Stress Analysis of Stirling Cryocooler Flexure Springs for Long Space Mission Lifetime

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Abstract. Cryocoolers in space instruments cool detectors and superconducting devices to cryogenic temperatures. Inside a cryocooler, a flexure spring ensures the cryocooler piston is axially free and radially restricted to prevent off-axis forces. Given that the flexure spring is mission-critical to the cryocooler operation yet prone to failure, its lifetime can determine the lifetime of the space mission. By improving the flexure spring design, space missions can be extended and present longer reliability. This study investigated the impact of various spring design parameters on a spiral flexure spring with spring arms in a characteristic spiral shape. These parameters include the number of arms, spring design, and thickness. Results show that the lowest maximum stress is observed at lower spring thickness values, moderate arm numbers, and teardrop sizes that vary for different arm numbers. Based on these results, an optimal design is proposed that maximizes flexure spring lifetime while operating at a resonant frequency to reduce the input power and increase lifetime. In this design, the design parameters are adjusted for optimal flexibility as a proof of concept, with additional designs inspired by kirigami proposed. In the latter design, a longer lifetime is demonstrated, extending to the lifespan of space missions that rely on cryocooler operation.

1 Introduction

Cryocoolers cool detectors and superconducting devices to cryogenic temperatures in space instruments ([1]). The Stirling cryocooler is one of the most established and reliable cryocoolers with the most flight heritage. A key factor responsible for its improved long lifetime is the Oxford-style compressors that use flexure bearing springs, which reduce off-axis forces in the compressor and prevent cryocooler failure ([2, 3]). This allows the cryocooler to operate millions of cycles over multiple years, typically designed for operation for up to 10 years. A cause of cryocooler failure is when the flexure spring fails due to oscillatory loading stress, resulting in piston-cylinder contact and failure ([**Bailey2019**]). Thus, extending the lifetime of the flexure spring is extremely important–the flexure spring vibrates rapidly and undergoes more fatigue and stress than any other element ([4, 5]).

In the past, fatigue analysis of flexure springs includes developing experimental testing setups and modeling specific spring systems ([**passive1**, 6]). However, not much work has been done to evaluate the influence of key design parameters on spring performance, including the shape, number, and thickness of the spring arm. This paper aims to evaluate the aforementioned parameters and their influence on spring performance and fatigue lifetime using SolidWorks as the Finite Element Analysis (FEA) tool ([7]). This paper includes a section on the relevant background theory, where flexure spring design and harmonic oscillatory loading theory is explored, followed by a section on the model setup in SolidWorks. Thereafter, the analysis and discussion of the investigations of the influence of the key parameters is detailed. This study will help understand the design requirements for cryocooler flexure springs to ensure long lifetime space missions.

