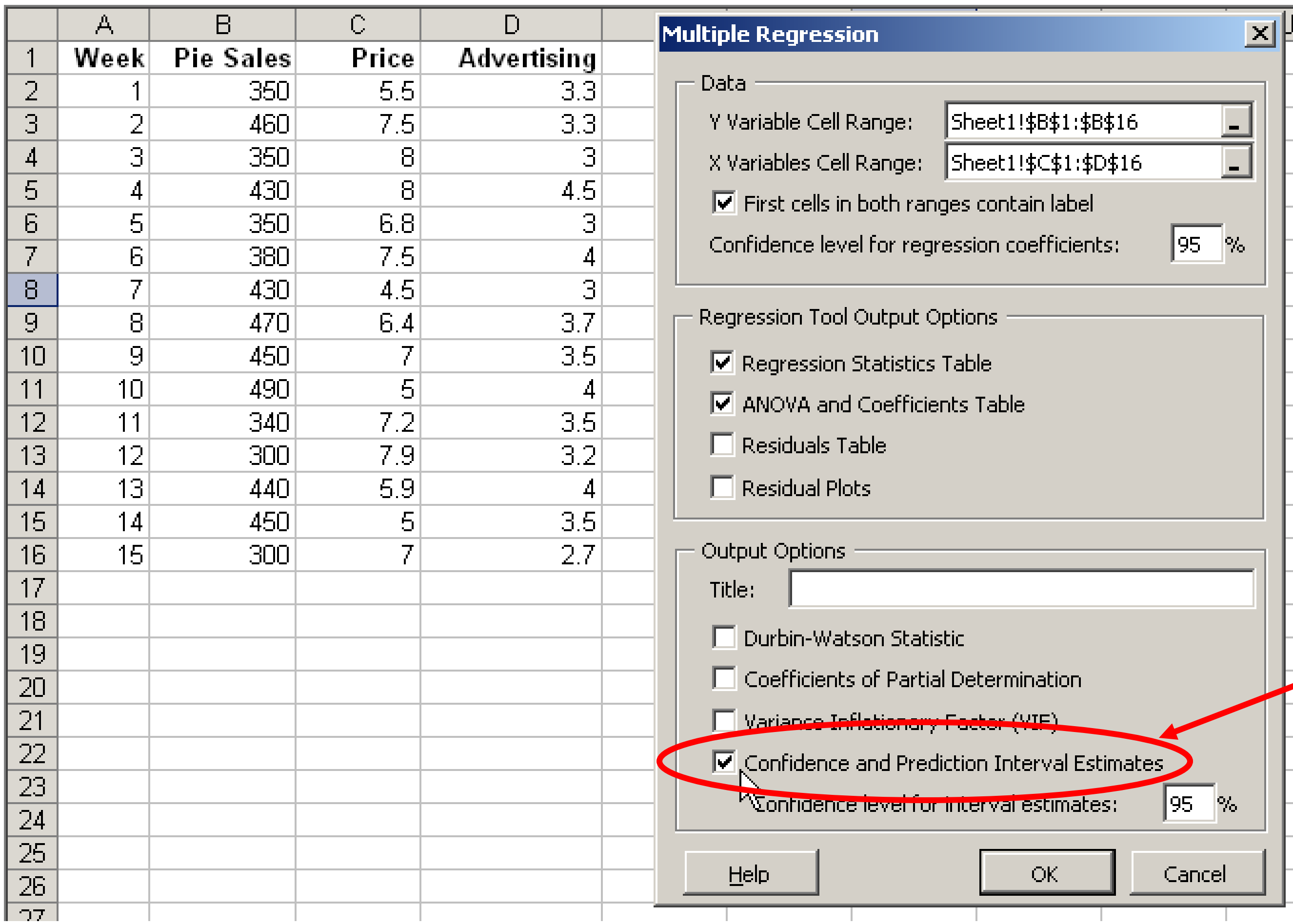
$$= 428.62$$

Predicted sales is 428.62 pies

Note that Advertising is in $100's, so $350 means that $x_2 = 3.5$

Predictions in PHStat

- PHStat | regression | multiple regression …



	A	B	C	D
1	Week	Pie Sales	Price	Advertising
2	1	350	5.5	3.3
3	2	460	7.5	3.3
4	3	350	8	3
5	4	430	8	4.5
6	5	350	6.8	3
7	6	380	7.5	4
8	7	430	4.5	3
9	8	470	6.4	3.7
10	9	450	7	3.5
11	10	490	5	4
12	11	340	7.2	3.5
13	12	300	7.9	3.2
14	13	440	5.9	4
15	14	450	5	3.5
16	15	300	7	2.7

Check the "confidence and prediction interval estimates" box

Predictions in PHStat

(continued)

	A	B
1	**Confidence and Prediction Estimate Intervals**	
2		
3	**Data**	
4	**Confidence Level**	**95%**
5		
6	**Price given value**	**5.5**
7	**Advertising given value**	**3.5**
8		
20	t Statistic	2.178813
21	**Predicted Y (YHat)**	**428.6216**
22		
23	**For Average Predicted Y (Yhat)**	
24	**Interval Half Width**	**37.50306**
25	**Confidence Interval Lower Limit**	**391.1185**
26	**Confidence Interval Upper Limit**	**466.1246**
27		
28	**For Individual Response Y**	
29	**Interval Half Width**	**110.0041**
30	**Prediction Interval Lower Limit**	**318.6174**
31	**Prediction Interval Upper Limit**	**538.6257**

