

# Speech Processing and Understanding

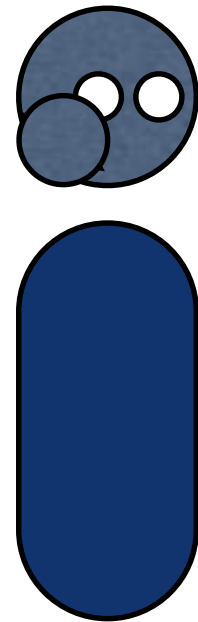
## CSC401 Assignment 3

# Agenda

- Background
  - Speech technology, in general
  - Acoustic phonetics
- Assignment 3
  - Speaker Recognition: Gaussian mixture models
  - Speech Recognition: Word-error rates with Levenshtein distance.

# Applications of Speech Technology

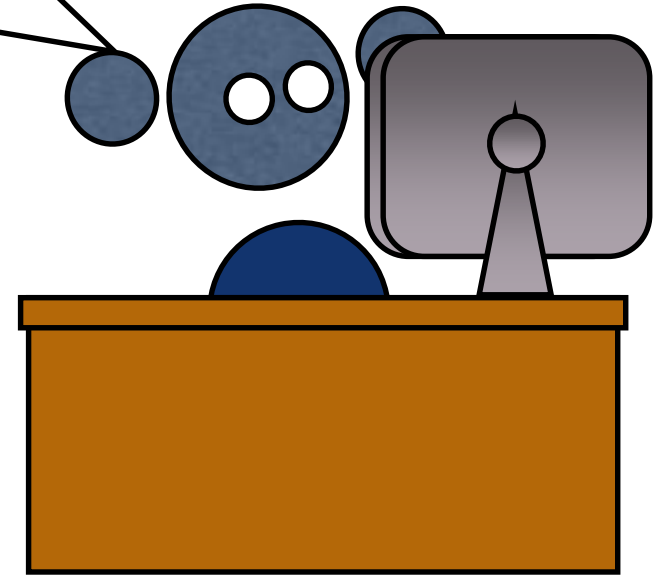
## Telephony



Buy ticket...  
AC490...  
yes

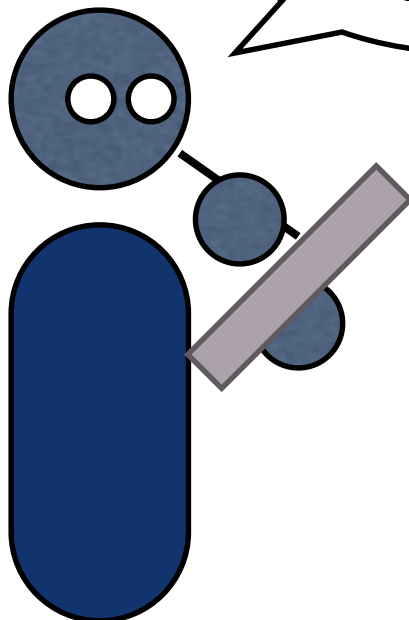
## Dictation

My hands are  
in the air.



## Multimodality & HCI

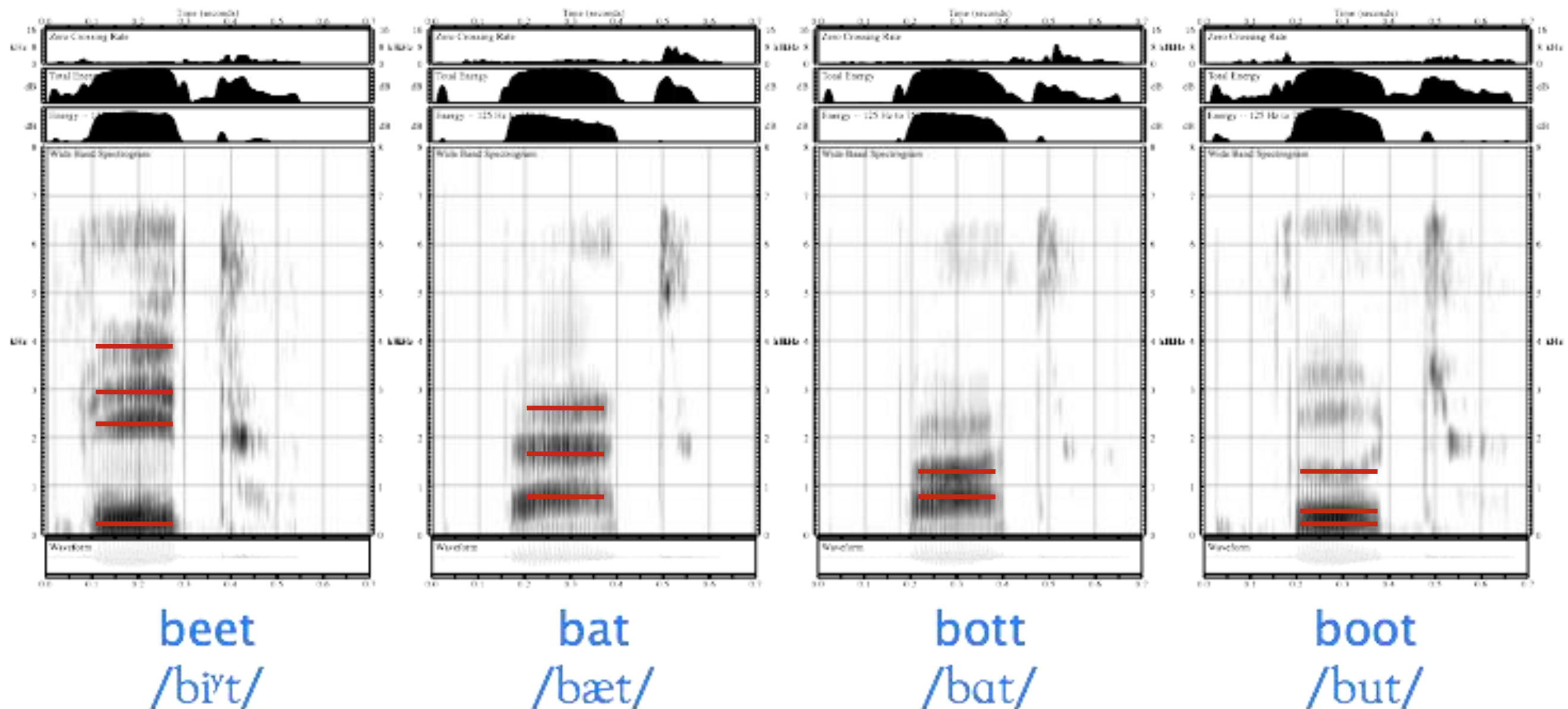
Put this  
there.



## Emerging...

- Data mining/indexing.
- Assistive technology.
- Conversation.

