AIP-Assignment 2-Q4

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1 Question 4

Algorithm : EWISTARS(Exponential Wavelet Iterative Shrinkage Thresholding Algorithm with Random Shift)
This uses following features :

1. ISTA
2. the sparse representation of exponent of wavelet transform(EWT)
3. the shift-invariance of random shifting (RS)

It is aimed at improving the reconstruction quality and accelerating computation.

Part A

1. **Reconstruction Model**

   (a) $y = Ux + e$,where U denotes the under sampling scheme in the k-space, namely, the incomplete Fourier transform.
   e is caused by either scanner imprecision's or the noise

   (b) $y = Q\omega + e$,where
   Q=UW,W is inverse sparse transform(EWT)
   ω denotes the sparsity coefficients

   (c) cost function defination
   S(ω)=$\| y - Q\omega \|_2^2 + \lambda \| \omega \|_1$

2. **Exponential Wavelet Transform** Pseudocode of exponential wavelet transform (EWT) is as follows:
   Step 1. Input the randomly under sampled MRI signal or image.
   Step 2. Repeat k times.
   Step 2.1. Carry out the wavelet transform (WT).
   Step 2.2. Normalize wavelet coefficients to [0 1].
   Step 3. Output the EWT coefficients.

   k is the number of exponential iterations and T_E is the exponential wavelet transform. A single (k=1) EWT is implemented by
   $T_E(x, 1) = \frac{exp(T_W(x))-1}{e-1}$ where,
   $T_W(x) = \omega$ here T_W represents the wavelet transform ($T_W = W^{-1}$)

3. The iterative thresholding algorithm (ISTA) presents a sequence of estimates ω_n, which gradually approximates to the optimal result w^*

4. **AGORITHM EWISTA**
   Input: ω_0
   $Y^{-1} \leftarrow diag(b)$
   Parameter Setting
   $t_0 = 1, n = 0, v_0 = \omega_0$
   Produce the sequence of EWT modified by RS
   $\{T_E(x, n)\}$
   Repeat
   $Q \leftarrow UW_n$, $A \leftarrow Q^H Q$, $a \leftarrow Q^H y$, {step 1}
   $\omega_{n+1} \leftarrow \Gamma_{\lambda b}(v_n + Y^{-1}(a - Av_n))$ {step 2}
   $t_{n+1} \leftarrow \frac{1+\sqrt{1+4t_n^2}}{2}$
   $v_{n+1} \leftarrow \omega_{n+1} + \frac{(t_n-1)}{t_{n+1}}(\omega_{n+1} - \omega_n)$ {step 3}
   $n \leftarrow n + 1$
   Until termination criterion are met
   Output: ω_{n+1}
   $x^* \leftarrow W\omega_{n+1}$

   Explanation:
   Step 2
   Γ is the shrinkage operator and b is the threshold:
   $\Gamma_b(z) = sgn(z)(|z| - min(b/2, |Z|))$(same as ISTA step)
   Step 3
   t_n increases with the number of iteration. It basically fasten the process of reaching the termination criterion. It takes newly calculated ω_{n+1} in addition with $\frac{(t_n-1)}{t_{n+1}}$ times the gap between newly calculated ω_{n+1} and ω_n.

Part b For paper link click here: EWISTARS

Part C Advantages over ISTA are :

1. It is aimed at improving the reconstruction quality(by using EWT) and accelerating computation(step 3).
2. EWISTARS algorithm takes advantage of the simplicity of ISTA, the sparse representation of EWT, and the shift-invariance of RS.
3. Applications in MRI

Part D Couldn't find an implementation of this algorithm.