Key

Math 1210 Fall 2018

Worksheet 4—Sections 2.7-3.1

Instructions: You will receive 10 completion points for this worksheet. In order to receive full credit, you must have made a valiant attempt at solving every problem. If you get stuck, or want to check your answers, please use the key which will be posted on Canvas after we have completed our group work. Please write neatly, show your work, label your answers where appropriate, and circle your final answers. This worksheet is intended to help you prepare for the exam. You should first complete the homework problems for any given section, and then attempt the worksheet problems. You may use notes, textbook, neighbor, tutor, worksheet key, calculator, or whatever else you feel is necessary in order to understand and complete the problems. However, if you cannot do a problem unaided, it is an indication that you are not prepared for the exam and you should work through several similar practice problems until you are sure you have grasped the concept correctly.

1. If a rock is thrown vertically upward from the surface of Mars with velocity 15 m/s, its height after t seconds is
$$h = 15t - 1.86t^2.$$

a) What is the velocity of the rock after 2 seconds?

$V(t) = 15 - 3.72t$

$V(2) = 15 - 3.72(2) = 7.56$ m/sec

b) What is the velocity of the rock when its height is 25 m on its way up? On its way down?

$h = 15t - 1.86t^2 \overset{\text{set}}{=} 25$

$-1.86t^2 + 15t - 25 = 0$

$t = \dfrac{-15 \pm \sqrt{15^2 - 4(-1.86)(-25)}}{2(-1.86)} = \dfrac{-15 \pm \sqrt{39}}{-3.72}$

$t \approx 2.35349$

$t \approx 5.71102$

$V(2.35) = 15 - 3.72(2.35349) \approx 6.24$ m/sec

$V(5.71) = 15 - 3.72(5.71) \approx -6.24$ m/sec

2. A particle moves with position function $s = t^4 - 4t^3 - 20t^2 + 20t \quad t \geq 0$

a) At what time does the particle have a velocity of 20 m/second?

$V(t) = 4t^3 - 12t^2 - 40t + 20 \overset{\text{set}}{=} 20$

$4t^3 - 12t^2 - 40t = 0$

$4t(t^2 - 3t - 10) = 0$

$4t(t+2)(t-5) = 0$

$t = 0$

$\cancel{t = -2}$ ← time can't be negative

$t = 5$

The velocity is 20 m/sec at time $t = 0$ & $t = 5$ seconds

b) At what time is the acceleration 0? What is the significance of this value of t?

$a(t) = 12t^2 - 24t - 40 \overset{\text{set}}{=} 0$

$t = \dfrac{24 \pm \sqrt{24^2 - 4(12)(-40)}}{24} = \dfrac{24 \pm \sqrt{2496}}{24}$

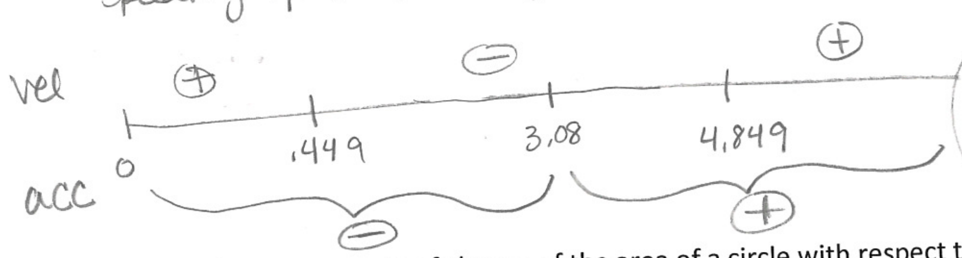
$t \approx 3.08$

$\cancel{t \approx -1.08}$

The velocity is constant here. The particle is neither speeding up nor slowing down at time $t = 3.08$ sec

c) When is the particle speeding up? When is it slowing down?

