



مدينة زويل للعلوم والتكنولوجيا
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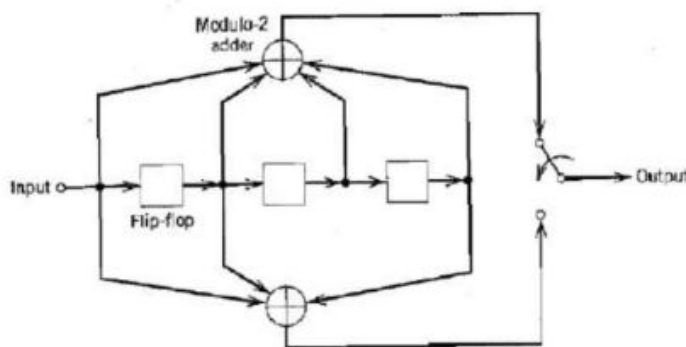
CIE 425 Assignment 4

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Question:

The figure below shows the encoder for a rate $r = \frac{1}{2}$, constraint length $K = 4$ convolutional code. Determine the encoder output produced by the message sequence 10111....



Verify your answer by implementing your Convolution Encoder function on MATLAB/Python/C/C++.

Answer:

Hand analysis:

$$m = 10111 \rightarrow m(x) = 1 + x^2 + x^3 + x^4$$

$$g_1 = 1111 \rightarrow g_1(x) = 1 + x + x^2 + x^3$$

$$g_2 = 1101 \rightarrow g_2(x) = 1 + x + x^3$$

$$C_1 = m(x)g_1(x) -$$

$$= (1 + x^2 + x^3 + x^4)(1 + x + x^2 + x^3)$$

$$= 1 + x + \cancel{x^2} + x^3 + \cancel{x^2} + \cancel{x^3} + \cancel{x^4} + x^5 + \cancel{x^3} + \cancel{x^4} + \cancel{x^5} + \cancel{x^6} + x^4 + \cancel{x^5} + \cancel{x^6} + x^7$$

$$= 1 + x + x^3 + x^4 + x^5 + x^7$$

$$\therefore C_1 = 11011101$$

$$C_2(x) = m(x)g_2(x)$$

$$= (1 + x^2 + x^3 + x^4)(1 + x + x^3)$$

$$= 1 + x + \cancel{x^3} + x^2 + \cancel{x^3} + \cancel{x^5} + x^3 + \cancel{x^4} + x^6 + \cancel{x^4} + \cancel{x^5} + x^7$$

$$= 1 + x + x^2 + x^3 + x^6 + x^7$$

$$\therefore C_2 = 11110011$$

$$\therefore C = [\underbrace{"11", "11", "01", "11", "10", "10", "01", "11"}, \underbrace{"11", "11", "01", "11", "10"}]$$

→ if m is the total message, we will consider the delays

$$C = ["11", "11", "01", "11", "10", "10", "01", "11"]$$

→ if m is not the total message, we will not consider the delays

$$C = ["11", "11", "01", "11", "10"]$$

Code implementation:

```
clc
clear

%inputs
m = [1 0 1 1 1];
g1 = [1 1 1 1];
g2 = [1 1 0 1];
r = 1/2;
%Notes: k = 4 -> 3 delays

%output after applying the convolution coding
c_final = convolution_coding(m,g1,g2);

%printing the output
%If m is the total message, we will consider the delays
fmt=['Output: ' repmat(' %s ',1,numel(c_final)) '\n'];
fprintf(fmt,c_final);
```

Output: 11 11 01 11 10 10 01 11

```
%If m is not the total message, we will not consider the delays
m_length = length(m);
required_length = 1/r*m_length/2;
fmt=['Output: ' repmat(' %s ',1,numel(c_final(1:required_length))) '\n'];
fprintf(fmt,c_final(1:required_length));
```

Output: 11 11 01 11 10

Comment: hand analysis results is the same as code results.

Functions [used in the code]

```
function equation_array = get_polynomial(bits_array)
%This function gets the polynomial function of stream of bits
%input: array of bits as integers (0s, 1s)
%output: polynomial function

syms x;
equation_array = [];
bits_length = length(bits_array);
for i = 1:bits_length
    if (bits_array(i) == 0)
        equation_array = [equation_array 0];
    else
        temp = x^(i-1);
        equation_array = [equation_array temp];
    end
end
```

```
end
```

```
function coefficients = get_coefficients(m_x,g_x)
```

```
%This function gets the coeffiecints of an equation resulted from multiplication of two polynor
```

```
%input: two polynomial equations
```

```
%output: array of integer coeffiecints
```

```
m_x_length = length(m_x);
```

```
g1_x_length = length(g_x);
```

```
c_x_array = [];
```

```
for i = 1:m_x_length
```

```
    for j = 1:g1_x_length
```

```
        temp = m_x(i)*g_x(j);
```

```
        c_x_array = [c_x_array temp];
```

```
    end
```

```
end
```

```
mult_equation = sum(c_x_array);
```

```
coefficients_flipped = sym2poly(mult_equation);
```

```
coefficients = fliplr(coefficients_flipped);
```

```
end
```

```
function c_bits_array = get_bits(c_coefficients)
```

```
%This function apply xor process (usong mod2) on a ploynomial equation using its coefficients
```

```
%input: array of integer coeffiecints
```

```
%output: array of integer (0s, 1s)
```

```
c_coefficients_length = length(c_coefficients);
```

```
c_bits_array = [];
```

```
for i = 1:c_coefficients_length
```

```
    if (mod(c_coefficients(i),2) == 0)
```

```
        c_bits_array = [c_bits_array 0];
```

```
    else
```

```
        c_bits_array = [c_bits_array 1];
```

```
    end
```

```
end
```

```
end
```

```
function c_final = get_final_output(c1_bits_array, c2_bits_array)
```

```
%This function create the final output using its 2 parts
```

```
%input: 2 arrays of integer coeffiecints
```

```
%output: array of strings
```

```
c_final = [];
```

```
c12_length = length(c1_bits_array);
```

```
for i = 1:c12_length
```

```
    temp = append(int2str(c1_bits_array(i)),int2str(c2_bits_array(i)));
```

```
    c_final = [c_final string(temp)];
```

```
end
```

```
end
```

```
function c_final = convolution_coding(m,g1,g2)
```

```
%This function apply convolution coding for a message with 2 generating functions
%input: 3 arrays of integers (0s, 1s)
%output: array of strings

m_x = get_polynomial(m);
g1_x = get_polynomial(g1);
g2_x = get_polynomial(g2);

c1_coefficients = get_coefficients(m_x,g1_x);
c2_coefficients = get_coefficients(m_x,g2_x);

c1_bits_array = get_bits(c1_coefficients);
c2_bits_array = get_bits(c2_coefficients);

c_final = get_final_output(c1_bits_array, c2_bits_array);
end
```