Input values

Predicted $\hat{y}$ value

Confidence interval for the mean $\hat{y}$ value, given these x's

Prediction interval for an individual $\hat{y}$ value, given these x's

Multiple Coefficient of Determination

- Reports the proportion of total variation in y explained by all x variables taken together

$$R^2 = \frac{SSR}{SST} = \frac{\text{Sum of squares regression}}{\text{Total sum of squares}}$$

Multiple Coefficient of Determination

(continued)

Regression Statistics	
Multiple R	**0.72213**
R Square	**0.52148**
Adjusted R Square	**0.44172**
Standard Error	**47.46341**
Observations	**15**

$$R^2 = \frac{SSR}{SST} = \frac{29460.0}{56493.3} = .52148$$

52.1% of the variation in pie sales is explained by the variation in price and advertising

ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	**2**	**29460.027**	**14730.013**	**6.53861**	**0.01201**
Residual	**12**	**27033.306**	**2252.776**		
Total	**14**	**56493.333**			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	**306.52619**	**114.25389**	**2.68285**	**0.01993**	**57.58835**	**555.46404**
Price	**-24.97509**	**10.83213**	**-2.30565**	**0.03979**	**-48.57626**	**-1.37392**
Advertising	**74.13096**	**25.96732**	**2.85478**	**0.01449**	**17.55303**	**130.70888**

Adjusted R^2

- R^2 never decreases when a new x variable is added to the model
  - This can be a disadvantage when comparing models
- What is the net effect of adding a new variable?
  - We lose a degree of freedom when a new x variable is added
  - Did the new x variable add enough explanatory power to offset the loss of one degree of freedom?

Adjusted R^2

(continued)

- Shows the proportion of variation in y explained by all x variables adjusted for the number of x variables used

$$R_A^2 = 1 - (1 - R^2)\left(\frac{n-1}{n-k-1}\right)$$

(where n = sample size, k = number of independent variables)

  - Penalize excessive use of unimportant independent variables
  - Smaller than R^2
  - Useful in comparing among models

Multiple Coefficient of Determination

(continued)

Regression Statistics	
Multiple R	**0.72213**
R Square	**0.52148**
Adjusted R Square	**0.44172**
Standard Error	**47.46341**
Observations	**15**

$$R_A^2 = .44172$$

44.2% of the variation in pie sales is explained by the variation in price and advertising, taking into account the sample size and number of independent variables

ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	**2**	**29460.027**	**14730.013**	**6.53861**	**0.01201**
Residual	**12**	**27033.306**	**2252.776**		
Total	**14**	**56493.333**			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
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Is the Model Significant?

- F-Test for Overall Significance of the Model
- Shows if there is a linear relationship between all of the x variables considered together and y
- Use F test statistic
- Hypotheses:
  - H_0: $\beta_1 = \beta_2 = \ldots = \beta_k = 0$ (no linear relationship)
  - H_A: at least one $\beta_i \neq 0$ (at least one independent variable affects y)

F-Test for Overall Significance

(continued)

- Test statistic:

$$F = \frac{\dfrac{SSR}{k}}{\dfrac{SSE}{n-k-1}} = \frac{MSR}{MSE}$$

where F has (numerator) $D_1 = k$ and
(denominator) $D_2 = (n - k - 1)$
degrees of freedom

F-Test for Overall Significance

(continued)

Regression Statistics	
Multiple R	**0.72213**
R Square	**0.52148**
Adjusted R Square	**0.44172**
Standard Error	**47.46341**
Observations	**15**

$$F = \frac{MSR}{MSE} = \frac{14730.0}{2252.8} = 6.5386$$

With 2 and 12 degrees of freedom

P-value for the F-Test

ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	**2**	**29460.027**	**14730.013**	**6.53861**	**0.01201**
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F-Test for Overall Significance

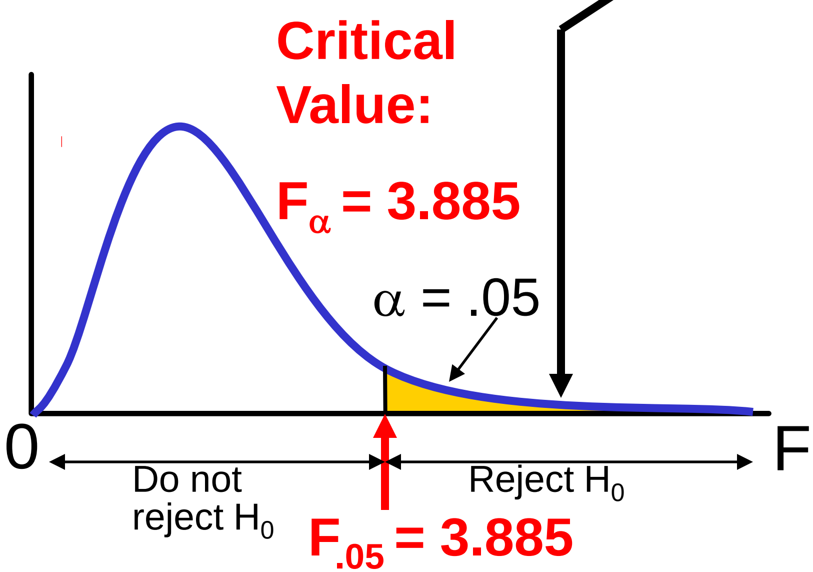
(continued)

