DC Motor Drive

Nicola Toscani

Electric Propulsion

A.Y. 2020/2021

Electrical Drive: Definition

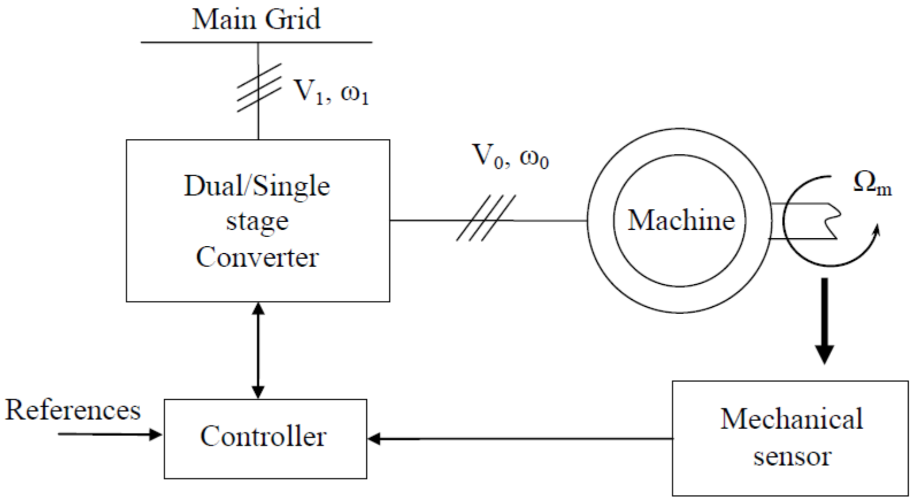
An electrical drive is a system which converts electric energy into mechanical one by making use of power electronics apparatuses, following an input function (**CEI 301-1**).

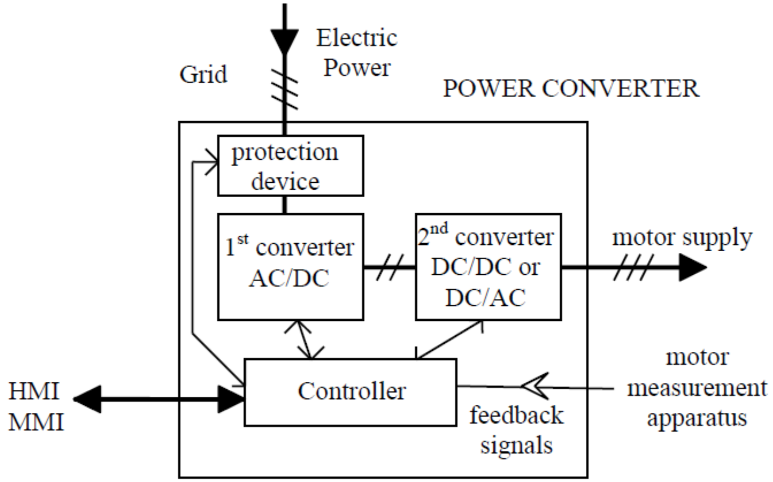
In industrial terminology, an electrical drive refers to both the power converter and the motor stage.

Typical applications of electrical drives:

- Traction (e-mobility, trains, subways, tramway vehicles, trolley busses, ships)
- Industrial applications (compressors, robotics, fluid handling,...)

Electrical Drive





PMDC Motor: Dynamical Model

Electrical equation:

$$v_{\mathrm{a}}(t) = L_{\mathrm{a}} \frac{\mathrm{d}i_{\mathrm{a}}(t)}{\mathrm{d}t} + R_{\mathrm{a}} i_{\mathrm{a}}(t) + e(t)$$

