Q-1)

a)

By defination of epipoles

$$e = P X_{o'} \quad \text{--- (1)}$$

$$e' = P X_o \quad \text{--- (2)}$$

where $X_o, X_{o'}$ are Camera centre.

Now,

$$P = K[I \mid 0]$$

$$P' = K'[R \mid t]$$

then,

$$X_o = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{--- (3)}$$

$$X_o' = \begin{bmatrix} -R^t t \\ 1 \end{bmatrix} \quad \text{--- (4)}$$

Putting (3) in (2) & (4) in (1)

$$e = P \begin{bmatrix} -R^T t \\ 1 \end{bmatrix} ; \quad e' = P' \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b)

Given $F = [e']_\times K' R K^{-1}$

$F = e' \times K' R K^{-1}$

Multiplying both sides by x.

$F x = e' \times K' R K^{-1} x$

Now from homography,

$x_2' = K_2 R_2 R_1^{-1} K_1^{-1} x_1$

Here

$R_1 = I$, $R_2 = R$, $K_2 = K'$, $K_1 = K$, $x_2 = x'$ & $x_1 = x$

$x' = K' R K^{-1} x$

Hence $\boxed{F x = e' \times x'}$

Q-2)

Ans)

Given $f = S_b R$

$x_K' = K_2^{-1} x'$

$x_K = K_1^{-1} x$

where x & x' are point in Image taken from Camera 1 & 2 respectively.

Now we know that (from question 1)

$e' \times x_K' = f x_K$

$e' \times x_K' = S_b R x_K$

$e' \times x_K' = b \times R x_K$

Comparing both sides, we get

$x_K' = R x_K$

$K_2^{-1} x' = R K_1^{-1} x$

$x' = K_2 R K_1^{-1} x$

which is true from homography.

Hence the last column of K_2, denoted by b, is one of the epipoles.

Q-3)

Given 4 equations,

$\tilde{x}_{ij} = x_{ij} - \bar{x}_i, \; \bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$ — ①

$\tilde{y}_{ij} = y_{ij} - \bar{y}_i, \; \bar{y}_i = \frac{1}{n}\sum_{j=1}^{n} y_{ij}$ — ②

$\frac{1}{n}\sum_{j=1}^{n} P_j = 0$ — ③

$$\left.\begin{aligned} x_{ij} &= i_i^T (P_j - t_i) \\ y_{ij} &= j_i^T (P_j - t_i) \end{aligned}\right\} \text{— ④}$$

where i is the frame, j is the co-ordinate in a particular frame

P_i be the projection of i^{th} frame

t_i be the translation of i^{th} frame.

Now from eq ①

$\tilde{x}_{ij} = x_{ij} - \bar{x}_i$

$x_{ij} = \tilde{x}_{ij} + \bar{x}_i$

$x_{ij} = i_i^T P_j + \bar{x}_i \quad \therefore \tilde{x}_{ij} = i_i^T P_j$

— ⑤

- Comparing eq's 4 & 5

$$\bar{x}_i = -t_i i_i^T$$

Similarly

$$\bar{y}_i = -t_i j_i^T$$

Hence,

$$t = (\bar{x}_1, \bar{x}_2 \cdots \bar{x}_f, \bar{y}_1, \bar{y}_2 \cdots \bar{y}_f)$$

Q4)

$x_1^T F_{13} x_3 = 0$

$x_2^T F_{23} x_3 = 0$

$$f_{ij} = \begin{bmatrix} f_{ij}^{11} & f_{ij}^{12} & f_{ij}^{13} \\ f_{ij}^{21} & f_{ij}^{22} & f_{ij}^{23} \\ f_{ij}^{31} & f_{ij}^{32} & f_{ij}^{33} \end{bmatrix}$$

$$x_1 = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \qquad x_2 = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \qquad x_3 = \begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix}$$

$$x_1^T f_{13} = \begin{bmatrix} x_1 f_{13}^{11} + y_1 f_{13}^{12} + f_{13}^{13} \\ x_1 f_{13}^{21} + y_1 f_{13}^{22} + f_{13}^{23} \\ x_1 f_{13}^{31} + y_1 f_{13}^{32} + f_{13}^{33} \end{bmatrix}$$

Similarly for $x_2^T f_{23}$

∴ Combining these eq's we can write as

$A \cdot x_3 = 0$

where A,

$$A = \begin{bmatrix} (x_1 f_{13}^{11} + y_1 f_{13}^{12} + f_{13}^{13}) & (x_1 f_{13}^{21} + y_1 f_{13}^{22} + f_{13}^{23}) & (x_1 f_{13}^{31} + y_1 f_{13}^{32} + f_{13}^{33}) \\ (x_2 f_{23}^{11} + y_2 f_{23}^{12} + f_{23}^{13}) & (x_2 f_{23}^{21} + x_2 f_{23}^{22} + f_{23}^{23}) & (x_2 f_{23}^{31} + y_2 f_{23}^{32} + f_{23}^{33}) \end{bmatrix}$$

Scanned by CamScanner

$\Rightarrow \{A_{2\times 3}\}[X_3]_{3\times 1} = 0$

A has rank 2 which is $\leq$ no. of unknowns

Therefore apart from "trivial soln" a unique soln exists.

We can also rewrite eqn

$Ax = b$

where

$$\overbrace{\begin{bmatrix} (x_1 f_{13}^{11} + y_1 f_{13}^{12} + f_{13}^{13}) & (x_1 f_{13}^{21} + y_1 f_{13}^{22} + f_{13}^{23}) \\ (x_2 f_{23}^{11} + y_2 f_{23}^{12} + f_{23}^{13}) & (x_2 f_{23}^{21} + y_2 f_{23}^{22} + f_{23}^{23}) \end{bmatrix}}^{A} \overset{X}{\begin{bmatrix} x_3 \\ y_3 \end{bmatrix}} =$$

$$\underbrace{\begin{bmatrix} -(x_1 f_{13}^{31} + y_1 f_{13}^{32} + f_{13}^{33}) \\ -(x_2 f_{23}^{31} + y_2 f_{23}^{32} + f_{23}^{33}) \end{bmatrix}}_{b}$$