Induction Motor Drive

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Electric Propulsion

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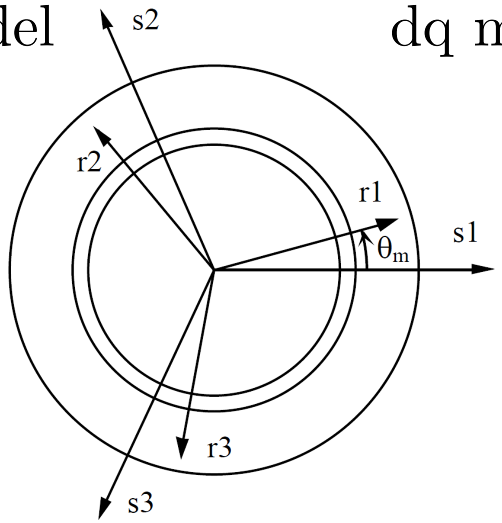
Modeling an Induction Motor

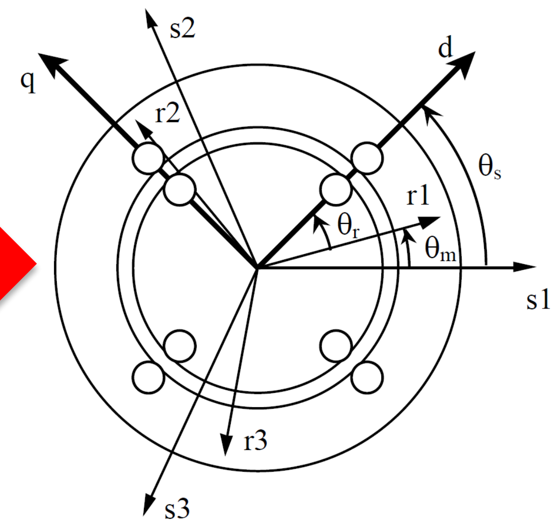
The modeling procedure for any induction motor foresees some steps:

- identification of the differential equations in the three-phase system (please see professor Castelli Dezza's notes);
- transformation of the machine model in a new reference frame. Space vector or matrix notations are typically used;
- the model can be translated from SI units into normalized quantities (per unit, p.u.)

abc model



dq model



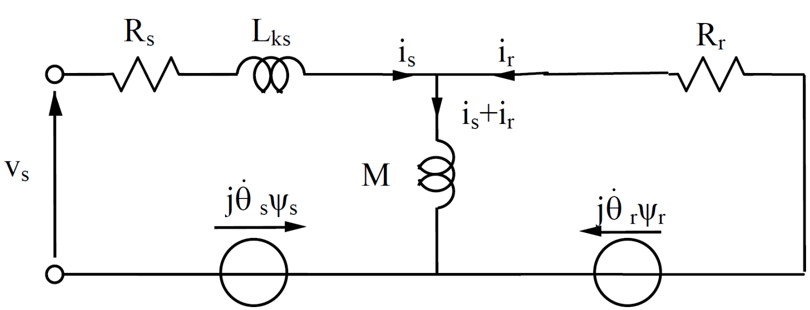
$$\boxed{\dot{\theta}_{\mathrm{s}} = \omega_{\mathrm{s}}}$$

$$\dot{\theta}_{\mathrm{m}} = \omega_{\mathrm{m}} = n_{\mathrm{p}}\Omega_{\mathrm{m}}$$

$$\dot{\theta}_{\mathrm{r}} = \omega_{\mathrm{r}} = \omega_{\mathrm{s}} - \omega_{\mathrm{m}}$$

$$s = \frac{\omega_{\mathrm{s}} - \omega_{\mathrm{m}}}{\omega_{\mathrm{s}}}$$

4-parameter model



Induction motor electrical model (space vector)

$$\begin{cases} \overline{v}_{\mathrm{s}} = R_{\mathrm{s}}\overline{i}_{\mathrm{s}} + p\overline{\psi}_{\mathrm{s}} + j\omega_{\mathrm{s}}\overline{\psi}_{\mathrm{s}} \\ \overline{v}_{\mathrm{r}} = R_{\mathrm{r}}\overline{i}_{\mathrm{r}} + p\overline{\psi}_{\mathrm{r}} + j\omega_{\mathrm{r}}\overline{\psi}_{\mathrm{r}} \\ \overline{\psi}_{\mathrm{s}} = L_{\mathrm{ks}}\overline{i}_{\mathrm{s}} + \overline{\psi}_{\mathrm{r}} \\ \overline{\psi}_{\mathrm{r}} = L_{\mathrm{m}}\left(\overline{i}_{\mathrm{s}} + \overline{i}_{\mathrm{r}}\right) \\ T_{\mathrm{e}} = n_{\mathrm{p}}\mathrm{Im}\left\{\overline{i}_{\mathrm{s}}\underline{\psi}_{\mathrm{s}}\right\} \end{cases}$$

Space Vector: Reference Frame

Stationary reference frame (Clarke transformation)

$$\theta_\mathrm{s} = 0,\ \omega_\mathrm{s} = 0$$

$$\overline{x}_{\alpha\beta} = \sqrt{\frac{2}{3}}\left(x_\mathrm{a} + \overline{\alpha}x_\mathrm{b} + \overline{\alpha}^2 x_\mathrm{c}\right) = x_\alpha + jx_\beta$$

$$\boxed{\begin{cases}\overline{\alpha} = e^{j\frac{2}{3}\pi} \\ \overline{\alpha}^2 = e^{j\frac{4}{3}\pi}\end{cases}}$$

Rotating reference frame (Park transformation)

$$\theta_\mathrm{s} = \int \omega_\mathrm{s}\mathrm{d}t,\ \omega_\mathrm{s} \neq 0 \quad \text{whatever speed can be chosen}$$

$$\overline{x}_\mathrm{dq} = \overline{x}_{\alpha\beta}e^{j\theta_\mathrm{s}} = \sqrt{\frac{2}{3}}\left(x_\mathrm{a} + \overline{\alpha}x_\mathrm{b} + \overline{\alpha}^2 x_\mathrm{c}\right)e^{j\theta_\mathrm{s}} = x_\mathrm{d} + jx_\mathrm{q}$$

$$\overline{x} = \overline{v}_\mathrm{s},\ \overline{v}_\mathrm{r},\ \overline{i}_\mathrm{s},\ \overline{i}_\mathrm{r},\ \overline{\psi}_\mathrm{s},\ \overline{\psi}_\mathrm{r}$$

Space Vector: Matrix Notation

The complex variables can be written as:

$$\overline{x} \rightarrow \boldsymbol{x} = \begin{bmatrix} x_{\mathrm{d}} \\ x_{\mathrm{q}} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\left\{\boldsymbol{x}\right\} \\ \mathrm{Im}\left\{\boldsymbol{x}\right\} \end{bmatrix}$$

$$\begin{cases} \boldsymbol{v}_{\mathrm{s}} = R_{\mathrm{s}}\boldsymbol{i}_{\mathrm{s}} + p\boldsymbol{\psi}_{\mathrm{s}} + \omega_{\mathrm{s}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \boldsymbol{\psi}_{\mathrm{s}} \\ \boldsymbol{v}_{\mathrm{r}} = R_{\mathrm{r}}\boldsymbol{i}_{\mathrm{r}} + p\boldsymbol{\psi}_{\mathrm{r}} + \omega_{\mathrm{r}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \boldsymbol{\psi}_{\mathrm{r}} \\ \boldsymbol{\psi}_{\mathrm{s}} = L_{\mathrm{ks}}\boldsymbol{i}_{\mathrm{s}} + \boldsymbol{\psi}_{\mathrm{r}} \\ \boldsymbol{\psi}_{\mathrm{r}} = L_{\mathrm{m}}\left(\boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}}\right) \\ T_{\mathrm{e}} = n_{\mathrm{p}}\mathrm{Im}\left\{\boldsymbol{i}_{\mathrm{s}} \times \boldsymbol{\psi}_{\mathrm{s}}\right\} \end{cases}$$