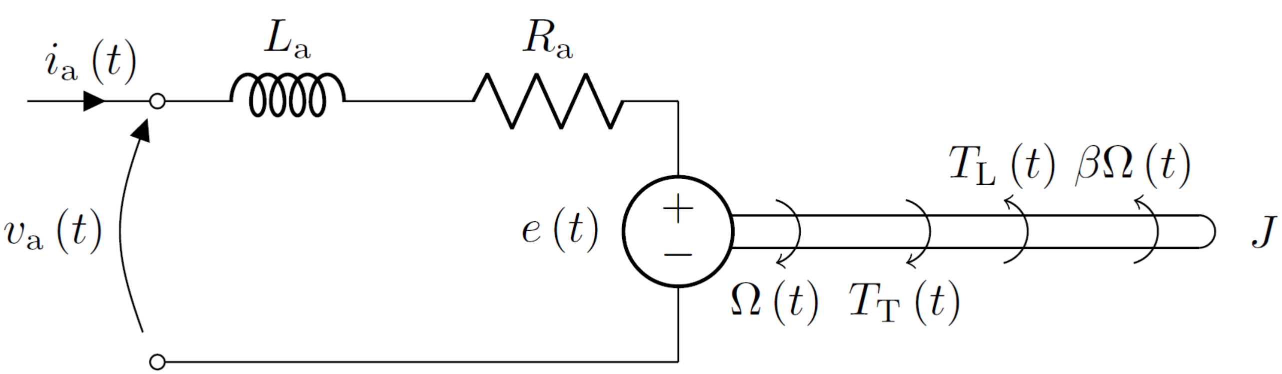
Mechanical equation:

$$T_{\mathrm{T}}(t) - T_{\mathrm{L}}(t) = J \frac{\mathrm{d}\Omega(t)}{\mathrm{d}t} + \beta\Omega(t)$$

Where:

$$e(t) = K\Omega(t)$$

$$T_{\mathrm{T}} = K i_{\mathrm{a}}(t)$$



PMDC Motor: Dynamical Model 2

State variable space:

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \Omega(t) & i_{\mathrm{a}}(t) \end{bmatrix}^{\mathrm{T}} \text{ state variables}$$

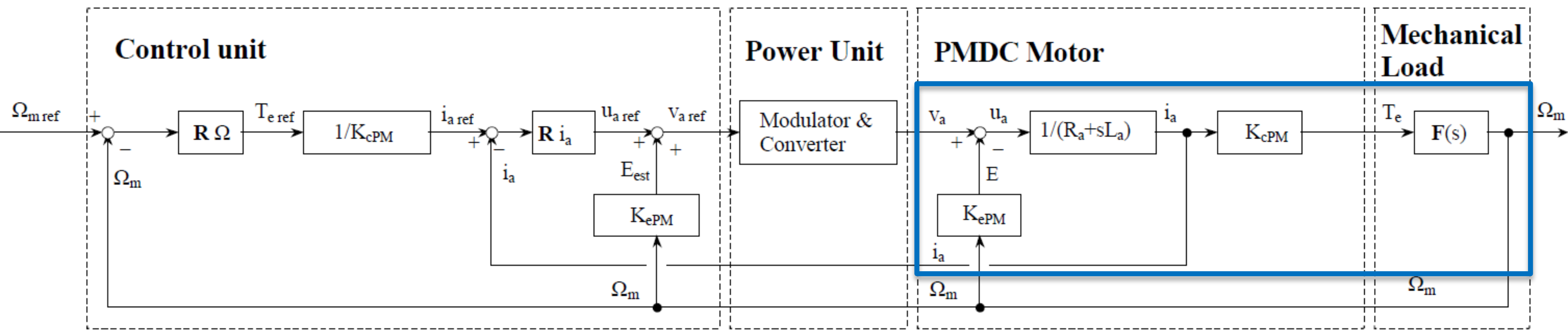
$$y(t) = x(t) \quad \text{output}$$

$$u(t) = \begin{bmatrix} T_{\mathrm{L}}(t) & v_{\mathrm{a}}(t) \end{bmatrix}^{\mathrm{T}} \text{ input}$$

$$\begin{cases} \dot{x}(t) = \underbrace{\begin{bmatrix} -\frac{\beta}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} -\frac{1}{J} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}}_{B} u(t) \\ y(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} x(t) \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = -\frac{\beta}{J} x_2(t) + \frac{K}{J} x_2(t) - \frac{1}{J} u_1(t) \\ \dot{x}_2(t) = -\frac{K}{L} x_1(t) - \frac{R}{L} x_2(t) + \frac{1}{L} u_2(t) \\ y_1(t) = x_1(t) \\ y_2(t) = x_2(t) \end{cases}$$

