CA200 – Quantitative Analysis for Business Decisions

File name: CA200_Section_03B_DecisionTheory

Table of Contents

3. **Decision theory**

3.1 Elements of a decision problem (See …3A …)

3.2 Decision making under risk (See …3A …)

3.3 Decision making under Uncertainty (See …3A …)

3.4 Sequential decisions – Decision Trees

Decision trees are made up of:

Square nodes – decision points

Round nodes – chance events (or outcome nodes)

Decision trees can be useful in dealing with problems involving "Imperfect Information" (such as Example 2 of section 3.2.3). Such problems involve the question of Information acquisition leading to decisions about

Choice of information source (to obtain the "imperfect information")

Choice of response to information (strategy)

Example 1:

An oil company must decide whether to drill (a_1) or not to drill (a_2) in a particular place in the Celtic sea. The well may turn out to be dry (θ_1), wet (θ_2), or soaking (θ_3), and on the basis of other drillings in the Celtic sea the company believes that the probabilities for these states are as follows:

$P(\theta_1) = 0.5$

$P(\theta_2) = 0.3$

$P(\theta_3) = 0.2$

The cost of drilling is \$70,000. If the well turns out to be wet the revenue will be \$120,000 and if it turns out to be soaking the revenue will be \$270,000. (There is no revenue for a dry well). Should the company drill or not?

(A) Solution without any more information: As usual, calculate the expected value of each action and choose the action that gives the larger expected value.

E(drilling) = -70,000 (0.5) + 50,000 (0.3) + 200,000 (0.2) = 20,000

E(not drilling) = 0

Therefore choose to drill.

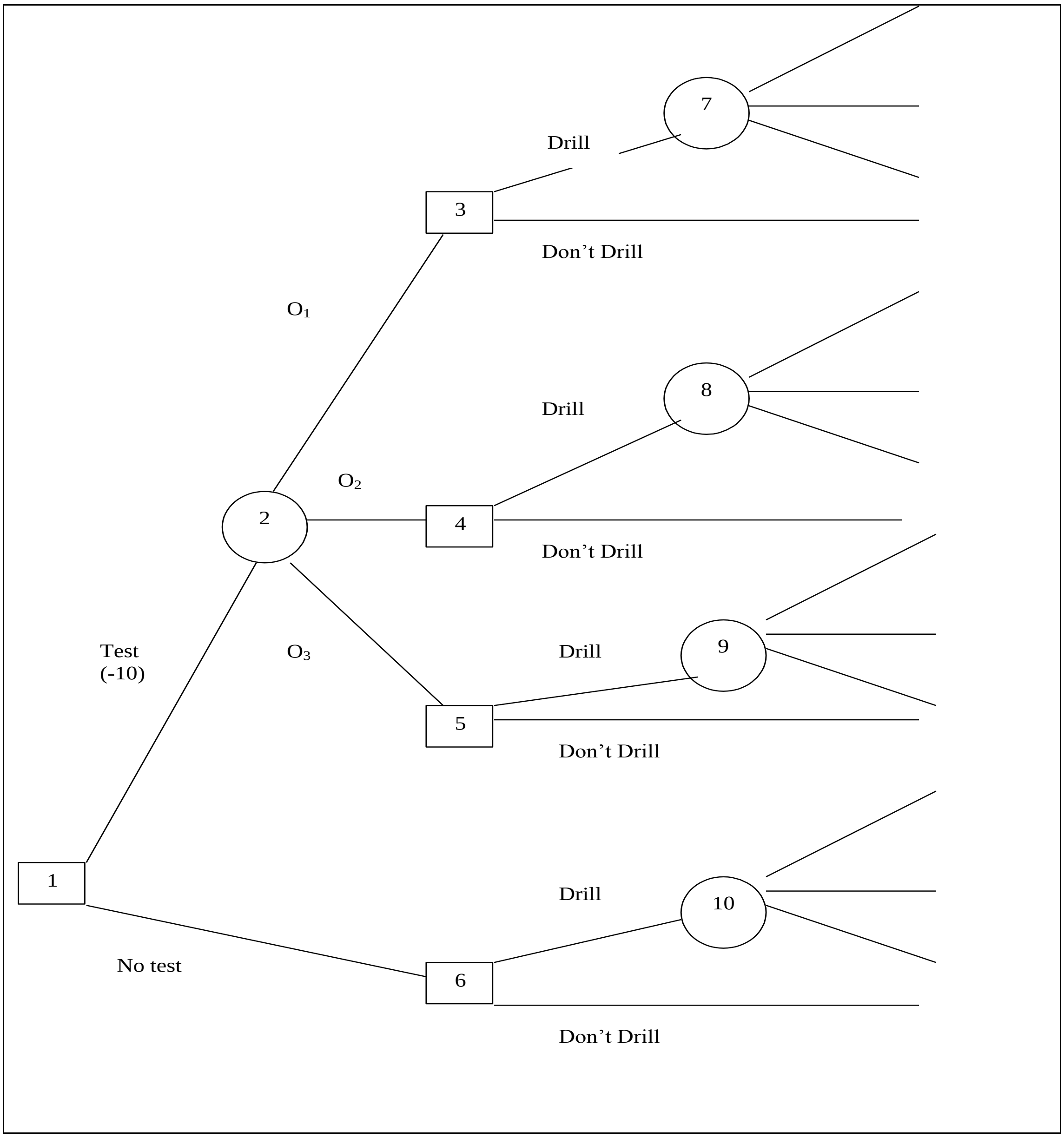
(B) <u>Solution for a situation where more (imperfect) information can be obtained</u>:

At a cost of $10,000 the company could take seismic soundings that will help to determine the underlying geological structure at the site. The soundings will show whether the ground has no structure (O_1), an open structure (O_2) or a closed structure (O_3). In the past, oil wells which have been dry, wet or soaking have had the following conditional probabilities associated with seismic test outcomes:

<u>Conditional probabilities – $P(O_k \mid \theta_j)$:</u>

Geological Structure **Well type**	O_1 *(None)*	O_2 *(Open)*	O_3 *(Closed)*
θ_1 *(Dry)*	0.6	0.3	0.1
θ_2 *(Wet)*	0.3	0.4	0.3
θ_3 *(Soaking)*	0.1	0.4	0.5

<u>Next we form a Decision Tree to represent the problem</u>. We will see that there are two phases (forward & backward) in drawing and adding data to such a tree.



<u>Information we have</u>:-

$P(\theta_j)$ $j = 1 \ldots 3$ *[What is likely well type?]*

$P(O_k | \theta_j)$ $k = 1 \ldots 3,\ j = 1 \ldots 3$ [*Structure if* well *type is given*]

<u>Information we need</u>:-

$P(O_k)$ $k = 1 \ldots 3$ *[What is likely structure?]*

$P(\theta_j | O_k)$ $k = 1 \ldots 3,\ j = 1 \ldots 3$ [*Well type if structure is given*]

We use our knowledge of <u>conditional probability</u> to get the information we need from what we know (same type of problem as encountered before).

<u>Note</u>: In the following we will use the common notation "$O_k \cap \theta_j$" to mean the joint event of "O_k *and* θ_j". For example, "$O_1 \cap \theta_2$" means the joint event of "No structure" and "Wet well".

From conditional probability rule,

$$P(O_k \mid \theta_j) = P(O_k \cap \theta_j) / P(\theta_i)$$

$$=> \quad P(O_k \cap \theta_j) = P(O_k \mid \theta_j) \times P(\theta_i) \qquad \text{[Items on right side are known]}$$

Then, we can find

$$P(O_k) = \sum P(O_k \cap \theta_j)$$

[e.g. p(open) = p(open|dry)+p(open|wet)+p(open|soaking)]

Therefore, again using the conditional probability rule,

$$P(\theta_j \mid O_k) = P(O_k \cap \theta_j) / P(O_k)$$

It is easiest to tabulate the calculations:

Joint probabilities: $P(O_k \cap \theta_j)$

<u>Well type</u>	O_1	O_2	O_3
θ_1	0.30	0.15	0.05
θ_2	0.09	0.12	0.09
θ_3	0.02	0.08	0.10
Column sums	$\boldsymbol{P(O_1)}$ **= 0.41**	$\boldsymbol{P(O_2)}$ **= 0.35**	$\boldsymbol{P(O_3)}$ **= 0.24**

