Econ 311: Problem Set #6

Due: Monday, December 15, 2008

Q.1 A random sample of 10 economists produced the following forecasts for percentage growth in real growth domestic product in the next year:

2.2 2.8 3.0 2.5 2.4 2.6 2.5 2.4 2.7 2.6

Use unbiased estimation procedures to find point estimates for:

a The population mean

Unbiased point estimator of the population mean is the sample mean: $\bar{X} = \frac{\sum_i^{n=10} X_i}{n} = 2.57.$

b The population variance

Unbiased point estimator of the population variance: $s^2 = \frac{1}{n-1}\sum_i^{n=10}(x_i - \bar{x})^2 = 0.0512$

c The variance of the sample mean

Unbiased point estimate of the variance of the sample mean $Var(\bar{X}) = s^2/n = 0.0512/10 = 0.00512$

d The population proportion of economists predicting growth of at least 2.5% in real domestic product

Unbiased estimate of the population proportion: $\hat{p} = \frac{x}{n} = \frac{7}{10} = 0.70$

e The variance of the sample proportion of economists predicting growth of at least 2.5% in real gross domestic product

Unbiased estimate of the variance of the sample proportion:$Var(\hat{p}) = \frac{\hat{p}(1-\hat{p})}{n} = \frac{0.7(1-0.7)}{10} = 0.021$

Q.2 A college admissions officer for an MBA program has determined that historically applicants have undergraduate grade point averages that are normally distributed with standard deviation with 0.45. From a random sample of 25 applications from the current year, the sample mean grade point average is 2.90.

a Find a 95% confidence interval for the population mean.

$n = 25$, $\bar{x} = 2.90$, $\sigma = 0.45$, $Z_{0.025} = 1.96$.
$\bar{x} \pm z(\sigma/\sqrt{n}) = 2.90 \pm 1.96(0.45/5)$ =2.7236 up to 3.0764.

Q.3 Times(in minutes) that a random sample of five people spend driving to work are 30 42 35 40 45

a Calculate the standard error.

$\bar{x} = 38.40$, $s = 5.94138$, so standard erro=$s/\sqrt{n} = 5.94138/\sqrt{5} = 2.6571$

b Find $t_{v,\alpha/2}$ for a 95% interval for the true population mean.

$t_{v,\alpha/2} = t_{4,0.025} = 2.776$

c Calculate width for a 95% confidence interval for the population mean time spent driving to work.
$W = 2ME = t_{n-1,\alpha/2}s/\sqrt{n} = 2 \times 2.776(5.94138/\sqrt{5}) = 14.752$

Q.4 A business school placement director wants to estimated the mean annual salaries five years after students graduate. A random sample of 25 such graduates found a sample mean of \$42,740 and a sample standard deviation of \$4,780. Find a 90% confidence interval for the population mean, assuming that the population distribution is normal.

$n = 25$, $\bar{x} = 42,740$, $s = 4,780$, $t_{24,0.05} = 1.711$,
so $42,740 \pm 1.711(4780/5) = \$41,104.28$ up to \$44,375.72.

Q.5 From a random sample of 400 registered voters in one city, 320 indicated that they would vote in favors of a proposed policy in an upcoming election.

a Calculate the LCL (Low confidence limit) for a 98% confidence interval estimates for the population proportion of this policy. $n = 400$, $\hat{p} = 320/400 = 0.80$, $z_{0.01} = 2.326$.
$LCL = \hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.80 - 2.326\sqrt{\frac{0.8(1-0.8)}{400}} = 0.75348$

b Calculate the width of a 90% confidence interval estimates for the population proportion in favor of this policy.
$w = 2ME = 2z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 2 \cdot 1.645\sqrt{\frac{0.8(1-0.8)}{400}} = 0.0658$