PMDC Motor: Control Scheme

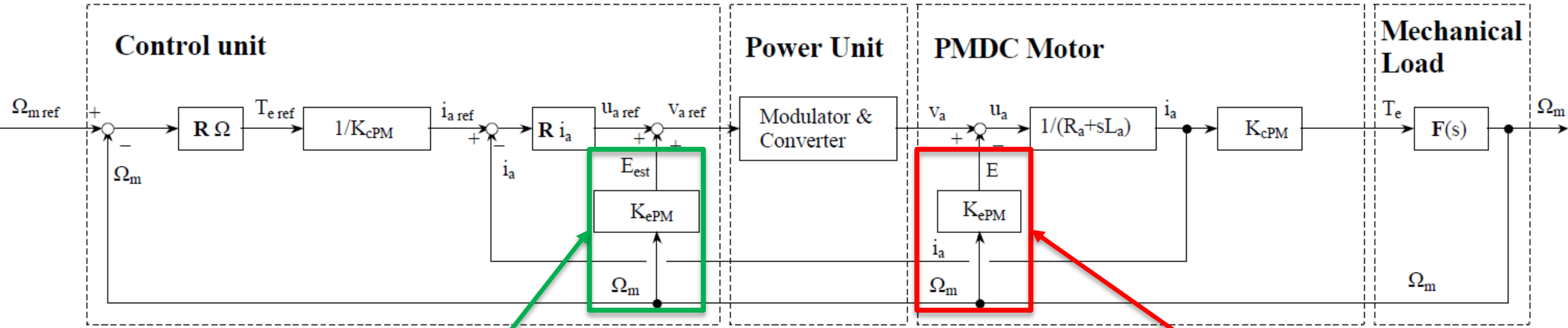


$T_{\mathrm{L}}(t)$

$$v_{\mathrm{a}}(t) \rightarrow \boxed{\begin{array}{c}\dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t)\end{array}} \rightarrow \begin{array}{l}\Omega(t) \\ i_{\mathrm{a}}(t)\end{array}$$

Coupling effects are hidden in the state variable space (matrices)

PMDC Motor: Decoupling



Decoupling through a compensation term is possible if a speed sensor or a good speed estimation is available $e_{\text{est}}(t)$

Electrical and mechanical dynamics are coupled through $e(t)$

PMDC Motor: Decoupling

Since the dynamics are decoupled, the two PI controllers can be designed independently

$$G_\Omega = \frac{1}{\beta + sJ} \qquad G_i = \frac{1}{R_\mathrm{a} + sL_\mathrm{a}}$$

Remember: the outer loop must be at least one decade slower than the inner loop (unitary gain) in order to tune the controller independently from the inner regulator (**cascade constraint**)

$$\Downarrow$$

$$10\omega_\Omega \le \omega_i$$

PMDC Motor: Preliminary Exercise

A PMDC motor equipped with a speed sensor has to follow a speed reference from 0 to 300 rad/s.

- Choose the bandwidth for the two control loops compliant with the cascade constraint. Consider a phase margin $\geq 60°$. Write a MATLAB script to compute kp and ki or use pidtool.

Motor parameters:

R [Ω]	L [mH]	K [Nm/A]	J [kgm^2]	β [Nms]
0.6	2	0.04	$6 \cdot 10^{-5}$	0.01

Separately Excited DC Motor

Electrical equations:

$$v_{\mathrm{a}}(t) = L_{\mathrm{a}}\frac{\mathrm{d}i_{\mathrm{a}}(t)}{\mathrm{d}t} + R_{\mathrm{a}}i_{\mathrm{a}}(t) + e(t) \qquad v_{\mathrm{e}}(t) = L_{\mathrm{e}}\frac{\mathrm{d}i_{\mathrm{e}}(t)}{\mathrm{d}t} + R_{\mathrm{e}}i_{\mathrm{e}}(t)$$