$H_0: \beta_1 = \beta_2 = 0$

$H_A: \beta_1$ and β_2 not both zero

$\alpha = .05$

$df_1 = 2 \qquad df_2 = 12$



Test Statistic:

$$F = \frac{MSR}{MSE} = 6.5386$$

Decision:

Reject H_0 at $\alpha = 0.05$

Conclusion:

The regression model does explain a significant portion of the variation in pie sales

(There is evidence that at least one independent variable affects y)

Are Individual Variables Significant?

- Use t-tests of individual variable slopes
- Shows if there is a linear relationship between the variable x_i and y
- Hypotheses:
  - H_0: $\beta_i = 0$ (no linear relationship)
  - H_A: $\beta_i \neq 0$ (linear relationship does exist between x_i and y)

Are Individual Variables Significant?

(continued)

H_0: $\beta_i = 0$ (no linear relationship)

H_A: $\beta_i \neq 0$ (linear relationship does exist between x_i and y)

Test Statistic:

$$\boxed{t = \frac{b_i - 0}{s_{b_i}}}$$

(df = n – k – 1)

Are Individual Variables Significant?

(continued)

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

t-value for Price is t = -2.306, with p-value .0398

t-value for Advertising is t = 2.855, with p-value .0145

ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
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Residual	12	27033.306	2252.776		
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Inferences about the Slope: t Test Example

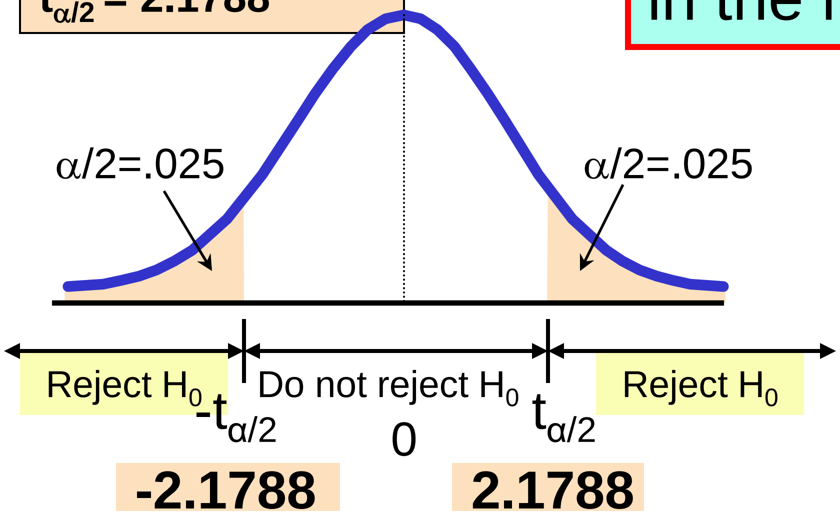
H_0: $\beta_i = 0$

H_A: $\beta_i \neq 0$

d.f. = 15-2-1 = 12

α = .05

$t_{\alpha/2}$ = 2.1788



From Excel output:

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Price	**-24.97509**	**10.83213**	**-2.30565**	**0.03979**
Advertising	**74.13096**	**25.96732**	**2.85478**	**0.01449**

The test statistic for each variable falls in the rejection region (p-values < .05)

Decision:

Reject H_0 for each variable

Conclusion:

There is evidence that both Price and Advertising affect pie sales at α = .05

Confidence Interval Estimate for the Slope

Confidence interval for the population slope β_1 (the effect of changes in price on pie sales):

$$b_i \pm t_{\alpha/2} S_{b_i}$$

where t has $(n - k - 1)$ d.f.

	Coefficients	*Standard Error*	…	*Lower 95%*	*Upper 95%*
Intercept	**306.52619**	**114.25389**	…	**57.58835**	**555.46404**
Price	**-24.97509**	**10.83213**	…	**-48.57626**	**-1.37392**
Advertising	**74.13096**	**25.96732**	…	**17.55303**	**130.70888**

Example: Weekly sales are estimated to be reduced by between 1.37 to 48.58 pies for each increase of $1 in the selling price

Standard Deviation of the Regression Model

- The estimate of the standard deviation of the regression model is:

$$s_\varepsilon = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{MSE}$$

- Is this value large or small? Must compare to the mean size of y for comparison

Standard Deviation of the Regression Model

(continued)

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

The standard deviation of the regression model is 47.46

ANOVA	*df*	*SS*	*MS*	*F*	*Significance F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
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Standard Deviation of the Regression Model

(continued)

- The standard deviation of the regression model is 47.46
- A rough prediction range for pie sales in a given week is $\pm 2(47.46) = 94.2$
- Pie sales in the sample were in the 300 to 500 per week range, so this range is probably too large to be acceptable. The analyst may want to look for additional variables that can explain more of the variation in weekly sales

Multicollinearity

- Multicollinearity: High correlation exists between two independent variables
- This means the two variables contribute redundant information to the multiple regression model