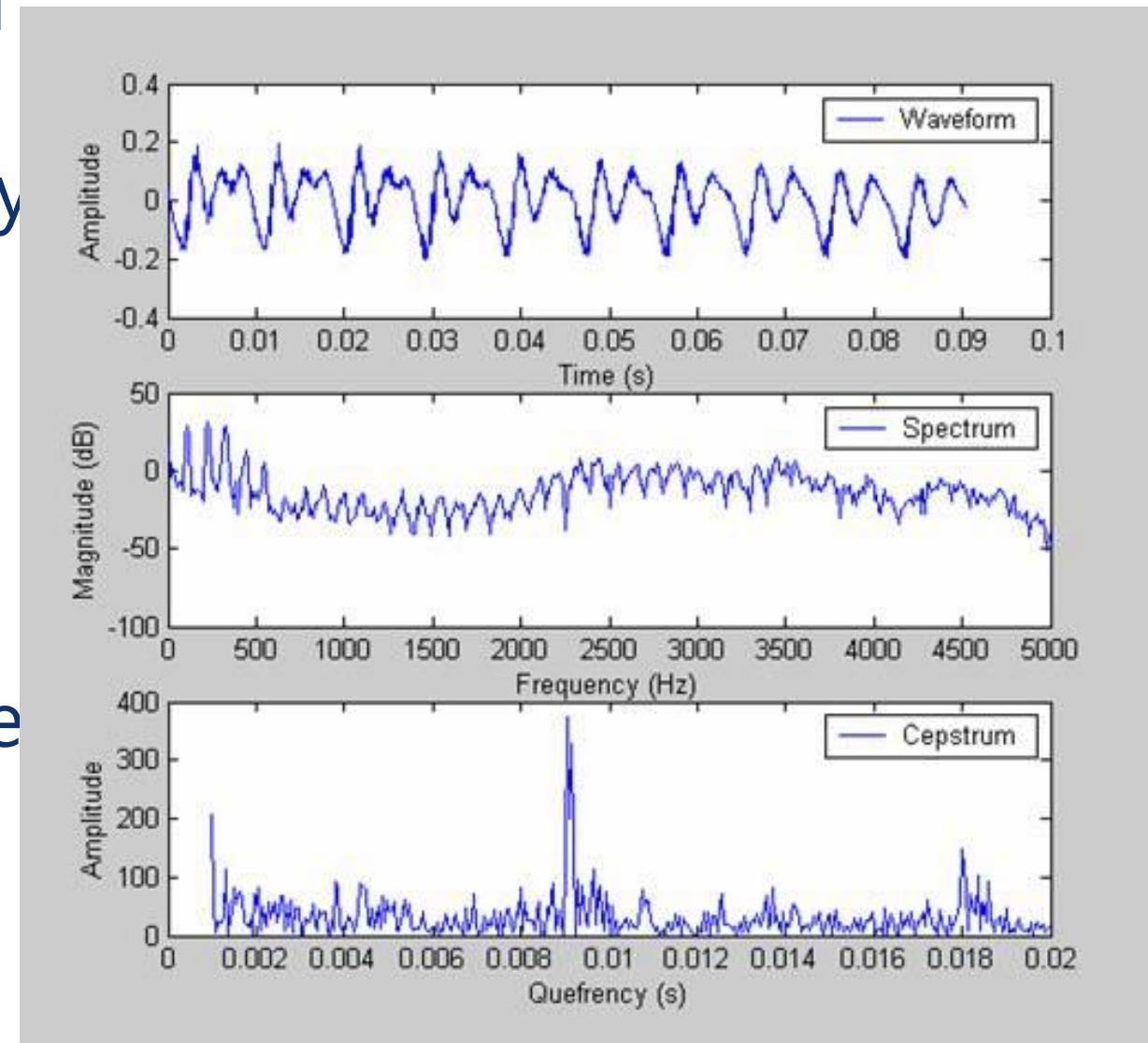
# Formants in sonorants



- However, formants are insufficient features for use in speech recognition generally...

# Mel-frequency cepstral coefficients

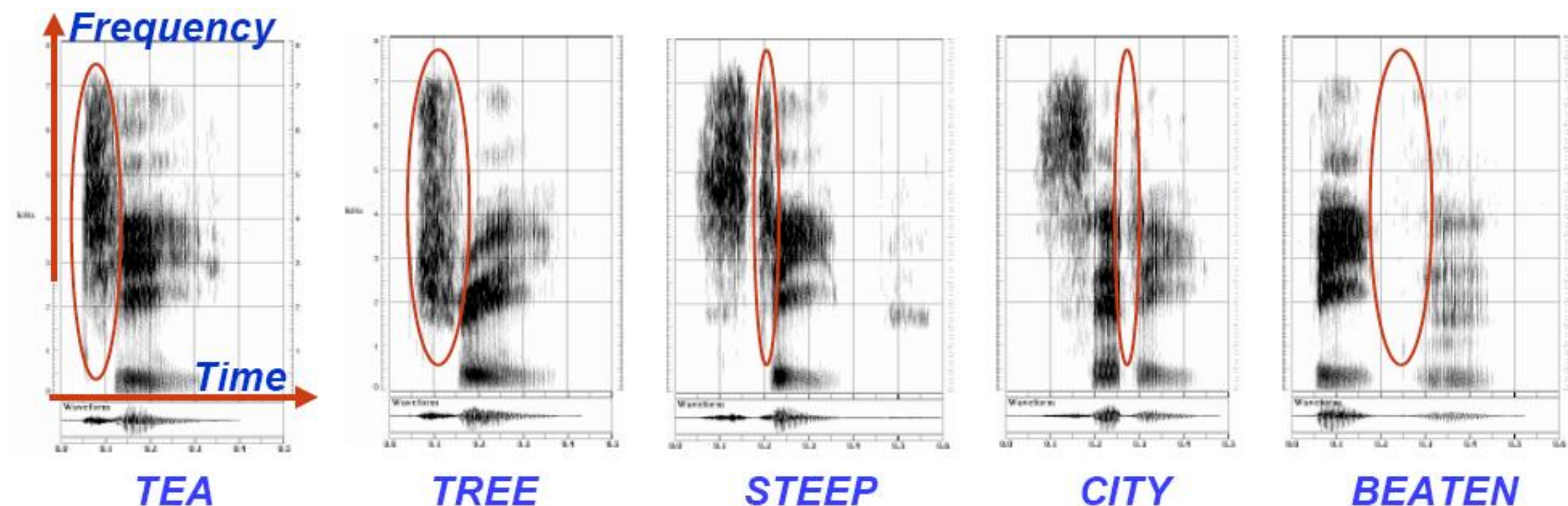
- In real speech data, the spectrogram is often transformed to a representation that more closely represents human auditory response and is more amenable to accurate classification.
- MFCCs are 'spectra of spectra'. They are the discrete cosine transform of the logarithms of the nonlinearly Mel-scaled powers of the Fourier transform of windows of the original waveform.





# Challenges in speech data

- Co-articulation and dropped phonemes.
- (Intra-and-Inter-) Speaker variability.
- No word boundaries.
- Slurring, disfluency (e.g., 'um').
- Signal Noise.
- Highly dimensional.



# Phonemes

- Words are formed by **phonemes** (aka 'phones'),  
e.g., 'pod' = /p aa d/
- Words have different pronunciations. and in practice we can never be certain of which phones were uttered, nor their start/stop points.

	Sentence														
	Verb phrase														
Syntactic	Verb				Noun phrase										
					Det	Modifier				Noun (plu)					
						Noun		Noun							
Lexical	open				the		pod		bay		doors				
Phonemic	ow	p	ah	n	dh	ah	p	aa	d	b	ey	d	ao	r	z

# Phonetic alphabets

- International Phonetic Association (IPA)
  - Can represent sounds in all languages
  - Contains non-ASCII characters
- ARPAbet
  - One of the earliest attempts at encoding English for early speech recognition.
- TIMIT/CMU
  - Very popular among modern databases for speech recognition.