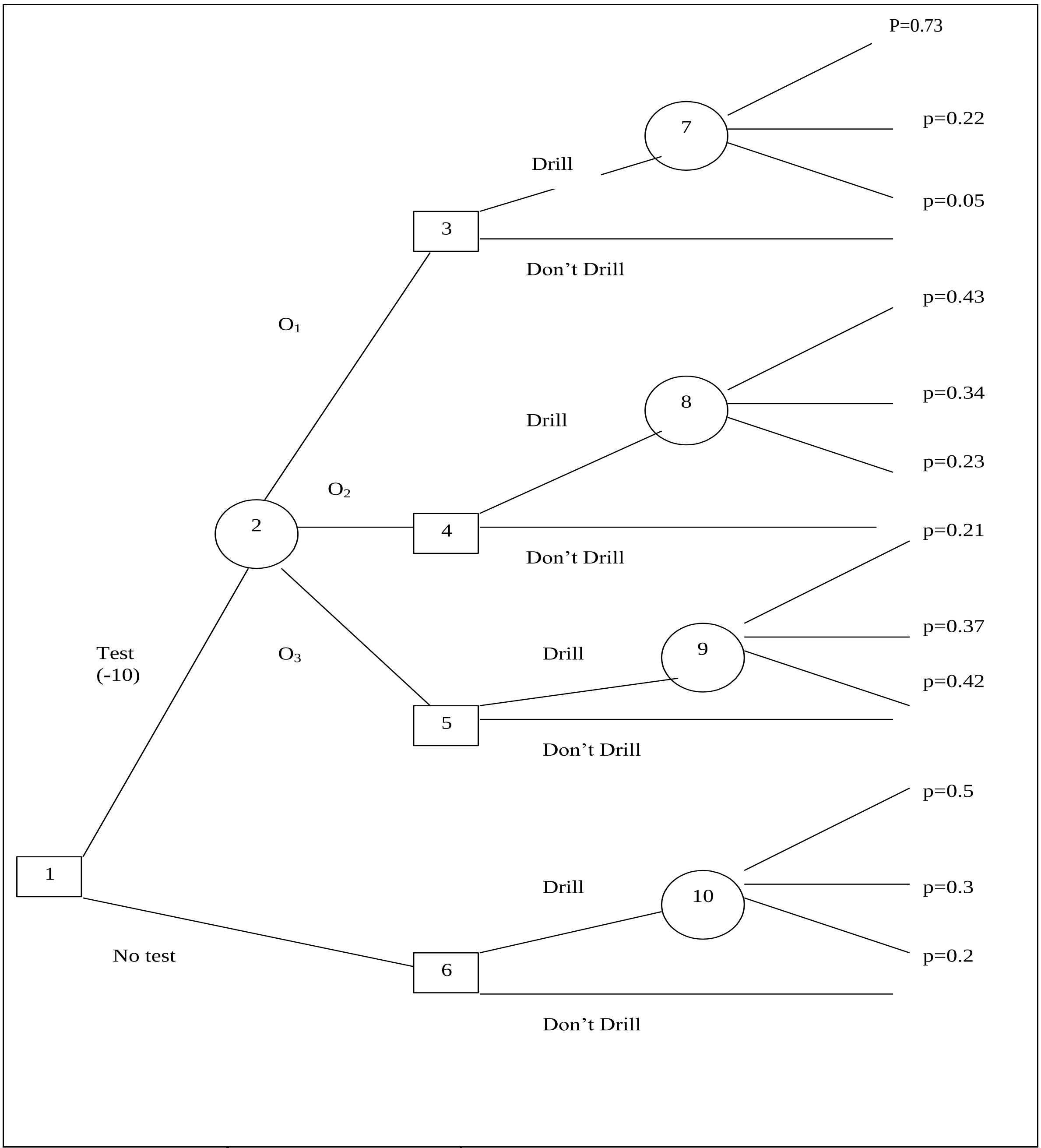
Conditional probabilities – $P(\theta_j \mid O_k)$

<u>Well type</u>	O_1	O_2	O_3
θ_1	0.73 (= *0.30/0.41*)	0.43	0.21
θ_2	0.22	0.34	0.37
θ_3	0.05	0.23	0.42 (=*0.10/0.24*)

Now we can add the probabilities and (hence) expected values at the nodes of the decision tree (backward induction).

Note: For convenience and clarity we will redraw the tree with various levels of data added, just to show how the process works. In practice, you would just draw the tree once and add data to it as you work through the problem.