Multicollinearity

(continued)

- Including two highly correlated independent variables can adversely affect the regression results
  - No new information provided
  - Can lead to unstable coefficients (large standard error and low t-values)
  - Coefficient signs may not match prior expectations

Some Indications of Severe Multicollinearity

- Incorrect signs on the coefficients
- Large change in the value of a previous coefficient when a new variable is added to the model
- A previously significant variable becomes insignificant when a new independent variable is added
- The estimate of the standard deviation of the model increases when a variable is added to the model

Detect Collinearity (Variance Inflationary Factor)

VIF_j is used to measure collinearity:

$$VIF_j = \frac{1}{1 - R_j^2}$$

R^2_j is the coefficient of determination when the j^{th} independent variable is regressed against the remaining $k - 1$ independent variables

If $VIF_j > 5$, x_j is highly correlated with the other explanatory variables

Detect Collinearity in PHStat

PHStat / regression / multiple regression …

Check the “variance inflationary factor (VIF)” box

Regression Analysis	
Price and all other X	
Regression Statistics	
Multiple R	0.030437581
R Square	0.000926446
Adjusted R Square	-0.075925366
Standard Error	1.21527235
Observations	15
VIF	**1.000927305**

Output for the pie sales example:

- Since there are only two explanatory variables, only one VIF is reported
  - VIF is < 5
  - There is no evidence of collinearity between Price and Advertising

Qualitative (Dummy) Variables

- Categorical explanatory variable (dummy variable) with two or more levels:
  - yes or no, on or off, male or female
  - coded as 0 or 1
- Regression intercepts are different if the variable is significant
- Assumes equal slopes for other variables
- The number of dummy variables needed is (number of levels - 1)

Dummy-Variable Model Example (with 2 Levels)

Let:

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2$$

y = pie sales

x_1 = price

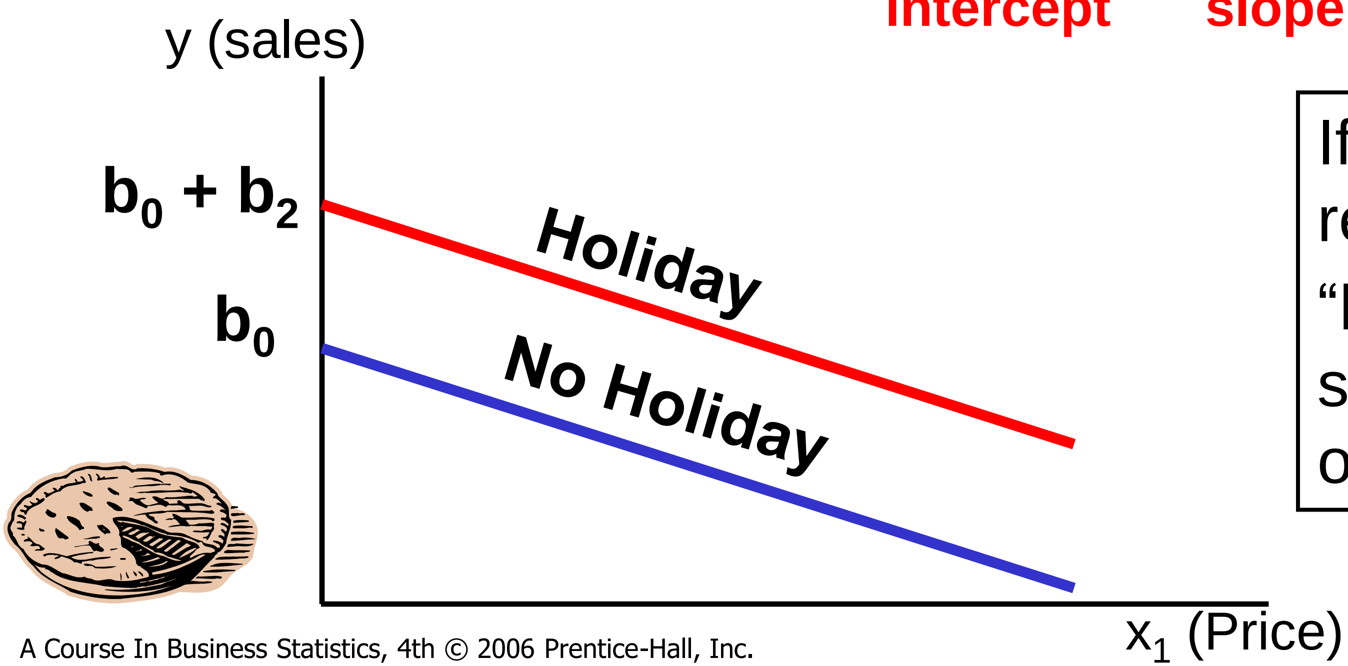
x_2 = holiday (X_2 = 1 if a holiday occurred during the week)
(X_2 = 0 if there was no holiday that week)

Dummy-Variable Model Example (with 2 Levels)

(continued)

$\hat{y} = b_0 + b_1 x_1 + b_2(1) = (b_0 + b_2) + b_1 x_1$	Holiday
$\hat{y} = b_0 + b_1 x_1 + b_2(0) = \quad b_0 \quad + b_1 x_1$	No Holiday