Space Vector: Simulink Model

Stationary reference frame (Clarke transformation) $\theta_\mathrm{s} = 0,\ \omega_\mathrm{s} = 0$

$$\begin{cases} \overline{v}_\mathrm{s} = R_\mathrm{s}\overline{i}_\mathrm{s} + p\overline{\psi}_\mathrm{s} \\ \overline{v}_\mathrm{r} = R_\mathrm{r}\overline{i}_\mathrm{r} + p\overline{\psi}_\mathrm{r} - j\omega_\mathrm{m}\overline{\psi}_\mathrm{r} \\ \overline{\psi}_\mathrm{s} = L_\mathrm{ks}\overline{i}_\mathrm{s} + \overline{\psi}_\mathrm{r} \\ \overline{\psi}_\mathrm{r} = L_\mathrm{m}\left(\overline{i}_\mathrm{s} + \overline{i}_\mathrm{r}\right) \\ T_\mathrm{e} = n_\mathrm{p}\mathrm{Im}\left\{\overline{i}_\mathrm{s}\underline{\psi}_\mathrm{s}\right\} \end{cases} \Rightarrow \begin{cases} \overline{\psi}_\mathrm{s} = \int \left(\overline{v}_\mathrm{s} - R_\mathrm{s}\overline{i}_\mathrm{s}\right) \mathrm{d}t \\ \overline{\psi}_\mathrm{r} = \int \left(\overline{v}_\mathrm{r} - R_\mathrm{r}\overline{i}_\mathrm{r} + j\omega_\mathrm{m}\overline{\psi}_\mathrm{r}\right) \mathrm{d}t \\ \overline{i}_\mathrm{s} = \dfrac{1}{L_\mathrm{ks}}\left(\overline{\psi}_\mathrm{s} - \overline{\psi}_\mathrm{r}\right) \\ \overline{i}_\mathrm{r} = \dfrac{\overline{\psi}_\mathrm{r}}{L_\mathrm{m}} - \overline{i}_\mathrm{s} \\ T_\mathrm{e} = n_\mathrm{p}\mathrm{Im}\left\{\overline{i}_\mathrm{s}\underline{\psi}_\mathrm{s}\right\} \end{cases}$$

Squirrel cage motors: $\overline{v}_\mathrm{r} = 0$

$$\boxed{j = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = e^{j\frac{\pi}{2}}}$$

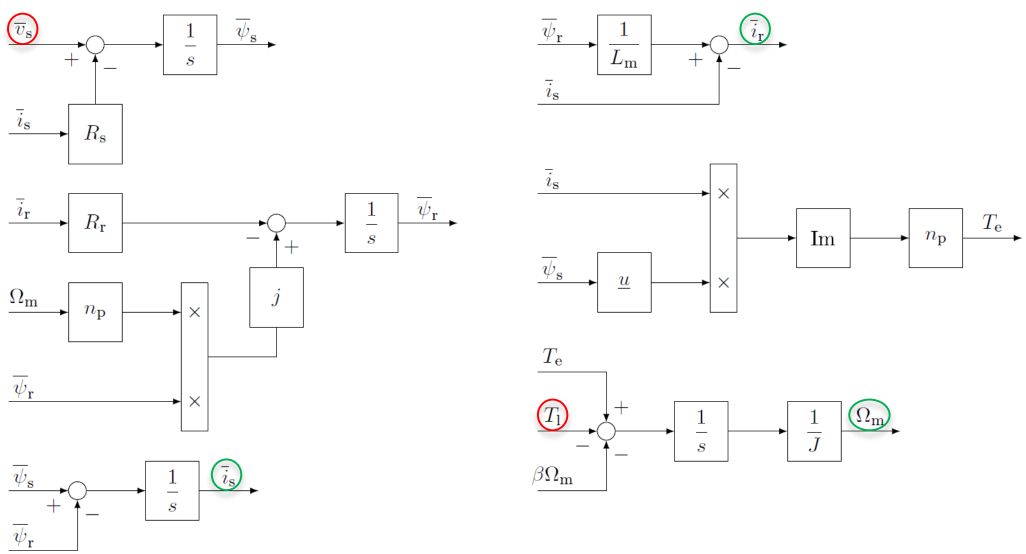
Mechanical equation: $T_\mathrm{e} - T_\mathrm{l} = J\dfrac{\mathrm{d}\Omega_\mathrm{m}}{\mathrm{d}t} + \beta\Omega_\mathrm{m}$

Space Vector: Simulink Model

Stationary reference frame (Clarke transformation) $\theta_\mathrm{s} = 0$, $\omega_\mathrm{s} = 0$

Squirrel cage motors: $\overline{v}_\mathrm{r} = 0$

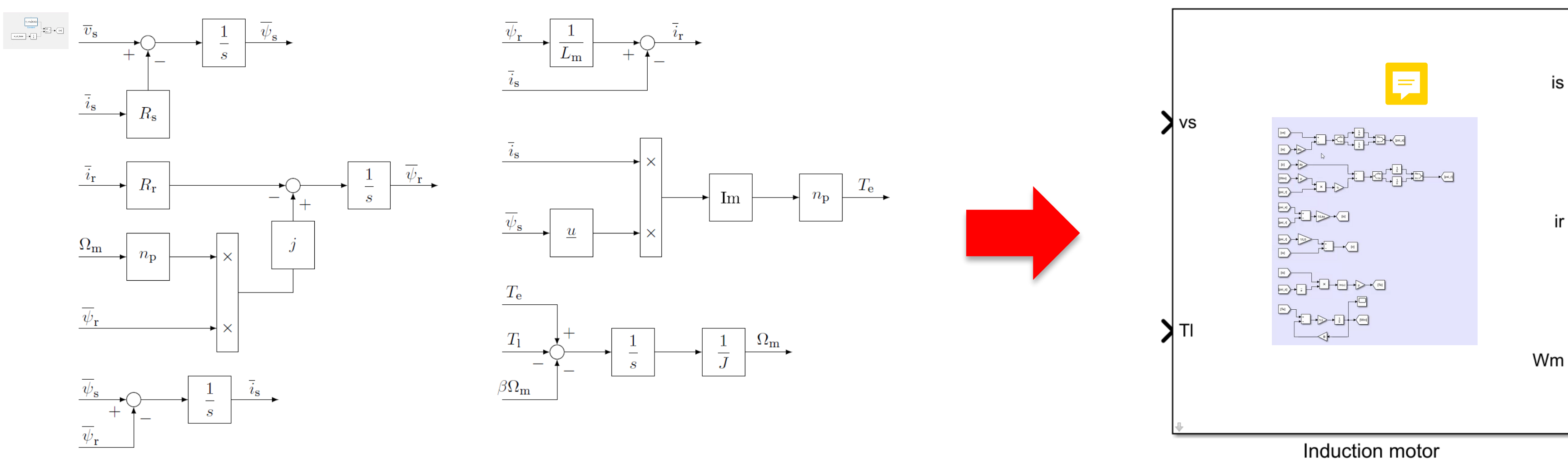
$$\begin{cases} \overline{\psi}_\mathrm{s} = \int \left(\overline{v}_\mathrm{s} - R_\mathrm{s}\overline{i}_\mathrm{s}\right) \mathrm{d}t \\ \overline{\psi}_\mathrm{r} = \int \left(-R_\mathrm{r}\overline{i}_\mathrm{r} + j\omega_\mathrm{m}\overline{\psi}_\mathrm{r}\right) \mathrm{d}t \\ \overline{i}_\mathrm{s} = \dfrac{1}{L_\mathrm{ks}} \left(\overline{\psi}_\mathrm{s} - \overline{\psi}_\mathrm{r}\right) \\ \overline{i}_\mathrm{r} = \dfrac{\overline{\psi}_\mathrm{r}}{L_\mathrm{m}} - \overline{i}_\mathrm{s} \\ \Omega_\mathrm{m} = \dfrac{1}{J} \int \left(T_\mathrm{e} - T_\mathrm{l} - \beta\Omega_\mathrm{m}\right) \mathrm{d}t \\ T_\mathrm{e} = n_\mathrm{p}\mathrm{Im}\left\{\overline{i}_\mathrm{s}\underline{\psi}_\mathrm{s}\right\} \end{cases}$$



Preliminary Exercise

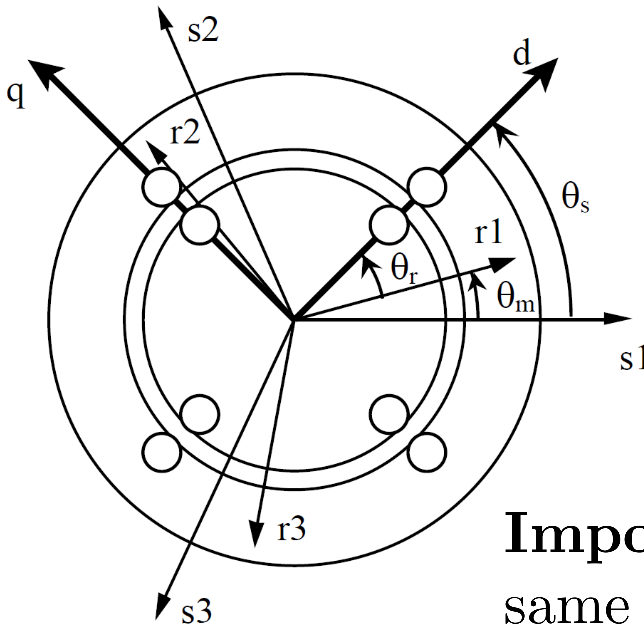
Build a Simulink model for a generic induction motor:

- create a subsystem in stationary reference frame (Clarke) for the motor;
- create a mask for the parameters of the motor



Rotor Field-Oriented Control (R-FOC)

The control is based on the same strategy adopted for DC motors. Namely, choosing a **suitable reference frame**, i_{sd} is used to control the magnitude of the rotor flux $\overline{\psi}_r$, whereas i_{sq} is exploited for varying the torque T_e.



