

# Chapter 6 Classical light propagation

## 6.1 The Lorentz oscillator model

The LOM describes the propagation of light in insulators by assuming that matter can be treated as a collection of *atomic*, *molecular* or *vibrational* oscillators. This provides a fundamental explanation for the complex refractive index and all properties that follow from that.

### 6.1.1 The atomic oscillator

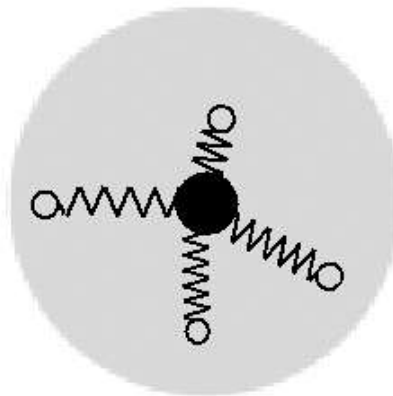


Figure 6.1: Concept of electrons as masses bound to nuclei by spring-like forces. ●

Assume the electrons and the nucleus form a dipole. During oscillations, Fig 6.1, the nucleus remains nearly stationary due to its heavy mass (compared to the electron); therefore, we start by assuming that only a single bound electron oscillates. If the electron has mass  $m_0$  and the ‘spring’ constant is  $K_s$  the natural frequency is:

$$\omega_0 = \sqrt{\frac{K_s}{m_0}} \text{ (rad/s)}. \quad (6.1)$$

Typical frequencies are  $10^{15}$  -  $10^{16}$  Hz (visible – UV). If  $K_s$  depends on the direction of vibration, then the material is *optically anisotropic*. The atomic oscillation produces a time varying dipole moment in addition to any pre-existing static atomic dipole moment.

Dipole oscillations lag the applied driving optical field giving rise to delayed re-emission and reduced phase velocity for transmitted light (thus giving a refractive index  $> 1$ ).

## 6.2 Dielectric Constant in the Lorentz Model

The magnitude of the time varying dipole moment in the atomic oscillator model is given by Eq. (6.2):

$$p(t) = -ex(t) \quad (6.2)$$

where  $x(t)$  is the displacement from the electron's equilibrium position. For one dipole, treated as a damped harmonic oscillator, the equation of motion is Eq.(6.3):

$$m_0 \frac{d^2 x}{dt^2} = -m_0 \gamma \frac{dx}{dt} - m_0 \omega_0^2 x - eE(t). \quad (6.3)$$

The three terms on the right hand side are a friction-like force with damping  $\gamma$ , the restoring force, and the optical driving force respectively.

The time dependence of the driving optical electric field is:

$$E(t) = E_0 \cos(\omega t) = E_0 \Re(\exp(-i\omega t)) \quad (6.4)$$

and hence we look for solutions of the form

$$x(t) = X_0 \Re(\exp(-i\omega t)). \quad (6.5)$$

Any phase difference between the driving field and  $x$  is included by allowing  $X_0$  to be complex.

We start from Eq.(6.6):

$$\begin{aligned} E(t) &= E_0 \exp(-i\omega t) \\ x(t) &= X_0 \exp(-i\omega t) \end{aligned} \quad (6.6)$$

Substituting into the equation of motion Eq.(6.3) we get:

$$-m_0 \omega^2 (X_0 e^{-i\omega t}) - im_0 \gamma \omega (X_0 e^{-i\omega t}) + m_0 \omega_0^2 (X_0 e^{-i\omega t}) = -e E_0 e^{-i\omega t}.$$

The solution for  $X_0$  is:

$$X_0 = \frac{-eE_0/m_0}{\omega_0^2 - \omega^2 - i\gamma\omega}. \quad (6.7)$$

If there are  $N$  oscillators per unit volume, the resulting induced polarisation of the medium is then:

$$P_{resonant} = Np = -NeX_0 = \frac{Ne^2}{m_0} \frac{1}{(\omega_0^2 - \omega^2 - i\gamma\omega)} E_0 \quad (6.8)$$

and the electric displacement  $D$  is (when a medium is present):

$$D = \varepsilon_0 E + P. \quad (6.9)$$

We can split the polarization into a non-resonant background term (perhaps due to other oscillators with resonances far from the frequency of the one that is our focus) and the resonant term just calculated for a single type of oscillator:

$$D = \varepsilon_0 E + P_{background} + P_{resonant} \quad (6.10)$$

The background polarisation must arise from some background electric susceptibility  $\chi_b$ :

$$D = \varepsilon_0 E + \varepsilon_0 \chi_b E + P_{resonant} = \varepsilon_r \varepsilon_0 E. \quad (6.11)$$