# Example phonetic alphabets

IPA	CMU	TIMIT	Example	IPA symbol name
[ɑ]	AA	aa	f <u>ath</u> er, h <u>o</u> t	script a
[æ]	AE	ae	h <u>a</u> d	digraph
[ə]	AH0	ax	sof <u>a</u>	schwa (common in unstressed syllables)
[ʌ]	AH1	ah	b <u>u</u> t	turned v
[ɔ:]	AO	ao	ca <u>u</u> ght	open o – Note, many speakers of Am. Eng. do not distinguish between [ɔ:] and [ɑ]. If your “caught” and “cot” sound the same, you do not.
[ɛ]	EH	eh	h <u>e</u> ad	epsilon
[ɪ]	IH	ih	h <u>i</u> d	small capital I
[i:]	IY	iy	h <u>ee</u> d	lowercase i
[ʊ]	UH	uh	h <u>oo</u> d, b <u>oo</u> k	upsilon
[u:]	UW	uw	b <u>oo</u> t	lowercase u
[aɪ]	AY	ay	h <u>i</u> de	
[aʊ]	AW	aw	h <u>ow</u>	
[eɪ]	EY	ey	to <u>d</u> ay	
[oʊ]	OW	ow	h <u>oe</u> d	
[ɔɪ]	OY	oy	jo <u>y</u> , ahoy	
[ər]	ER0	axr	h <u>e</u> rself	schwar (schwa changed by following r)
[ɜr]	ER1	er	b <u>ir</u> d	reverse epsilon right hook

IPA	CMU	TIMIT	Example	IPA symbol name
[ŋ]	NG	ng	s <u>ing</u> s <u>ong</u>	eng or angma
[ʃ]	SH	<u>sh</u>	<u>s</u> heet, w <u>ish</u>	esh or long s
[tʃ]	CH	<u>ch</u>	<u>ch</u> ease	
[j]	Y	y	y <u>e</u> llow	lowercase j
[ʒ]	ZJ	zh	vi <u>s</u> ion	long z or yogh
[dʒ]	JH	jh	ju <u>d</u> ge	
[ð]	DH	dh	<u>th</u> ee, <u>th</u> is	eth

- The other consonants are transcribed as you would expect
  - i.e., p, b, m, t, d, n, k, g, s, z, f, v, w, h

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# Assignment 3

- Two parts:
  - Speaker identification: Determine which of 30 speakers an unknown test sample of speech comes from, given Gaussian mixture models you will train for each speaker.
  - Speech recognition: Compute word-error rates for speech recognition systems using Levenshtein distance.

# Speaker Data

- 32 speakers (e.g., S-3C, S-5A).
- Each speaker has up to 12 training utterances.
  - e.g., `/u/csc401/A3/data/S-3C/0.wav`
- Each utterance has 3 files:
  - `*.wav` : The original wave file.
  - `*.mfcc.npy` : The MFCC features in NumPy format
  - `*.txt` : Sentence-level transcription.

# Speaker Data (cont.)

- All you need to know: A speech utterance is an  $N \times d$  matrix
  - Each row represents the features of a  $d$ -dimensional point in time.
  - There are  $N$  rows in a sequence of  $N$  frames.
  - The data is in numpy arrays \* `.mfcc.npy`
  - To read the files: `np.load('1.mfcc.npy')`

		data dimension			
		1	2		d
time ↓	frames	1	$X_1[1]$	$X_1[2]$	$X_1[d]$
		2	$X_2[1]$	$X_2[2]$	$X_2[d]$
			...	...	...
		N	$X_N[1]$	$X_N[2]$	$X_N[d]$

# Speaker Data (cont.)

- You are given human transcriptions in `transcripts.txt`
- You are also given Kaldi and Google transcriptions in `transcripts.*.txt`.
- Ignore any symbols that are not words.



# Agenda

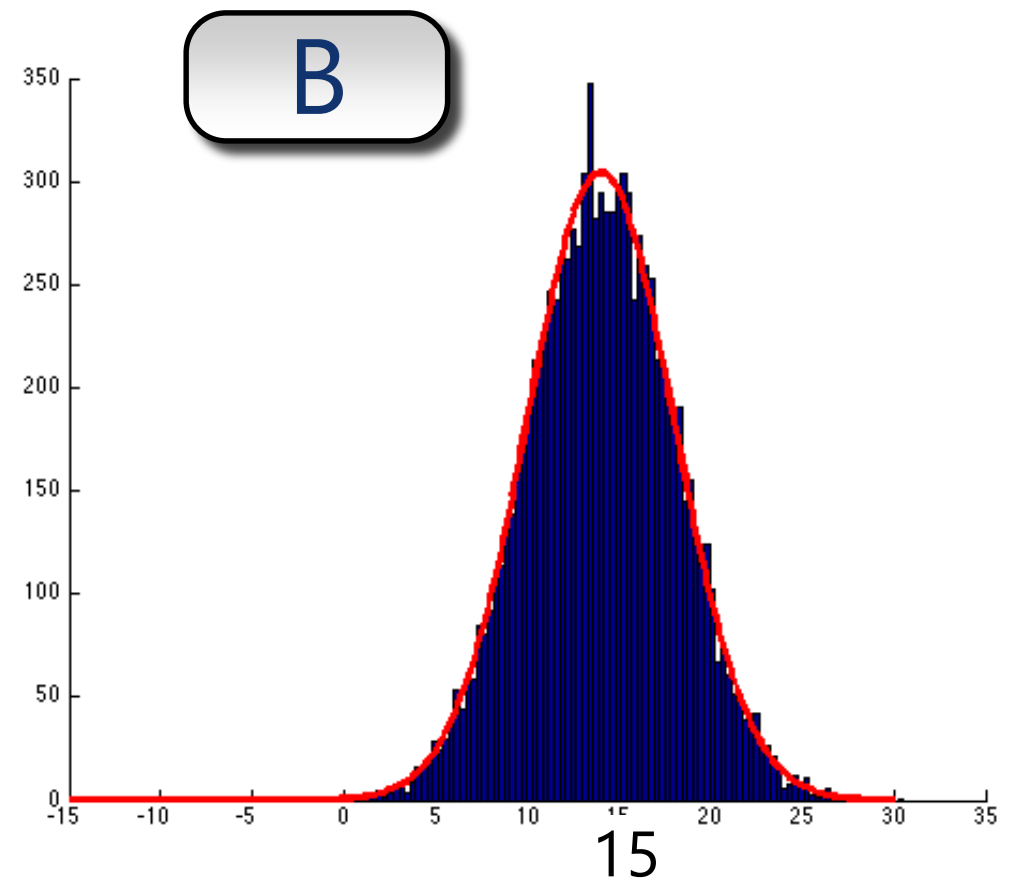
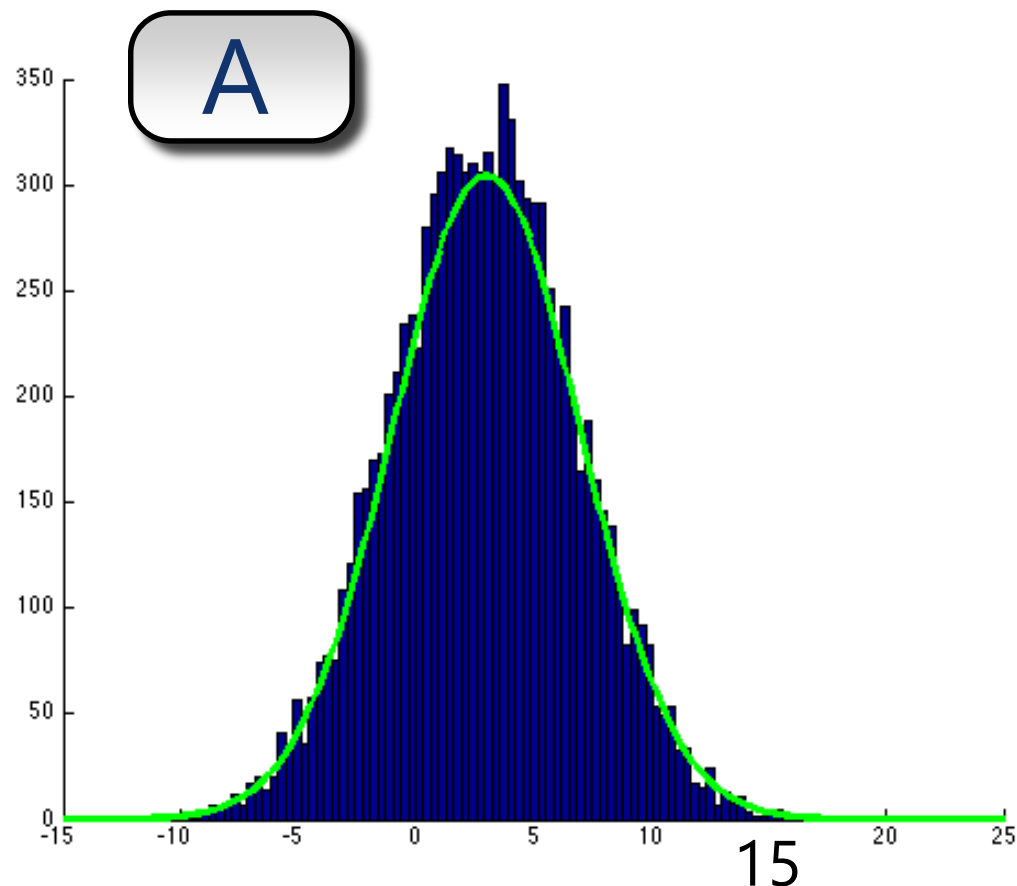
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# Speaker Recognition