Speeding up when V & a have same sign



Speeding up when $.449 < t < 3.08$ and $t > 4.849$

Slowing down when $0 < t < .449$

$3.08 < t < 4.849$

3. a) Find the average rate of change of the area of a circle with respect to its radius r as r changes from

i) 2 to 3 ii) 2 to 2.5 iii) 2 to 2.1

Average rate of change is just $\dfrac{\Delta y}{\Delta x}$ & $A = \pi r^2$

i) $A(2) = 4\pi$

$A(3) = 9\pi$

$\dfrac{\Delta y}{\Delta x} = \dfrac{9\pi - 4\pi}{3-2} = 5\pi$

ii) $A(2.5) = 6.25\pi$

$\dfrac{\Delta y}{\Delta x} = \dfrac{6.25\pi - 4\pi}{2.5 - 2} = \dfrac{2.25\pi}{.5} = 4.5\pi$

iii) $A(2.1) = 4.41\pi$

$\dfrac{\Delta y}{\Delta x} = \dfrac{4.41\pi - 4\pi}{2.1 - 2} = \dfrac{.41\pi}{.1} = 4.1\pi$

the derivative!

b) Find the instantaneous rate of change when $r = 2$

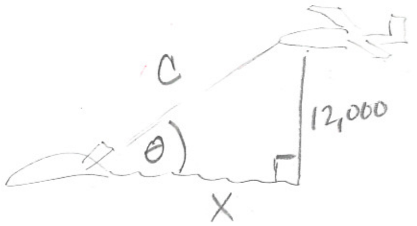
$A = \pi r^2$

$A' = 2\pi r$

$A'(2) = 2 \cdot \pi \cdot 2 = 4\pi$

Note: the rate of change of the Area of a circle w.r.t. r is equal to the circumference of the circle... :)

4. A jet is flying at a constant altitude of 12,000 feet above sea level as it approaches a Pacific island. The aircraft comes within the direct line of sight of a radar station located on the island, and the radar indicates the initial angle between sea level and its line of sight to the aircraft is 30 degrees. How fast (in miles per hour) is the aircraft approaching the island when first detected by the radar instrument if the radar instrument is turning upward (counterclockwise) at the rate of $\frac{2}{3}$ deg/sec in order to keep the aircraft within its direct line of sight.



Given: $b = 12{,}000$ Find: $\frac{dx}{dt}$ When: $\theta = 30^\circ \cdot \frac{\pi}{180} = \frac{\pi}{6}$ rads

$\frac{d\theta}{dt} = \frac{2}{3} \cdot \frac{\pi}{180} = \frac{\pi}{270}$ rad/sec

Relate: $\tan\theta = \frac{12{,}000}{x}$

$\tan\theta = 12{,}000\,x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{12{,}000}{x^2}\frac{dx}{dt}$

$\frac{4}{3} \cdot \frac{\pi}{270} = \frac{-12{,}000}{(\sqrt{3} \cdot 12{,}000)^2} \cdot \frac{dx}{dt}$

$\left(\sec\frac{\pi}{6}\right)^2 = \left(\frac{1}{\cos\pi/6}\right)^2 = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$

$\tan\frac{\pi}{6} = \frac{12{,}000}{x}$

$\frac{1}{\sqrt{3}} = \frac{12{,}000}{x} \Rightarrow x = \sqrt{3} \cdot 12{,}000$

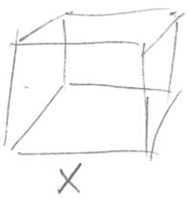
$\rightarrow \frac{2\pi}{405} = \frac{-1}{36{,}000}\frac{dx}{dt}$

$\frac{-72{,}000\pi}{405} = \frac{dx}{dt}$

$\frac{dx}{dt} \approx -380.79$ mph

Speed is 380 mph

5. Suppose the original 24 m edge length x of a cube decreases at the rate of 5 m/min. When $x = 3$ m, ~~at what rate~~ does the cube's surface area and volume change?



Given: $\frac{dx}{dt} = -5$ m/min Find: $\frac{dS}{dt}$ & $\frac{dV}{dt}$ when: $x = 3$

Relate S & x: $S = 6x^2$

$\frac{dS}{dt} = 12x\frac{dx}{dt}$

$\frac{dS}{dt} = 36(-5)$

$= -180$ m²/min

$V = x^3 \leftarrow$ Relate V & x

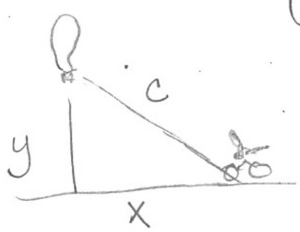
$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$\frac{dV}{dt} = 3(3^2)(-5)$

$= 27(-5)$

$= -135$ m³/min

6. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft./sec. Just when the balloon is 65 ft. above the ground, a bicycle moving at a constant rate of 17 ft./sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 sec later?



