

Chapter 7 Lorentz model in media

Here, we extend our discussion of the Lorentz model by considering multiple resonators within a material. We will look at:

- Multiple resonances
- Interband transitions
- Dispersion
- Local field correction (the Clausius-Mossotti relation)

7.1 Multiple resonances

A real optical material will have many characteristic resonance frequencies (e.g. different electronic transitions, lattice vibrations). We can add these resonances into the Lorentz model by simple summation.

$$\vec{P}_{res} = \left(\frac{Ne^2}{m_0} \sum_j \frac{1}{(\omega_j^2 - \omega^2 - i\gamma_j\omega)} \right) \vec{E}_0 \quad (7.1)$$

which gives

$$\varepsilon_r(\omega) = 1 + \frac{Ne^2}{\varepsilon_0 m_0} \sum_j \frac{1}{(\omega_j^2 - \omega^2 - i\gamma_j\omega)} \quad (7.2)$$

Note that we now do not need to include a separate background susceptibility, because it is potentially present in our summation.

Compare this result for ε_r with the phenomenological Sellmeier equation for n (and see Problem sheet 2):

$$n^2 = 1 + \sum_j \frac{A_j \lambda^2}{(\lambda^2 - B_j)}. \quad (7.3)$$

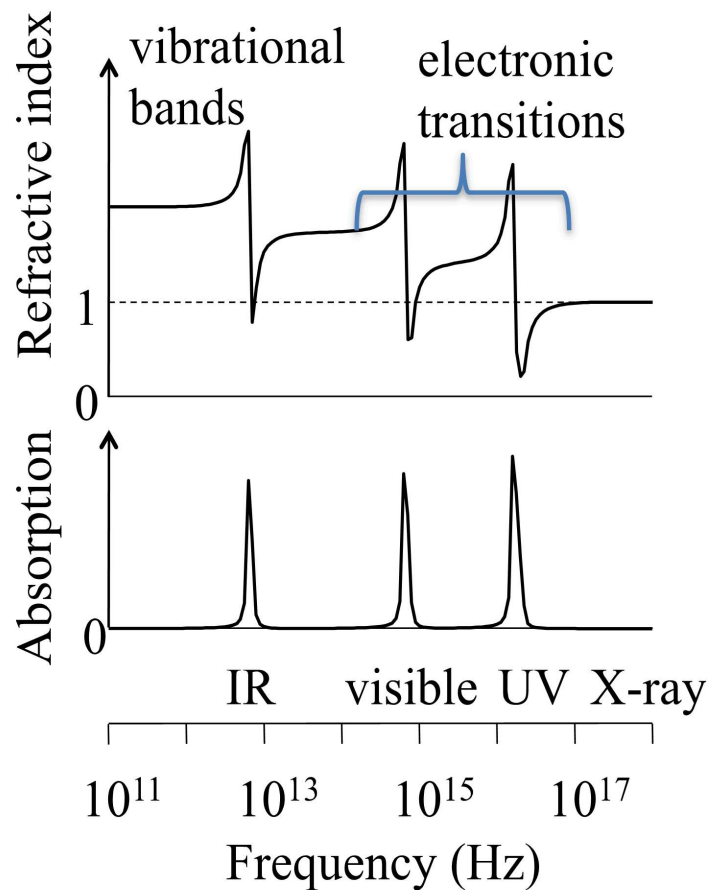


Figure 7.1: Hypothetical solid with three well separated resonances. •

Fig.7.1 shows several regions:

- at very high frequencies, the electrons cannot respond to the field. There is no polarization and $n = 1$;
- the highest frequency electronic resonance is for the inner (core) electrons of atoms, in the X-ray region;
- at lower frequencies (UV-visible), electronic transitions of outer atomic electron (valence) shells are involved;
- In the IR there are resonances at the vibrational frequencies of a solid (phonons) or molecular vibrations (gas/liquid).

There are some points to note:

There is an absorption peak at each resonance and the refractive index “wiggles” across the resonance. The steps in refractive index are due to $\varepsilon_{st} > \varepsilon_{\infty}$ for each resonance. As previously defined, ε_{st} and ε_{∞} are relative to a single resonance: their values for a system of many resonances would be different. Between the resonances, the medium is transparent - the absorption coefficient is very small and the refractive index is almost, but not completely, constant.

In real materials, different resonances have different absorption coefficients; these can be measured experimentally. The model can account for this by assigning a phenomenological oscillator strength, f_j , to each resonance. We then obtain Eq.(7.4):

$$\varepsilon_r(\omega) = 1 + \frac{Ne^2}{\varepsilon_0 m_0} \sum_j \frac{f_j}{(\omega_j^2 - \omega^2 - i\gamma_j \omega)}. \quad (7.4)$$

It can be shown from quantum mechanics that we must have $\sum_i f_i = 1$ for any single electron. We can understand this if a particular electron involved in several transitions has its absorption strength divided between the transitions. In quantum mechanics the oscillator strength is determined by the transition probability (see 5.4).