- The data is randomly split into training and testing utterances. We don't know which speaker produced which test utterance.
- Every speaker occupies a characteristic part of the acoustic space.
- We want to learn a probability distribution for each speaker that describes their acoustic behaviour.
  - Use those distributions to identify the speaker-dependent features of some unknown sample of speech data.

# Some background: fitting to data

- Given a set of observations  $X$  of some random variable, we wish to know how  $X$  was generated.
- Here, we assume that the data was sampled from a Gaussian Distribution (validated by data).
- Given a new data point ( $x=15$ ), It is more likely that  $x$  was generated by B.



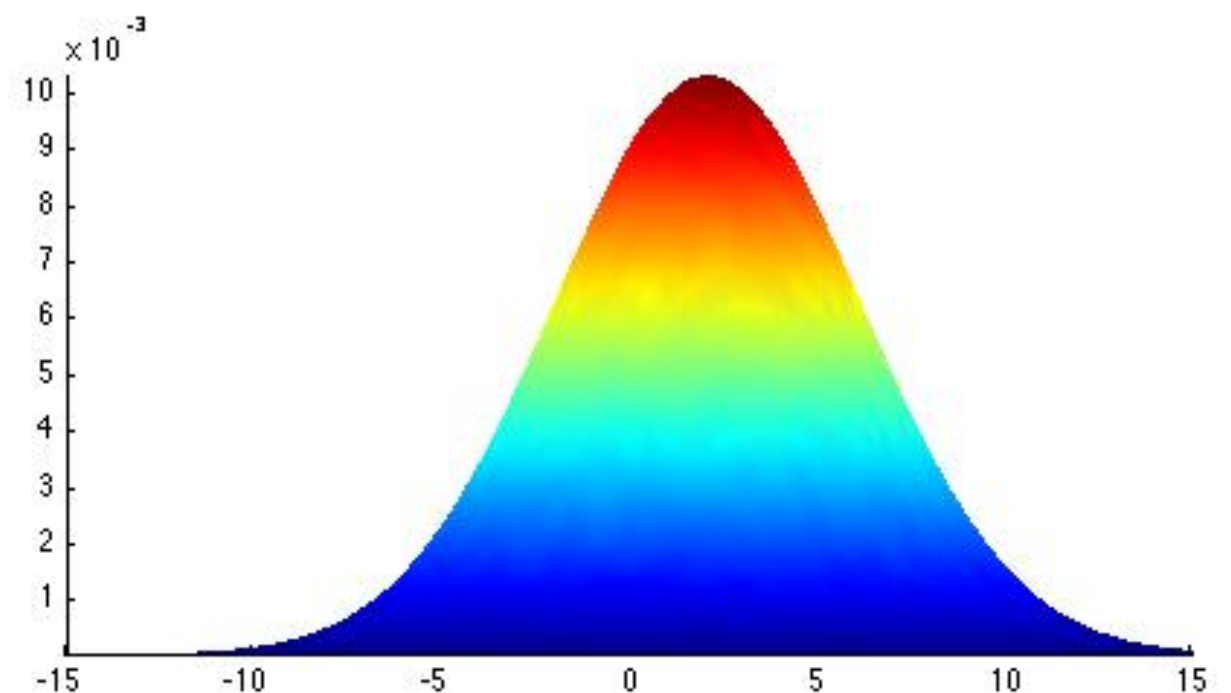
# Finding parameters: 1D Gaussians

- Often called *Normal* distributions

$$p(x) = \frac{\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}$$

$$N(\mu, \sigma)$$

$$\mu = E(x) = \int xp(x)dx$$



$$\sigma^2 = E((x - \mu)^2) = \int (x - \mu)^2 p(x) dx$$

- The parameters we can adjust to fit the data are  $\mu$  and  $\sigma^2$ :

$$\theta = \langle \mu, \sigma \rangle$$

# Maximum likelihood estimation

- Given data:  $X = \{x_1, x_2, \dots, x_n\}$
- and Parameter set:  $\theta$
- Maximum likelihood attempts to find the parameter set that maximizes the likelihood of the data.

$$L(X, \theta) = p(X \mid \theta) = p(x_1, x_2, \dots, x_n \mid \theta) = \prod_{i=1}^n p(x_i \mid \theta)$$

- The likelihood function  $L(X, \theta)$  provides a surface over all possible parameterizations. In order to find the Maximum Likelihood, we set the derivative to zero:

$$\frac{\partial}{\partial \theta} L(X, \theta) = 0$$

# MLE - 1D Gaussian

- Estimate  $\hat{\mu}$

$$L(X, \mu) = p(X \mid \mu) = \prod_{i=1}^n p(x_i \mid \mu) = \prod_{i=1}^n \frac{\exp\left(-\frac{(x_i - \mu)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma}$$

$$\log L(X, \mu) = -\frac{\sum_i (x_i - \mu)^2}{2\sigma^2} - n \log \sqrt{2\pi}\sigma$$

$$\frac{\partial}{\partial \mu} \log L(X, \mu) = \frac{\sum_i (x_i - \mu)}{\sigma^2} = 0$$

$$\hat{\mu} = \frac{\sum_i x_i}{n}$$

- A similar approach gives the MLE estimate of  $\hat{\sigma}^2$  :

