Math 1210 Fall 2018

Worksheet 5—Sections 3.2-3.5

Instructions: In order to receive full credit, you must have made a valiant attempt at solving every problem. If you get stuck, or want to check your answers, please use the key which will be posted on Canvas after we have completed our group work. Please write neatly, show your work, label your answers where appropriate, and circle your final answers. This worksheet is intended to help you prepare for the exam. You should first complete the homework problems for any given section, and then attempt the worksheet problems. You may use notes, textbook, neighbor, tutor, worksheet key, calculator, or whatever else you feel is necessary in order to understand and complete the problems. However, if you cannot do a problem unaided, it is an indication that you are not prepared for the exam and you should work through several similar practice problems until you are sure you have grasped the concept correctly.

1. Find the value or values of c that satisfy the conclusion of the Mean Value Theorem for the function $f(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2, 5]$.

$a = 2$ $b = 5$ $f(2) = 32 - 32 + 14 - 2 = 12$

$f(5) = 500 - 200 + 35 - 2 = 333$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{f(5) - f(2)}{5 - 2} = \frac{333 - 12}{3} = 107$$

$C = 3.62$

$$f'(z) = 12z^2 - 16z + 7$$

$$f'(c) = 12c^2 - 16c + 7 \overset{set}{=} 107$$

$$12c^2 - 16c - 100 = 0$$

$$c = \frac{16 \pm \sqrt{16^2 - 4(12)(-100)}}{24} = \frac{16 \pm \sqrt{5056}}{24} = \begin{matrix} 3,62 \\ \cancel{-2,296} \end{matrix}$$

2. Let $f(x) = x - 6\sqrt{x - 1}$. Identify the intervals on which f is increasing or decreasing. Find the local and absolute extreme values.

$$f(x) = x - 6(x-1)^{1/2}$$

$$f'(x) = 1 - 3(x-1)^{-1/2}$$

$$= 1 - \frac{3}{\sqrt{x-1}} \overset{set}{=} 0$$

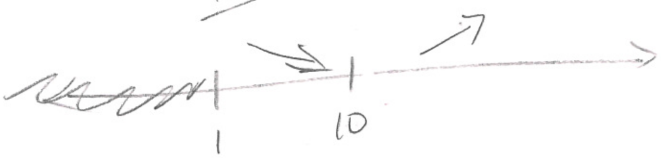
$$1 = \frac{3}{\sqrt{x-1}}$$ ← undefined for $x < 1$

$$\sqrt{x-1} = 3$$

$$x - 1 = 9$$

$$\boxed{x = 10}$$

- decreasing from $(1, 10)$
- Increasing from $(10, \infty)$
- Local & ABS minimum at $x = 10$
  Value is $f(10) = -8$ $(10, -8)$



$$f'(2) = -2 < 0$$

$$f'(11) = .05 > 0$$

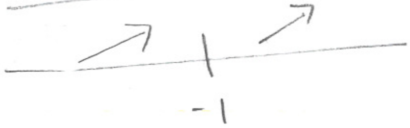
3. Let $k(x) = x^3 + 3x^2 + 3x + 1$. Identify the intervals on which k is increasing or decreasing. Find the local and absolute extreme values. Determine where k is concave up and where it is concave down. Find all points of inflection. Sketch the graph of k.

$k'(x) = 3x^2 + 6x + 3$

$3(x^2 + 2x + 1) \overset{\text{set}}{=} 0$

$(x+1)(x+1) = 0$

$\boxed{x = -1}$ 1 cr. pt. ← not a local max or min



$k'(0): + \cdot + = +$

$k'(-2): - \cdot - = +$

$k(-1) = -1 + 3 - 3 + 1 = 0$

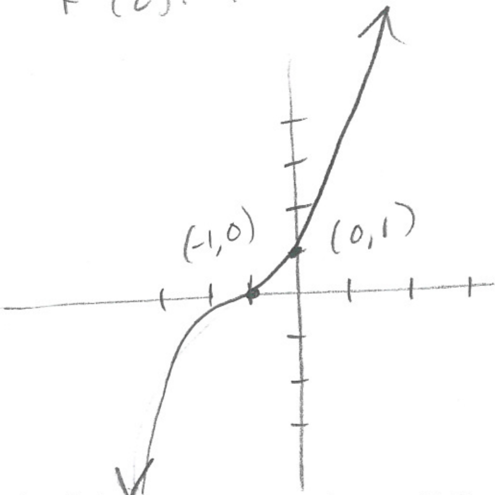
y-int: 1

$k''(x) = 6x + 6 \overset{\text{set}}{=} 0$

$\boxed{x = -1}$ ← 2nd Der. Test fails
1 poss Inf. pt.