Different intercept **Same slope**



If $H_0: \beta_2 = 0$ is rejected, then "Holiday" has a significant effect on pie sales

Interpretation of the Dummy Variable Coefficient (with 2 Levels)

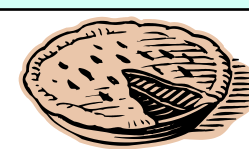
Example: $\text{Sales} = 300 - 30(\text{Price}) + 15(\text{Holiday})$

Sales: number of pies sold per week

Price: pie price in $

$$\text{Holiday:} \begin{cases} 1 & \text{If a holiday occurred during the week} \\ 0 & \text{If no holiday occurred} \end{cases}$$

$b_2 = 15$: on average, sales were 15 pies greater in weeks with a holiday than in weeks without a holiday, given the same price



Dummy-Variable Models (more than 2 Levels)

- The number of dummy variables is **one less than the number of levels**
- Example:

  y = house price ; x_1 = square feet

- The style of the house is also thought to matter:

  $$\text{Style} = \underbrace{\text{ranch, split level, condo}}$$

Three levels, so two dummy variables are needed

Dummy-Variable Models (more than 2 Levels)

(continued)

Let the default category be "condo"

$$x_2 = \begin{cases} 1 \text{ if ranch} \\ 0 \quad \text{if not} \end{cases} \qquad x_3 = \begin{cases} 1 \text{ if splitlevel} \\ 0 \quad \text{if not} \end{cases}$$

$$\boxed{\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3}$$

b_2 shows the impact on price if the house is a ranch style, compared to a condo

b_3 shows the impact on price if the house is a split level style, compared to a condo

Interpreting the Dummy Variable Coefficients (with 3 Levels)

Suppose the estimated equation is

$$\hat{y} = 20.43 + 0.045x_1 + 23.53x_2 + 18.84x_3$$

For a condo: $x_2 = x_3 = 0$

$$\hat{y} = 20.43 + 0.045x_1$$

For a ranch: $x_3 = 0$

$$\hat{y} = 20.43 + 0.045x_1 + 23.53$$

For a split level: $x_2 = 0$

$$\hat{y} = 20.43 + 0.045x_1 + 18.84$$

With the same square feet, a ranch will have an estimated average price of 23.53 thousand dollars more than a condo

With the same square feet, a ranch will have an estimated average price of 18.84 thousand dollars more than a condo.

Nonlinear Relationships

- The relationship between the dependent variable and an independent variable may not be linear
- Useful when scatter diagram indicates non-linear relationship
- Example: Quadratic model
  - $$y = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \varepsilon$$
  - The second independent variable is the square of the first variable

Polynomial Regression Model

General form:

$$y = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \ldots + \beta_p x_j^p + \varepsilon$$

- where:

  β_0 = Population regression constant

  β_i = Population regression coefficient for variable x_j : j = 1, 2, …*k*

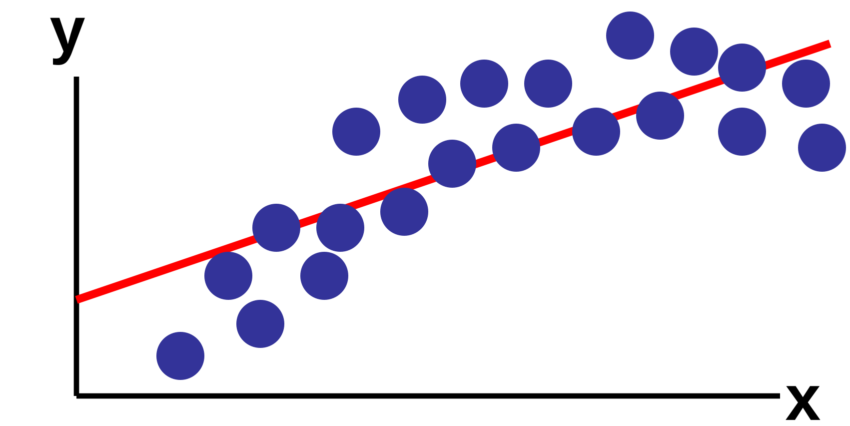
  p = Order of the polynomial

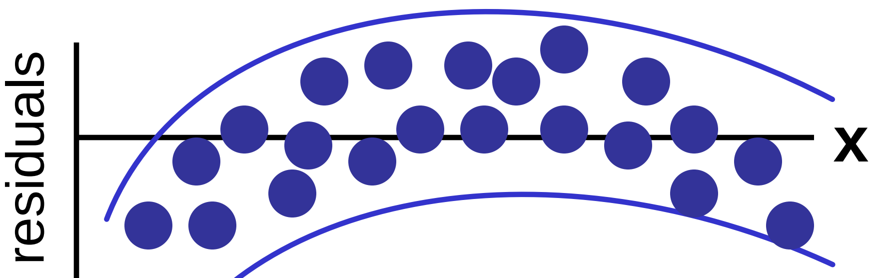
  ε_i = Model error

If p = 2 the model is a quadratic model:

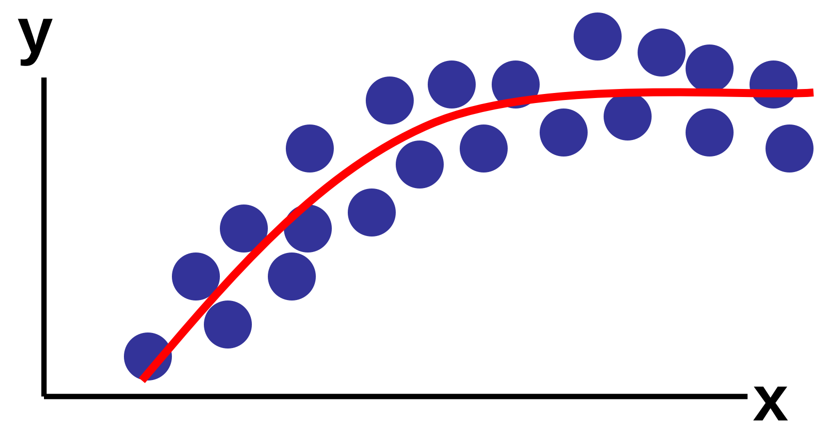
$$y = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \varepsilon$$

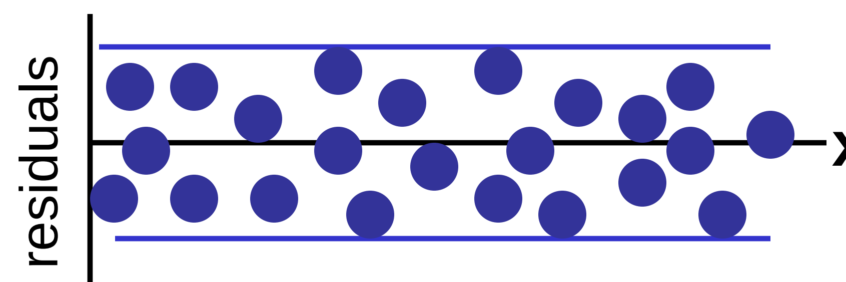
Linear vs. Nonlinear Fit





Linear fit does not give random residuals



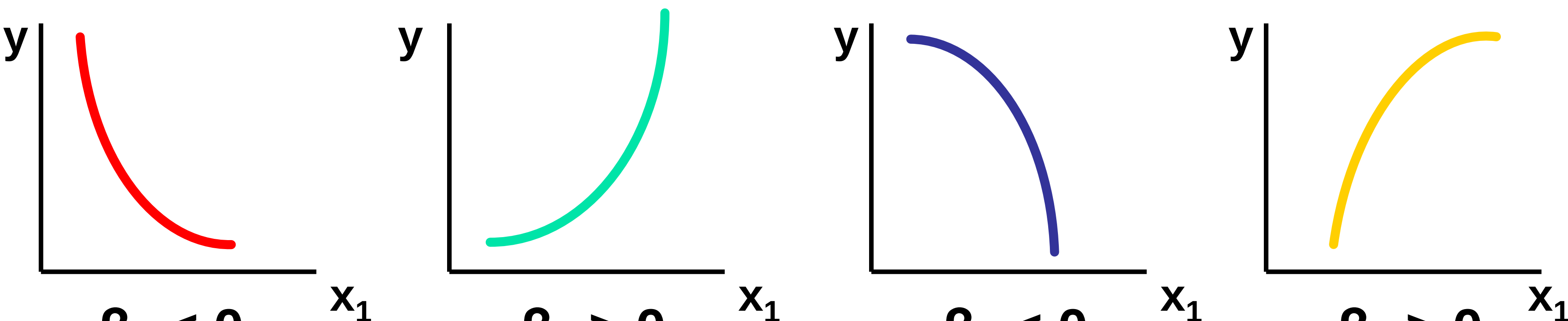


Nonlinear fit gives random residuals

Quadratic Regression Model

$$y = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \varepsilon$$

Quadratic models may be considered when scatter diagram takes on the following shapes:



$\beta_1 < 0$	$\beta_1 > 0$	$\beta_1 < 0$	$\beta_1 > 0$
$\beta_2 > 0$	$\beta_2 > 0$	$\beta_2 < 0$	$\beta_2 < 0$

β_1 = the coefficient of the linear term
β_2 = the coefficient of the squared term

Testing for Significance: Quadratic Model

- Test for Overall Relationship
  - F test statistic = $\frac{MSR}{MSE}$
- Testing the Quadratic Effect
  - Compare quadratic model

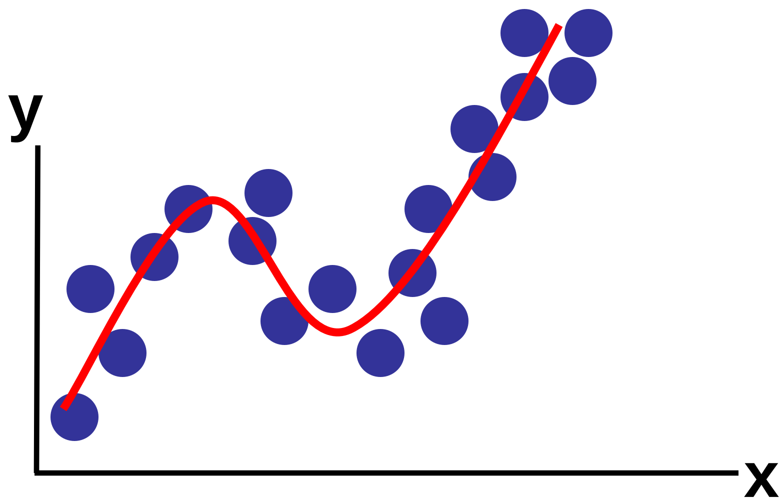
$$\boxed{y = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \varepsilon}$$

with the linear model

$$\boxed{y = \beta_0 + \beta_1 x_j + \varepsilon}$$

  - Hypotheses
    - $H_0: \beta_2 = 0$ (No 2nd order polynomial term)
    - $H_A: \beta_2 \neq 0$ (2nd order polynomial term is needed)

