Chapter 8 Optical properties of plasmas

We now apply ideas from the Lorentz oscillator model to free charges in a plasma or metal, linking to the Drude theory of conduction in metals. We'll consider:

- Free electron plasma and its dielectric function
- Optical conductivity
- Ideal plasma reflectivity
- Infrared reflectivity of real metals

8.1 Dielectric constant of a plasma

A neutral gas of charged particles is called a *plasma*, arguably a fourth state of matter. Plasma properties play important roles in semiconductor devices, material processing, fusion research and astrophysics.

Metals, doped semiconductors and ionized gases can be treated as *single component plasmas* in which ions are slow-moving or immobile and charge carriers are free to move. In metals, charge carriers come from valence electrons whereas, in semiconductors, they come mostly from donors (n-type) or acceptors (p-type).

Two-component plasmas are ones where both positive and negative charges are mobile, e.g. the extremely hot deuterium and tritium plasmas in a fusion reactor, where there are both both free ions and electrons with very different plasma frequencies.

We take the case of free electrons in a metal (number density N) as an example, and aim to work out the dielectric constant. The equation of motion of a single electron is:

$$m_0 \frac{d^2x}{dt^2} + m_0\gamma\frac{dx}{dt} = -eE(t) = -eE_0e^{-i\omega t} \tag{8.1}$$

where γ is the damping rate. This equation is very similar to the one used in chapter 4 to derive the Lorentz oscillator model, Eq. (6.3), but here there is no term representing a restoring force, since the electrons are free. We do not need to solve this equation from the beginning: removing the restoring force is equivalent to setting $\omega_0 = 0$ in Eq. (6.3) and everything derived from it.

In this way, we obtain a new version of the expression for the dielectric function Eq. (6.12) (note, the background term has also been dropped):

$$\tilde{\varepsilon}_r(\omega) = 1 - \frac{\omega_p^2}{(\omega^2 + i\gamma\omega)}. \tag{8.2}$$

where ω_p is called the **plasma frequency** and is given by:

$$\omega_p = \sqrt{\frac{Ne^2}{\varepsilon_0 m_0}}. \tag{8.3}$$

It is usual to replace the damping rate by the inverse electron scattering time τ:

$$\tau = 1/\gamma. \tag{8.4}$$

With this change, we obtain:

$$\tilde{\varepsilon}_r(\omega) = 1 - \frac{\omega_p^2}{(\omega^2 + i\,\gamma\,\omega)} \;\rightarrow\; \tilde{\varepsilon}_r(\omega) = 1 + \frac{i\tau\omega_p^2}{\omega(1 - i\tau\,\omega)}. \tag{8.5}$$

The real and imaginary parts are:

$$\varepsilon_1 = 1 - \frac{\omega_p^2\tau^2}{(1 + \omega^2\tau^2)} \tag{8.6}$$

$$\varepsilon_2 = \frac{\omega_p^2\tau}{\omega(1 + \omega^2\tau^2)}. \tag{8.7}$$

From ε_1 and ε_2 we can find n and κ using the standard equations and, hence, find the absorption coefficient and reflectivity.

8.2 Relationship between dielectric constant and

conductivity

In the Drude model, the DC electrical conductivity is σ_0:

$$\sigma_0 = \frac{Ne^2\tau}{m_0} = \omega_p^2 \varepsilon_o \tau \tag{8.8}$$

and the AC or 'optical' conductivity is

$$\tilde{\sigma}(\omega) = \frac{\sigma_0}{1 - i\omega\tau}. \tag{8.9}$$

Here, N is the number of free electrons per unit volume and τ is the momentum scattering time. For metals and highly doped semiconductors $\tau \sim 10^{14} - 10^{13}$s, so that $\tilde{\sigma}(\omega)$ doesn't vary much from the static value until the frequency is in the infrared.

Comparison of previous expressions for $\tilde{\varepsilon}(\omega)$ with that for $\tilde{\sigma}(\omega)$ shows that

$$\tilde{\varepsilon}_r(\omega) = 1 + i\frac{\tilde{\sigma}(\omega)}{\varepsilon_o \omega}. \tag{8.10}$$

To check this, substitute Eq. (8.9) into Eq. (8.10) to obtain Eq. (8.5).