$k''(-2): -$

$k''(0): +$



- No local or Absolute extrema
- Increasing on $(-\infty, \infty)$
- Concave Down $(-\infty, -1)$
- concave up $(-1, \infty)$

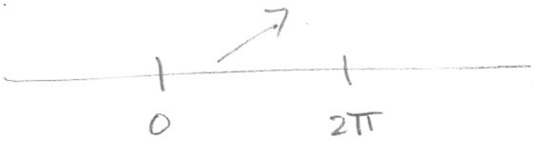
4. Let $w(x) = x - \sin x$ for $0 \leq x \leq 2\pi$. Find the local and absolute extreme values and inflection points. Graph the function.

$w'(x) = 1 - \cos x \overset{\text{set}}{=} 0$

$1 = \cos x$

$\boxed{x = 0, 2\pi}$ critical pts are at ends

y-int: (0,0)



$w'(\pi) = 2 > 0$

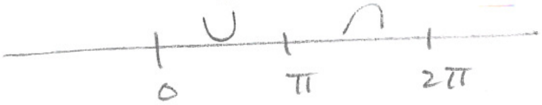
$w(0) = 0$

$w(2\pi) = 2\pi - \sin 2\pi = 2\pi$

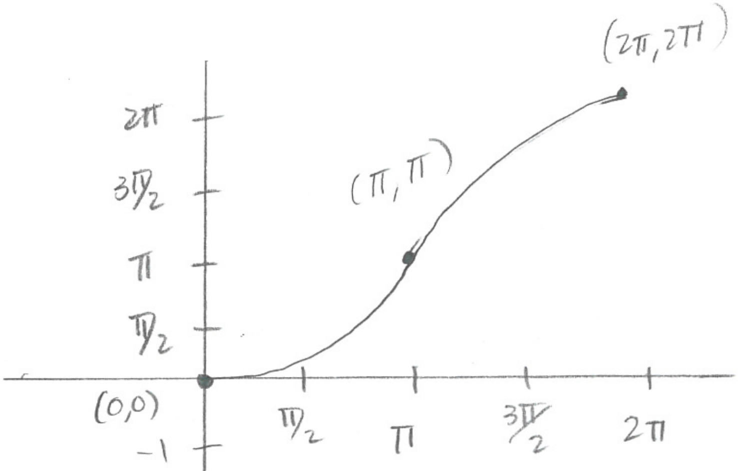
$w(\pi) = \pi - \sin \pi = \pi$

$w''(x) = \sin x \overset{\text{set}}{=} 0$ Inflection pt.

$x = 0, \pi, 2\pi$



$w''(\pi/2) = 1 > 0$ $w''(3\pi/2) = -1 < 0$



Inf. pt at (π,π)

ABS min at x = 0 (0,0)
ABS max at x = 2π (2π, 2π)

$f = x \quad g = (9-x^2)^{1/2}$

$f' = 1 \quad g' = \frac{1}{2}(9-x^2)^{-1/2}(-2x)$

5. Let $s(x) = x\sqrt{9-x^2}$ for $\boxed{-3 \le x \le 3}$. Find the local and absolute extreme values and inflection points. Graph the function.

$s'(x) = (9-x^2)^{1/2} + x\left(\frac{1}{2}\right)(9-x^2)^{-1/2}(-2x)$

$= \sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}} \overset{\text{set}}{=} 0$

undefined for $x = \pm 3$

$\sqrt{9-x^2} = \frac{x^2}{\sqrt{9-x^2}}$

$9 - x^2 = x^2$

$9 = 2x^2$

$x^2 = \frac{9}{2}$

$x = \pm\frac{3}{\sqrt{2}} \approx \pm 2.12$

$\boxed{x = -3, -2.12, 2.12, 3}$

2 critical points
(Notice 2 are end points)

$\boxed{\text{Continued at back}}$

$s'(x) = \frac{(9-x^2) - x^2}{\sqrt{9-x^2}} = \frac{9-2x^2}{(9-x^2)^{1/2}}$

$s''(x) = \frac{(9-x^2)^{1/2}(-4x) - (9-2x^2)\left(\frac{1}{2}\right)(9-x^2)^{-1/2}(-2x)}{9-x^2}$

$= \frac{-4x(9-x^2)^{1/2} + x(9-2x^2)(9-x^2)^{-1/2}}{9-x^2}$

$= \frac{-4x}{(9-x^2)^{1/2}} + \frac{x(9-2x^2)}{(9-x^2)^{3/2}}$

$= \frac{-4x(9-x^2) + x(9-2x^2)}{(9-x^2)^{3/2}}$

$= \frac{-36x + 4x^3 + 9x - 2x^3}{(9-x^2)^{3/2}} = \boxed{\frac{2x^3 - 27x}{(9-x^2)^{3/2}}}$