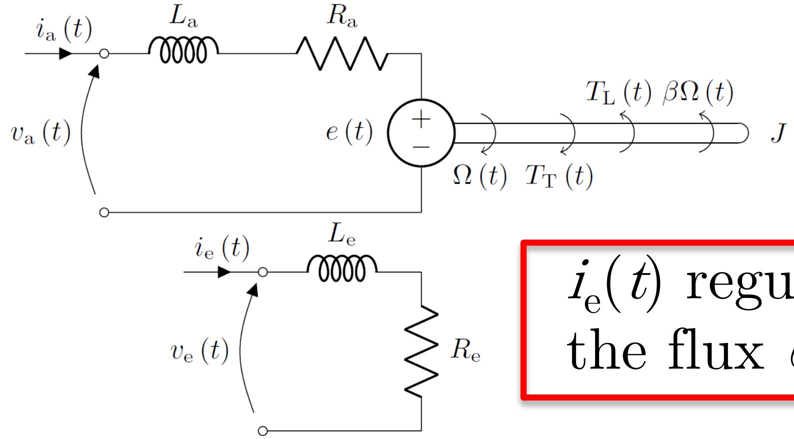
Mechanical equation:

$$T_{\mathrm{T}}(t) - T_{\mathrm{L}}(t) = J\frac{\mathrm{d}\Omega(t)}{\mathrm{d}t} + \beta\Omega(t)$$

Where:

$$e(t) = \boxed{K_{\mathrm{S}}i_{\mathrm{e}}(t)}\,\Omega(t)$$

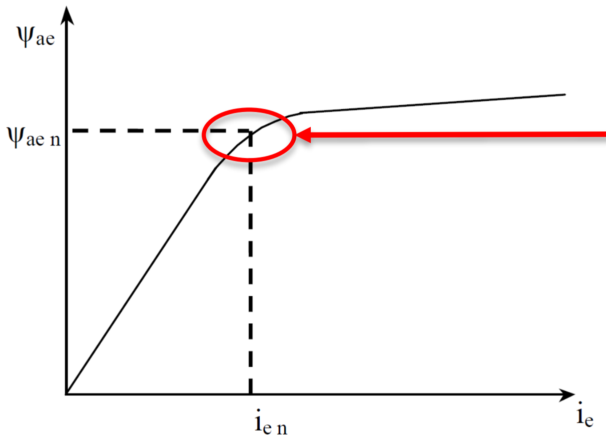
$$T_{\mathrm{T}} = \boxed{K_{\mathrm{S}}i_{\mathrm{e}}(t)}\,i_{\mathrm{a}}(t)$$



$i_{\mathrm{e}}(t)$ regulates the flux $\psi_{\mathrm{ae}}(t)$

SEDC Motor: Excitation

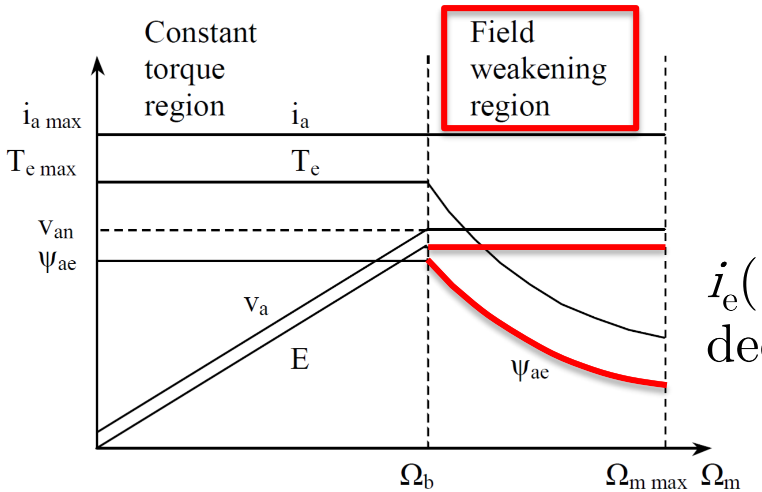
The relationship between $i_e(t)$ and $\psi_{ae}(t)$ is nonlinear in general (ferromagnetic material)