8.3 Undamped plasma reflectivity

If the damping is negligible, $\gamma = 0$ and Eq.(8.2) shows that

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \tag{8.11}$$

Recall from Eq.(2.1) that:

$$\tilde{n} = n + i\kappa = \sqrt{\tilde{\varepsilon}_r}. \tag{8.12}$$

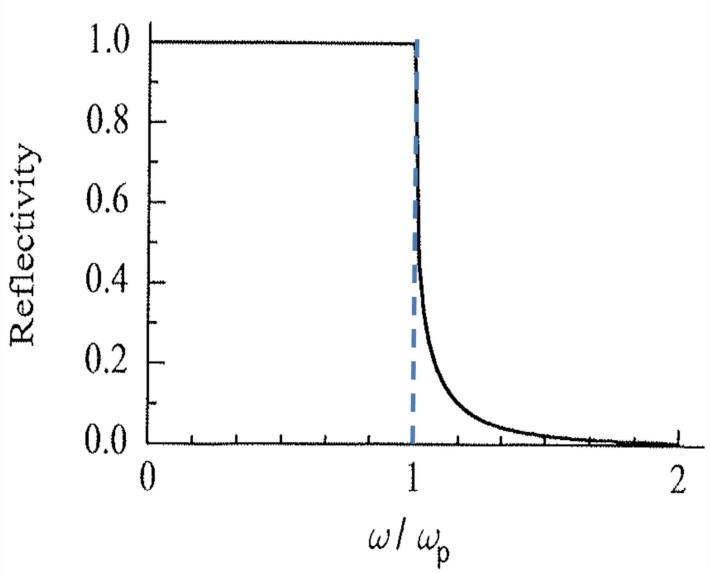

Figure 8.1: Reflectivity of an ideal undamped plasma. ●

The reflectivity, plotted above in Fig.8.1 is given as before by:

$$R = \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2} \tag{8.13}$$

and the shape of the plot is easily understood. If $\omega < \omega_p$ then $\tilde{\varepsilon}_r$ is negative and $\tilde{n}$ is purely imaginary, so that $n = 0$. The reflectivity below ω_p is therefore:

$$R = \frac{1 + \kappa^2}{1 + \kappa^2} = 1. \tag{8.14}$$

The physical interpretation of this is that, when $\omega < \omega_p$, the electrons can move fast enough to screen the optical field, so that the light hardly penetrates the surface, and the reflectivity is high.

For $\omega > \omega_p$, $\tilde{\varepsilon}_r$ is positive and $\tilde{n}$ is real, so that $R < 1$. The electrons now do not move fast enough to screen the field completely, and the electric field penetrates significantly, with more loss and lower reflectivity.

In metals, the plasma frequency is typically in the range 1-5 $\times 10^{15}$Hz (energies 4-20 eV) and so metals become transparent in the deep UV range.

8.4 Low frequency reflectivity with damping

For metals at very low frequencies (e.g. microwave and far infrared) where $\omega\tau << 1$, we can approximate as follows:

$$\varepsilon_1 = 1 - \frac{\omega_p^2\tau^2}{(1+\omega^2\tau^2)} \approx 1 - \omega_p^2\tau^2 \approx -\omega_p^2\tau^2 \tag{8.15}$$

where the last step is justified because ω_p is of order 10^{15}Hz, but τ is only around 10^{-13}s:

$$\omega_p >> 1/\tau. \tag{8.16}$$

$$\varepsilon_2 = \frac{\omega_p^2\tau}{\omega(1+\omega^2\tau^2)} \approx \frac{\omega_p^2\tau}{\omega} \tag{8.15}$$

Comparing ε_1 and ε_2,

$$\varepsilon_1/\varepsilon_2 = -\omega\tau << 1, \tag{8.17}$$

we can see that we are in the limit of strong absorption and so:

$$n \approx \kappa = \sqrt{\frac{\varepsilon_2}{2}} \tag{8.18}$$

with both $n >> 1$ and $\kappa >> 1$.