7.1.1 Comparison with experiment: pure silica glass

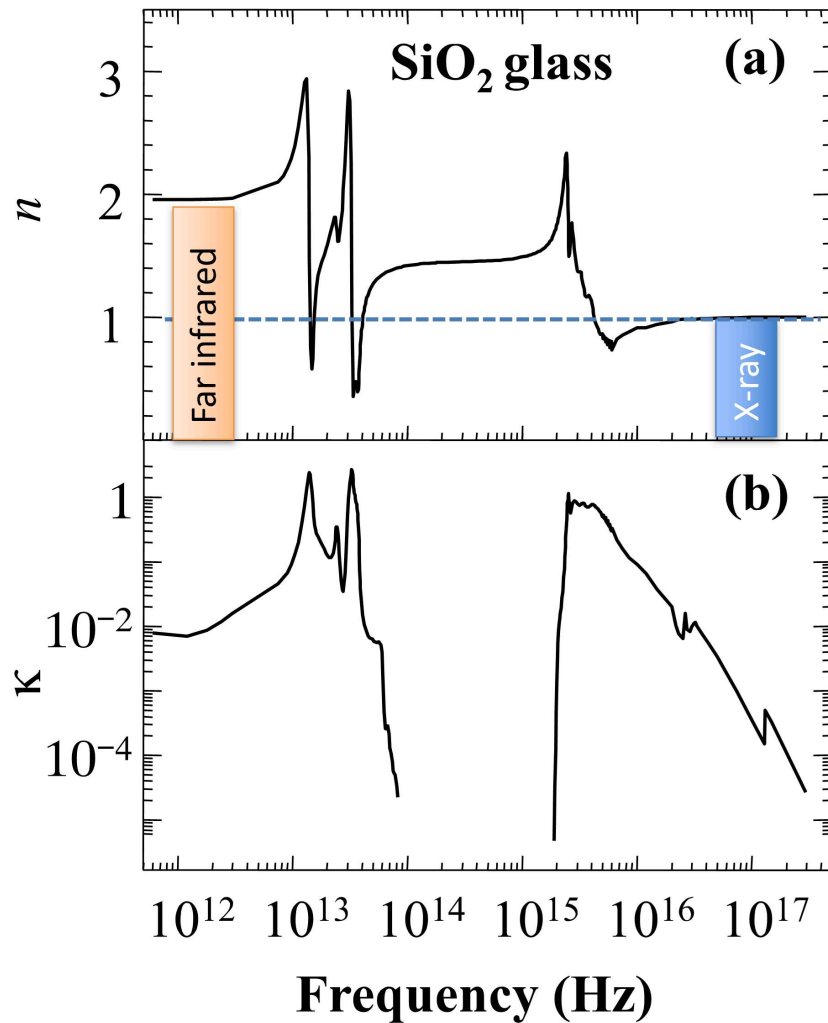


Figure 7.2: Example of multiple resonance response: amorphous SiO₂. •

There is strong absorption in the infrared from two vibrational modes of the SiO₂ molecular unit – there is no long-range order so that vibrations are localised.

In the ultraviolet, there are electronic transitions across the bandgap of around 10 eV.

Between these frequency ranges, there is a wide region in the near-IR, visible and near-UV ($10^{14} - 2 \times 10^{15}$ Hz) where silica glass is transparent and $\kappa \ll n$.

7.2 Lorentz model and interband transitions

The Lorentz oscillator model provides a good description of transitions involving electrons localised on atoms or molecules, but a poorer description of interband transitions which involve delocalised electrons.

For example, the absorption spectrum of a gas consists of a series of well defined peaks whereas interband absorption in a solid is characterised by an absorption edge near the band gap E_g . If the resonant interband frequency is identified with E_g/\hbar then n varies qualitatively as expected around this frequency but κ does not – see the data for SiO_2 in section 5.1.1. An accurate description of interband transitions requires a quantum mechanical model.