- d-axis aligned with the rotor flux

$$\begin{cases} \psi_{rd} = \left|\overline{\psi}_r\right| \\ \psi_q = 0 \end{cases}$$

- the reference frame rotates synchronously with the rotor flux

Important: i_{sd} and i_{sq} are components of the same transformed three-phase system

R-FOC: Working Principles

The model to be adopted for the exercise is obtained applying the previous assumptions:

$$
\begin{cases}
\overline{v}_{\mathrm{s}} = R_{\mathrm{s}}\overline{i}_{\mathrm{s}} + p\overline{\psi}_{\mathrm{s}} \\
\overline{v}_{\mathrm{r}} = R_{\mathrm{r}}\overline{i}_{\mathrm{r}} + p\overline{\psi}_{\mathrm{r}} - j\omega_{\mathrm{m}}\overline{\psi}_{\mathrm{r}} \\
\overline{\psi}_{\mathrm{s}} = L_{\mathrm{ks}}\overline{i}_{\mathrm{s}} + \overline{\psi}_{\mathrm{r}} \\
\overline{\psi}_{\mathrm{r}} = L_{\mathrm{m}}\left(\overline{i}_{\mathrm{s}} + \overline{i}_{\mathrm{r}}\right) \\
T_{\mathrm{e}} - T_{\mathrm{l}} = J\dfrac{\mathrm{d}\Omega_{\mathrm{m}}}{\mathrm{d}t} + \beta\Omega_{\mathrm{m}} \\
T_{\mathrm{e}} = n_{\mathrm{p}}\mathrm{Im}\left\{\overline{i}_{\mathrm{s}}\underline{\psi}_{\mathrm{s}}\right\}
\end{cases}
$$

$$
\begin{cases}
v_{\mathrm{sd}} = R_{\mathrm{ks}}i_{\mathrm{sd}} + L_{\mathrm{ks}}pi_{\mathrm{sd}} - \dfrac{R_{\mathrm{r}}}{L_{\mathrm{m}}}\psi_{\mathrm{rd}} - \omega_{\mathrm{s}}L_{\mathrm{ks}}i_{\mathrm{sq}} \\
v_{\mathrm{sq}} = R_{\mathrm{ks}}i_{\mathrm{sq}} + L_{\mathrm{ks}}pi_{\mathrm{sq}} + \omega_{\mathrm{m}}\psi_{\mathrm{rd}} + \omega_{\mathrm{s}}L_{\mathrm{ks}}i_{\mathrm{sd}} \\
p\psi_{\mathrm{rd}} = R_{\mathrm{r}}i_{\mathrm{sd}} - \dfrac{R_{\mathrm{r}}}{L_{\mathrm{m}}}\psi_{\mathrm{rd}} \\
0 = R_{\mathrm{r}}i_{\mathrm{sq}} - \omega_{\mathrm{r}}\psi_{\mathrm{rd}} \\
T_{\mathrm{e}} - T_{\mathrm{l}} = J\dfrac{\mathrm{d}\Omega_{\mathrm{m}}}{\mathrm{d}t} + \beta\Omega_{\mathrm{m}} \\
T_{\mathrm{e}} = n_{\mathrm{p}}\psi_{\mathrm{rd}}i_{\mathrm{sq}}
\end{cases}
$$

R-FOC: Working Principles

Namely, in canonical form:

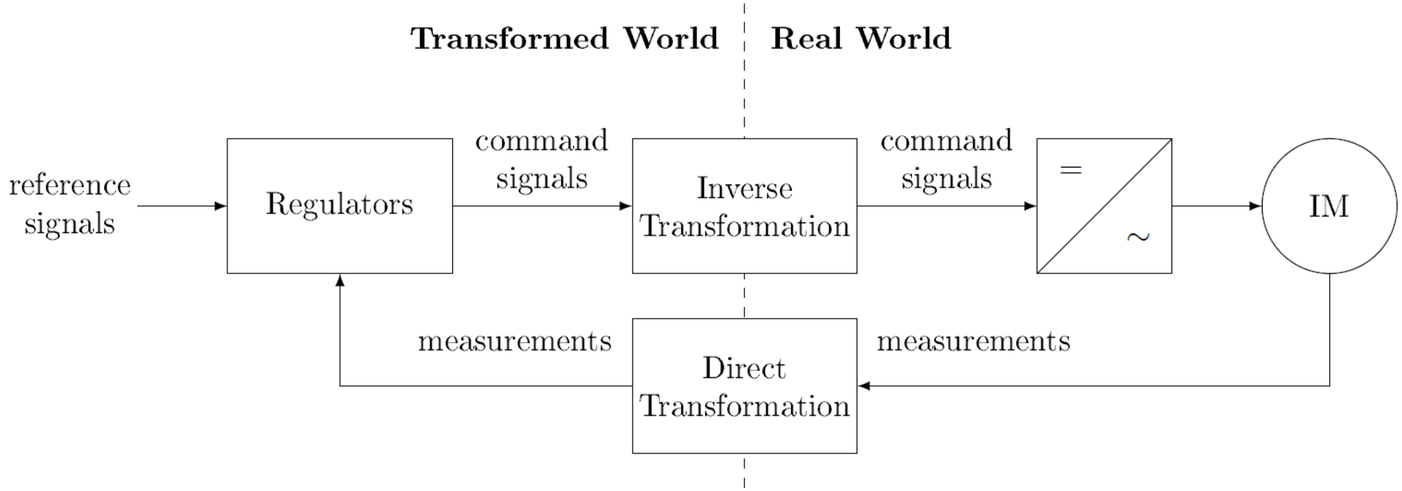
$$
\begin{cases}
p i_{\mathrm{sd}} = \dfrac{1}{L_{\mathrm{ks}}}\left(v_{\mathrm{sd}} - R_{\mathrm{ks}} i_{\mathrm{sd}} + \dfrac{R_{\mathrm{r}}}{L_{\mathrm{m}}}\psi_{\mathrm{rd}} + \omega_{\mathrm{s}} L_{\mathrm{ks}} i_{\mathrm{sq}}\right) \\
p i_{\mathrm{sq}} = \dfrac{1}{L_{\mathrm{ks}}}\left(v_{\mathrm{sq}} - R_{\mathrm{ks}} i_{\mathrm{sq}} - \omega_{\mathrm{m}}\psi_{\mathrm{rd}} - \omega_{\mathrm{s}} L_{\mathrm{ks}} i_{\mathrm{sd}}\right) \\
\boxed{p\psi_{\mathrm{rd}} = R_{\mathrm{r}} i_{\mathrm{sd}} - \dfrac{R_{\mathrm{r}}}{L_{\mathrm{m}}}\psi_{\mathrm{rd}}} \quad \text{the flux depends on } i_{\mathrm{sd}} \text{ only} \\
0 = R_{\mathrm{r}} i_{\mathrm{sq}} - \omega_{\mathrm{r}}\psi_{\mathrm{rd}} \\
p\Omega_{\mathrm{m}} = \dfrac{1}{J}\left(T_{\mathrm{e}} - T_{\mathrm{l}} - \beta\Omega_{\mathrm{m}}\right) \\
\boxed{T_{\mathrm{e}} = n_{\mathrm{p}}\psi_{\mathrm{rd}} i_{\mathrm{sq}}} \quad \text{if } \psi_{\mathrm{rd}} = \text{cost, the torque depends on } i_{\mathrm{sq}} \text{ only}
\end{cases}
$$

Vector Control

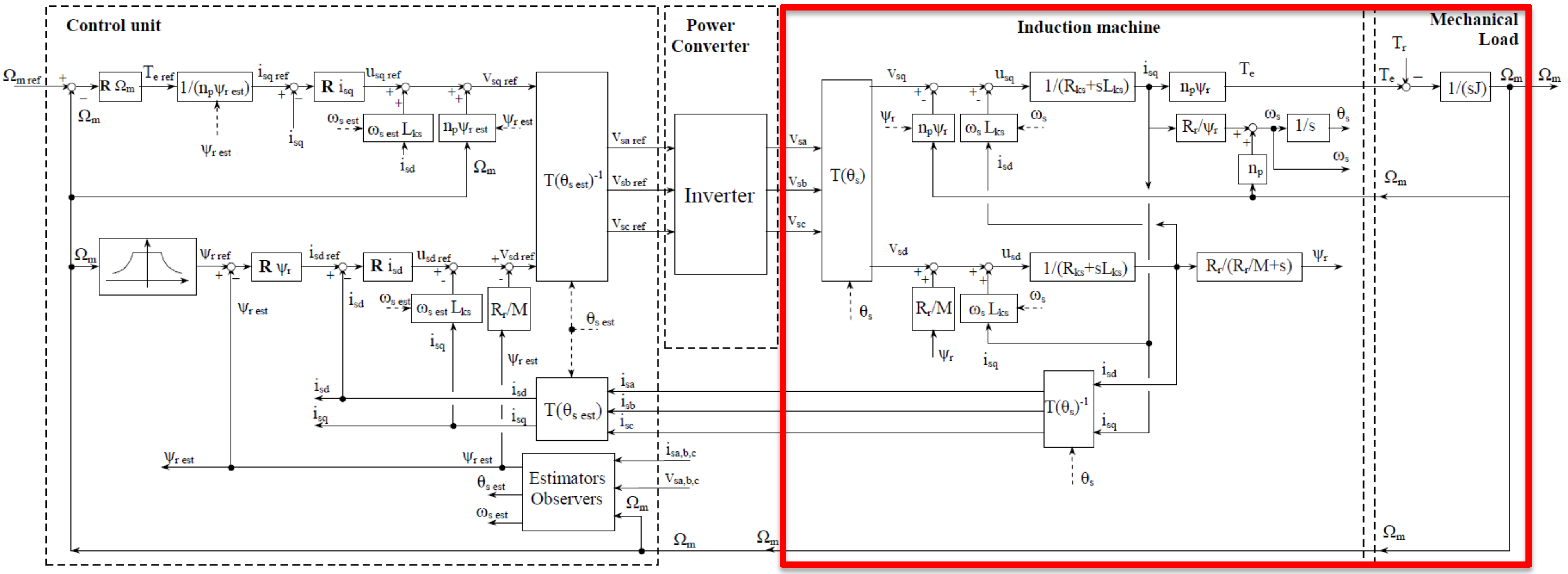
The control is based on an **equivalent transformed motor**