8.5 Reflectivity (normal incidence)

We now derive the reflectivity under these conditions, starting from the most general expression.

$$R = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2 = \frac{(n-1)^2+\kappa^2}{(n+1)^2+\kappa^2} \tag{8.19}$$

$$= \frac{n^2-2n+1+\kappa^2}{n^2+2n+1+\kappa^2}$$

$$= 1 - \frac{4n}{n^2+2n+1+\kappa^2}$$

$$\approx 1 - \frac{2}{n}, n, \kappa \gg 1$$

To proceed, we require the value of n:

$$n^2 = \frac{\varepsilon_2}{2} = \frac{\sigma_0}{2\varepsilon_0\omega(1+\omega^2\tau^2)} \approx \frac{\sigma_0}{2\varepsilon_0\omega}, \omega\tau \ll 1$$

$$n \approx \sqrt{\frac{\sigma_0}{2\varepsilon_0 \omega}}$$

$$R = 1 - 2\sqrt{\frac{2\varepsilon_o \omega}{\sigma_o}} = 1 - 2\sqrt{\frac{2\varepsilon_o m_o \omega}{Ne^2\tau}} \tag{8.20}$$

This is the **Hagen-Rubens** relation for the infrared reflectivity of metals.

8.6 Absorption coefficient

Recall our basic definitions from Chapter 2, and the fact that we are in the strong absorption limit, Eq.(8.18), which with Eq.(8.20) gives us a value for κ.

$$\alpha = \frac{2\kappa\omega}{c} = \sqrt{2\sigma_o \omega \mu_o} \text{ using } c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \tag{8.21}$$

We introduce the concept of the **skin depth** δ, the length scale over which the electric field in a conductor decays. The field decays exponentially from the surface

$$E(z) = E_0 e^{-z/\delta} \tag{8.22}$$

and so δ is given by:

$$\delta = \frac{2}{\alpha} = \sqrt{\frac{2}{\sigma_o \omega \mu_o}}. \tag{8.23}$$

At higher frequencies, when $\omega\tau << 1$ is no longer true, these approximations break down, as is the case for near infrared absorption due to free carriers in semiconductors (below).

8.7 Reflectivity and interband absorption in real metals

Light penetrates only a short way into the surface of a good conductor (remember the skin depth concept). Interband transitions can cause absorption of light that has penetrated the surface and the reflectivity then falls below 1 when $\omega < \omega_p$.

As an example, we take aluminium, Fig. 8.2. The prediction of the plasma frequency edge (dotted line) is roughly correct (and agreement can be improved by taking m^*, rather than m_0, to calculate ω_p). The reflectivity is less than 100% below ω_p, but adding damping only has a small effect (dashed line).

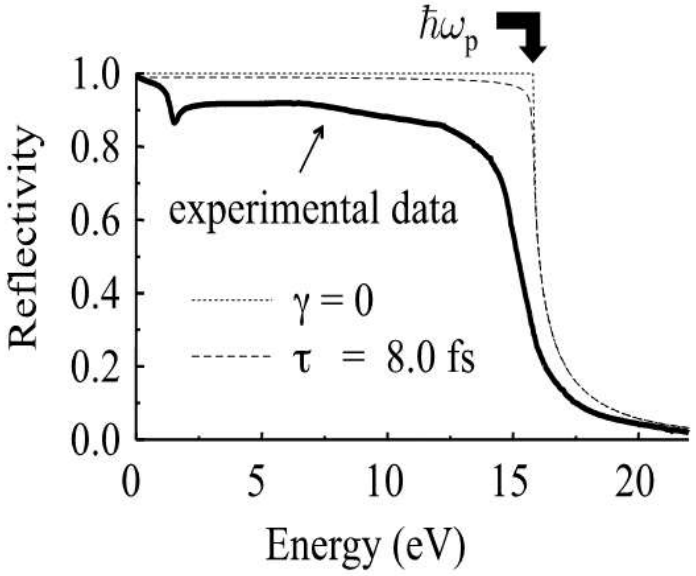

Figure 8.2: Experimental reflectivity of aluminium. ●

The main reduction in reflectivity is due to interband transitions with an onset at 1.5 eV. The reflectivity tends to 100% below 1.5 eV (as there are no interband transitions there).

As a second example, we consider copper. The eleven valence electrons of Cu fill up the $3d$ band and half-fill the $4s$ band. The Fermi energy therefore lies within the $4s$ band. An optical transition can excite an electron from an occupied $3d$ state to an empty state at or above the Fermi level. The high density of $3d$ states makes this a likely process.