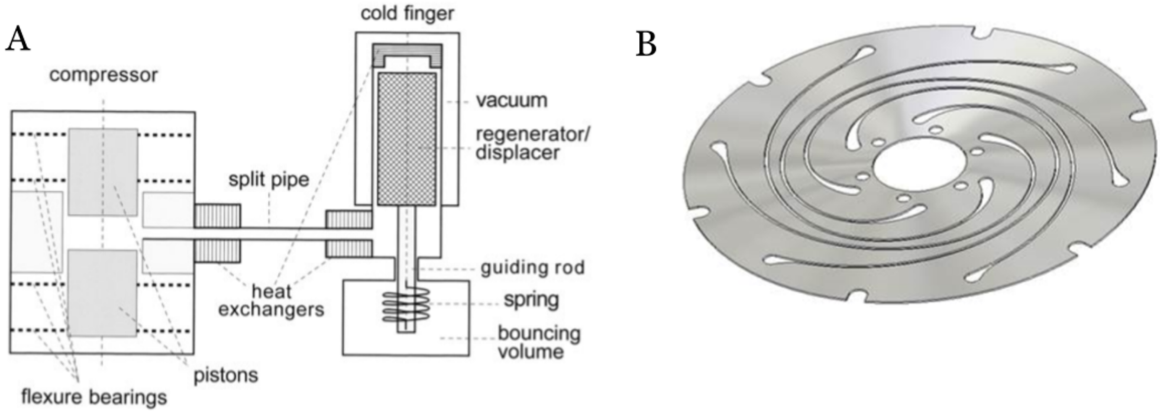

Figure 1: (a) Schematic diagram of a split-pair Stirling refrigerator. The cooling power is supplied to the heat exchanger of the cold finger. Usually, the heat flows are so small that there is no need for physical heat exchangers around the split pipe. The flexure bearing springs are indicated in the cryocooler compressor (reproduced from ([11])), **(b)** a typical cryocooler flexure bearing spring with spiral spring arms (reproduced from ([12])).

2 Background Theory

2.1 Space cryocoolers

Space cryocoolers cool spacecraft detectors to cryogenic temperatures via thermodynamic cycles that manipulate heat and work pathways to achieve a warm end and a cold end. Typically, this process involves expanding a gas at a low pressure and then compressing at a high pressure. For Stirling cryocoolers, this is achieved through the Stirling refrigeration cycle ([1]). Repeating this cycle many times makes it possible to achieve low cryogenic temperatures at which thermal noise in the detector at the cold end is significantly reduced, rendering it sensitive to faint signals in space. Moreover, cryocoolers are used to render a material in its superconducting state for detection for detectors or instruments that exploit quantum-limited sensitivities through superconducting effects. Common types of cryocoolers are Stirling, Joule-Thomson, Brayton, Gifford-McMahon, and pulse tube cryocoolers ([8]). Each type has its benefits and drawbacks, such as operating efficiencies, frequency, vibration, lifetime, operating temperature, and input power ([9, 10]). Figure 1 presents a schematic of a typical split-pair Oxford-style Stirling cryocooler that utilizes flexure bearing springs in the compressor.

2.2 Flexure springs

As previously mentioned, cryocoolers achieve cryogenic temperatures through the controlled compression and expansion of a gas, typically helium. In a Stirling cryocooler, this process is undergone inside the linear compressor.

Flexure springs keep the compressor piston's movement axially free and radially stiff. This element prevents the piston from creating friction with the compressor through contact and is critical to cryocooler operation. Figure 1 shows the design of a typical cryocooler flexure-bearing spring with spiral spring arms.

Two primary methods are used to assess the fatigue limit of a flexure spring: laboratory experiments and computational simulation. In the first method, a flexure spring is set up to oscillate at a specified amplitude. The number of successful loading cycles before failure occurs is measured. After 10^7 cycles, the spring is considered to not fail at that given stroke, and the stroke is increased. A fatigue limit for the specific spring is established upon failure following an incremental increase in the stroke. In computer simulations, FEA is used to calculate the stress, and hence, fatigue life of a such flexure spring ([13]).

When developing flexure springs, many parameters influence the harmonic operation and function of the spring and are noteworthy for optimization. These include the number of spring arms, arm profile, shape, material, thickness, and slot width, which can be modified for optimized performances with varying requirements. In previous studies, it has been found that the spiral profile has the most significant impact on the spring performance ([14]).

The resonant frequency of the compressor is determined by the effective moving mass of the compressor, the axial stiffness of the flexures, and the effective gas spring stiffness from the compression process.

Operating at resonance minimises the force needed to drive the compressor, and hence minimises the compressor drive current and input power.

3 Methods

SolidWorks is a software that permits computer-aided design (CAD) modeling, solid modeling, structural engineering, and complex simulations through FEA modeling, an analysis method for the effect of forces on real-world bodies. For this study, SolidWorks 'Simulation Studies' was utilized to study the regional stress of the flexure spring, and SolidWorks 'Fatigue Studies' to investigate the life and damage of flexure spring designs operating under SHM ([7]). In this study, FEA is performed on flexure bearing springs ([13]).

