Problem 3. [4 marks]

(a) Give, with brief justification, an expression for the number of clauses of ϕG in terms of m (the number of edges of G) and t (the number of triangles in G). For full marks, the number of clauses produced by your awk program (prob2.awk) must be correct as well as equalling the expression you give here.

The expression for the number of clauses ϕG is m + t by adding up the clauses to satisfy the vertex cover condition and the clauses to satisfy the triangle free condition. Every edge in the graph contributes to an OR clause because it is either one vertex or another that will hold true for a vertex cover graph. Every triangle in the graph also contributes to a negative OR clause because there should be a vertex that is false so that the graph does not contain edges that form a triangle.

(b) Give, with brief justification, an expression for an upper bound on the number of triangles t in G in terms of m. (You don't need to find the tightest bound possible, but it needs to be tight enough to support your argument in part (c).)

The expression for an upper bound on the number of triangles t = mC3 = m(m-1)(m-2)…1/6. A triangle is formed by three edges, and the maximum number of triangles occurs when every possible combination of three edges forms a triangle. The concept of mC3 is to calculate the maximum number of ways to choose three edges that can form a triangle.

(c) Based on your answers to (a) and (b), give with brief justification an upper bound on the number of clauses of ϕG, expressed only in terms of m.

The upper bound on the number of clauses of ϕG is m + t = m + mC3. By combining the answers in a and b, m + mC3 is sufficient to cover the number of clauses of ϕG that tries to construct a triangle free vertex cover, both conditions that concern a and b respectively.