Assembling this all together, we find:

$$\varepsilon_r(\omega) = 1 + \chi_b + \frac{Ne^2}{\varepsilon_0 m_0} \frac{1}{(\omega_0^2 - \omega^2 - i\gamma\omega)}. \quad (6.12)$$

Now we can extract the real ( $\varepsilon_1$ ) and imaginary ( $\varepsilon_2$ ) parts of the dielectric function, and these are plotted in Fig 6.2:

$$\varepsilon_1(\omega) = 1 + \chi_b + \frac{Ne^2}{\varepsilon_0 m_0} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} \quad (6.13)$$

$$\varepsilon_2(\omega) = \frac{Ne^2}{\varepsilon_0 m_0} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} \quad (6.14)$$

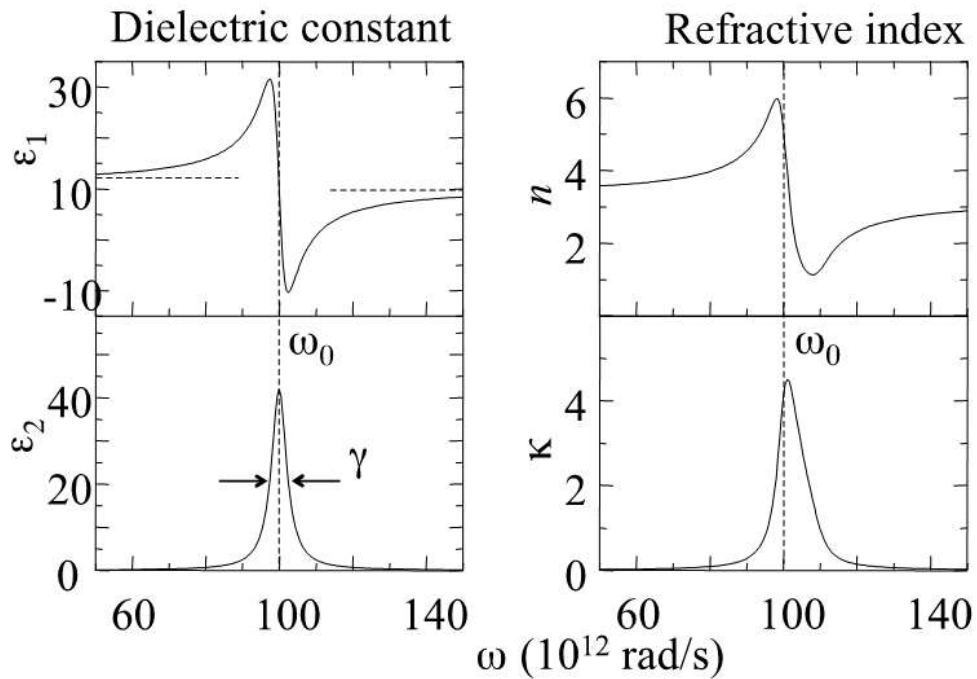


Figure 6.2: Dielectric properties of the Lorentz oscillator. •

The low (static  $\varepsilon_{st}$ ) and high frequency ( $\varepsilon_{\infty}$ ) limits are:

$$\varepsilon_r(0) \equiv \varepsilon_{st} = 1 + \chi_b + \frac{Ne^2}{\varepsilon_0 m_0 \omega_0^2} \quad (6.15)$$

$$\varepsilon_r(\infty) \equiv \varepsilon_{\infty} = 1 + \chi_b \quad (6.16)$$

## 6.2.1 Notable features

There is clearly a resonance in  $\varepsilon_2$  at  $\omega = \omega_0$ . It has a full width at half maximum equal to  $\gamma$  ( $\varepsilon_2$  is half its maximum value when  $\gamma = \omega_0 \pm \gamma/2$  – see Problem sheet 2).

Across a resonance region, there is a step change in  $\varepsilon_1$  (and also  $n$ ). Its size is determined by  $N$  and  $\omega_0$  and it affects a frequency range *much* wider than the absorption peak.

$\varepsilon_1$  and  $n$  are higher on the low frequency (long wavelength) side of a resonance than on high frequency (short wavelength) side.

## 6.3 Molecular oscillators

In a polar medium, the electron charge cloud of some or all of the bonds sits closer to one atom type than the other(s). Ionic molecules are very polar; purely covalent materials, in which identical atoms share electrons, are not polar.

The analysis is the same as for the atomic oscillator.  $\omega_0$  will be lower because the oscillating mass (reduced mass  $\mu = Mm/(M + m)$ ) is larger than  $m_0$  by a factor  $\sim 10^4 - 10^5$ . Typically  $\omega_0 \sim 10^{12} - 10^{14}$  Hz (near-infrared to far infrared).