$2x^3 - 27x \overset{\text{set}}{=} 0$

$x(2x^2 - 27) = 0$

$\boxed{x = 0} \quad x^2 = \frac{27}{2} \Rightarrow \boxed{\cancel{x = \pm\sqrt{\frac{27}{2}}} \approx \pm 3.7}$

3 inflection points but 2 are outside the interval

6. The function g is defined by $g(x) = \frac{4x^2+3}{x^2-4}$ for $x \neq -2, x \neq 2$

a) Find $\lim_{x\to\infty} \frac{4x^2+3}{x^2-4}$

$\lim_{x\to\infty} \frac{4x^2+3}{x^2-4} = \lim_{x\to\infty} \frac{4 + 3/x^2}{1 - 4/x^2} = \boxed{4}$

b) Find $\lim_{x\to 2^+} \frac{4x^2+3}{x^2-4}$

VA at $x = 2$

$\lim_{x\to 2^+} \frac{\overset{\oplus}{4x^2+3}}{\underset{+}{x^2-4}} = \boxed{\infty}$

c) Identify the horizontal and vertical asymptotes. Write a sentence explaining how you determined the existence of each asymptote.

H.A. at $y = 4$ – found by taking $\lim_{x\to\infty}$

V.A. at $x = \pm 2$, – zeros of the denominator – also verify by taking left & right hand limits at $x = \pm 2$.

7. Evaluate the following limit and make sure you understand the difference between the graph of a function that contains a hole and one that contains a vertical asymptote. $\lim\limits_{x\to3^+}\frac{2x^2-5x-3}{x^2-5x+6}$

$$\lim_{x\to3^+}\frac{2x^2-5x-3}{x^2-5x+6}=\lim_{x\to3^+}\frac{(2x+1)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}=\lim_{x\to3^+}\frac{2x+1}{x-2}$$

$$=\frac{6+1}{3-2}=\frac{7}{1}=\boxed{7}$$

-6

$\wedge$

$+1\ -6$

$(2x^2-6x)+(x-3)$

$2x(x-3)+1(x-3)$

$(2x+1)(x-3)$

There is a hole at $x=3$, NO V.A.

8. Define (write) a rational function that has the following characteristics:

1. $x=-4$ is a vertical asymptote of the graph of $y=f(x)$ ← must be a factor in den.
2. There is a hole in the graph of $y=f(x)$ when $x=1$ ← must be a factor in BOTH den & Num
3. The x-axis is a horizontal asymptote of the graph of $y=f(x)$ ← degree of den is greater than degree of Num
4. $f(3)=0$ ← means there is a factor in Num: $(x-3)$

$$\boxed{f(x)=\frac{(x-3)(x-1)}{(x+4)^2(x-1)}}$$

There are multiple functions that meet this criteria. This is one of them

9. Investigate the following limits and identify any horizontal, vertical or oblique asymptotes.

a. $\lim\limits_{x\to\infty}\frac{2x^3+7}{x^3-x^2+x+7}$

$$=\lim_{x\to\infty}\frac{2x^3/x^3+7/x^3}{x^3/x^3-x^2/x^3+x/x^3+7/x^3}=\lim_{x\to\infty}\frac{2+7/x^3}{1-1/x+1/x^2+7/x^3}=\frac{2+0}{1-0+0+0}=\boxed{2}$$

← HA at $y=2$

Use desmos or calculator to find the zero of the denominator. There is a V.A. at $x\approx-1.48$

b. $\lim\limits_{x\to\infty}\frac{3x^2+5}{x^3+9}$

$$=\lim_{x\to\infty}\frac{3x^2/x^3+5/x^3}{x^3/x^3+9/x^3}=\lim_{x\to\infty}\frac{3/x+5/x^3}{1+9/x^3}=\frac{0+0}{1+0}=0$$

← H.A at $y=0$

V.A. where denominator equals 0....

$x^3+9\overset{set}{=}0$

$x^3=-9$

$x=\sqrt[3]{-9}$

So there is a VA at $x=\sqrt[3]{-9}\approx-2.08$

c. $\lim_{x\to\infty} \frac{x^5+13}{x^3+27}$

$$= \lim_{x\to\infty} \frac{x^5/x^3 + 13/x^3}{x^3/x^3 + 27/x^3} = \lim_{x\to\infty} \frac{\overset{\nearrow\infty}{x^2} + \overset{\nearrow 0}{13/x^3}}{1 + \underset{\searrow 0}{27/x^3}} = \infty$$

No H.A.