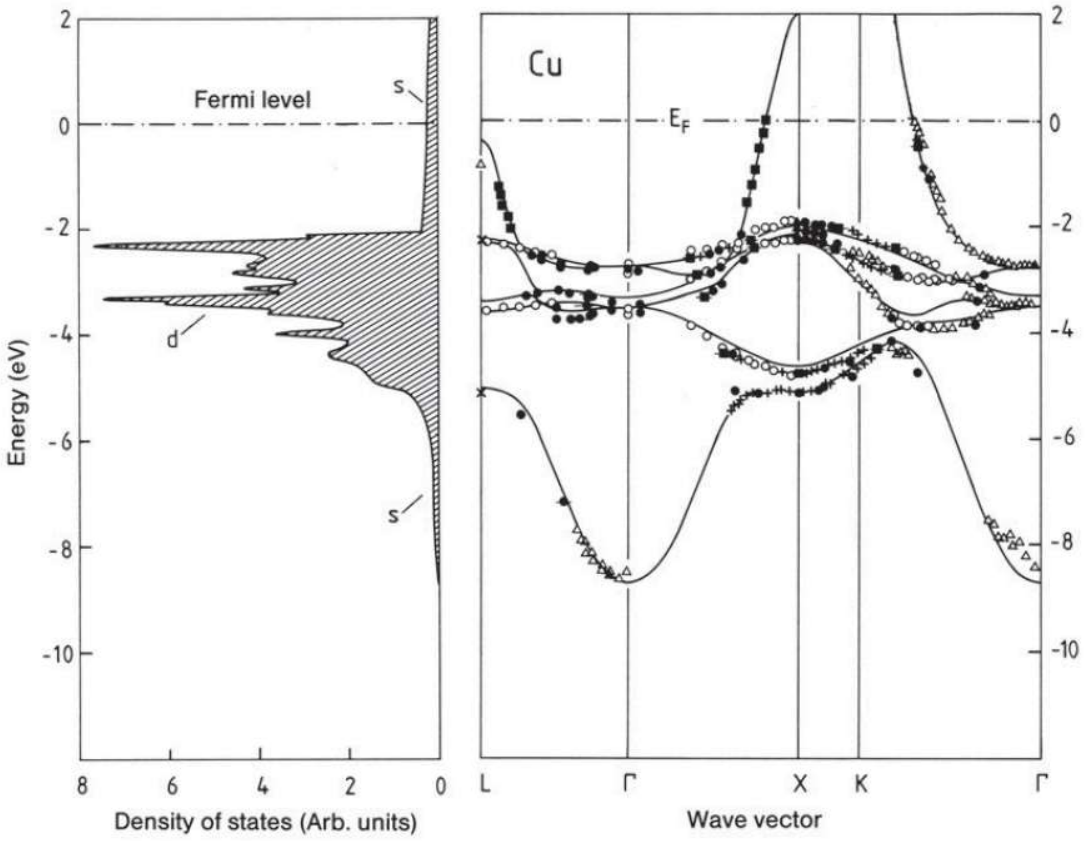

Figure 8.3: Band structure and density of states for copper showing the s states at the Fermi energy and the very high density of d states around 2 eV below the Fermi energy. (Courths and Hüfner (1984)) ●

Optical transitions from $3d$ to $4s$ are possible with a sharp threshold at 2.2 eV (560 nm). The plasma frequency corresponds to an energy of 10.8 eV but the reflectivity between 2 and 10 eV is dominated by the effect of interband transitions. Copper reflects light with $\lambda > 600$ nm well. It therefore has a reddish colour.

