Introduction to Image and Video Processing

Coronaproject 2: frequency domain filtering, processing

Selim Gilon* **- i6192074**

*Maastricht University - Department of Data Science and Knowledge Engineering

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1 Periodicity, Frequency Filtering

- Create a two dimensional periodic signal with one frequency (e.g. sin, cos...).
  The signal I created is $sin(2 * pi * (x * fr + y * fr))$ where fr is the frequency.

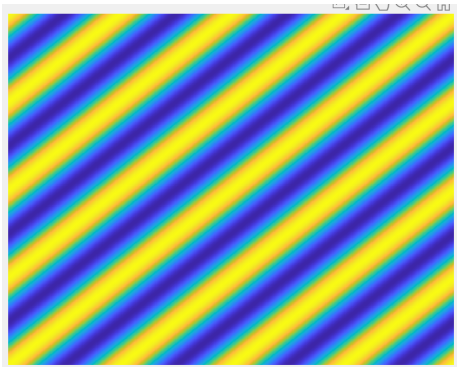

Figure 1: Signal

(a) Calculate its 2D FT analytically ("on paper" with math) and by applying the 2D FFT. I computed using the fft2 matlab function. See the code for more details.

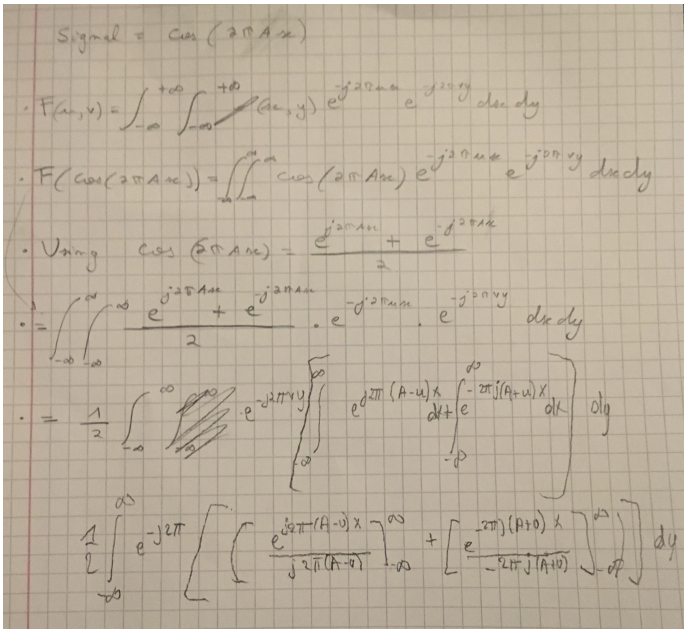

Figure 2: FT of signal

I started the computation by hand but didn't have enough time to finish it unfortunately.

(b) Depict its 2D magnitude and phase after shifting the center of frequency coordinates to the center of your image. What do you observe?

After having shifted the center of frequency coordinates to the center of your image: We can see on the magnitude plot the 2 small white points, they correspond to the signal created. We can only see those 2 small points because there is only one frequency forming the image shown above. While when we analyze a "real" image, we can see that there is a lot of white spot on the magnitude spectrum because the image is decomposed by a lot of frequencies. This is from Fourrier's theory. Of course, it is symmetric. We can also observe it on the 1D power spectrum and we can find the frequency. The 2 peaks are the frequency of the signal generated. On the phase angle, we can observe a diagonal pattern (lines) in the opposite direction of the signal. The signal is very simple so this is why only see 2 dots, because it only consists of 1 function.