The R-FOC allows to obtain two **decoupled** dynamics:

- slow transients (rotor flux, i_{sd})
- fast transients (torque, i_{sq})

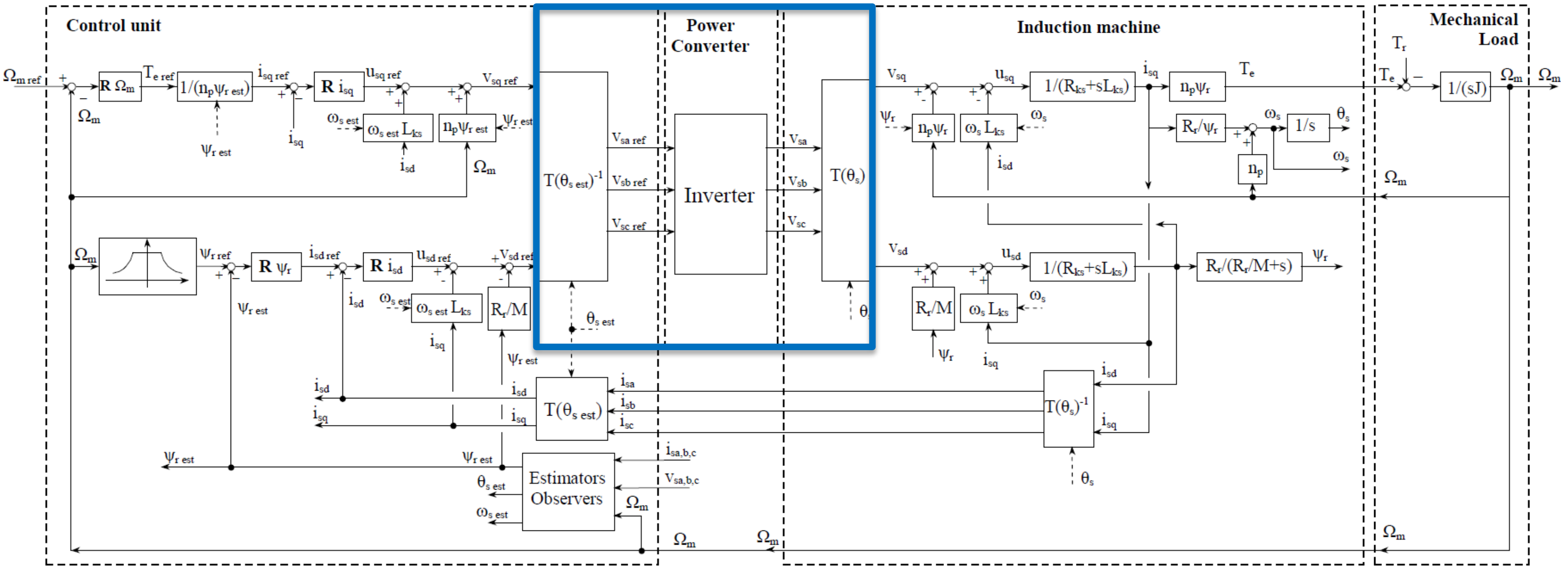


R-FOC: Control Scheme



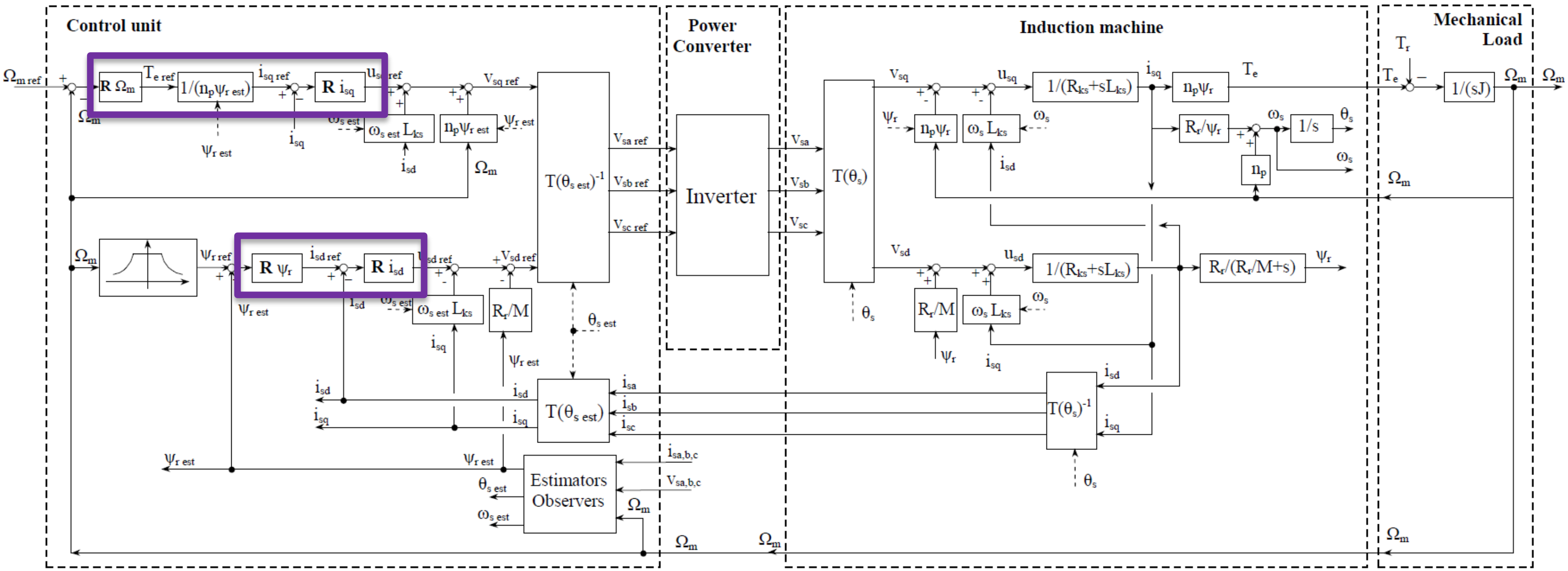
Motor model: use this scheme to test each regulator. Then, move to the model with stationary reference frame for the complete simulation

R-FOC: Control Scheme



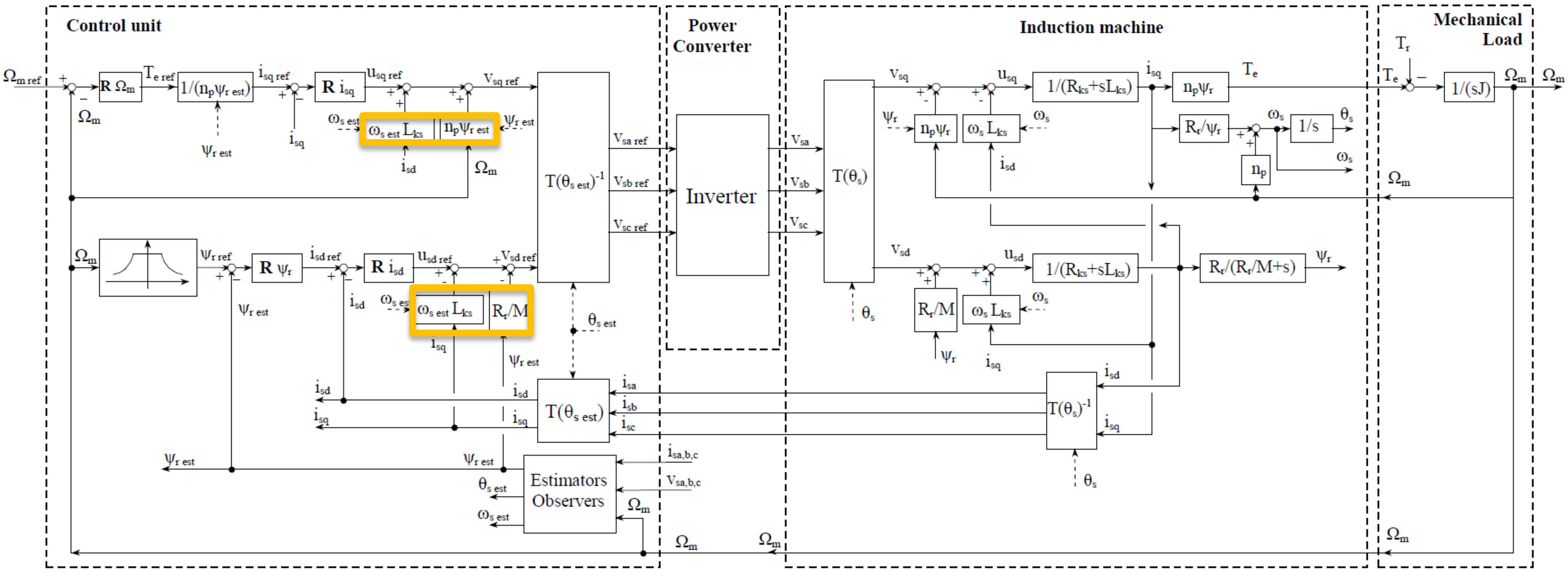
Inverter model: approximate it as a unitary gain. Do not consider the complete transformations and anti-transformations. Only angles can be used to move from the control to the motor model.