Nominal operating point at the knee of the magnetizing curve

Operating regions: field weakening when $\Omega > \Omega_b$

$$e(t) = K_S i_e(t)\,\Omega(t) = E_n$$



$i_e(t)$ must decrease

SEDC Motor: Field Weakening

$$\Omega < \Omega_{\mathrm{b}} \qquad e\left(t\right) = K_{\mathrm{S}} i_{\mathrm{e}}\left(t\right) \Omega\left(t\right) \rightarrow i_{\mathrm{e}}\left(t\right) = I_{\mathrm{e,n}}$$

Thus:

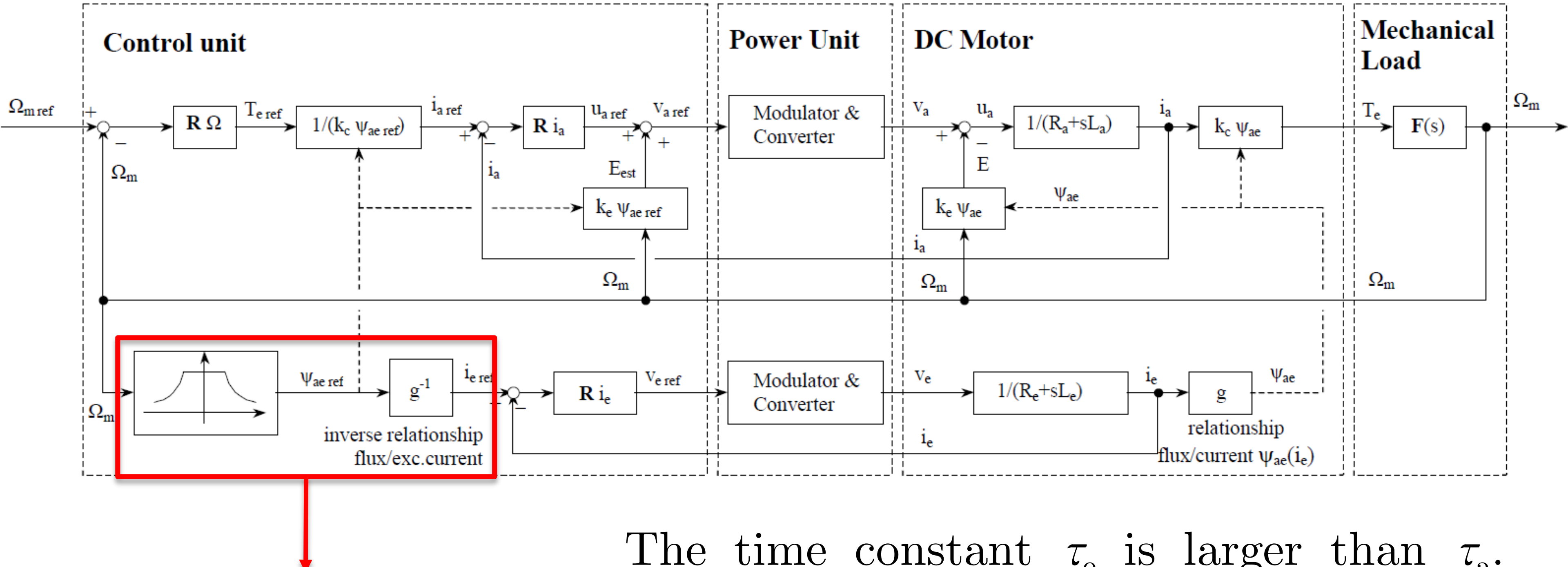
$$e\left(t\right) = \underbrace{K_{\mathrm{S}} I_{\mathrm{e,n}}}_{K} \Omega\left(t\right)$$

$$\Omega > \Omega_{\mathrm{b}} \qquad e\left(t\right) = K_{\mathrm{S}} i_{\mathrm{e}}\left(t\right) \Omega\left(t\right) = E_{\mathrm{n}}$$

