

# Triangle Cavity Flow

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# Introduction

In this study I have looked at steady 2-D incompressible flow inside a cavity driven triangle. This seemingly simple geometry gives interesting flows for different Reynolds number, and the implementation is rather easy to check with published papers. I used [?] and [2] as a reference to make mesh and to look at formation of eddies. The goal will be to compare results with [1] and [2] , and discuss results. Blabla et al. (2009) sadflkjaskøfjasdølfkjaskdflkj

## Methods

### Mesh

I used gmsh to make the all the meshes. The equilateral triangle I made has 3 corner points  $a = (-\sqrt{3}, 1)$ ,  $b = (\sqrt{3}, 1)$ ,  $c = (0, -2)$ , and then I made a point between each these so i could get a finer mesh in the corners since this is where most of the interesting effects happen and also the place where the program might diverge. 73541 vertices and 145664 cells. [1] used a mesh of  $(512 \times 512)/2$  gridpoints giving 131072 cells.

The isocles triangle has points  $a = (-1, 0)$ ,  $b = (1, 0)$ ,  $c = (0, -4)$  with the same idea with points on the line. 57843 vertices and 113176 cells. The mesh is also finer at the corners and super fine at the bottom as we will discuss later.

### Software

I programmed in Python, using the FEniCS software with Oasis, which solves the Navier-Stokes equations both transient and steady.

## Numerics

This problem was solved with a steady state solver of the N-S equations. I set the velocity  $u(x,y) = (1,0)$  at the top and set no slip on the sides. To look at eddies forming I computed the stream function:

$$\nabla^2 \psi = -\omega, \quad \omega = \nabla \times u$$

where I set  $\psi = 0$  on the boundary.

The streamfunction was programmed the following way:

```
bc0 = DirichletBC(V, (0, 0), nos)
bc1 = DirichletBC(V, (1.0, 0), top)
uu = project(u_, V, bcs=[bc0, bc1])
psi = Function(Q)
p = TrialFunction(Q)
q = TestFunction(Q)
solve(inner(grad(p), grad(q))*dx == inner(curl(u_), q)*dx, psi, \
```

```

    bcs=[DirichletBC(Q, 0, "on_boundary")]
    sort = psi.vector().array().argsort()
    xx = interpolate(Expression("x[0]"), Q)
    yy = interpolate(Expression("x[1]"), Q)
    xm_1 = xx.vector()[sort[0]]
    ym_1 = yy.vector()[sort[0]]
    xm_2 = xx.vector()[sort[-1]]
    ym_2 = yy.vector()[sort[-1]]
    print "Center main-eddy: x: %.4f, %.4f " %(xm_1, ym_1)
    print "Stream function value at main-eddy: %.4f " %(psi(xm_1,ym_1))
    mycurl = project(curl(u_), Q, bcs=[DirichletBC(Q, 0, "on_boundary")])
    v_value = mycurl(xm_1,ym_1)
    print "Vorticity value at main-eddy: %.4f " %(v_value)

```

## Results

First I considered an equilateral Triangle with corner points:

$$a = (-\sqrt{3}, 1), b = (\sqrt{3}, 1), c = (0, -2)$$

Where the Reynolds number is defined as  $Re = \frac{U}{\mu}$ , where I set  $U = 1$  and controlled Re by changing  $\mu$

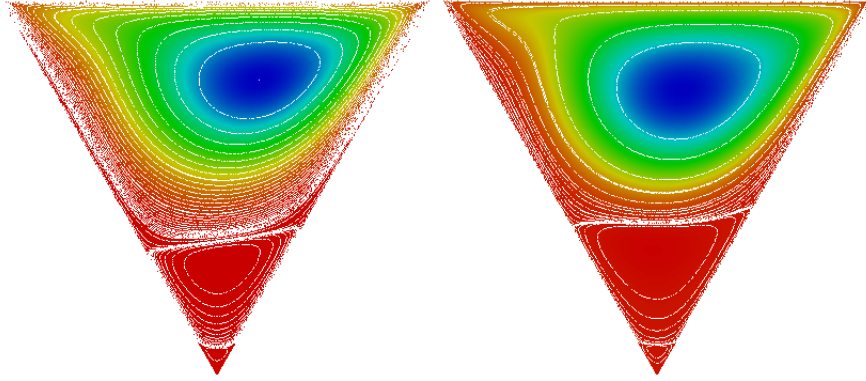
Table 1 looks at the position of the center eddy and the value of the streamfunction and vorticity at this point.