R-FOC: Control Scheme



Cascade control approach: inner loops must be much faster than the outer ones (at least 10 times). This constraint allows to design the PI independently.

R-FOC: Control Scheme



Decoupling terms

R-FOC: Decoupling Terms

Considering the differential equations for the two currents

$$p i_{\mathrm{sd}} = \frac{1}{L_{\mathrm{ks}}} \left(v_{\mathrm{sd}} - R_{\mathrm{ks}} i_{\mathrm{sd}} + \boxed{\frac{R_{\mathrm{r}}}{L_{\mathrm{m}}} \psi_{\mathrm{rd}} + \omega_{\mathrm{s}} L_{\mathrm{ks}} i_{\mathrm{sq}}} \right)$$

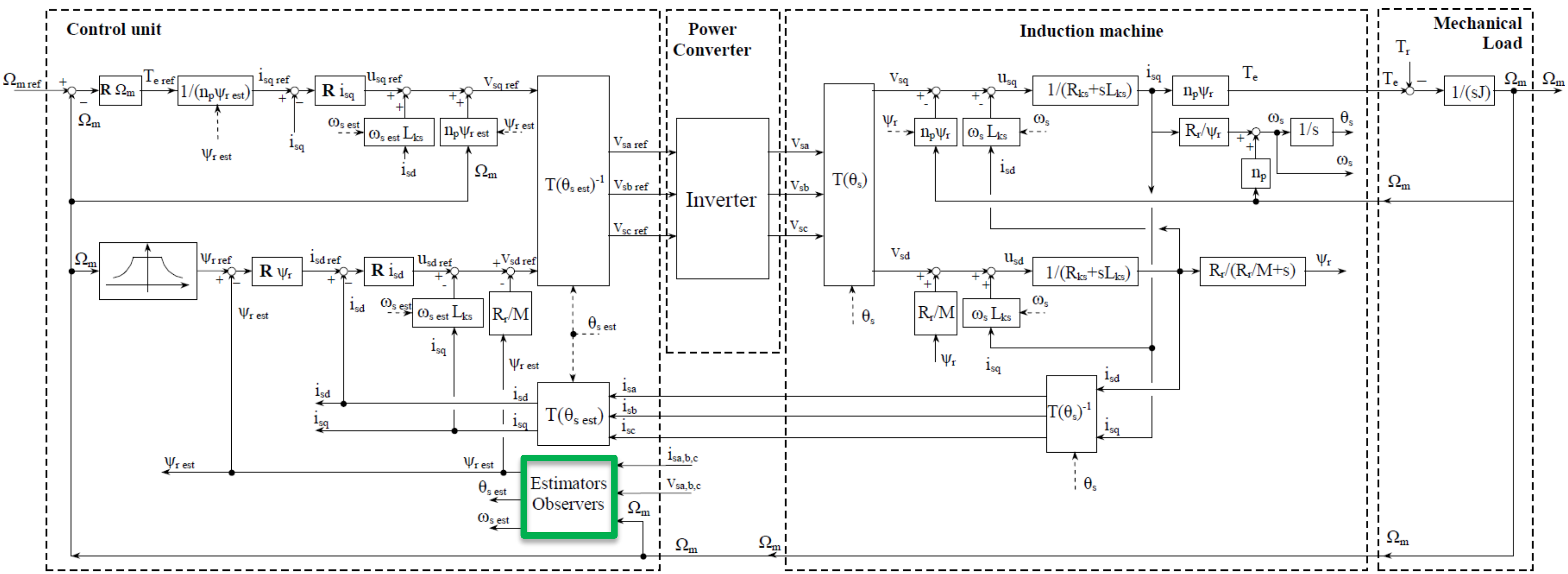
$$p i_{\mathrm{sq}} = \frac{1}{L_{\mathrm{ks}}} \left(v_{\mathrm{sq}} - R_{\mathrm{ks}} i_{\mathrm{sq}} - \boxed{\omega_{\mathrm{m}} \psi_{\mathrm{rd}} - \omega_{\mathrm{s}} L_{\mathrm{ks}} i_{\mathrm{sd}}} \right)$$

The two controls for d and q axis are coupled.

Coupling terms must be compensated. Then, the two current loops can be designed separately considering the same transfer function.

$$G(s) = \frac{1}{R_{\mathrm{ks}} + s L_{\mathrm{ks}}}$$

R-FOC: Control Scheme



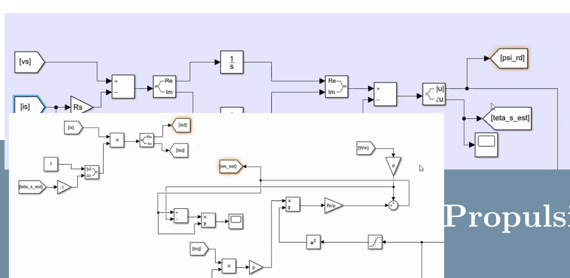
Estimators

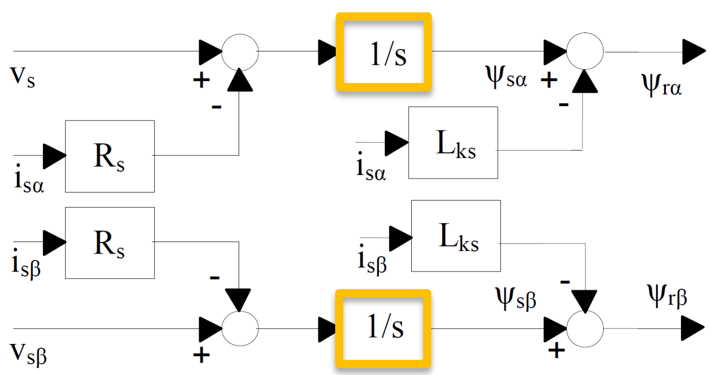
R-FOC: Estimators

The R-FOC needs an estimator to compute the rotor flux starting from the voltage and current measurements. Moreover, angular speed ω_s must be provided as well.

VI estimator computes both the rotor and the stator flux on stationary reference frame starting from the measured voltage and current. Torque can be estimated as well if needed.

$$\begin{cases} \overline{\psi}_s = \int \left(\overline{v}_s - R_s \overline{i}_s \right) dt \\ \overline{\psi}_r = \overline{\psi}_s - L_{ks} \overline{i}_s \end{cases}$$