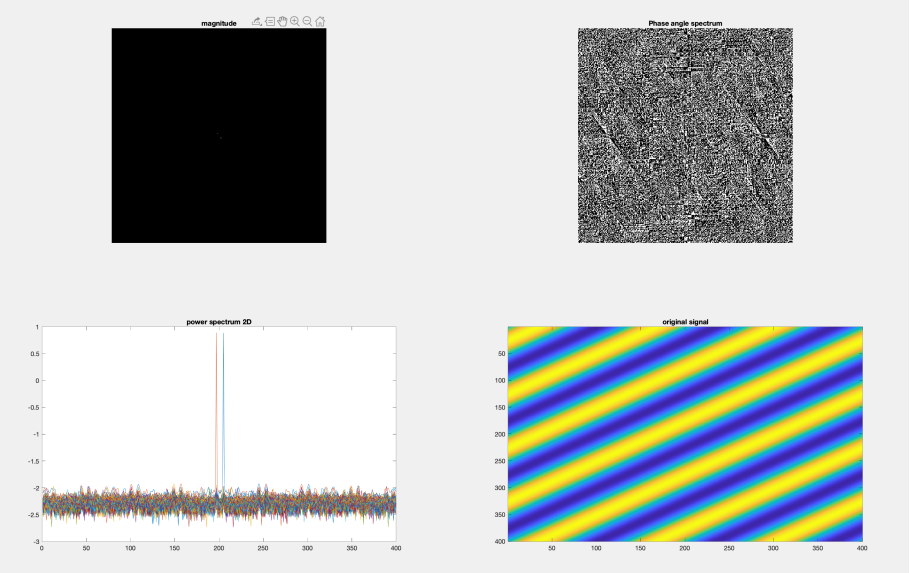

Figure 3: Signal analysis

- Create or find a clearly periodic image with a repeating pattern in the x, y or both directions. This should be a more complex 2D periodic function than in the previous question.

  (a) Depict its 2D magnitude and phase after shifting the center of frequency coordinates to the center of your image.What do you observe?

  On the magnitude, we can clearly see the repeating patterns in x and y directions. As we can see on the magnitude plot, the image is composed of many frequencies and there are quite a few strong ones. We can also see that the images is formed by 2 major directions: a vertical and an horizontal one (this is logic since the brick are rectangular). This shows the repeating pattern on the image. The vertical line is brighter and this is because it represents the horizontal lines in the original image and those are longer than the vertical ones. And since the original image has only 2 different directions of edges, the magnitude spectrum has only 2 lines. Observing the phase spectrum, we can again, see the horizontal and vertical patterns from the image. Indeed, they are represented by horizontal and vertical lines on this plot. In general, I believe not much information can be found on the phase plot.

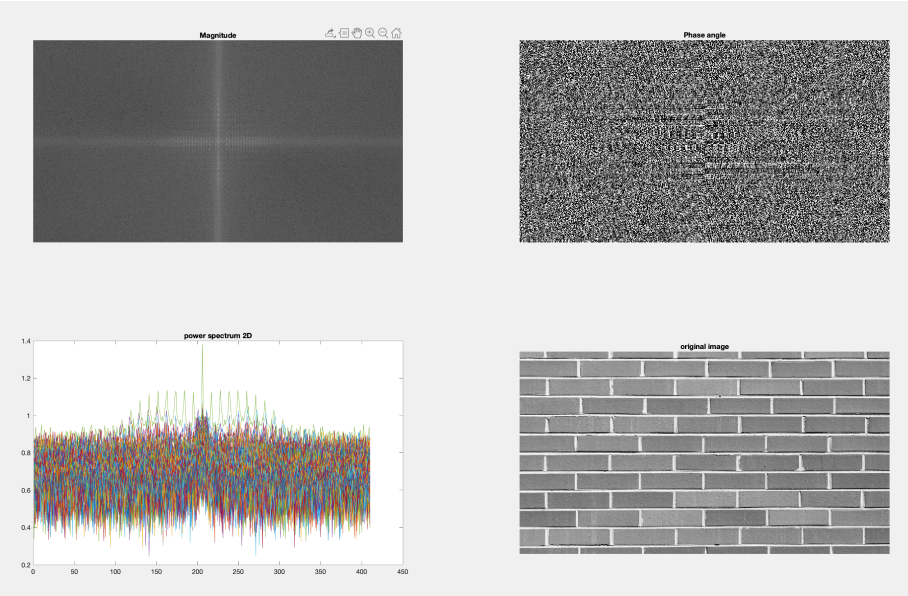

Figure 4: image with repeating pattern in x and y direction

  (b) Remove the strongest frequency from its FT and then find the inverse 2D FFT. Depict the resulting image and discuss your results.

  As we can see on the 1D power spectrum, the strongest frequency has been removed (the highest peak is not there anymore). By definition, the strongest one is the one at the center after having shifted the zero frequencies in FT. I remove it by setting its value to 0. That can also be done with a notch filter for example with a very small radius (I tried this way as well). It is obvious that the image changed a lot and this is pretty straight forward since the frequency which was the most "responsible" for this image has been removed. Therefore, the image is not the same as the original one.