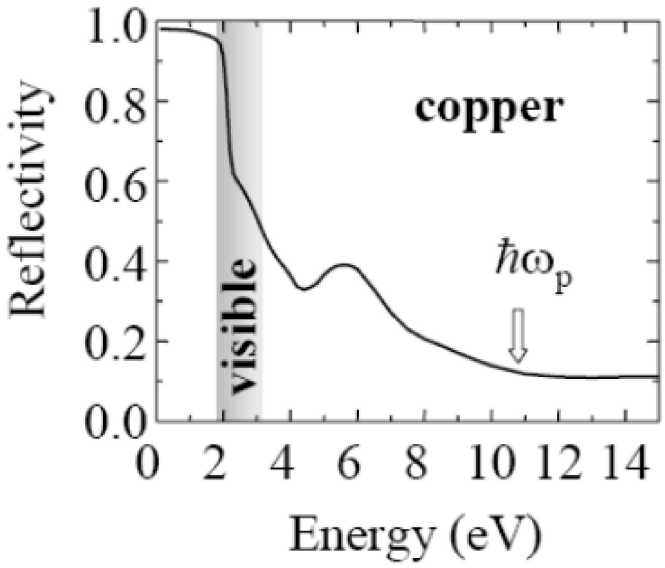

Figure 8.4: Experimental reflectivity of copper, showing the pronounced effect of interband transitions. ●

Gold provides a similar example: the interband absorption threshold between the d bands and the Fermi level occurs at 2.4 eV, (516 nm), a slightly higher energy than copper. The plasma edge is at 9 eV. Gold reflects red, orange and yellow light more than green and blue, and this accounts for its characteristic yellowish colour.

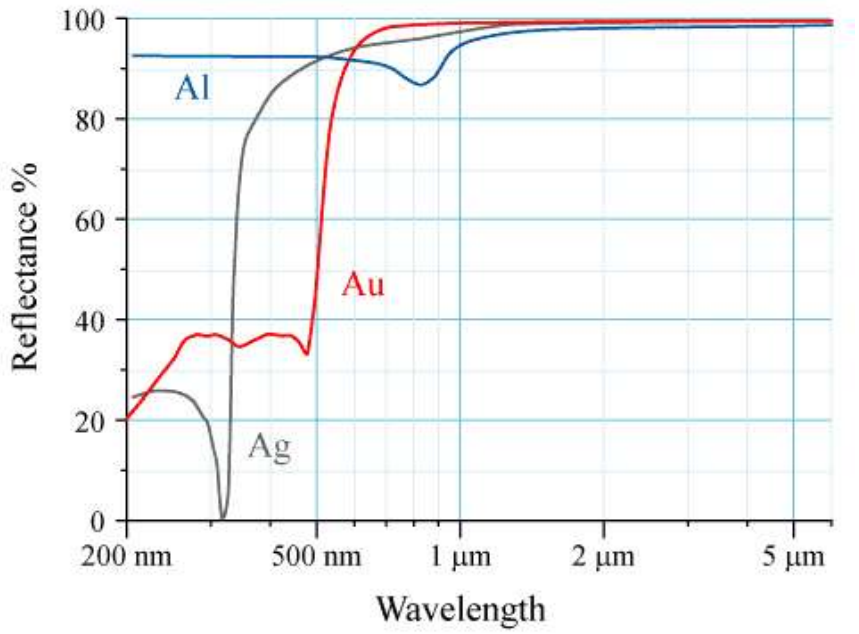

Figure 8.5: Reflectivity of gold and silver (from ZME Science). ●

Silver is rather different: the interband absorption edge is in the UV at ~4eV. The reflectivity remains high for the entire visible spectrum, accounting for its lack of colour and its use in high quality mirrors.

8.8 Doped semiconductors

Doping introduces mobile charge carriers into semiconductors at finite temperatures and these charge carriers can exhibit metal-like behaviour. They can be described by a modified Drude-Lorentz model:

- The effective mass of the carriers is modifed: m_0 is replaced by m^* in the expression for ω_p
- the background permittivity of the semiconductor lattice is large and ε_0 must be replaced by $\varepsilon_{opt}\varepsilon_0$ where ε_{opt} is the value of ε_r in the transparent region below the interband absorption edge in the undoped material.

- The free carrier density N can be controlled over a huge range, by built-in doping or by electrical “gating” in electronic devices.

The plasma frequency is therefore modified:

$$\tilde{\varepsilon}_r(\omega) = \varepsilon_{opt}(1 - \frac{\omega_p^2}{(\omega^2 + i\,\gamma\,\omega)}),\ \omega_p = \sqrt{\frac{Ne^2}{\varepsilon_{opt}\varepsilon_o m^*}} \tag{8.24}$$

The plasma reflectivity edge is in the mid to far infrared where most intrinsic (i.e, undoped) semiconductors are transparent.