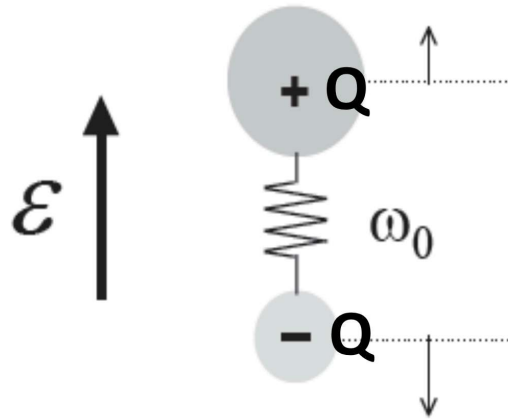


Figure 6.3: Molecular oscillator: a polar molecule in which the atoms are positively and negatively charged. •

Some gases and most organic compounds can be characterised by the frequencies of their infrared absorption peaks.

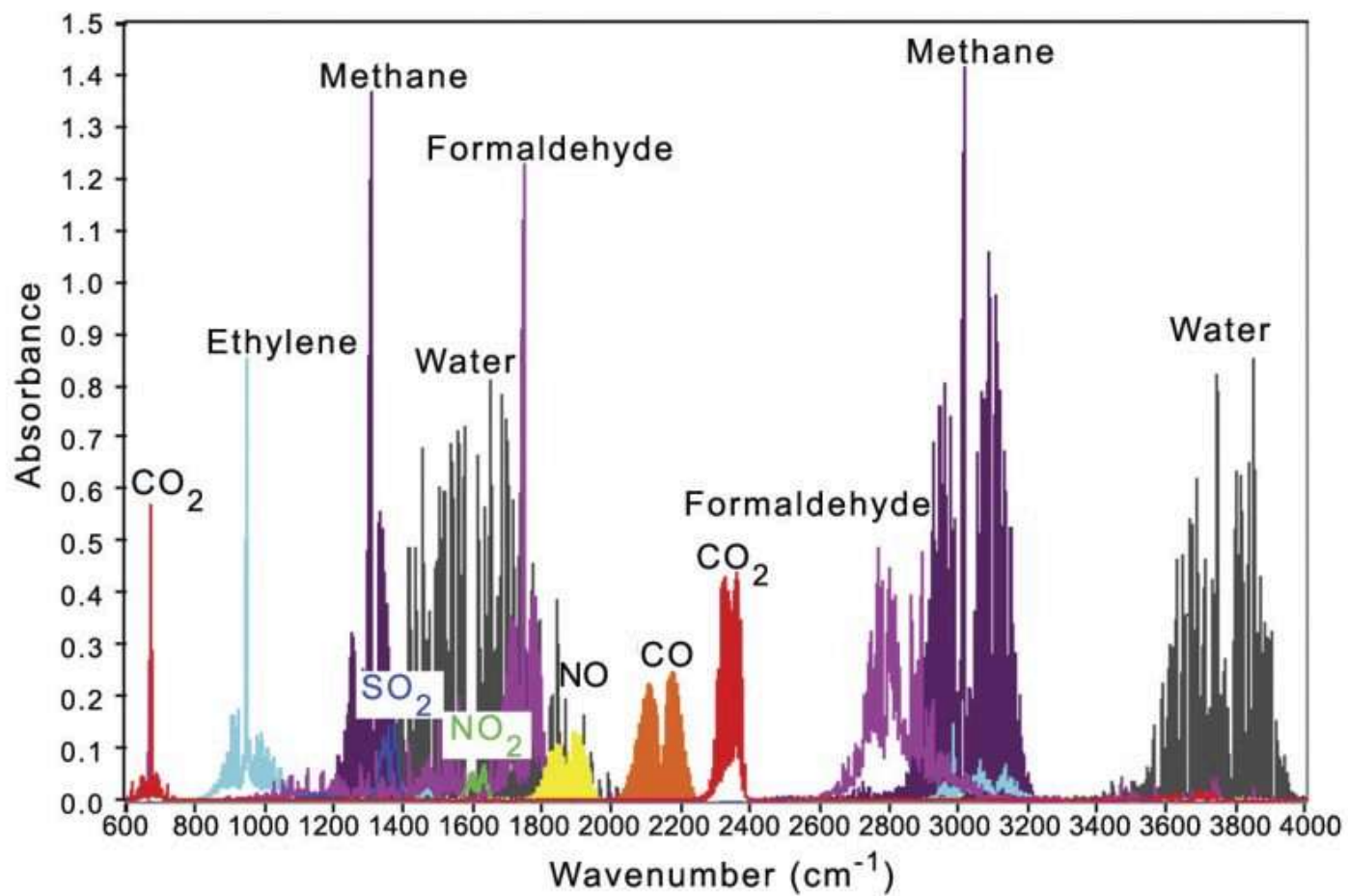


Figure 6.4: IR absorbance of various gases: every sharp line is a real, distinct spectral feature. ●