Low-pass filter are used in practice instead of integrators

R-FOC: Estimators

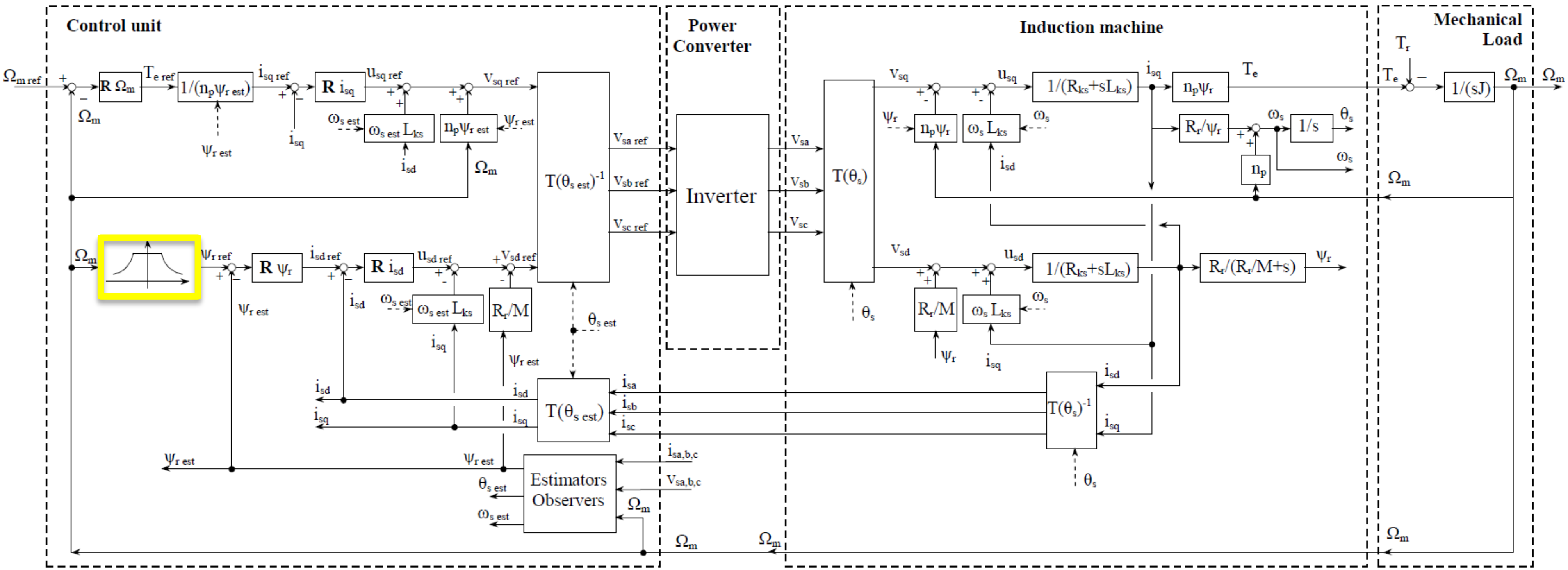
In principle, ω_s can be estimated from the VI estimator as well. Indeed, the angle θ_s is computed knowing real and imaginary part of the rotor flux. The speed can be obtained from the derivative of this angle.

However, it is good practice to avoid computations of any derivative, since this kind of operation is not numerically stable. Therefore, an approximation can be obtained considering the following equations at steady-state:

$$\begin{cases} 0 = R_r i_{rq} + \omega_r \psi_{rd} \quad \text{rotor equation} \\ \psi_{rq} = 0 = L_m \left(i_{sq} + i_{rq} \right) \rightarrow i_{rq} = -i_{sq} \quad \text{rotor flux equation} \end{cases}$$

$$\omega_r = R_r \frac{i_{sq}}{\psi_{rd}} \rightarrow \boxed{\omega_s = \omega_r + \omega_m}$$

R-FOC: Control Scheme



Operating regions and field weakening

R-FOC: Field Weakening

If $\Omega_{\mathrm{m}} < \Omega_{\mathrm{b}}$ (use of the iron at the knee of the magnetizing curve)

$$\psi_{\mathrm{rd}} = \mathrm{const}$$

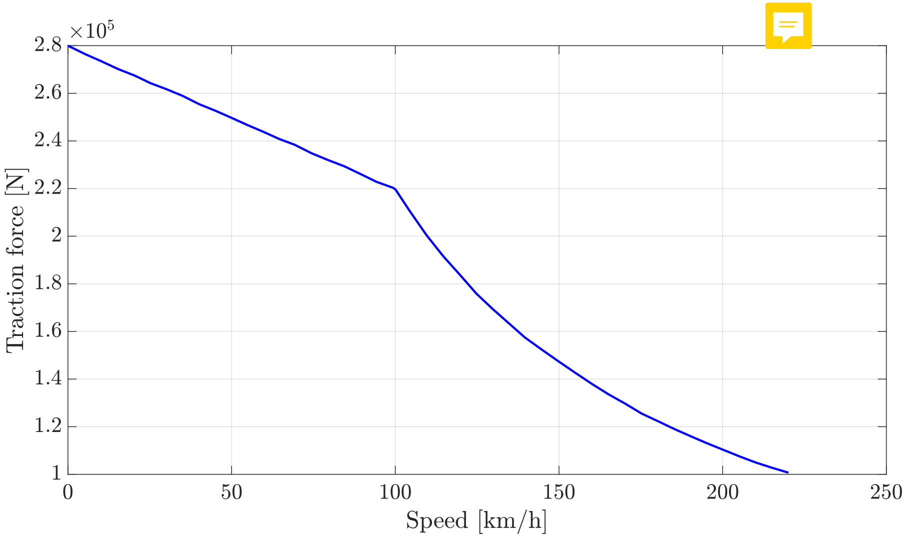
If $\Omega_{\mathrm{m}} > \Omega_{\mathrm{b}}$ (field weakening)

$$i_{\mathrm{sq}} = \mathrm{const}$$

$$\psi_{\mathrm{rd}} = \frac{R_{\mathrm{r}} i_{\mathrm{sq}}}{\omega_{\mathrm{r}}}$$

Exercise for Report

The E.402B is a locomotive model built from 1996 up to 2000 by Ansaldo company and it is typically exploited by Ferrovie dello Stato in Intercity trains. It has 4 induction motors for an overall power of 5.6 MW. The main characteristics of the locomotive and of the overall train are reported in the following slides.

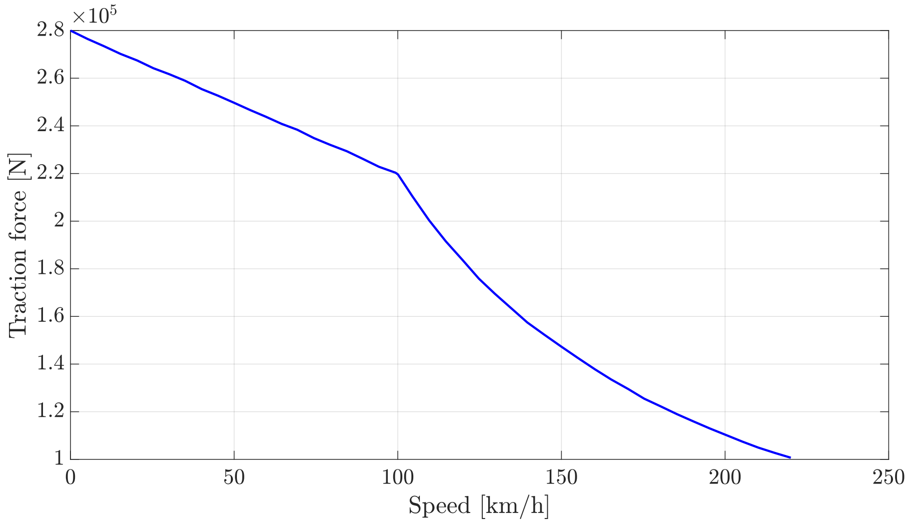


Exercise for Report

- Rated voltage of each motor: 1860 V
- Efficiency of each motor: 0.9
- Pole pairs of each motor: 2
- Power factor of each motor: $\cos\varphi = 0.85$
- Stator resistance: 0.01 pu
- Rotor resistance: 0.01 pu
- Locked rotor inductance: 0.1 pu
- No load inductance: 3.5 pu
- Diameter of the wheels: 1250 mm
- Transmission ratio: 23/91

Exercise for Report

- Weight of the locomotive: 89 t
- Weight of each coach: 47 t
- Number of coaches: 11
- Average number of passengers on each coach: 45
- Maximum speed of the train: 200 km/h
- Traction diagram available on beep



Exercise for Report

Assume a standard weight of 80 kg for each passenger and that each of them carries a luggage of 15 kg. In addition, assume that the four motors behave like one equivalent motor with the same number of poles, efficiency, power factor, rated voltage and parameters in pu of each electrical machine.

Based on this data:

- compute all the parameters of the system to control;
- find the operating regions for the rotor flux and current i_{sd};
- design and simulate a control system for the train. Use ramps with saturations or step blocks as generators for reference signals. Do not overcome the maximum speed of the train.

Exercise for Report: Hints

- The nominal power of an electrical machine refers to the power delivered to a load. Thus, P_{r} is a mechanical power for a motor. The corresponding absorbed electrical power is:

$$P_{\mathrm{el}} = \frac{P_{\mathrm{r}}}{\eta}$$

- The relationship between speed of the vehicle and rotational speed of the motor is:

$$v_{\mathrm{m/s}} = \Omega_{\mathrm{rad/s}} \rho \frac{d}{2}$$

- The equivalent inertia seen from the motor shaft can be computed as:

$$J_{\mathrm{eq}} = m \frac{v_{\mathrm{m/s}}^2}{\Omega_{\mathrm{rad/s}}^2} = m \frac{\rho^2 d^2}{4}$$