$$\hat{\sigma}^2 = \frac{\sum_i (x_i - \hat{\mu})^2}{n}$$



# Multidimensional Gaussians

- When your data is d-dimensional, the input variable is

$$\vec{x} = \langle x[1], x[2], \dots, x[d] \rangle$$

the mean vector is

$$\vec{\mu} = E(\vec{x}) = \langle \mu[1], \mu[2], \dots, \mu[d] \rangle$$

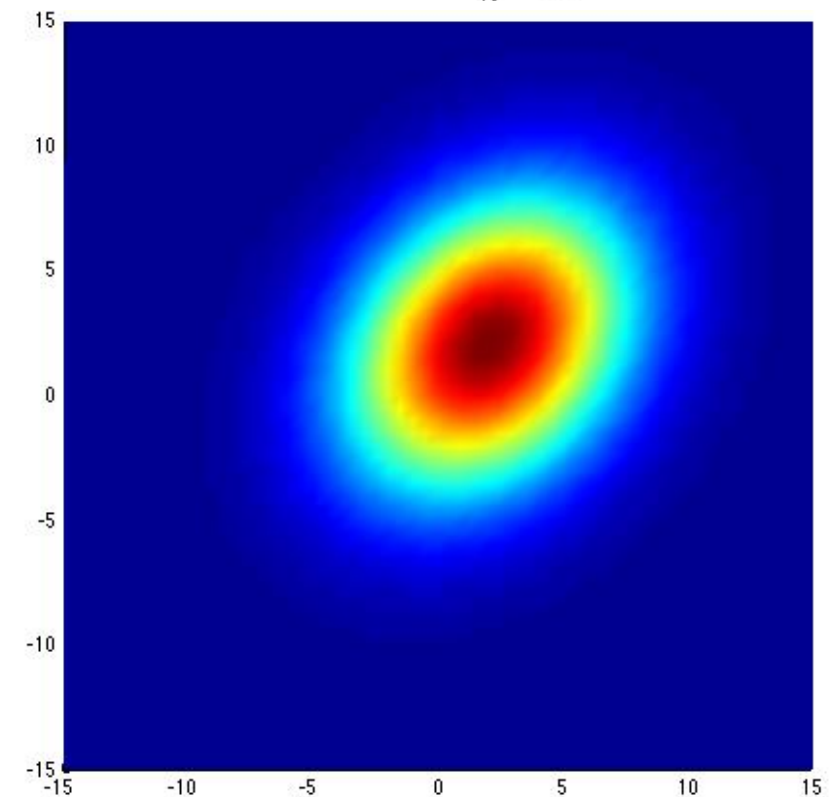
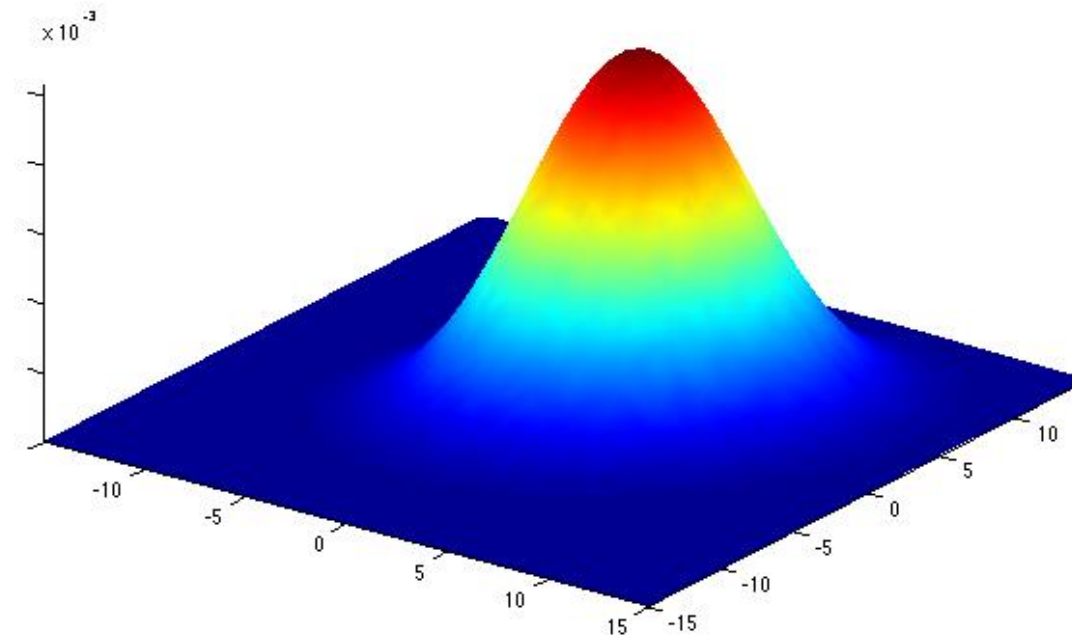
the covariance matrix is