First, we present the tree with the probability data added:

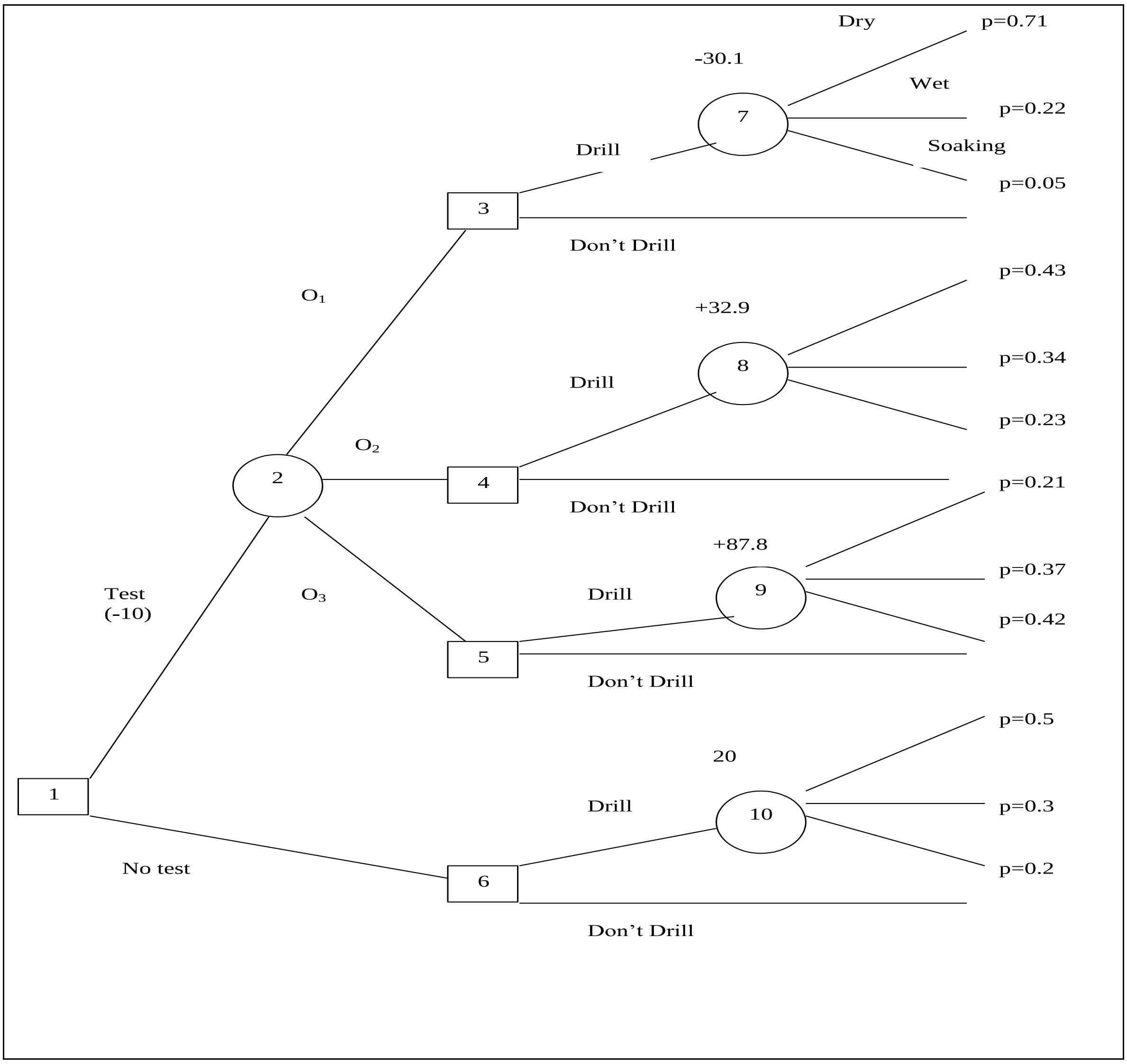


For example, $P(\theta_1 | O_1)=0.73, \ldots, P(\theta_3 | O_3)=0.42$.
Also, as give, $P(\theta_1)=0.5, \ldots, P(\theta_3)=0.22$.