SolidWorks 'Simulation Studies' simulated the basic properties of the flexure spring, such as stress for varying displacements. Then, these 'Simulation Studies' were analyzed in 'Design Studies,' where we parameterized the 'Simulation Studies' to achieve a results table for varying parameter values. The 'Simulation Studies' were also used in the 'Fatigue Studies,' to analyze the effects cyclic stress on the flexure spring.

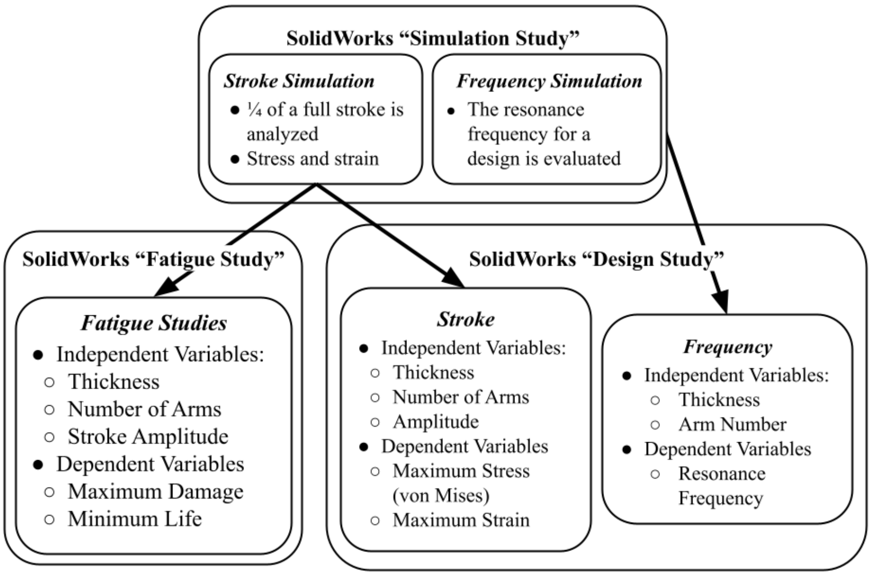

Figure 2: SolidWorks Study Design Flowchart. A flowchart showing the research analysis process, including SolidWorks 'Simulation Studies,' 'Design Studies,' and 'Fatigue Studies'.

Firstly, we used two types of 'Simulation Studies': stroke and frequency. The former analyzed stress for varying displacements. The latter calculated the resonance frequency of the flexure spring. Next, to understand how the stress varied in accordance with parameters, we used 'Design Studies' to parameterize variables such as the number of spiral arms, size of teardrops, thickness, and amplitude, the results of which will be discussed in the following analysis section. The von Mises Stress is assessed to analyze the stress induced upon a flexure spring.The stress used to evaluate the designs was the von Mises stress, which is used to evaluate when a material yields ([15]). The subsequent analysis of the flexure spring lifetime will be conducted primarily using von Mises stress. Finally, SolidWorks 'Fatigue Studies' allowed us to run 'Simulation Studies' for cycles on the order of 10^6 and 10^7 to analyze how damage and life scale in response to cyclic fatigue.

4 Results and Discussion

4.1 Design parameters

The parameters we investigated and relevant ranges are shown in Table 1. These are spring thickness, arm number, teardrop size, and displacement. Note that the resonance frequency is not parameterized according to spring displacement. The resonant frequency calculated is that of a single spring without the addition of any structural mass.

Table 1: The parameters used for parametric analysis, including the range and step size.