This study was done with 73541 vertices and 145664 cells. [1] used 131072 cells. The velocity was calculated in quadratic space.

We see that most of the streamfunction values, vorticity and position are similar to  $10^{-2}$ . As we increase Re the streamfunction value stays very similar, but the vorticity starts to become a little different but similar to  $10^{-1}$ . The position of the main eddy is very similar to Re = 1250, and changes a bit at Re = 1500.

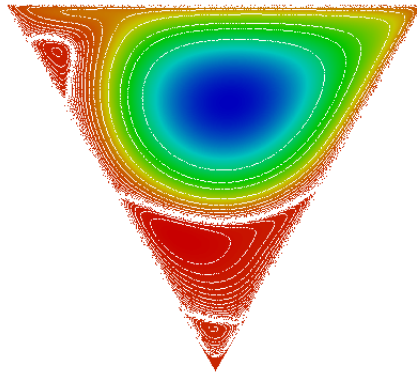
Erturk and Gockol				Me		
Re	$\psi$	$\omega$	(x,y)	$\psi$	$\omega$	(x,y)
1	-0.2329	-1.3788	(0.0101, 0.4668)	-0.2329	-1.3658	(0.0109, 0.4617)
50	-0.2369	-1.4689	(0.3484, 0.4434)	-0.2367	-1.4708	(0.3525, 0.4467)
100	-0.2482	-1.3669	(0.3315, 0.3555)	-0.2476	-1.3599	(0.3339, 0.3576)
200	-0.2624	-1.2518	(0.2030, 0.2734)	-0.2613	-1.2459	(0.1978, 0.2758)
350	-0.2724	-1.1985	(0.1556, 0.2383)	-0.2708	-1.1887	(0.1605, 0.2412)
500	-0.2774	-1.1791	(0.1319, 0.2207)	-0.2754	-1.1666	(0.1395, 0.2237)
750	-0.2818	-1.1668	(0.1150, 0.2031)	-0.2793	-1.1504	(0.1160, 0.2056)
1000	-0.2844	-1.1629	(0.1116, 0.1973)	-0.2814	-1.1427	(0.1066, 0.1944)
1250	-0.2861	-1.1624	(0.1049, 0.1973)	-0.2826	-1.1382	(0.1066, 0.1944)
1500	-0.2872	-1.1639	(0.1015, 0.1914)	-0.2834	-1.1354	(0.1022, 0.1828)
1750	-0.2881	-1.1675	(0.1015, 0.1914)	-0.2839	-1.1336	(0.1022, 0.1828)

Tabell 1: Eddies of equilateral Triangle



Figur 1:  $Re = 100$

Figur 2:  $Re = 200$



Figur 3:  $Re = 1000$

Figur 4: Streamfunctions of equilateral triangles

Next i studied what happens to the position of the main eddy when we refine the mesh. I used  $Re = 100$  and matched the results with [1]  $(x,y) = (0.3315, 0.3555)$

Next I considered an Isosceles Triangle with corner points:

$a = (-1, 0)$ ,  $b = (1, 0)$ ,  $c = (0, -4)$ , with 64051 vertices and 126444 cells

Again I looked at the position of the main eddy in Table 4. Mesh:

The next thing i looked at was where the second eddy was. This eddy is spinning the opposite direction so i had to look for the highest value of the stream function:

As we see in figure 2 and 3 the second eddy moved up and to the left, which

Re	$ \Delta\psi $	$ \Delta\omega $
1	0.0	$1.3 * 10^{-3}$
50	$2.0 * 10^{-4}$	$1.9 * 10^{-3}$
100	$6.0 * 10^{-4}$	$7.0 * 10^{-3}$
200	$1.1 * 10^{-3}$	$5.9 * 10^{-3}$
350	$1.6 * 10^{-3}$	$9.8 * 10^{-3}$
500	$2.0 * 10^{-3}$	$12.5 * 10^{-3}$
750	$2.5 * 10^{-3}$	$16.4 * 10^{-3}$
1000	$3.0 * 10^{-3}$	$20.2 * 10^{-3}$
1250	$3.5 * 10^{-3}$	$24.2 * 10^{-3}$
1500	$3.6 * 10^{-3}$	$28.5 * 10^{-3}$
1750	$4.2 * 10^{-3}$	$33.9 * 10^{-3}$

Tabell 2: Absolute difference of  $\psi$  and  $\omega$  vs Erturk and Gockol

Vertices	$\Delta x$	$\Delta y$	(x,y) = (0.3315, 0.3555)
273	0.052	0.067	(0.3837, 0.4225)
1064	0.070	0.01	(0.4042, 0.3742)
3946	0.042	0.008	(0.3728, 0.3641)
15134	0.0019	0.007	(0.3515, 0.3633)
59217	0.006	0.003	(0.3375, 0.3594)

Tabell 3: Refinement study with Re = 100

correlates with table 3.