7.3 optical dispersion

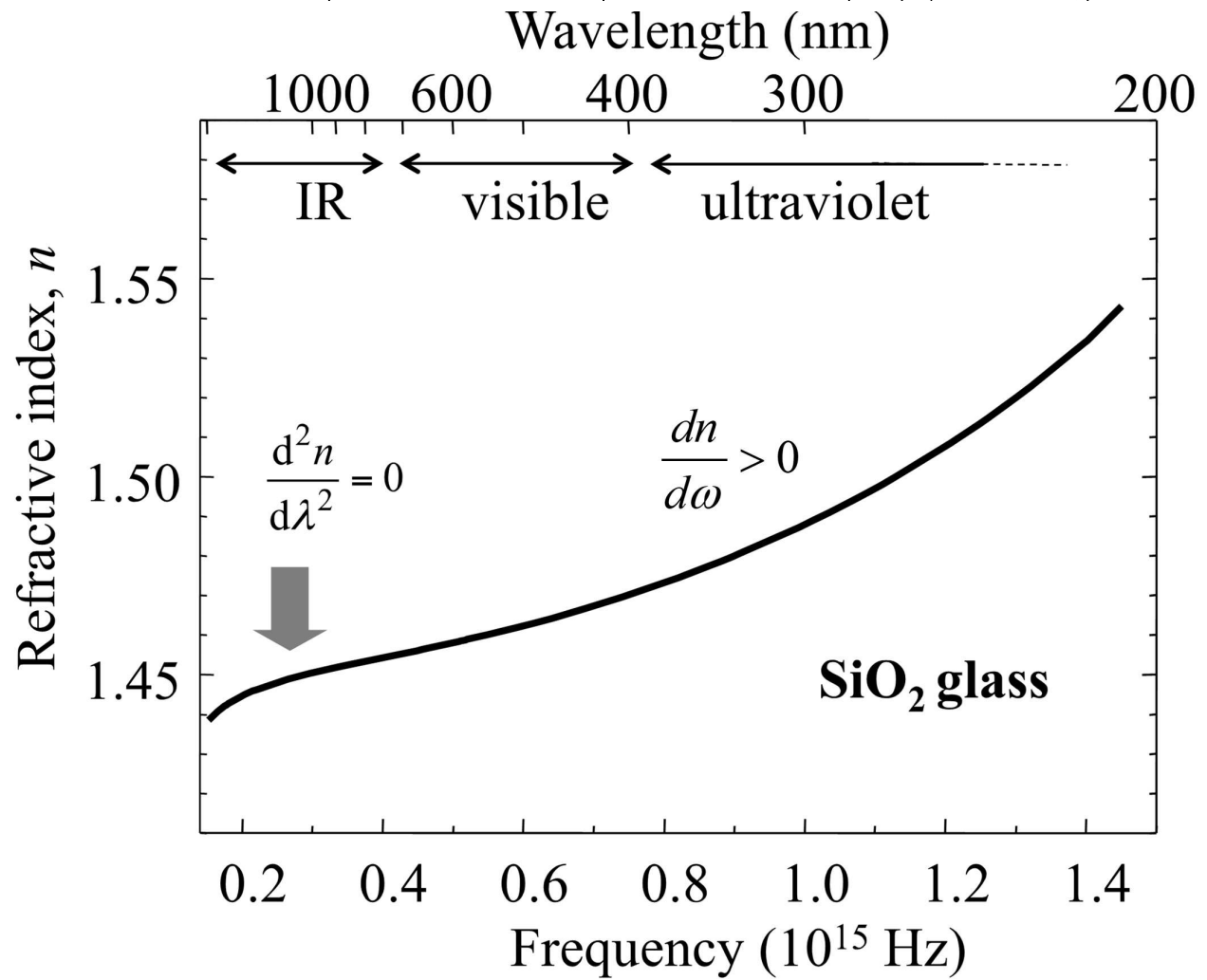
In the transparent region the refractive index increases with increasing frequency – this is called *normal dispersion*. In the highly absorbing regions n decreases with increasing frequency – this dispersion is described as *anomalous*.

Dispersion in the visible part of the spectrum arises from the high frequency wing of infrared absorption and the low frequency wing of UV absorption.

Note that $n < 1$ at some frequencies and phase velocity $v_{ph} = c/n$ is greater than c . This does not violate relativity because signals travel at the group velocity, which is less than v_{ph} :

$$v_g = \frac{d\omega}{dk} = v_{ph} / \left(1 + \frac{\omega}{n} \frac{dn}{d\omega} \right) \text{ and } \frac{dn}{d\omega} \text{ generally } > 0 \quad (7.5)$$

Fig.7.3 completes our picture of silica glass with its dispersion in its transparency range. The group velocity dispersion (GVD) is proportional to $\frac{d^2n}{d\lambda^2}$ and determines, for example, pulse spreading (in time) in optical fibres. The GVD goes to zero near $1.3\mu\text{m}$, giving this wavelength range enormous technological significance.

Figure 7.3: Example of dispersion: amorphous SiO_2 . •