Stochastic Machine Learning
01 - Introduction

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1. Introduction → Machine learning basics

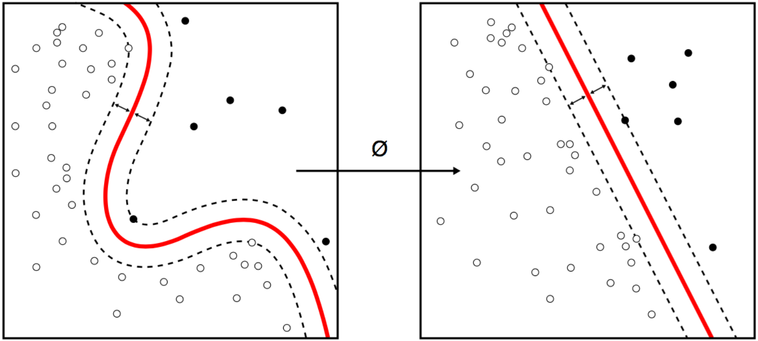
Types of machine learning:

- **Supervised learning:** The data consists of datapoints and associated labels, i.e. we start from the dataset
$$(x_i, y_i)_{i \in I}.$$
We give some examples:
- **Image recognition** (face recognition) where the images come with labels, i.e. cats / dogs or the person to which the image is associated to.
- **Spam filter** the training set contains emails together with the label spam / no spam.
- **Speech recognition** here sample speech files comes together with the content of the sentences. It is clear, that some sort of grammar understanding helps to break up the sentences into smaller pieces, i.e. words.
- **Ratings** here, to a creditor we assign the credit quality (AAA, ...) A typical finance application.

- **Unsupervised learning:** In this case the data just comes at it is, i.e.

$$(x_i)_{i \in I}$$

and one goal would be to identify a certain structure from the data itself. In this sense the machine learning algorithm shall itself find a characteristics which divides the data into suitable subsets.



Picture by: Alisneaky, svg version by User:Zirguezi - Own work,
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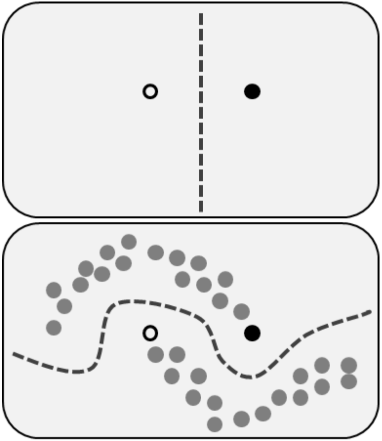
Some examples

- Analysis of genomic data
- Density estimation
- Clustering
- Principal component analysis

- **Semi-supervised learning:** only a few data are labelled and many are unlabelled.
- Labelling typically is quite expensive and the additional use of unlabelled data might improve the performance. However, some assumptions need to be made, such that this procedure works through.



Picture by: Techerin - Own work,

Questions

- What is (semi-/un-/-supervised) learning ?
- Give examples
- The examples you have been chosing in the first part, please classify them

Dynamic contexts

- It is apparent, that the above questions have been static
- Many applications are dynamic !
- To drive a car
- To manage a portfolio
- To predict future evolutions from a time-series

This will require different methods which we will meet in the course. We give one example

Reinforcement learning

is quite different from the above examples.

- First: time matters, the problem depends on time ! Observations accumulate over time.
- There is no supervisor but a reward signal measuring the quality of the decision.
- The approach utilizes a probabilistic framework: **Markov decision processes.**
- Examples are: drive a car, optimally manage a portfolio, program a roboter, playing a game (Pong, breakout for example) ...

- A **stochastic process** is simply a collection of rvs: $(X_t)_{t\in\mathbb{N}}$.
- It is called **Markovian**, if for all Borel sets A, the transition probabilities do not depend on the full history, but only on the last value from the history:

$$P(X_{t+1} \in A | X_s : s \leq t) = P(X_{t+1} \in A | X_t), \quad t \in \mathbb{N}.$$

- A **Markov decision process** consists of a process X and decisions $(d_t)_{t\in\mathbb{N}}$, such that the transition probalities may depend on d. The behaviour of the process is described by the *transition probabilities*

$$P(X_{t+1} \in A | X_t, d_t), \qquad t \in \mathbb{N}$$

  and the *policy*

$$P(d_t \in A | X_t), \quad t \in \mathbb{N}.$$

- In the reinforcement learning, the probabilities are **not** known and have to be learned!
- In a nutshell, we proceed iteratively through time.
- At time t, we *observe* X_t, get a reward $U(X_t)$ and are able to make a decision d_t which influences the state at time $t+1$, X_{t+1}.
- A *policy* describes the decision given the state. It can be stochastic or deterministic.
- While initially the environment is unknown, the system gathers information through its interactions with the environment and improves its policy.

A quite related area is **Statistical Learning**. This new area of statistics is quite related to machine learning and we will study a number of relevant problems[10].

- Formally, we have an observation given by pairs (x_i, y_i), $i \in I$ and randomness is modelled with an (unknown) probability distribution
- The task is to predict y based on x.
- From all functions f in some set $\mathscr{H}$ we want to choose f so that the *expected risk*
  $$E[L(f(X), Y)]$$
  is minimal. Here, L is some chosen loss function.
- because the probability is unknown, one estimates the expected risk with the *empricial risk*
  $$\frac{1}{n}\sum_{i=1}^{n} L(f(x_i), y_i).$$

Popular and well-known examples are

- **Regression** in the simple least-squares regression, $f(x) = m + nx$ and $L(\cdot) = \cdot^2$
- **Classification** also falls into this framework: here Y takes only finitely many values, like $\{A, B, C, ...\}$ and possibly a step-function is chosen as loss function.

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[10] There is a lot of interesting literature in this area: e.g. T. Hastie, R. Tibshirani, and J. Friedman (2009). **The Elements of Statistical Learning**. Springer Series in Statistics. Springer New York Inc. URL: https://statweb.stanford.edu/~tibs/ElemStatLearn/, Vladimir Vapnik (2013). **The nature of statistical learning theory**. Springer science & business media.

Questions

- What is the essential difference of static and dynamic settings?
- What is a stochastic process. What is a Markov process. Do you know a process which is not Markovian?
- What is the difference to a Markov decision process.
- Simulate a Markov process in Python.
- Simulate a Markov decision process in Python.
- Choose a target, a loss function (or a reward) and try to find the optimal decision process.
- What is statistical learning.