Then, *proceeding backwards*, we can calculate the **expected values** at nodes 7 to 10, corresponding to chance events:
Node 7 = (- 70 x 0.73) + (50 x 0.22) + (200 x 0.05) = - 30.1

Node 8 = + 32.9, Node 9 = + 87.8, Node 10 = + 20.0 [***Check***]

These are added to the tree as follows:



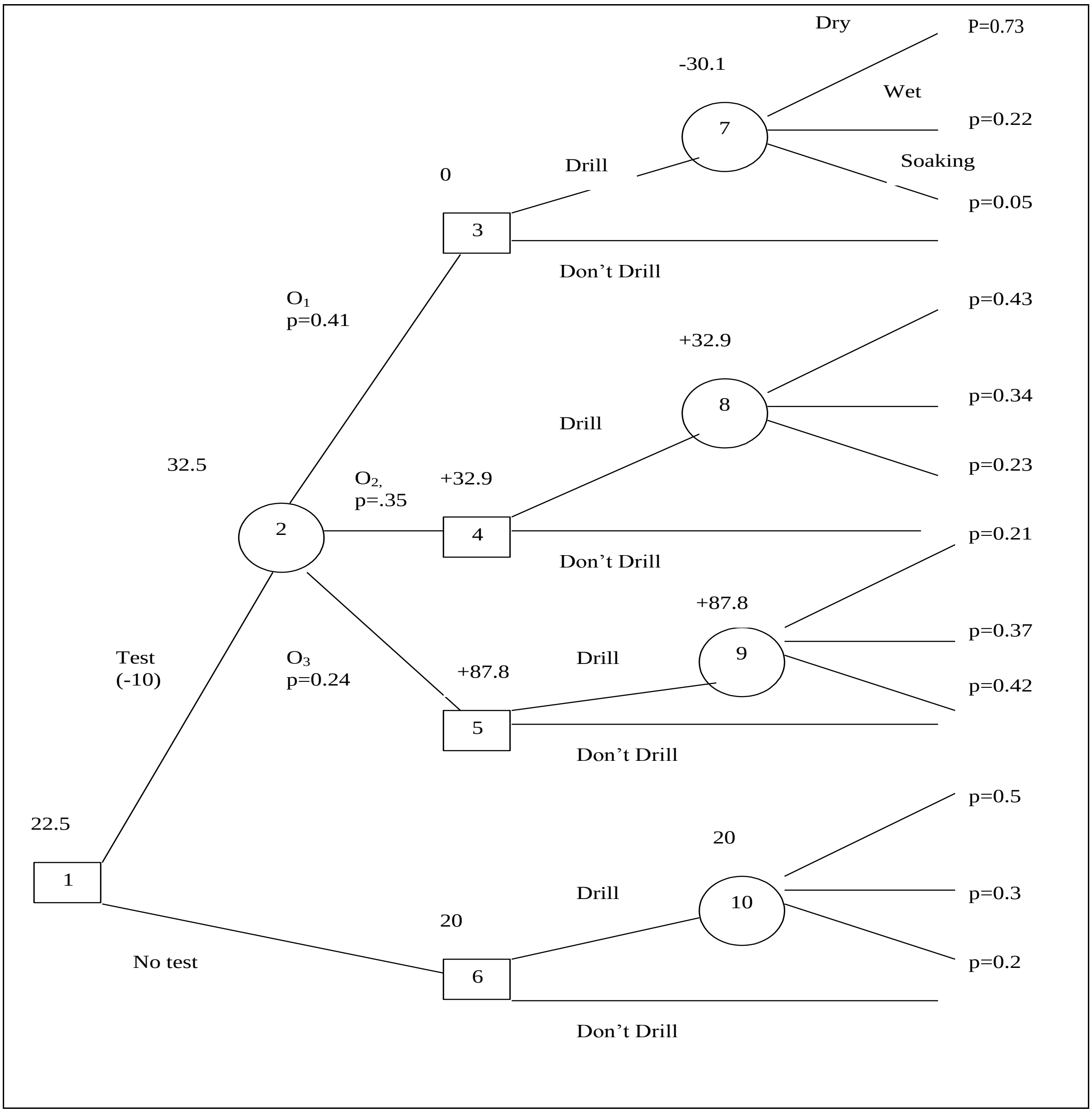
Now we can calculate the expected values at the "decision" events 3 to 6 where we choose the largest expected value of the available options (again proceeding backwards from right to left);

Node 3: Choose don't drill – expected value = 0

Node 4: Choose Drill – expected value = 32.9

Node 5: Choose Drill – expected value = 87.8

Node 6: Choose Drill – expected value = 20.0



Next, we can calculate the expected value of Node 2 (chance event):

Node 2: Expected value = 0 x 0.41 + (32.9 x 0.35) + (87.8 x 0.24) = 32.5

Finally, therefore at node 1 (decision point):

node 1: Expected value is 32.5-20 = 12.5

=> **Test should be carried out. Then, unless the result of the test shows "no structure", the decision should be to drill.**

Example 2: A firm is considering whether to launch a new product. The success of the idea depends on the ability of a competitor to bring out a competing product (estimated at 60%) and the relationship of the competitor's price to the firm's price. Table A shows the profits for each price range that could be set by the firm related to the possible competing prices.