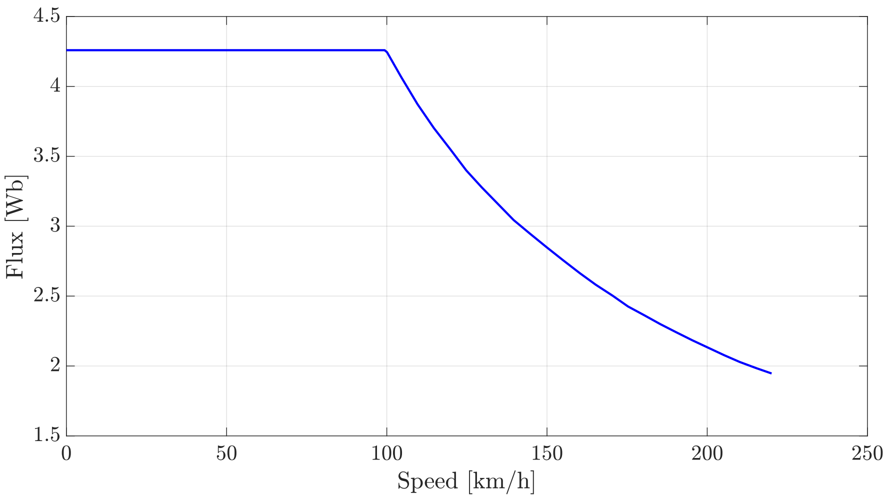
Exercise for Report: Hints

- Estimate the friction coefficient knowing that the friction force F_{fr} is 62 kN at 200 km/h. Remember that:

$$T_{\text{fr}} = F_{\text{fr}} \frac{d}{2} \rho$$

- The maximum flux can be estimated as:

$$\psi_{\text{r,max}} = \frac{V_{\text{r}}}{\omega_{\text{b}}} 0.8$$



Exercise for Report: Hints

- Simulate the motor supplied at rated speed in no load conditions. Evaluate both the starting transients (look at the settling times) and the steady states for:
  - stator and rotor currents;
  - Stator and rotor flux;
  - torque and speed.
- Design and simulate all the regulators considering the cascade constraint. It is possible to test the behavior of each regulator through simplified simulations.

Exercise for Report: Suggested Procedure

- compute all the machine parameters;
- determine the operating regions for i_{sd} and the rotor flux;
- build the motor model in stationary reference frame (Clarke);
- evaluate its dynamical behavior in no-load conditions;
- exploit the no-load simulation for implementing the estimators;
- design the regulators;
- test the control loops through simplified simulations;
- include the operating regions in the simplified simulations;
- simulate the complete system: control + motor on Clarke axis;
- **extra**: include expressions for computation of friction forces.

Exercise for Report: Extra

A practical equation can be used for the estimation of the so called ordinary friction forces, i.e., rolling and air friction forces:

$$f_{\text{fr}}(v) = 1.625 + 2.05\left(\frac{v}{100}\right)^2$$

where f_{fr} is a force in N per unit weight in KN and v is the speed in km/h. The overall friction force F_{fr} at a specified speed v is obtained by multiplying f_{fr} by the weight of the whole train.

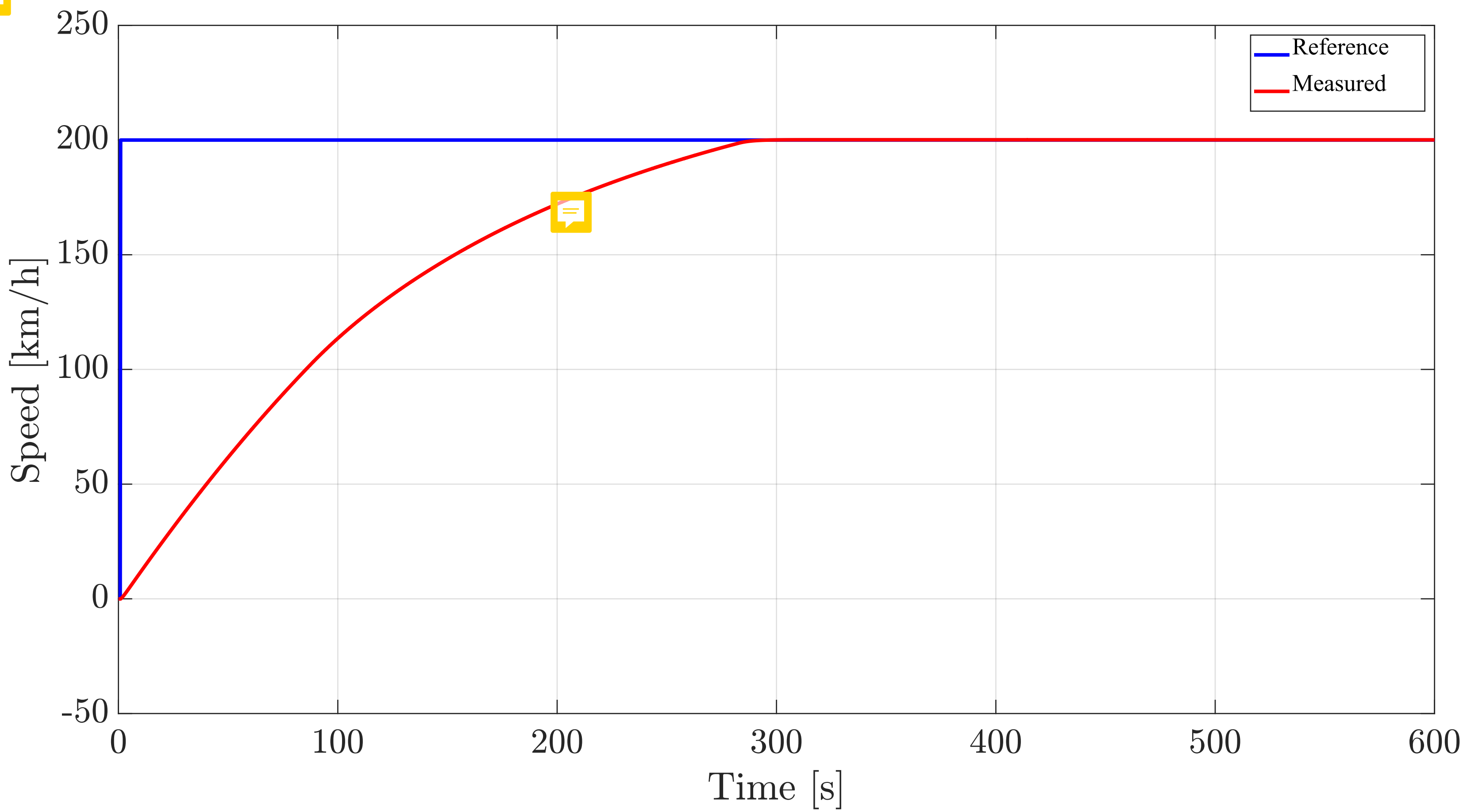
Exercise for Report: Extra

Thus, the mechanical equation of the system cab be rewritten as:

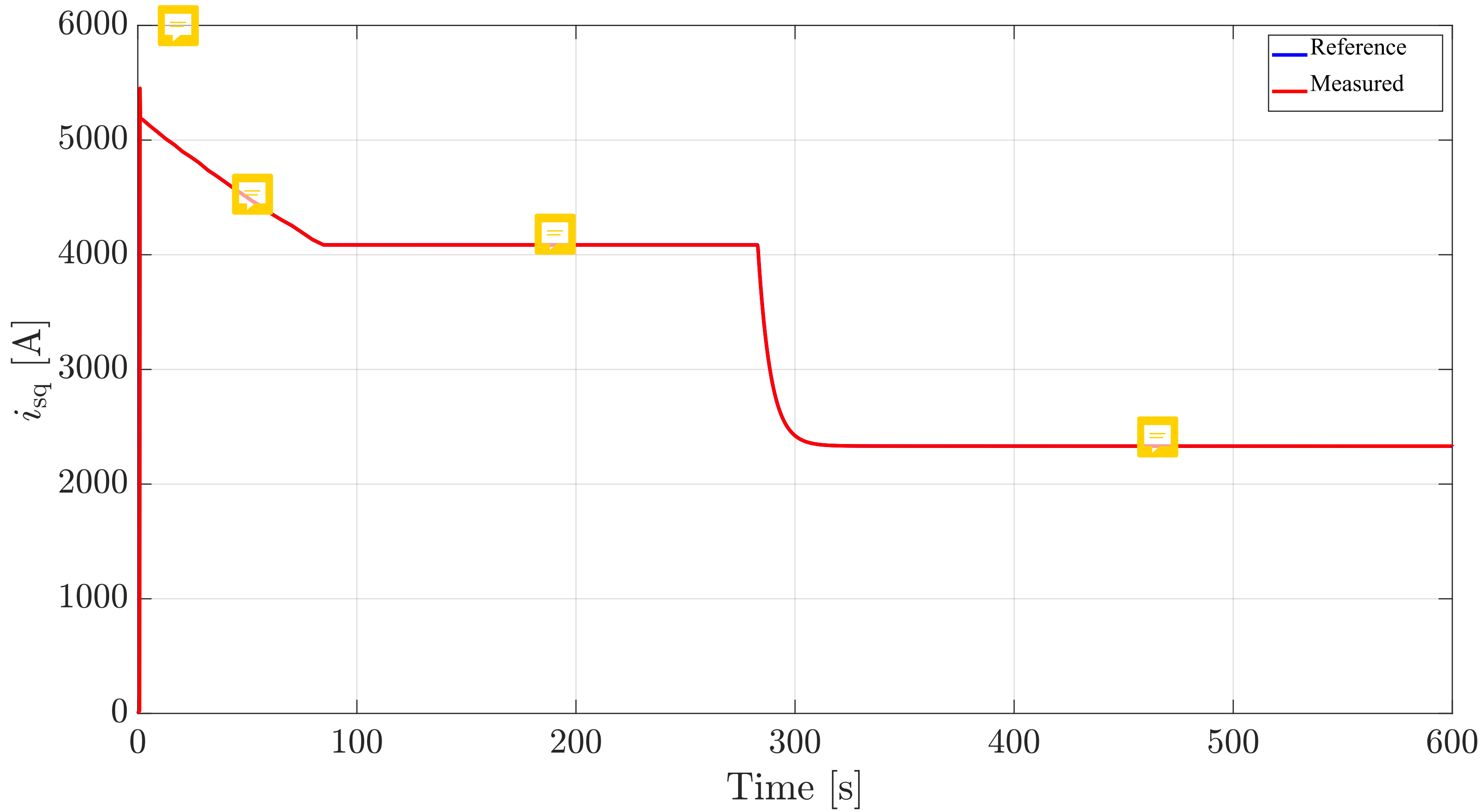
$$F_{\mathrm{tr}} - F_{\mathrm{fr}} - M_{\mathrm{train}}\left(1 + 0.075\right)\frac{\mathrm{d}v}{\mathrm{d}t}$$

Where M is the mass of the overall train in kg and v is the speed in m/s.