Higher Order Models



If p = 3 the model is a cubic form:

$$y = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \beta_3 x_j^3 + \varepsilon$$

Interaction Effects

- Hypothesizes interaction between pairs of x variables
  - Response to one x variable varies at different levels of another x variable
- Contains two-way cross product terms

$$\boxed{y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1^2 x_2}$$

Basic Terms

Interactive Terms

Effect of Interaction

- Given:

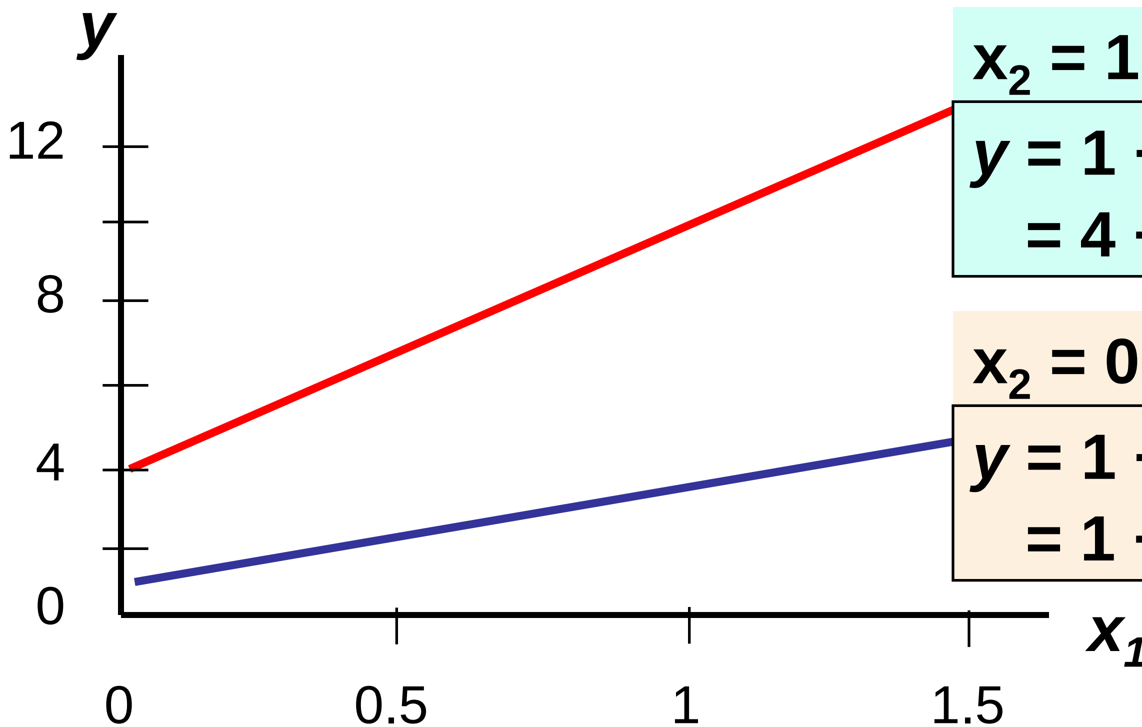
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \varepsilon$$

- Without interaction term, effect of x_1 on y is measured by β_1
- With interaction term, effect of x_1 on y is measured by $\beta_1 + \beta_3 x_2$
- Effect changes as x_2 increases

Interaction Example

$$y = 1 + 2x_1 + 3x_2 + 4x_1x_2$$

where x_2 = 0 or 1 (dummy variable)



$x_2 = 1$

$$y = 1 + 2x_1 + 3(1) + 4x_1(1) \\ = 4 + 6x_1$$

$x_2 = 0$

$$y = 1 + 2x_1 + 3(0) + 4x_1(0) \\ = 1 + 2x_1$$

Effect (slope) of x_1 on y does depend on x_2 value

Interaction Regression Model Worksheet

Case, i	y_i	x_{1i}	x_{2i}	$x_{1i}\, x_{2i}$
1	1	1	3	3
2	4	8	5	40
3	1	3	2	6
4	3	5	6	30
:	:	:	:	:

multiply x_1 by x_2 to get x_1x_2, then run regression with y, x_1, x_2 , x_1x_2

Evaluating Presence of Interaction

- Hypothesize interaction between pairs of independent variables

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \varepsilon$$

- Hypotheses:
  - H_0: $\beta_3 = 0$ (no interaction between x_1 and x_2)
  - H_A: $\beta_3 \neq 0$ (x_1 interacts with x_2)