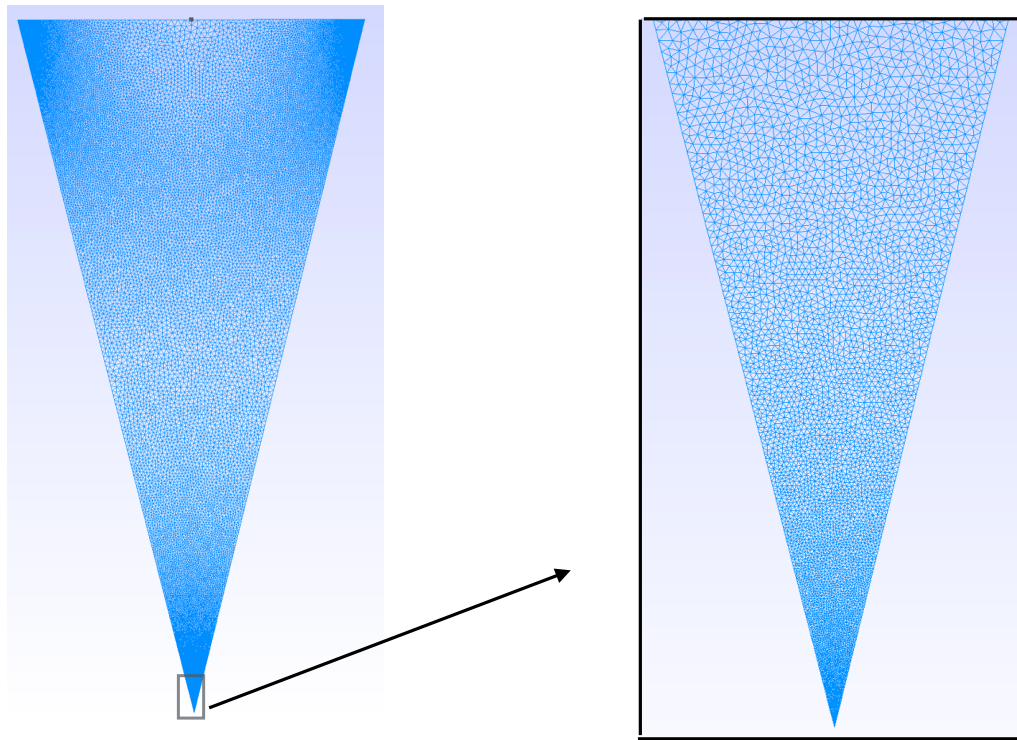
The next thing i looked at was how many eddies i could find. According to [2] every eddy formed will be 406 times weaker then the previous. So the velocities and the very bottom eddies will be very small. To find these eddies I looked at the absolute velocities in the horizontal direction on a line straight through the mesh from the top at (0,0) to (0,-4). As we can see when the plot has a dip is where the velocities have changed direction and where we can find the eddies center. I used a very fine mesh at the bottom to catch as many eddies as a could. After a careful count, i got 9 eddies. This was done with Re=1, and with a mesh consisting of 57843 vertices and 113176 cells.

We can see that near the bottom we get disturbances. As we see in Figure 7 and ? that the mesh if very fine near the bottom corner. So the disturbances is probably caused by the fact that the computer can only represent numbers down to  $10^{-17}$  [Ref]. One could argue that with a fine enough mesh and a computer able to represent infinitesimal numbers, that the number of eddies would go to infinity as we go down in the mesh.

If we compare to [2] in figure 5, where they have used a reynolds number of 0 (Stokes flow), we see that we get very similar results.

Representere tall i python, i pcen. ish 10e-18 eln. Sjekke mat-inf komenpdiet Mulig at vi kan bestemme steppet"metode - lage meshet. steg for steg POWER-POINT lage figurer, export i pdf letter å lese tables

større diff med høyere Re . evnt



Figur 5: Mesh of isosceles triangle

forfining av mesh med ett Re, med diff etc

diskutere mesh fra tidligere studier

vise hvor fint meshet er nederst.