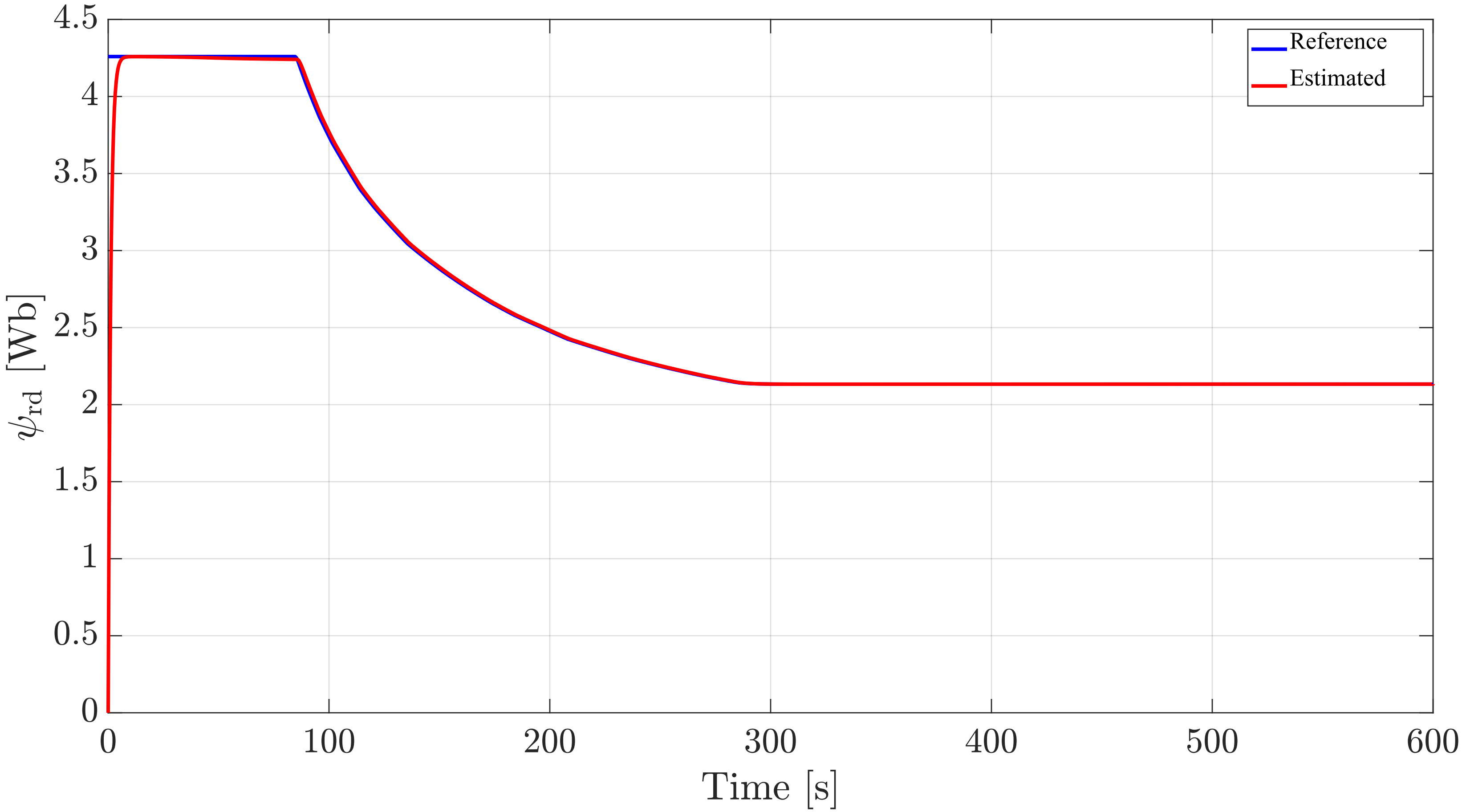
Exercise for Report: Results



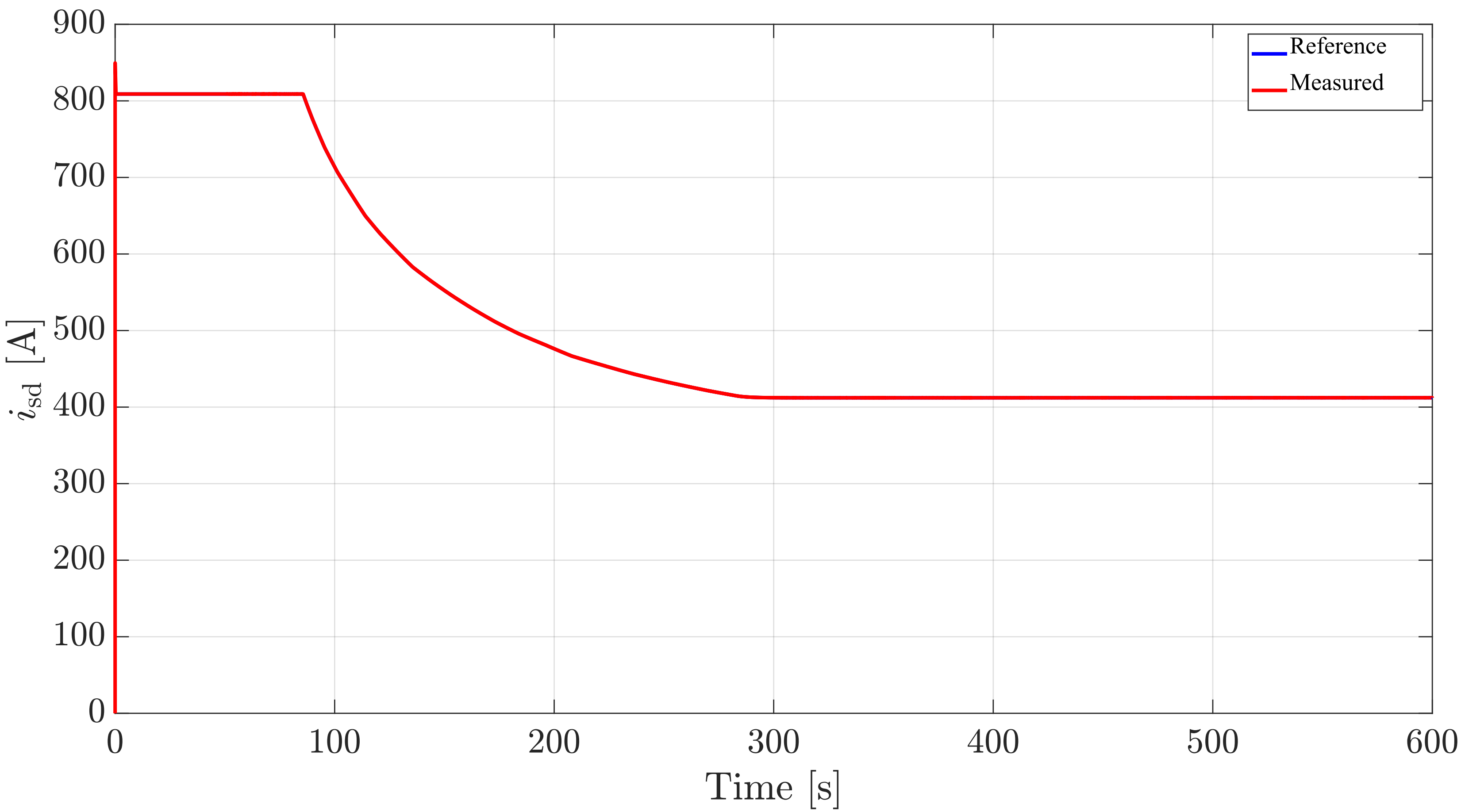
Exercise for Report: Results



Exercise for Report: Results



Exercise for Report: Results



Curiosity: Coasting Mode