Practice Exam 2—2.7-3.5

1. At a time t in seconds, the position, s, in meters, of a body moving along a coordinate line is given by $s(t) = 3t^2 - t^3,\ 0 \leq t \leq 3$.

a) What is the velocity of the particle at 1 second?

b) What is the acceleration of the particle at 2 seconds?

c) When is the particle moving forwards? Backwards?

d) What is the total distance traveled by the particle?

e) When is the particle speeding up? Slowing down?

2. At time t, the position of a particle moving along a vertical line is given by $s(t) = t^3 - 21t^2 + 144t$. Find the body's acceleration each time the velocity is zero.

3. A ball dropped from the top of a building has a height of $h = 55.123 - 4.9t^2$ meters after t seconds. What is the velocity of the ball at the moment the ball strikes the ground?

4. When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

5. The power P (watts) of an electric circuit is related to the circuit's resistance R (ohms) and the current I (amperes) by the equation $P = RI^2$. If the power in an electric circuit remains at a constant level of 2200 watts and the current is decreasing at a constant rate of $\frac{1}{2}$ ampere per second, at what rate is the resistance changing at the moment the circuit contains 20 amperes of current?

6. A 20-foot ladder is leaning against a house when its base starts to slide away from the house. The base of the ladder is sliding away from the house at a rate of 2 ft/sec.

a) When the base is 12 feet from the house, at what rate is the top of the ladder sliding down the wall?

b) When the base is 12 feet from the house, at what rate is the angle between the ladder and the ground changing?

7. Find the absolute maximum and absolute minimum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[-3, 3]$.

8. Find the absolute max and absolute min values of the function $g(x) = \sqrt{3 + 2x - x^2}$ on the interval $[0, 3]$.

9. Find the absolute maximum and absolute minimum values of the function $h(x) = \sin x \cos x$ on the interval $[0, \pi]$.

10. $f(x) = x^3 + 3x^2 + 4$

a) Find the intervals on which the function is increasing or decreasing.

b) Find the relative (local) extrema of the function.

c) Find the intervals on which the function is concave up or concave down.

11. $f(x) = x^4 - 8x^2 + 1$

a) Use the second derivative test to find the relative extrema of this function.

b) Find the intervals on which the function is concave up or concave down.

12. $f(x) = \frac{x}{16+x^3}$

a) Find the relative extrema of the function using any method.

b) Find any horizontal asymptotes using methods of calculus.

c) Find any vertical asymptotes and describe the behavior of the function at each asymptote. Use ∞ and $-\infty$ whenever appropriate.

13. $f(x) = 2x^3 - x^2 - 7x + 5$

a) Find the local maximum(s) and minimum(s) using the first derivative test.

b) Find the inflection points and determine the intervals of concavity.

c) Graph a rough sketch of the function. Be sure to label all local extrema, inflection points, and determine correct end behavior.

14. Let f be a function that is continuous for all real numbers and has exactly three critical numbers: $x = -2, x = 0, x = 2$. Use the values of $f'(x)$ and $f''(x)$ that are shown below to answer the following questions a, b, c.

$f'(-5) = 525$ $f'(-3) = 45$ $f'(-1) = -3$ $f'(1) = -3$

$f''(-4) = -224$ $f''(-2) = -16$ $f''(0) = 0$ $f''(2) = 16$ $f''(4) = 224$

a) Is $f(-2)$ a local max, a local min, or neither? Explain how you came to the conclusion that you did.

b) Is $f(0)$ a local max, a local min, or neither? Explain how you came to the conclusion that you did.

c) Is $f(2)$ a local max, a local min, or neither? Explain how you came to the conclusion that you did.

15) True or False: If f is a continuous function, and $f'(3) = 0$ and $f''(3) = -2$, then $f(3)$ is a local minimum?

16) True or False: If f is a continuous function, and $f'(3) = 0$ and $f''(3) = 0$, then $f(3)$ cannot be either a local maximum or local minimum?

17) True or False: If f is a continuous function for all real numbers, and $f(-3)$ is a local max, then $f'(-3) = 0$ or $f'(-3)$ is undefined.

18) True or False: If $f''(2) = 0$ then the point $(2, f(2))$ is an inflection point.

19) If $f''(2) = 0$ and $f''(x) > 0$ for all $x < 2$ and $f''(x) < 0$ for all $x > 2$ then what can you conclude about the function f? Draw the graph of a function that satisfies the conditions expressed.

20) True or False: It is impossible for $f'(2) < 0$ and $f''(2) > 0$.
Can you visualize an example of a function where this is possible?

21) Draw the graph of a function that satisfies the following conditions:

1) $f(-3) = 0$, $f(3) = 3$
2) $f'(x) \geq 0$ for all $x < 5$, $f'(0) = 0$
3) $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$
4) $\lim_{x-5} f(x) = \infty$
5) $\lim_{x \to \infty} f(x) = 0$

22) Find the value(s) of c that is/are guaranteed by the Mean Value Theorem for the function $f(x) = 3x^2 + 2x - 1$ on the interval $[-2, 2]$

23) Find the value(s) of c that is/are guaranteed by the Mean Value Theorem for the function $f(x) = x^3 - 3x^2$ on the interval $[-3, 3]$

24. Let $f(x) = 3 + x^{\frac{2}{5}}$ at $a = 32$

a) Find the linearization of $f(x)$ at $a = 32$.

b) Use the linearization in part (a) to approximate $f(31.9)$

25. Find the following limits or show that they do not exist.

a) $\lim\limits_{x \to \infty} 2x^3 - x^4$

b) $\lim\limits_{x \to -\infty} x^3(x+2)^2(x-1)$

c) $\lim\limits_{x \to \infty} x^2(x^2-1)(x+2)$

d) $\lim_{x \to \infty} \frac{x+3x^2}{4x-1}$

26. Find the horizontal asymptotes of the curve and use them, together with concavity and intervals of increase and decrease, to sketch the curve.

a) $f(x) = \frac{1+2x^2}{1+x^2}$

b) $y = \frac{x}{\sqrt{x^2+1}}$