$$\Sigma = E((\vec{x} - \vec{\mu})(\vec{x} - \vec{\mu})^T)$$

with  $\Sigma[i, j] = E(x[i]x[j]) - \mu[i]\mu[j]$

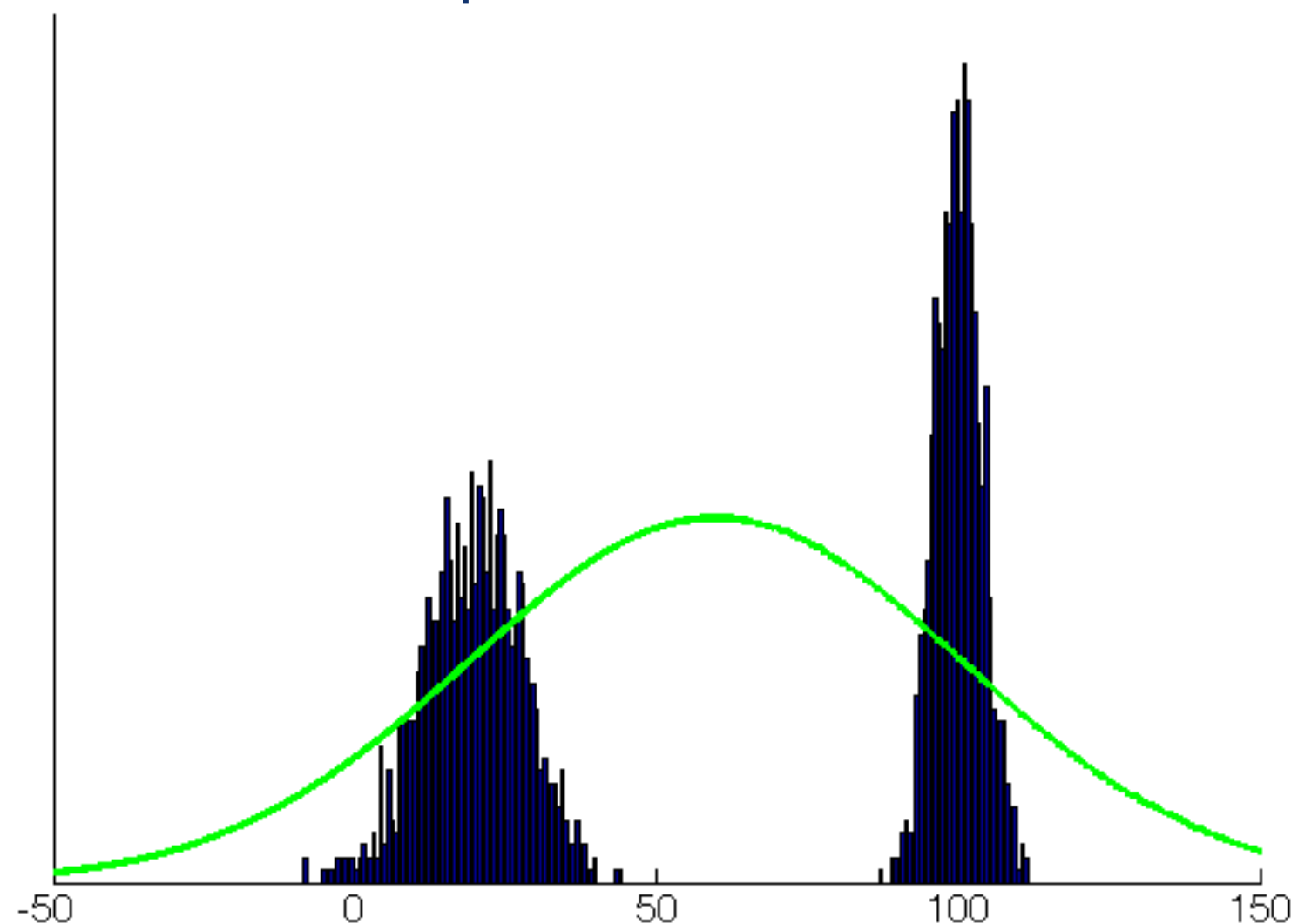
and

$$p(\vec{x}) = \frac{\exp\left(-\frac{(\vec{x} - \vec{\mu})^T \Sigma^{-1} (\vec{x} - \vec{\mu})}{2}\right)}{(2\pi)^{d/2} |\Sigma|^{1/2}}$$



# Non-Gaussian data

- Our speaker data does not behave unimodally.
  - i.e., we can't use just 1 Gaussian per speaker.
- E.g., observations below occur mostly bimodally, so fitting 1 Gaussian would not be representative.

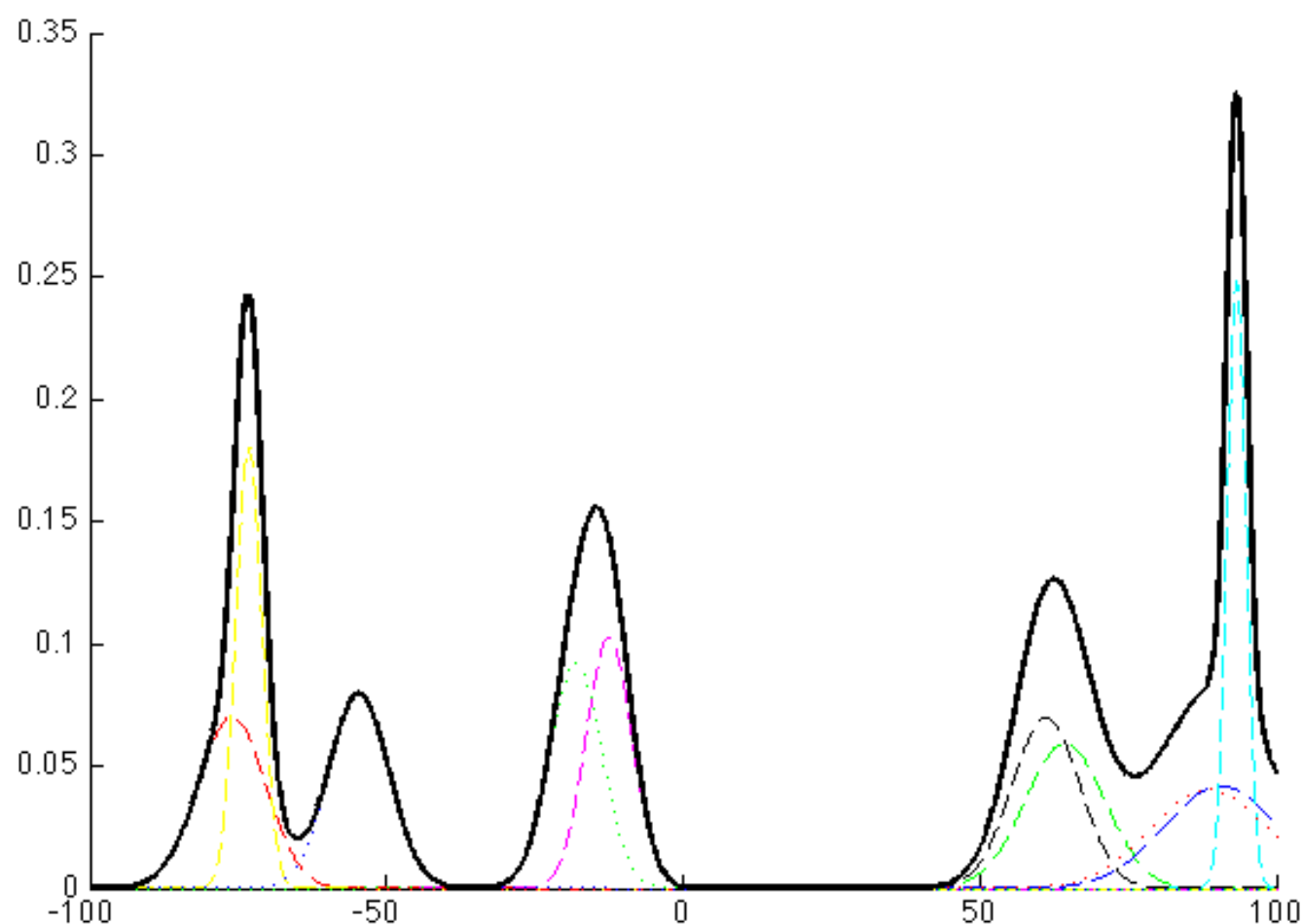


# Gaussian mixtures

- Gaussian mixtures are a weighted linear combination of  $M$  component gaussians.

$$\langle \Gamma_1, \dots, \Gamma_M \rangle$$

$$p(\vec{x}) = \sum_{j=1}^M p(\Gamma_j) p(\vec{x} \mid \Gamma_j)$$



# MLE for Gaussian mixtures

- For notational convenience,  $\omega_m = p(\Gamma_m)$ ,  $b_m(\vec{x}_t) = p(\vec{x}_t | \Gamma_m)$

- So

$$p_{\Theta}(\vec{x}_t) = \sum_{m=1}^M \omega_m b_m(\vec{x}_t), \quad \Theta = \langle \omega_m, \vec{\mu}_m, \Sigma_m \rangle, \quad m = 1, \dots, M$$

$$b_m(\vec{x}_t) = \frac{\exp\left(-\frac{1}{2} \sum_{i=1}^d \frac{(x_t[i] - \mu_m[i])^2}{\sigma_m^2[i]}\right)}{(2\pi)^{d/2} \left(\prod_{i=1}^d \sigma_m^2[i]\right)^{1/2}}$$

- To find  $\hat{\Theta}$ , we solve  $\nabla_{\Theta} \log L(X, \Theta) = 0$  where

$$\log L(X, \Theta) = \sum_{t=1}^N \log p_{\Theta}(\vec{x}_t) = \sum_{t=1}^N \log \left( \sum_{m=1}^M \omega_m b_m(\vec{x}_t) \right)$$