Parameter	Units	Lower Bound	Upper Bound	Step
Spring Thickness	mm	0.25	5	0.25
Spring Arm Number		1	20	1
Spring Teardrop Size	mm	1	3	0.2
Spring Displacement	mm	0.25	10	0.25

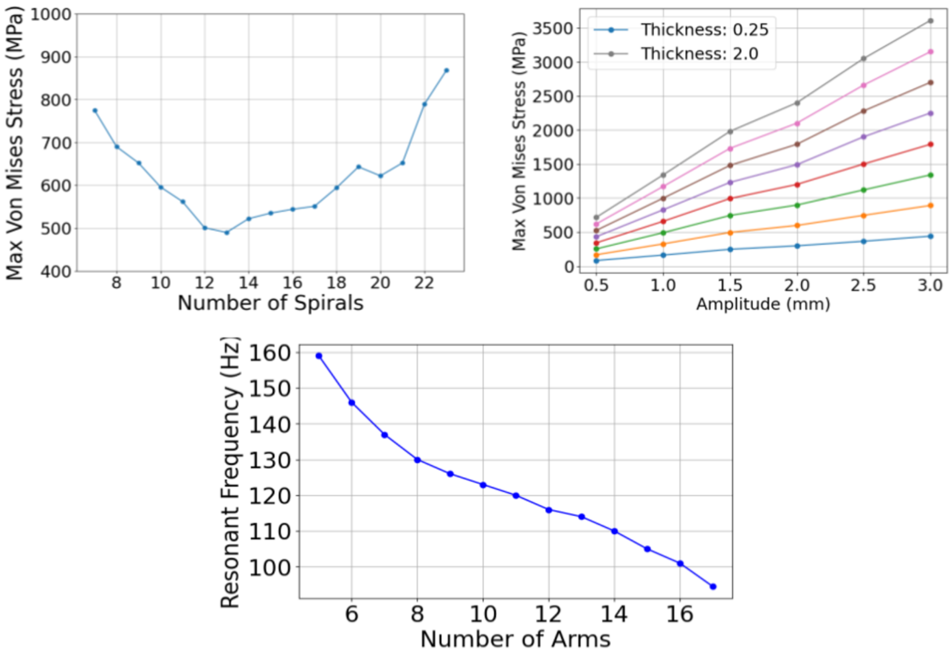

Figure 3: Top left: The variation of maximum stress according to spring thickness and stroke amplitude. **Top right:** The variation of maximum stress according to the number of spiral arms. **Bottom:** The resonance frequency variation according to the number of spiral arms.

4.2 Spring thickness

Figure 3 shows the variation of maximum stress based on spring thickness and stroke amplitude. Thickness is varied from a range of 0.5 mm to 5.0 mm with a step of 0.5 mm. Amplitude is varied from a range of 0.25 mm to 10 mm with a step of 0.25 mm. As the Figure shows, the maximum von Mises stress is proportional to the spring thickness. For higher amplitudes, higher thicknesses result in significantly higher stresses. This plot shows that lower flexure spring thicknesses can achieve the lowest maximum stress.

4.3 Spring arm number

We analyzed both the variation of maximum stress and frequency with respect to the number of spiral spring arms. Figure 3 shows how stress and frequency changes with number of spirals.

In Figure 3, it can be seen that the maximum stress sharply decreases between 4 and 10 arms. Arm numbers below four were omitted for a better graph window. For 10-18 arms, the stress stays around 550 MPa. Beyond 18 arms, stress increases. The minimum stress is 490 MPa at 13 arms, balancing flexibility and strength. Thicker, fewer arms provide higher strength and lower flexibility, while thinner arms provide lower strength and higher flexibility.

In Figure 3, the resonance frequency sharply drops from 2 arms to 5 arms, from roughly 320 Hz to 160 Hz. Beyond five arms, the resonance frequency slowly declines from 150 Hz to 100 Hz.