(1) Given: $\frac{dx}{dt} = 17$ ft/sec

$\frac{dy}{dt} = 1$ ft/sec

(2) Find: $\frac{dc}{dt}$

(3) When: 3 seconds AFTER $y = 65$!

then $\boxed{y = 68}$

$\boxed{x = 3(17) = 51}$

(4) Relate x, y & c: $x^2 + y^2 = c^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}c\frac{dc}{dt}$

(5) When $y = 68$ & $x = 51$ what is c?

$x^2 + y^2 = c^2$

$51^2 + 68^2 = c^2$

$7225 = c^2$

$\boxed{c = 85}$

(6) $51 \cdot 17 + 68 \cdot 1 = 85 \cdot \frac{dc}{dt}$

$935 = 85\frac{dc}{dt}$

(7) $\frac{dc}{dt} = 11$ ft/sec

http://tutorial.math.lamar.edu/Problems/CalcI/LinearApproximations.aspx

7. Find the linear approximation to the function at the given point: $h(t) = t^4 - 6t^3 + 3t - 7$ at $t = -3$ **Solution**

$L(t) = h(a) + h'(a)(t-a)$

$L(t) = h(-3) + h'(-3)(t+3)$

$= 227 - 267(t+3)$

$= 227 - 267t - 801$

$= -574 - 267t$

$h(-3) = (-3)^4 - 6(-3)^3 + 3(-3) - 7$

$= 81 + 162 - 9 - 7 = 227$

$h'(t) = 4t^3 - 18t^2 + 3$

$h'(-3) = 4(-3)^3 - 18(-3)^2 + 3$

$= -108 - 162 + 3 = -267$

8. a) Find the linear approximation to $g(z) = \sqrt[4]{z}$ at $z = 2$

$L(z) = g(a) + g'(a)(z-a)$

$= g(2) + g'(2)(z-2)$

$= 2^{1/4} + \frac{1}{4}2^{-3/4}(z-2)$

$g(2) = \sqrt[4]{2} = 2^{1/4}$

$g'(z) = \frac{1}{4}(z)^{-3/4}$

$g'(2) = \frac{1}{4}2^{-3/4}$

b) Use the linear approximation to approximate the value of $\sqrt[4]{3}$ and $\sqrt[4]{10}$. Compare the approximated values to the exact values. **Solution** Let $z = 3$ & Let $z = 10$

$g(3) \approx L(3) = 2^{1/4} + \frac{1}{4}2^{-3/4}(3-2)$

$= 2^{1/4} + \frac{1}{4}2^{-3/4}$

≈ 1.33786

$g(3) = \sqrt[4]{3} = \boxed{1.31607}$

$g(10) \approx L(10) = 2^{1/4} + \frac{1}{4}2^{-3/4}(10-2)$

$= 2^{1/4} + \frac{1}{4}2^{-3/4} \cdot 8$

$= 2^{1/4} + 2^{1/4}$

$= 2(2^{1/4}) \approx 2.37841$

$g(10) = \sqrt[4]{10}$

$= \boxed{1.77827}$

9. a) Find the linear approximation to $f(t) = \cos 2t$ at $t = \frac{1}{2}$.

$L(t) = f(a) + f'(a)(t-a)$

$L(t) = f(\frac{1}{2}) + f'(\frac{1}{2})(t - \frac{1}{2})$

$= \cos 1 - 2\sin 1\,(t - \frac{1}{2})$

$f(\frac{1}{2}) = \cos 2 \cdot \frac{1}{2} = \cos 1$

$f'(t) = -\sin 2t\,(2) = -2\sin 2t$

$f'(\frac{1}{2}) = -2\sin 2(\frac{1}{2}) = -2\sin 1$

b) Use the linear approximation to approximate the value of $\cos 2$ and $\cos 18$. Compare the approximated values to the exact values. **Solution**

Find $f(1)$ & $f(9)$

$f(1) \approx L(1) = \cos 1 - 2\sin(1)(1 - \frac{1}{2})$

$= \cos 1 - 2\sin(1)(\frac{1}{2})$

$= \cos 1 - \sin 1$

$= -.301169$

$f(9) \approx L(9) = \cos 1 - 2\sin 1\,(9 - \frac{1}{2})$

$= \cos 1 - 2\sin 1\,(8.5)$

$= \cos 1 - 17\sin 1$

$= -13.7647$

10. Find the absolute maximum and minimum of the function defined by $f(x) = 4 - x^3$ on $[-2,1]$.

$f'(x) = -3x^2 \overset{\text{set}}{=} 0$ The derivative is defined everywhere.