	If competitor's price is			
If firm's price is	Low	Medium	High	Profit if no competitor
Low	30	42	45	50
Medium	34	45	49	70
High	10	30	53	90

Table A: Profit in €'000s

The firm must set its price first because its product will be on the market earlier so that the competitor will be able to react to the price. Estimates of the probability of a competitor's price are shown in Table B.

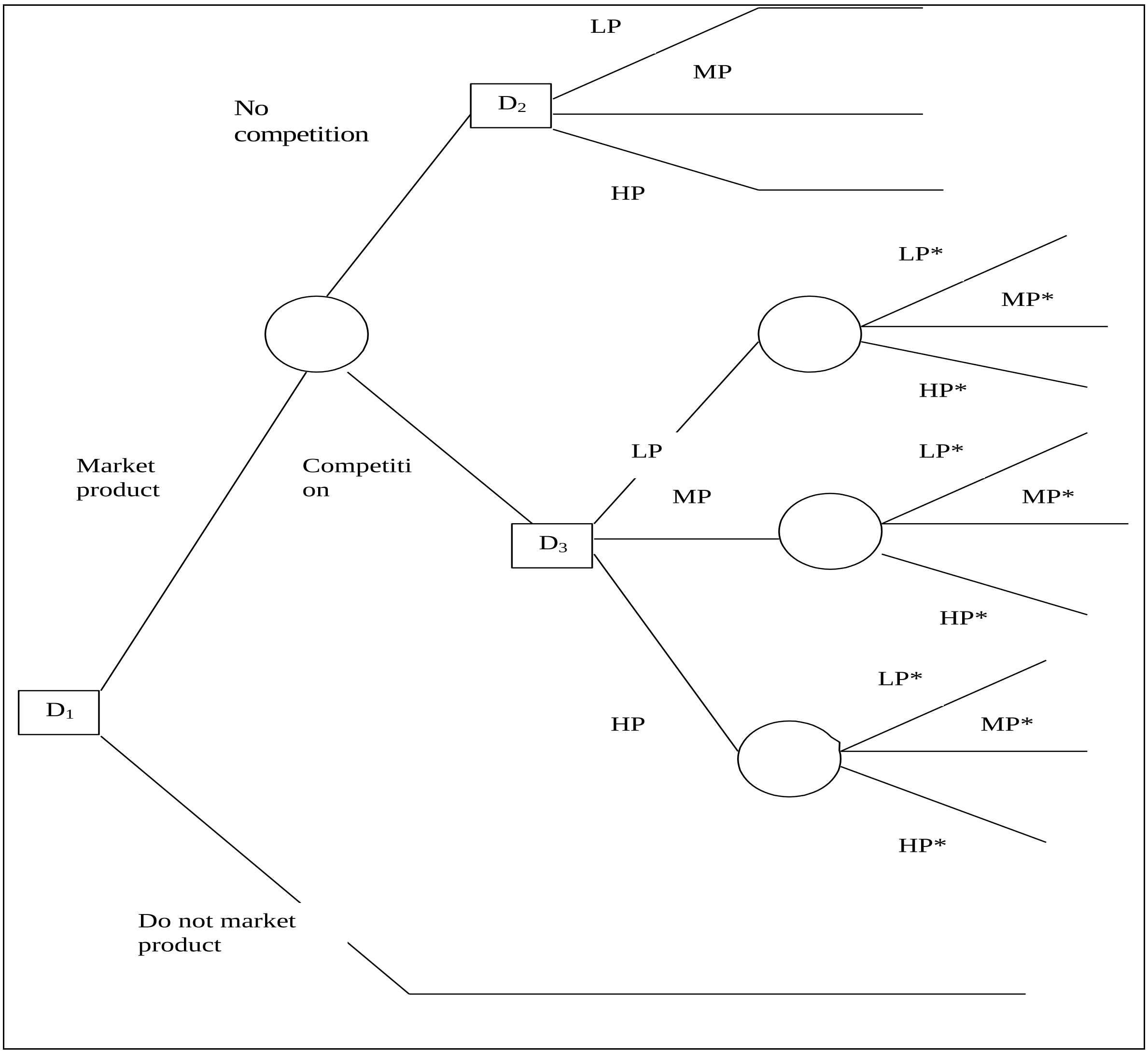
	Competitor's price expected to be		
If firm's price is	Low	Medium	High
Low	0.8	0.15	0.05
Medium	0.2	0.70	0.10
High	0.05	0.35	0.60