V.A.: $x^3+27 \overset{set}{=} 0 \Rightarrow x^3 = -27$

$x = -3$ ← V.A. at $x = -3$

d. $\lim_{x\to\infty} \frac{3x^2+5}{x+1}$

$$= \lim_{x\to\infty} \frac{3x^2/x + 5/x}{x/x + 1/x} = \lim_{x\to\infty} \frac{\overset{\to\infty}{3x} + \overset{\to 0}{5/x}}{1 + \underset{\searrow 0}{1/x}} = \infty$$

~~No H.A.~~

BUT there is a slant Asymptote at: $y = 3x - 3$

$$\begin{array}{r} 3x - 3 \\ x+1 \overline{)\,3x^2+0x+5} \\ 3x^2+3x \\ \hline -3x+5 \\ -3x-3 \\ \hline 8 \end{array}$$

← Remainder

V.A.: $x+1 \overset{set}{=} 0$

$x = -1$ ← V.A.

e. $\lim_{x\to 1^-} \frac{2}{x^2-1}$ ← VA at $x = 1$

$$\lim_{x\to 1^-} \frac{\overset{(+)}{2}}{\underset{(-)}{x^2-1}} = -\infty$$

10. Finish sketching the graphs of any of the functions we didn't finish during the lecture(s).

11. Make sure you know the difference between these four tests and when/how they can be used: The increasing/decreasing test, First Derivative Test, Test for Concavity, Second Derivative Test. You will be quizzed/tested on these! (i.e. I might ask you to use the First Derivative test to determine local extrema. Or, I could ask you to use the Second Derivative test to locate them. You will have to use the correct test to get credit!)

#5. continued...

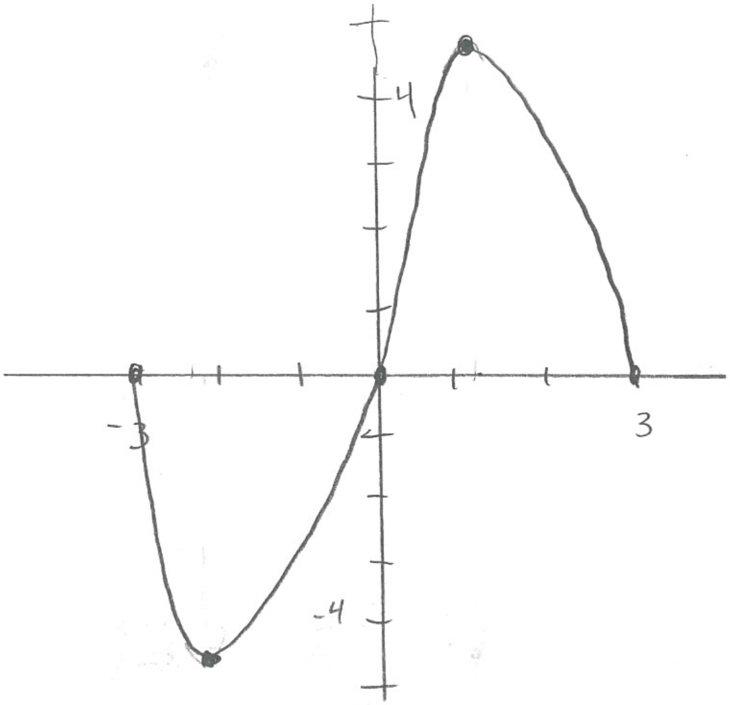
write s'' as

$$\frac{x(2x^2-27)}{(9-x^2)^{3/2}}$$

second derivative & Inf. pts

$s''(-1) = \frac{- \cdot -}{+} = + > 0$

$s''(1) = \frac{+ \cdot -}{+} = - < 0$



$s(-3) = 0$

$s(3) = 0$

$s(0) = 0$

$s\left(-\frac{3}{\sqrt{2}}\right) = -\frac{3}{\sqrt{2}}\sqrt{9-\left(-\frac{3}{\sqrt{2}}\right)^2} = -\frac{3}{\sqrt{2}}\sqrt{9-\frac{9}{2}} = -\frac{3}{\sqrt{2}}\cdot\frac{3}{\sqrt{2}} = \boxed{-\frac{9}{2}}$ min

$s\left(\frac{3}{\sqrt{2}}\right) = \boxed{\frac{9}{2}}$ max

- local & ABS min at $x = -\frac{3}{\sqrt{2}}$ value is $-\frac{9}{2}$
- local & ABS max at $x = \frac{3}{\sqrt{2}}$ value is $\frac{9}{2}$
- Inflection point at $(0,0)$