...see Appendix for more

# MLE for Gaussian mixtures (pt. 2)

- Given 
$$\frac{\partial \log L(X, \Theta)}{\partial \mu_m[n]} = \sum_{t=1}^N \frac{1}{p_{\Theta}(\vec{x}_t)} \left[ \frac{\partial}{\partial \mu_m[n]} \omega_m b_m(\vec{x}_t) \right]$$
- Since 
$$\frac{\partial}{\partial \mu_m[n]} b_m(\vec{x}_t) = b_m(\vec{x}_t) \frac{x_t[n] - \mu_m[n]}{\sigma_m^2[n]}$$
- We obtain  $\hat{\mu}_m[n]$  by solving for  $\mu_m[n]$  in :

$$\frac{\partial \log L(X, \Theta)}{\partial \mu_m[n]} = \sum_{t=1}^N \frac{\omega_m}{p_{\Theta}(\vec{x}_t)} b_m(\vec{x}_t) \frac{x_t[n] - \mu_m[n]}{\sigma_m^2[n]} = 0$$

and:

$$b_m(\vec{x}_t) = p(\vec{x}_t \mid \Gamma_m)$$
$$p(\Gamma_m \mid \vec{x}_t, \Theta) = \frac{\omega_m}{p_{\Theta}(\vec{x}_t)} b_m(\vec{x}_t)$$

$$\hat{\mu}_m[n] = \frac{\sum_t p(\Gamma_m \mid \vec{x}_t, \Theta) x_t[n]}{\sum_t p(\Gamma_m \mid \vec{x}_t, \Theta)}$$

# Recipe for GMM ML estimation

- Do the following for each speaker individually. Use all the frames available in their respective **Training** directories

1.	<b><u>Initialize</u></b> : Guess $\Theta = \langle \omega_m, \vec{\mu}_m, \Sigma_m \rangle$ , $m = 1, \dots, M$ with M random vectors in the data, or by performing M-means clustering.	
2.	<b><u>Compute likelihood</u></b> : Compute $b_m(\vec{x}_t)$ and $P(\Gamma_m   \vec{x}_t, \Theta)$	
3.	<b><u>Update parameters</u></b> :	$\hat{\omega}_m = \frac{1}{T} \sum_{t=1}^T p(\Gamma_m   \vec{x}_t, \Theta)$
	$\hat{\sigma}_m^2 = \frac{\sum_t p(\Gamma_m   \vec{x}_t, \Theta) \vec{x}_t^2}{\sum_t p(\Gamma_m   \vec{x}_t, \Theta)} - \hat{\vec{\mu}}_m^2$	$\hat{\vec{\mu}}_m = \frac{\sum_t p(\Gamma_m   \vec{x}_t, \Theta) \vec{x}_t}{\sum_t p(\Gamma_m   \vec{x}_t, \Theta)}$
		$\log p(X   \hat{\Theta}_{i+1}) - \log p(X   \hat{\Theta}_i) < \epsilon$
	Repeat 2&3 until converges	



# Cheat sheet

$$b_m(\vec{x}_t) = p(\vec{x}_t \mid \Gamma_m)$$

$$b_m(\vec{x}_t) = \frac{\exp\left(-\frac{1}{2} \sum_{i=1}^d \frac{(x_t[i] - \mu_m[i])^2}{\sigma_m^2[i]}\right)}{(2\pi)^{d/2} \left(\prod_{i=1}^d \sigma_m^2[i]\right)^{1/2}}$$

Probability of observing  $x_t$  in the  $m^{\text{th}}$  Gaussian

$$\omega_m = p(\Gamma_m)$$

Prior probability of the  $m^{\text{th}}$  Gaussian

$$p(\Gamma_m \mid \vec{x}_t, \Theta) = \frac{\omega_m}{p_{\Theta}(\vec{x}_t)} b_m(\vec{x}_t)$$

Probability of the  $m^{\text{th}}$  Gaussian, given  $x_t$

$$p_{\Theta}(\vec{x}_t) = \sum_{m=1}^M \omega_m b_m(\vec{x}_t)$$

Probability of  $x_t$  in the GMM

# Initializing theta

$$\Theta = \langle \omega_1, \mu_1, \Sigma_1, \omega_2, \mu_2, \Sigma_2, \dots, \omega_M, \mu_M, \Sigma_M \rangle$$

- Initialize each  $\mu_m$  to a random vector from the data.
- Initialize  $\Sigma_m$  to a 'random' diagonal matrix (or identity matrix).
- Initialize  $\omega_m$  randomly, with these constraints:

$$0 \leq \omega_m \leq 1$$

$$\sum_m \omega_m = 1$$

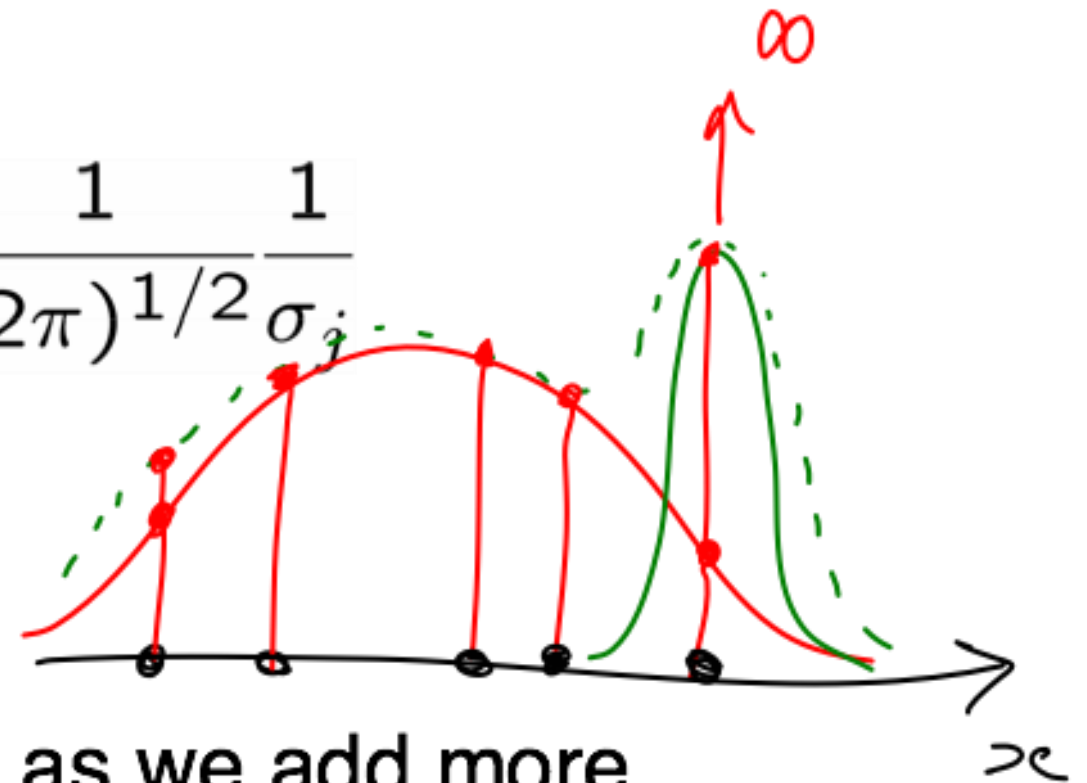
- A good choice would be to set  $\omega_m$  to  $1/M$

# Over-fitting in Gaussian Mixture Models

- Singularities in likelihood function when a component ‘collapses’ onto a data point:

$$\mathcal{N}(\mathbf{x}_n | \mathbf{x}_n, \sigma_j^2 \mathbf{I}) = \frac{1}{(2\pi)^{1/2}} \frac{1}{\sigma_j}$$

then consider  $\sigma_j \rightarrow 0$



- Likelihood function gets larger as we add more components (and hence parameters) to the model
  - not clear how to choose the number  $K$  of components

Solutions:

- Ensure that the variances don't get too small.
- Bayesian GMMs

# Your Task

- For each speaker, train a GMM, using the EM algorithm, assuming diagonal covariance.
- Identify the speaker of each test utterance.
- Experiment with the number of mixture elements in the models, the improvement threshold, number of possible speakers, etc.
- Comment on the results

# Practical tips for MLE of GMMs

- We assume diagonal covariance matrices. This reduces the number of parameters and can be sufficient in practice given enough components.
- Numerical Stability: Compute likelihoods in the log domain (especially when calculating the likelihood of a sequence of frames).

$$\log b_m(\vec{x}_t) = - \sum_{n=1}^d \frac{(\vec{x}_t[n] - \vec{\mu}_m[n])^2}{2\sigma_m^2[n]} - \frac{d}{2} \log 2\pi - \frac{1}{2} \log \prod_{n=1}^d \sigma_m^2[n]$$

- Here,  $\vec{x}_t$ ,  $\vec{\mu}_m$  and  $\sigma_m^2$  are d-dimensional vectors.