Table B: Probabilities of expected competitor prices

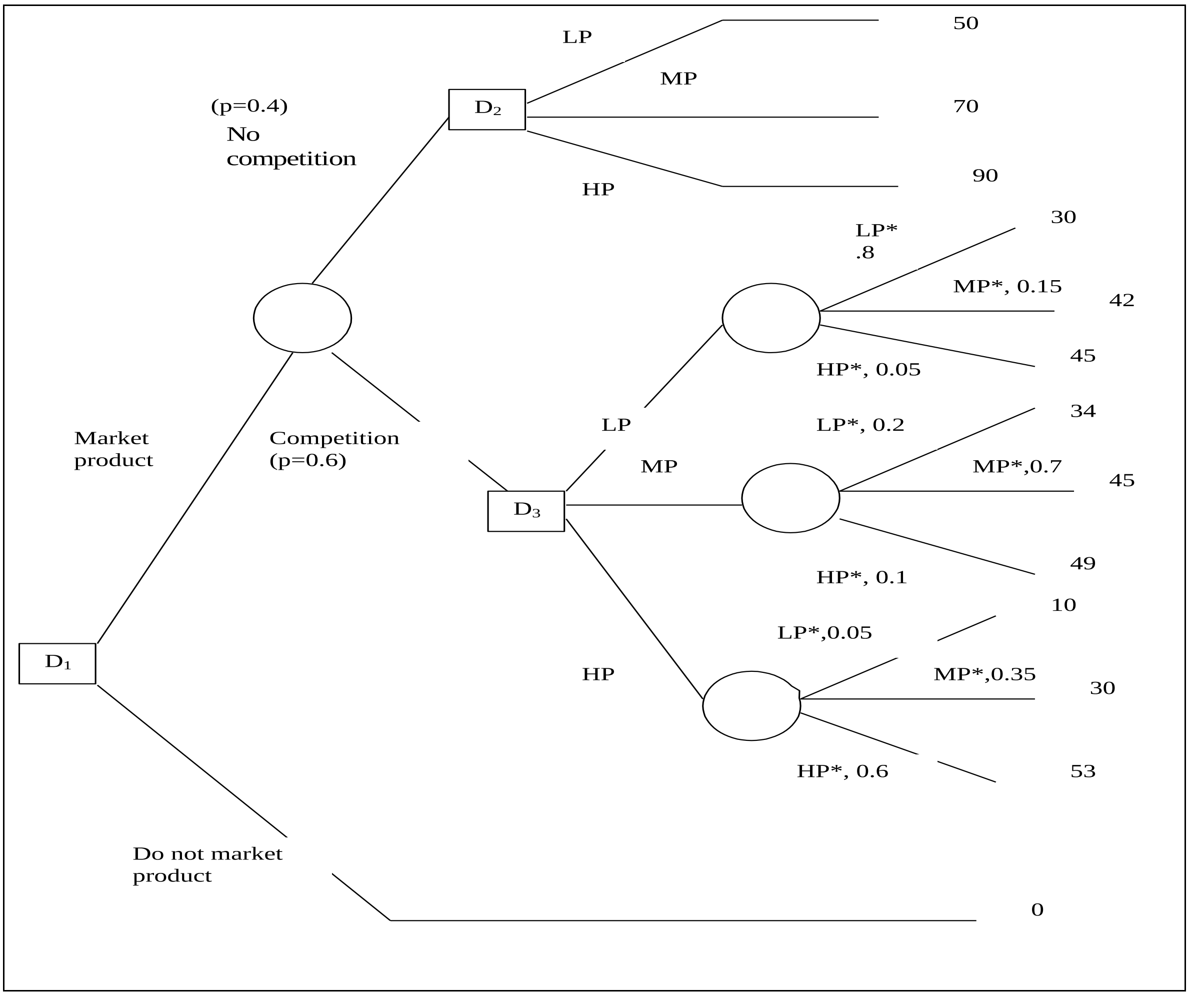
(a) Draw a decision tree and analyse the problem.

(b) Recommend what the company should do.