Model Building

- Goal is to develop a model with the best set of independent variables
  - Easier to interpret if unimportant variables are removed
  - Lower probability of collinearity
- Stepwise regression procedure
  - Provide evaluation of alternative models as variables are added
- Best-subset approach
  - Try all combinations and select the best using the highest adjusted R^2 and lowest s_ε

Stepwise Regression

- Idea: develop the least squares regression equation in steps, either through forward selection, backward elimination, or through standard stepwise regression

- The coefficient of partial determination is the measure of the marginal contribution of each independent variable, given that other independent variables are in the model

Best Subsets Regression

- Idea: estimate all possible regression equations using all possible combinations of independent variables

- Choose the best fit by looking for the highest adjusted R^2 and lowest standard error s_ε

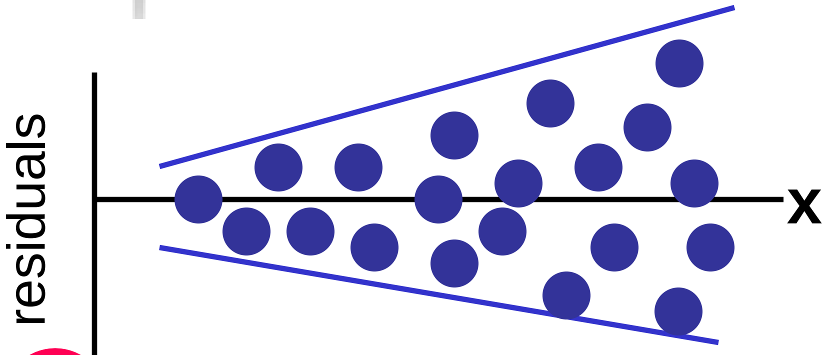
Stepwise regression and best subsets regression can be performed using PHStat, Minitab, or other statistical software packages

Aptness of the Model

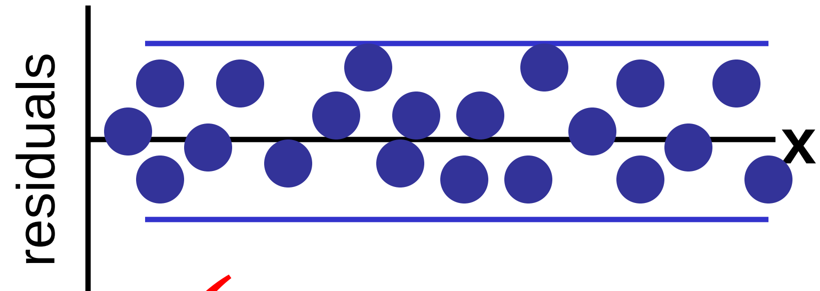
- Diagnostic checks on the model include verifying the assumptions of multiple regression:
  - Each x_i is linearly related to y
  - Errors have constant variance
  - Errors are independent
  - Error are normally distributed

Errors (or Residuals) are given by $\boxed{e_i = (y - \hat{y})}$

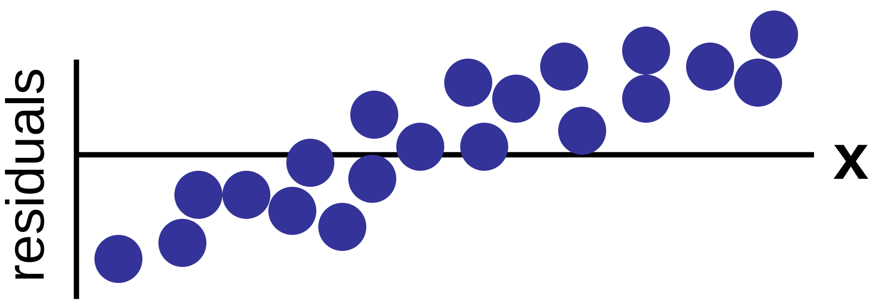
Residual Analysis



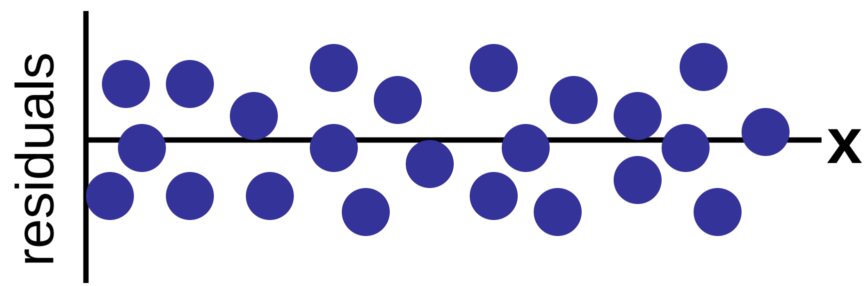
Non-constant variance



Constant variance



Not Independent



Independent

The Normality Assumption

- Errors are assumed to be normally distributed
- Standardized residuals can be calculated by computer
- Examine a histogram or a normal probability plot of the standardized residuals to check for normality

Chapter Summary

- Developed the multiple regression model
- Tested the significance of the multiple regression model
- Developed adjusted R^2
- Tested individual regression coefficients
- Used dummy variables
- Examined interaction in a multiple regression model

Chapter Summary

(continued)

- Described nonlinear regression models
- Described multicollinearity
- Discussed model building
  - Stepwise regression
  - Best subsets regression
- Examined residual plots to check model assumptions