$$y = f(x)$$

$$v^2 = v_x^2 + (v_y + S(x))^2$$

$$v_y = f'(x)\, v_x \qquad T = \int_0^T dt = \int_0^D \frac{dx}{v_x(x)}$$

$$\frac{dy}{dt} = \frac{df}{dx}\frac{dx}{dt}$$

$$v^2 = v_x^2 + (g v_x + S)^2$$

$$v^2 = v_x^2 + (g v_x)^2 + 2 g v_x S + S^2$$

$$v_x^2 + (g v_x)^2 + 2 g v_x S + S^2 - v^2 = 0$$

$$\underbrace{(1 + g)}_{a} v_x^2 + \underbrace{(2gS)}_{b} v_x + \underbrace{(S^2 - v^2)}_{c} = 0$$

$$v_x = \frac{-\cancel{2}gS \pm \sqrt{(\cancel{2}gS)^2 - \cancel{4}(g^2+1)(S^2 - v^2)}}{\cancel{2}(g^2+1)}$$

$$v_x = \frac{-gS \pm \sqrt{g^2 S^2 - (g^2+1)(S^2 - v^2)}}{(g^2+1)}$$

$$v_x = \frac{-gS \pm \sqrt{g^2 S^2 - (g^2 S^2 - g^2 v^2 + S^2 - v^2)}}{(g^2+1)}$$

$$v_x = \frac{-gS + \sqrt{g^2 v^2 - S^2 + v^2}}{(g^2+1)}$$

$$v_x = \frac{-gS + \sqrt{g^2 v^2 - S^2 + v^2}}{(g^2+1)}$$

$$T[g;S] = \int_0^D \frac{1+g^2}{\sqrt{(g^2+1)v^2 - S^2} - gS}\, dx$$