- Skru på krylov løser for ett reynolds tall
- Gå fra LU løser til Krylov
- Bilde av høyere reynoldstall – check
- linear vs kvadratisk element –
- bilde av mesh ved siden av hverandre

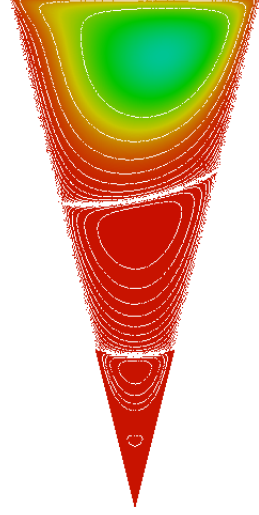


Figure 6: Re= 100

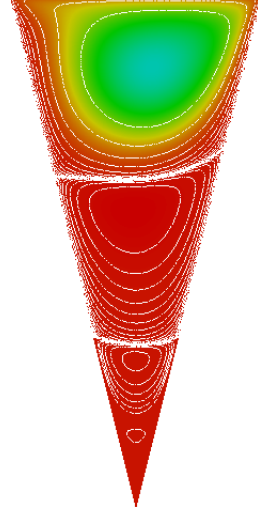


Figure 7: Re = 200

Figure 8: Streamfunctions of isoceles triangles

	Erturk and Gockol	Me
Re	(x,y)	(x,y)
12.5	(0.059, -0.391)	(0.0587, -0.4001 )
25	(0.115,-0.398)	(0.1101, -0.3999 )
100	(0.213, -0.477)	(0.2124, -0.4767 )
200	(0.129, -0.563)	(0.1275, -0.5608 )

Tabell 4: Position of main eddy, isoceles triangle

	Me		
Re	$\psi$	$\omega$	(x,y)
12.5	0.0002	0.0108	(-0.0018, -2.1924)
100	0.0020	0.0699	(0.0490, -1.8199)
200	0.0073	0.2183	(-0.0237, -1.6809)

Tabell 5: Position of second eddy,isoceles

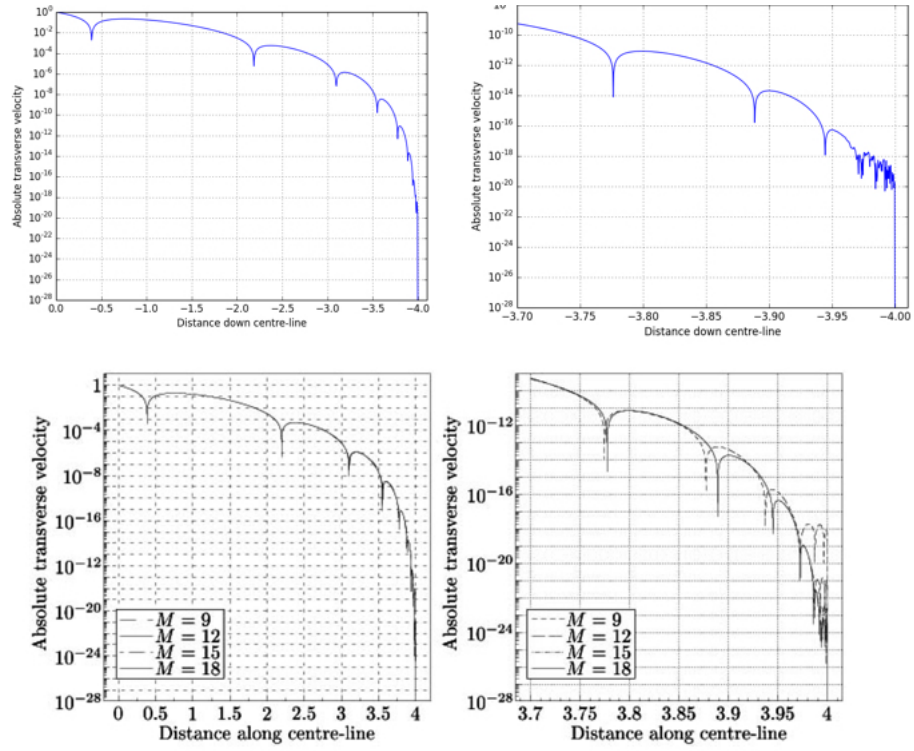


FIG. 1.4. The centre-line transverse velocity as a function of perpendicular height from the top of the wedge shown in Fig. 1.3. (Expansion order  $P = M - 1$ .)

Figure 9: Absolute transverse velocity, my study and



## Bibliografi