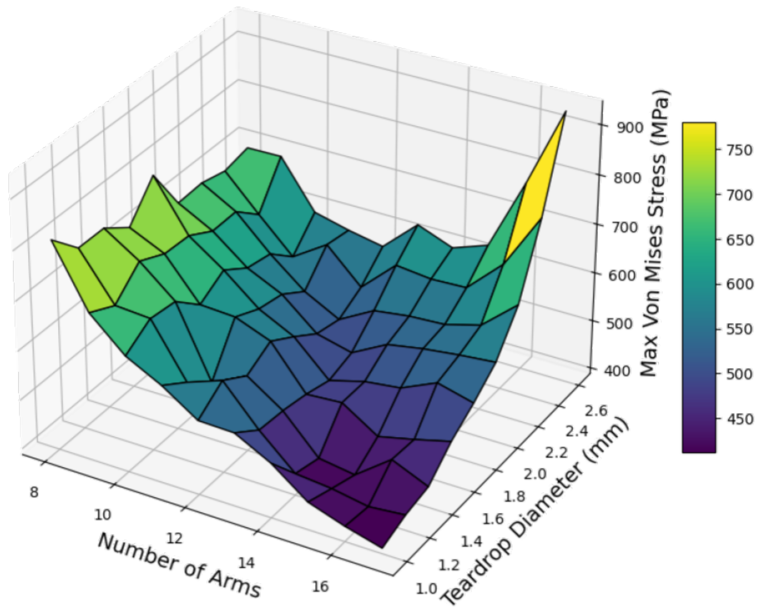

Figure 4: Stress as a function of number of spring arms and teardrop size.

4.4 Teardrop Size

Figure 4 shows a 3-dimensional graph of the maximum stress for a 4 mm amplitude with varying number of arms and teardrop diameter. The highest stresses are near low arm numbers with low teardrop diameters and high arm number with high teardrop diameter. The teardrop diameter where the maximum stress varies the least as the number of arms changes is around 2 mm, and the number of arms where the maximum stress varies the least as the number of arms changes is around 12. The lowest maximum stress is 398.6 MPa, at 17 arms and 1 mm teardrop diameter.

4.5 Spring design

The principle of design optimization can be extended to other spring designs. For example, consider a kirigami-inspired design without spirals but rather concentric slits with equally spaced bridges, as shown in Figure 5. This spring design offers different performance and tunable parameters than the spiral arm design. For example, tunable parameters include slit size, number of bridges, number of concentric circles, and circle spacing. Although the design is fundamentally different, a similar procedure can be undertaken to fine-tune each parameter using simulation testing in SolidWorks. This design yielded a lower reaction force and stress than the prior design. This shows that the spiral design is comparatively superior for stiffness and structural integrity, giving a more extended lifetime flexure spring design.

Overall, the kirigami-inspired spring design has much lower stiffness, both axially and radially. Further work includes comparing stresses in the case where the kirigami string has the same axial and/or radial stiffness as the spiral arms design. Another advantage of the kirigami design is that it does not have rotation with displacement, unlike the spiral arms design, though rotation is not the most important factor in flexure spring performance.

4.6 Discussion

Based on the results, lower spring thicknesses tend to result in lower maximum stresses. In thicker springs, the deformation from strokes creates a higher bending moment and shear force, increasing the spring material's stress. In contrast, thin springs result in lower maximum stress because the bending moment for the same displacement is the same. The thickness of the spring also dramatically affects the restoring force and thus resonant frequency. Therefore, additional investigation should be done to investigate the proportionality between reaction force and stress for various thicknesses, as reaction force is likely related to stress.

Furthermore, even though decreasing the thickness of the spring continuously decreases the maximum stress, it is unlikely that damage scales down similarly, or else the minimum damage would be when