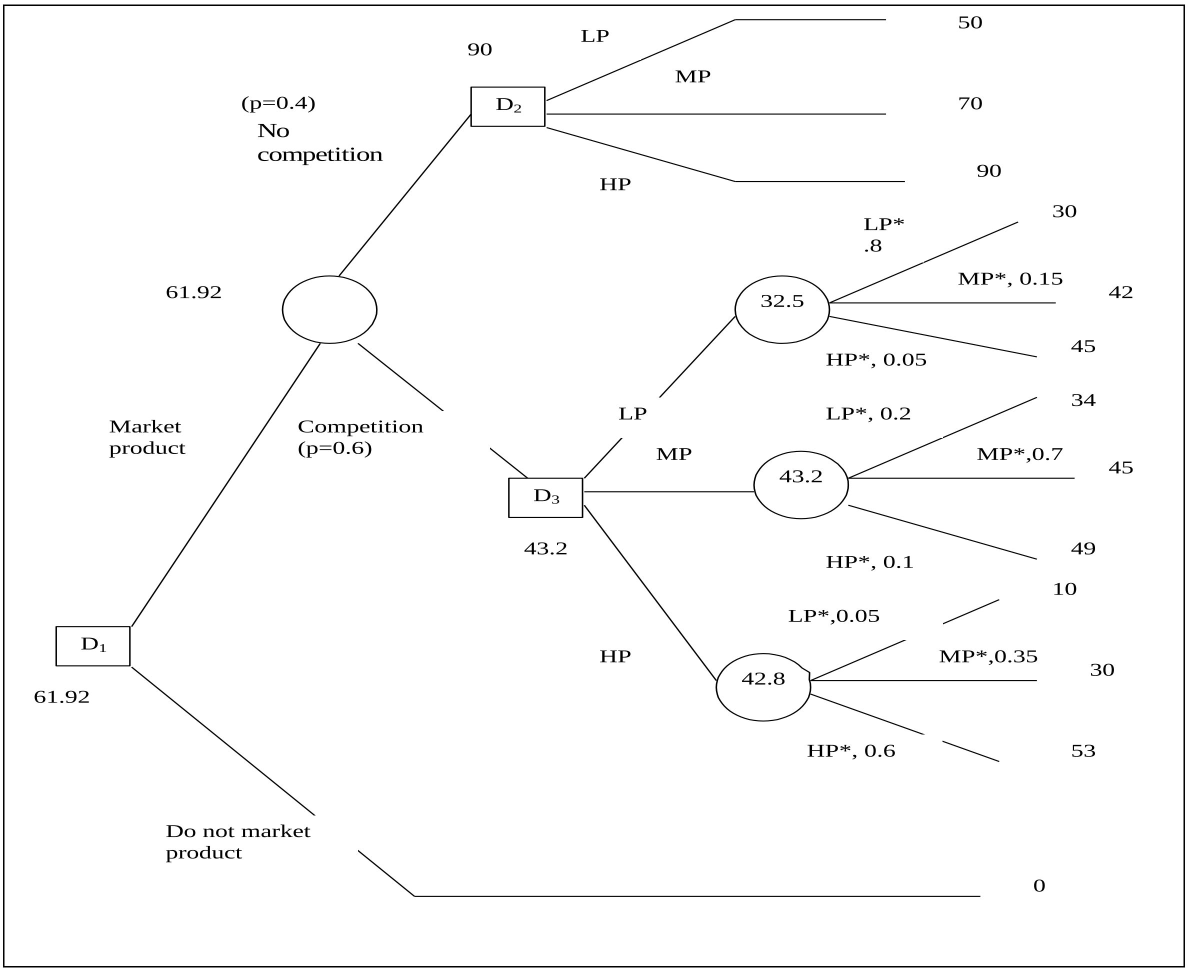
Solution: The first stage is to draw the tree, starting from the left, showing the various decision points and outcome nodes. Concentrate first on the logic of the problem and don't bother with the values and probabilities to start with. This results in the following figure, where * is used to indicate "competitor". This stage is called the **forward pass**.



The next step is to fill in the outcome values and probabilities from Tables A and B.



The final stage uses the probabilities and outcome values and calculates the **expected values** at various points so that the correct decisions are highlighted. This stage works from right to left and is known as the **backward pass**. This is shown in the following figure.



<u>Recommendation</u>: The company should market the product. If there is no competition then the product should be set to the high price. If there is competition then the product should be set to the medium price (for which the EV is slightly greater than the low price and considerably greater than for the high price).

Note: This was somewhat easier than Example 1 in that we did not have to apply the conditional probability rule.

3.5 Exercises

These will be specified separately