# Practical tips (pt. 2)

- Efficiency: Pre-compute terms not dependent on  $\vec{x}_t$

$$\begin{aligned} \log b_m(\vec{x}_t) = & - \sum_{n=1}^d \left( \frac{1}{2} \vec{x}_t[n]^2 \sigma_m^{-2}[n] - \mu_m[n] \vec{x}_t[n] \sigma_m^{-2}[n] \right) \\ & - \left( \sum_{n=1}^d \frac{\mu_m[n]^2}{2\sigma_m^2[n]} + \frac{d}{2} \log 2\pi + \frac{1}{2} \log \prod_{n=1}^d \sigma_m^2[n] \right) \end{aligned}$$



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# Word-error rates

- If somebody said  
REF: how to recognize speech  
but an ASR system heard  
HYP: how to wreck a nice beach  
how do we measure the error that occurred?
- One measure is  $\frac{\text{\#CorrectWords}}{\text{\#HypothesisWords}}$   
e.g., 2/6 above
- Another measure is  $\frac{S+I+D}{\text{\#ReferenceWords}}$ 
  - S: # Substitution errors (one word for another)
  - I: # Insertion errors (extra words)
  - D: # Deletion errors (words that are missing).

# Computing Levenshtein Distance

- In the example  
REF: how to recognize speech.  
HYP: how to wreck a nice beach  
How do we count each of S, I, and D?
- If “wreck” is a substitution error, what about “a” and “nice”?

# Computing Levenshtein Distance

- In the example  
REF: how to recognize speech.  
HYP: how to wreck a nice beach  
How do we count each of S, I, and D?  
If “wreck” is a substitution error, what about “a” and “nice”?

- Levenshtein distance:  
Initialize  $R[0,0] = 0$ , and  $R[i,j] = \infty$  for all  $i=0$  or  $j=0$   
for  $i=1..n$  (#ReferenceWords)  
  for  $j=1..m$  (#Hypothesis words)  
     $R[i,j] = \min(\begin{matrix} R[i-1,j] + 1 & \text{(deletion)} \\ R[i-1,j-1] & \text{(only if words match)} \\ R[i-1,j-1]+1 & \text{(only if words differ)} \\ R[i,j-1] + 1 & \text{(insertion)} \end{matrix})$   
  
Return  $100 * R(n,m) / n$

# Levenshtein example

		how	to	wreck	a	nice	beach
	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
how	$\infty$	0	1	2	3	4	5
to	$\infty$						
recognize	$\infty$						
speech	$\infty$						

# Levenshtein example

		how	to	wreck	a	nice	beach
	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
how	$\infty$	0	1	2	3	4	5
to	$\infty$	1	0	1	2	3	4
recognize	$\infty$						
speech	$\infty$						

Diagram illustrating the Levenshtein distance calculation between the words "how" and "to". The table shows the distance between each pair of words. The distance between "how" and "to" is 1, indicated by an arrow pointing from the cell (how, how) to the cell (to, to). The distance between "to" and "to" is 0, indicated by an arrow pointing from the cell (to, to) to the cell (to, to).

# Levenshtein example

		how	to	wreck	a	nice	beach
	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
how	$\infty$	0	1	2	3	4	5
to	$\infty$	1	0	1	2	3	4
recognize	$\infty$	2	1	1	2	3	4
speech	$\infty$						

# Levenshtein example

		how	to	wreck	a	nice	beach
	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
how	$\infty$	0	1	2	3	4	5
to	$\infty$	1	0	1	2	3	4
recognize	$\infty$	2	1	1	2	3	4
speech	$\infty$	3	2	2	2	3	4

Word-error rate is  $4/4 = 100\%$

2 substitutions, 2 insertions



# Appendices

# Multidimensional Gaussians, pt. 2

- If the  $i^{\text{th}}$  and  $j^{\text{th}}$  dimensions are statistically independent,

$$E(x[i]x[j]) = E(x[i])E(x[j])$$

and

$$\Sigma[i, j] = 0$$

- If all dimensions are statistically independent,  $\Sigma[i, j] = 0, \forall i \neq j$  and the covariance matrix becomes diagonal, which means

$$p(\vec{x}) = \prod_{i=1}^d p(x[i])$$

where

$$p(x[i]) \sim N(\mu[i], \Sigma[i, i])$$
$$\Sigma[i, i] = \sigma^2[i]$$

# MLE example - dD Gaussians

- The MLE estimates for parameters  $\Theta = \langle \theta_1, \theta_2, \dots, \theta_d \rangle$  given i.i.d. training data  $X = \langle \vec{x}_1, \dots, \vec{x}_n \rangle$  are obtained by maximizing the joint likelihood

$$L(X, \Theta) = p(X \mid \Theta) = p(\vec{x}_1, \dots, \vec{x}_n \mid \Theta) = \prod_{i=1}^n p(\vec{x}_i \mid \Theta)$$

- To do so, we solve  $\nabla_{\Theta} L(X, \Theta) = 0$  where

$$\nabla_{\Theta} = \left\langle \frac{\partial}{\partial \theta_1}, \dots, \frac{\partial}{\partial \theta_d} \right\rangle$$

- Giving

$$\hat{\vec{\mu}} = \frac{\sum_{t=1}^n \vec{x}_t}{n} \quad \hat{\Sigma} = \frac{\sum_{t=1}^n \left( \vec{x}_t - \hat{\vec{\mu}} \right) \left( \vec{x}_t - \hat{\vec{\mu}} \right)^T}{n}$$

# MLE for Gaussian mixtures (pt1.5)

- Given  $\log L(X, \Theta) = \sum_{t=1}^N \log p_{\Theta}(\vec{x}_t)$  and  $p_{\Theta}(\vec{x}_t) = \sum_{m=1}^M \omega_m b_m(\vec{x}_t)$
- Obtain an ML estimate,  $\hat{\mu}_m$ , of the mean vector by maximizing  $\log L(X, \mu_m)$  w.r.t.  $\mu_m[n]$

$$\frac{\partial \log L(X, \Theta)}{\partial \mu_m[n]} = \sum_{t=1}^N \frac{\partial}{\partial \mu_m[n]} \log p_{\Theta}(\vec{x}_t) = \sum_{t=1}^N \frac{1}{p_{\Theta}(\vec{x}_t)} \left[ \frac{\partial}{\partial \mu_m[n]} \omega_m b_m(\vec{x}_t) \right]$$

- Why?

d of sum = sum of d

d rule for log<sub>e</sub>

d wrt  $\mu_m$  is 0 for all other mixtures in the sum in  $p_{\Theta}(\vec{x}_t)$