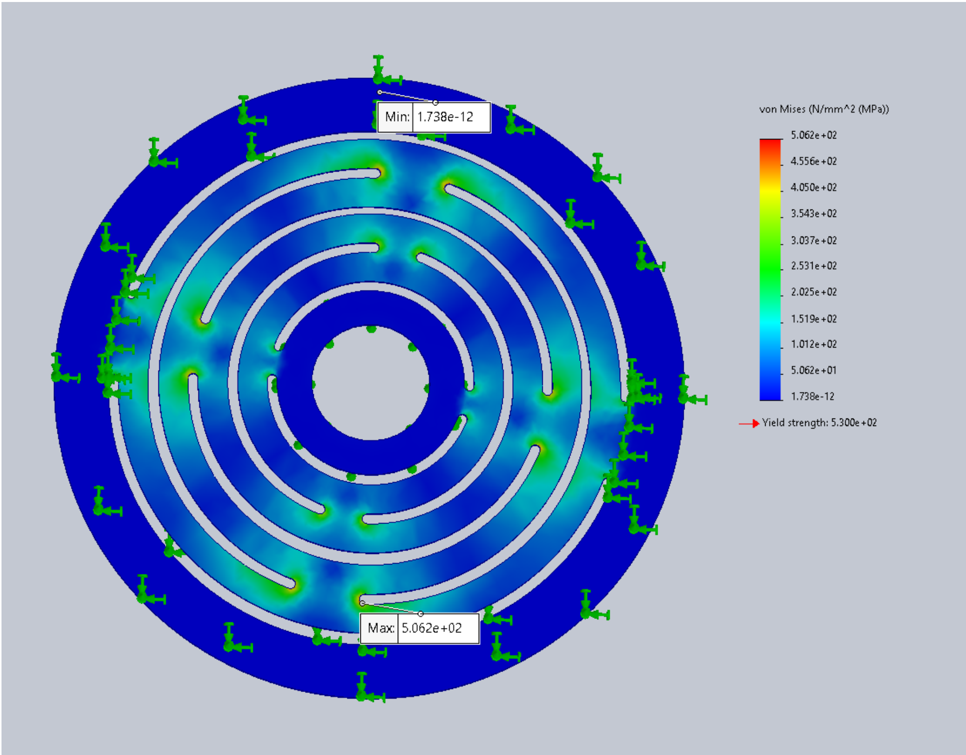

Figure 5: An example kirigami-inspired spring design

the spring has zero thickness. Further investigation should also be undertaken to find the relationship between maximum damage as thickness and amplitude vary. The geometric spiral used (r proportional to θ also is not the optimal spiral shape, so other spiral shapes should be explored. In this study, the geometric spiral was chosen to reduce the number of variables.

The result is that the optimal spring arm number is 13-14 arms. This would mean that each arm is relatively thin, especially near the teardrops. Thinner arms intuitively mean higher flexibility and lower rigidity, corresponding to lower stress. This is the case for a low number of arms. As the number of arms becomes exceptionally high, the increase in flexibility becomes less critical compared to the decrease in arm width. This also corresponds to increased maximum stress, measured by force per unit area. As a result of these observations, the maximum stress decreases sharply, then increases again after 13-14 arms, as expected. However, 13-14 arms may leave the spring susceptible to radial instabilities, which are not accounted for by the simulation. The result is, therefore, that the optimal number of arms would be in the range of 13-14 arms but likely lower. Furthermore, the frequency changes slightly depending on the number of spiral arms. Depending on the needs of particular cryocooler applications, the number of arms can be chosen to achieve the proper resonance frequency to achieve higher efficiency in cryocooler driving.

Finally, the maximum stress varies in different ways for varying teardrop diameters. For a high number of arms, a high teardrop diameter drastically increases the maximum stress. For a low number of arms, the high teardrop diameter decreases the maximum stress because the teardrops are areas of maximum stress. The lowest maximum stress is around the mid to high range of several arms with a small teardrop size. This is because the high number of arms increases flexibility and reduces rigidity, as previously discussed. At the same time, the lower teardrop size ensures that no part of the arm is excessively thin. The highest amounts of maximum stress are at low arms with small teardrop diameters and high numbers with high teardrop diameters, which correspond to high rigidity and excessively thin arms, respectively. Therefore, a moderate teardrop size and a relatively high number of arms should be chosen to minimize stress.

5 Conclusions

Lower thicknesses of springs generally decrease the maximum stress. High numbers of spring arms result in lower maximum stress because of increased flexibility and decreased rigidity. It also results in a lower resonant frequency because of a decreased restoring force resulting from decreased rigidity. The teardrop size affects the results differently, but the optimal solution is a moderate to low teardrop diameter. The optimal spring design involves a moderately high number of spring arms, lower thickness, and a low teardrop diameter to optimize the spring's flexibility.

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