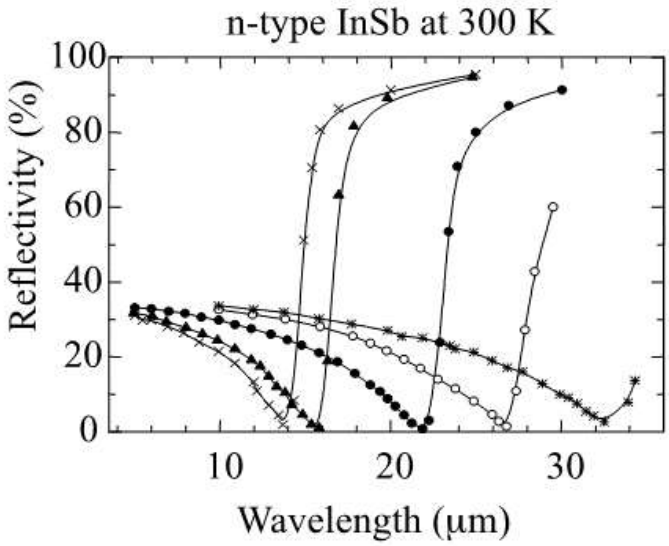

Figure 8.6: Reflectivity of InSb for different levels of doping (see main text). ●

As Eq.(8.24) predicts and Fig.8.6 shows, the plasma frequency depends on the doping level. In Fig.8.6, the doping levels range from 4.0×10^{24} m^{-3} (symbol $\times$) to 3.5×10^{23} m^{-3} (symbol *).

8.9 Plasma oscillations

In a medium with no net charge, Gauss’s law states that:

$$\nabla \cdot \vec{D} = \nabla \cdot (\tilde{\varepsilon}_r \varepsilon_0 \vec{E}) = 0. \tag{8.25}$$

Consider a wave solution of the form:

$$\vec{E}(\vec{r}, t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}. \tag{8.26}$$

Substituting this into Eq.(8.25) gives

$$\tilde{\varepsilon}_r\varepsilon_0\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)=\tilde{\varepsilon}_r\varepsilon_0(k_xE_x+k_yE_y+k_zE_z)\ \Rightarrow \vec{k}.\vec{E}=0 \qquad (8.27)$$

if $\tilde{\varepsilon}_r \neq 0$; the condition $\vec{k}.\vec{E}=0$ defines a **transverse** wave.

However, if $\tilde{\varepsilon}_r = 0$, then $\vec{k}.\vec{E}$ does not have to equal zero, and the material can support **longitudinal** electromagnetic waves.

We have seen in lecture 7 that for an undamped system of free electrons,

$$\varepsilon_r(\omega)=1-\frac{\omega_p^2}{\omega^2}. \qquad (8.28)$$

Clearly, at the plasma frequency, $\tilde{\varepsilon}_r = 0$, so that we can have longitudinal EM waves at ω_p.

These waves are associated with collective longitudinal oscillations of the electrons relative to the background of the positive ion lattice and are called bulk or volume **plasmons**, Fig.8.7. Light in vacuum is a transverse wave and so cannot directly excite bulk plasmons.

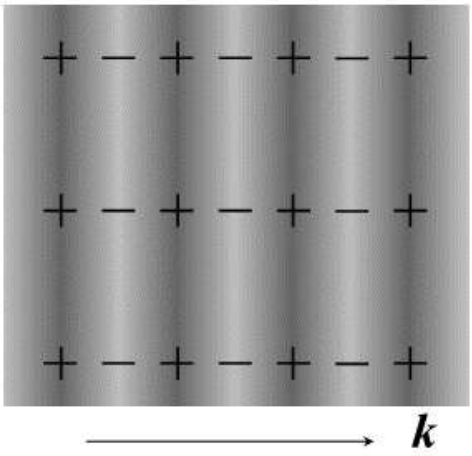

Figure 8.7: Schematic view of a longitudinal bulk plasmon, where the shading represents the density of free charge and the electric field is parallel to k. ●

The plasma frequency ω_p is therefore both the frequency above which conductors become transparent, Eq. (8.3), and the natural oscillation frequency of the electron gas relative to the ion cores.

References

Courths, R., and S. Hüfner. 1984. “Photoemission Experiments on Copper.” *Physics Reports* 112 (2): 53–171. https://doi.org/https://doi.org/10.1016/0370-1573(84)90167-4.