$x = 0 \leftarrow$ 1 critical point

Check:

$f(0) = 4$

$f(-2) = 4 - (-2)^3 = 4 + 8 = 12 \leftarrow$ ABS max is 12 at $x = -2$

$f(1) = 4 - 1^3 = 3 \leftarrow$ ABS min is 3 at $x = 3$

11. Find the absolute maximum and minimum of the function defined by $g(x) = e^{-x^2}$ on $[-2,1]$.

on $[0,2]$

12. Determine all critical points for the function defined by $h(x) = (x-1)^2(x-3)^3$.

$f \quad g$

$f' = 2(x-1)$

$g' = 3(x-3)^2$

$h'(x) = (x-3)^3(2)(x-1) + (x-1)^2(3)(x-3)^2$ ← defined everywhere...

$= 2(x-1)(x-3)^3 + 3(x-1)^2(x-3)^2 \overset{set}{=} 0$

$= (x-1)(x-3)^2[2(x-3)+3(x-1)] = 0$

$= (x-1)(x-3)^2[2x-6+3x-3] = 0$

$= (x-1)(x-3)^2(5x-9) = 0$

No need to test, it's not in the interval

$\boxed{x=1}$ $\cancel{x=3}$ $x = \boxed{\frac{9}{5}}$ ← 3 critical points

ABS max is 0 at $x=1$ $(1,0)$

ABS min is -27 at $x=0$ $(0,-27)$

Check:

$f(1) = (1-1)^2(1-3)^3 = \boxed{0}$

$f(\frac{9}{5}) = (\frac{9}{5}-1)^2(\frac{9}{5}-3)^3 = (\frac{4}{5})^2(-\frac{6}{5})^3 = \frac{16}{25} \cdot \frac{-216}{125} = \frac{-3456}{3125} \approx \boxed{-1.1059}$

$f(0) = (-1)^2(-3)^3 = 1 \cdot (-27) = \boxed{-27}$

$f(2) = (2-1)^2(2-3)^3 = 1 \cdot (-1) = \boxed{-1}$

13. Determine all critical points for the function defined by $u(x) = x^2\sqrt{3-x}$.

on the interval $[-1,3]$

$u(x) = x^2(3-x)^{1/2}$

$f \quad g$

$f' = 2x$

$g' = \frac{1}{2}(3-x)^{-1/2}(-1)$

$= -\frac{1}{2}(3-x)^{-1/2}$

$u'(x) = (3-x)^{1/2}(2x) + x^2(-\frac{1}{2})(3-x)^{-1/2} \overset{set}{=} 0$

$= 2x\sqrt{3-x} - \frac{x^2}{2\sqrt{3-x}} = 0$ ← Not defined for $x=3$

$\sqrt{3-x} \cdot 2x\sqrt{3-x} = \frac{x^2}{2\cancel{\sqrt{3-x}}} \cdot \cancel{\sqrt{3-x}}$

$2 \cdot 2x(3-x) = \frac{x^2}{2} \cdot 2$

$4x(3-x) = x^2$

$12x - 4x^2 = x^2$

$-5x^2 + 12x = 0$

$x(-5x+12) = 0$

$\boxed{x=0}$ $\boxed{x = \frac{12}{5}}$

3 critical points

$f(0) = 0$

$f(\frac{12}{5}) = (\frac{12}{5})^2\sqrt{3-\frac{12}{5}} \approx 4.46167$

$f(3) = 9 \cdot 0 = 0$

$f(-1) = (-1)^2\sqrt{3+1} = 1 \cdot \sqrt{4} = 2$

ABS max is 4.46 at $x = 12/5 \approx 2.4$

ABS min is 0 at $x=0$ & $x=-1$