Thus:

$$i_{\mathrm{e}}\left(t\right) = \frac{E_{\mathrm{n}}}{K_{\mathrm{S}} \Omega\left(t\right)}$$

SEDC Motor: Field Weakening



$$\begin{cases} i_e(t) = I_{e,n} & \Omega < \Omega_b \\ i_e(t) = \dfrac{E_n}{K_S \Omega(t)} & \Omega > \Omega_b \end{cases}$$

The time constant τ_e is larger than τ_a. Therefore, a speed reference command must be set after the excitation circuit reaches its steady-state.

Exercise for Report

Tramway vehicles “Carelli 1928” by ATM company (still in operation in Milan and San Francisco are moved by four DC motors with series excitation. Their main characteristics are reported in the following slide.

Exercise for Report

- line voltage: $V_{\mathrm{DC}} = 600$ V
- rated power of each motor: $P_{\mathrm{r}} = 21$ kW
- maximum speed of the vehicle: $v_{\mathrm{max}} = 42$ km/h
- rated speed of the motor: $\Omega_{\mathrm{r}} = \Omega_{\mathrm{b}} = 970$ rpm
- efficiency: $\eta = 0.9$
- armature circuit time constant: $\tau_{\mathrm{a}} = 10$ ms
- mass of the vehicle at no load: $m_{\mathrm{T}} = 15$ t
- maximum loading capacity: 130 passengers;
- diameter of the wheel: $d = 680$ mm
- gearbox ratio (motor-to-wheels): $\rho = 13/74$;
- The equivalent friction force on the shaft is proportional to Ω and, at rated speed, it is 1/10 of the traction force.

Exercise for Report

For simplicity, consider that the four motors behave like one equivalent DC motor with separate excitation, with:

- excitation rated voltage: $V_{\mathrm{er}} = 60$ V
- excitation rated current: $I_{\mathrm{er}} = 5$ A
- excitation time constant: $\tau_{\mathrm{e}} = 0.1$ s

Based on this data:

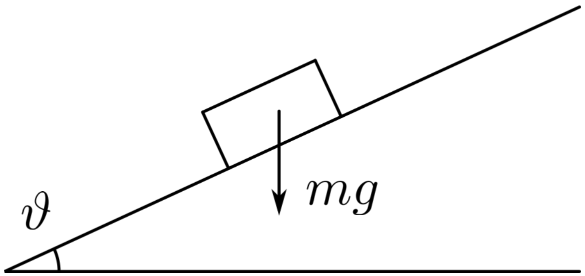
- compute the parameters of the system to control;
- design and simulate a speed control for covering the 10 km track reported in the next slide.

Exercise for Report

distance [km]	**slope %**	**speed**
$0-1$	0	$v_{\mathrm{r}}/2$
$1-3$	0	v_{r}
$3-4$	5	v_{r}
$4-6$	0	v_{max}
$6-8$	0	v_{r}
$8-9$	-5	v_{r}
$9-10$	0	$v_{\mathrm{r}}/2$

The slope is:

$$slope\ \% = 100 \tan(\vartheta)$$



Exercise for Report: Hints

- The nominal power of an electrical machine refers to the power delivered to a load. Thus, P_r is a mechanical power for a motor. The corresponding absorbed electrical power is:

$$P_\mathrm{el} = \frac{P_\mathrm{r}}{\eta}$$

- The relationship between speed of the vehicle and rotational speed of the motor is:

$$v_{\mathrm{m/s}} = \Omega_{\mathrm{rad/s}} \rho \frac{d}{2}$$

- The equivalent inertia seen from the motor shaft can be computed as:

$$J_\mathrm{eq} = m \frac{v_{\mathrm{m/s}}^2}{\Omega_{\mathrm{rad/s}}^2} = m \frac{\rho^2 d^2}{4}$$

Exercise for Report: Expected Result

Assume no coasting occurs