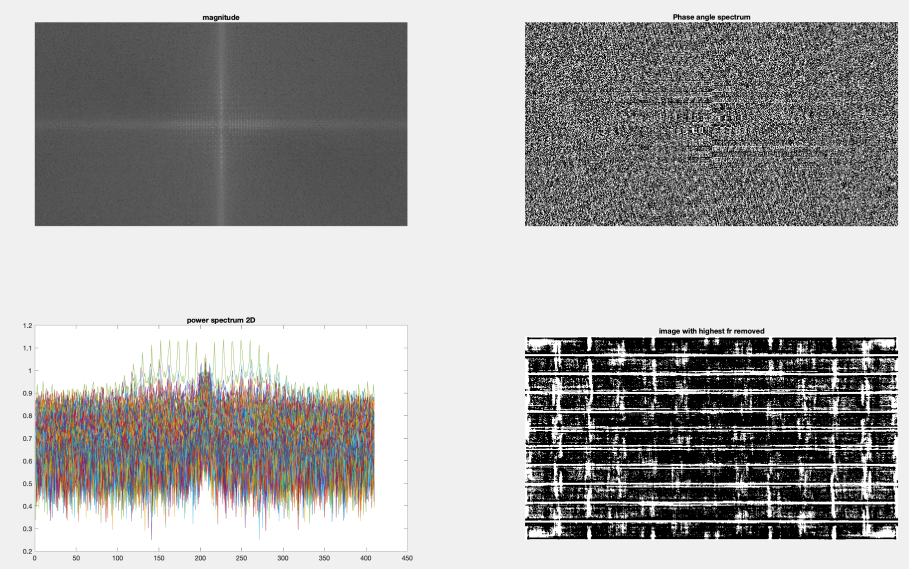

Figure 5: image strongest frequency removed

2 Periodic noise removal

- Choose an image of your liking and add periodic noise to it. You have to decide what kind of periodic noise you want to add. For the rest of this exercise you should assume you have been given only the noisy image and that this noise is unknown to you.

  I chose the deer image because I like it. I added periodic noise to it using a sinusoidal function with a certain frequency and a coefficient in order to increase to increase the effect of the noise on the image. To do so, I first created a matrix of the same size as the image which contains periodic noise. Afterwards, I summed the values of the image with the value of this noisy matrix. This is $g = f + n$, where n is the noise, f is the original image and g is the noisy image.

  In the illustration below, the frequency of the noise is 40 and the coefficient is 20. All of this was done in the spatial domain.

Figure 6: original image and image with periodical noise

- Calculate the 2D FFT of the noisy image(using an inbuilt function like fft2, numpy.fft. fft2). Display the noisy image's power spectrum in 1D, 2D, 3D and comment on it. What does it reveal about the noise?

I then computed the 2D Fourrier transorm of the image using the fft2 function and then shifted it to the center using fftshift. Afterwards, I simply computed and plotted the power spectrums by taking the absolute values of the FT and squaring them.

We can see 2 peaks next to the center of the power spectrum. Those represent the noise that we added. Since we can see that the peaks are at frequency 171 and 251, we can compute the difference between the highest frequency (at the center) and the 2 peaks is $211 - 171 = 40$.

On the power spectrum 2D, we can not really see the noise but this is because the coefficient is not very big. Indeed, I conducted some experiments with other coeff/frequencies and we can clearly see the noise in some cases (see the example below).

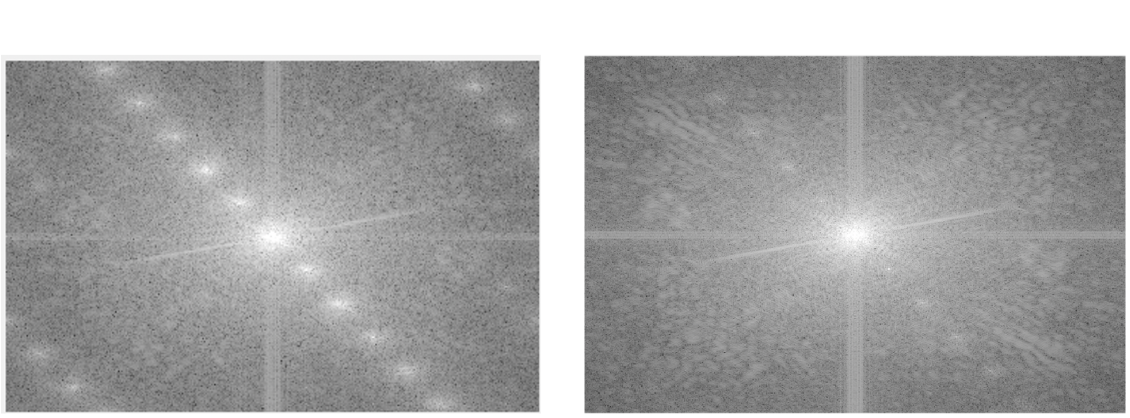

Figure 7: Examples with other values for the periodic noise

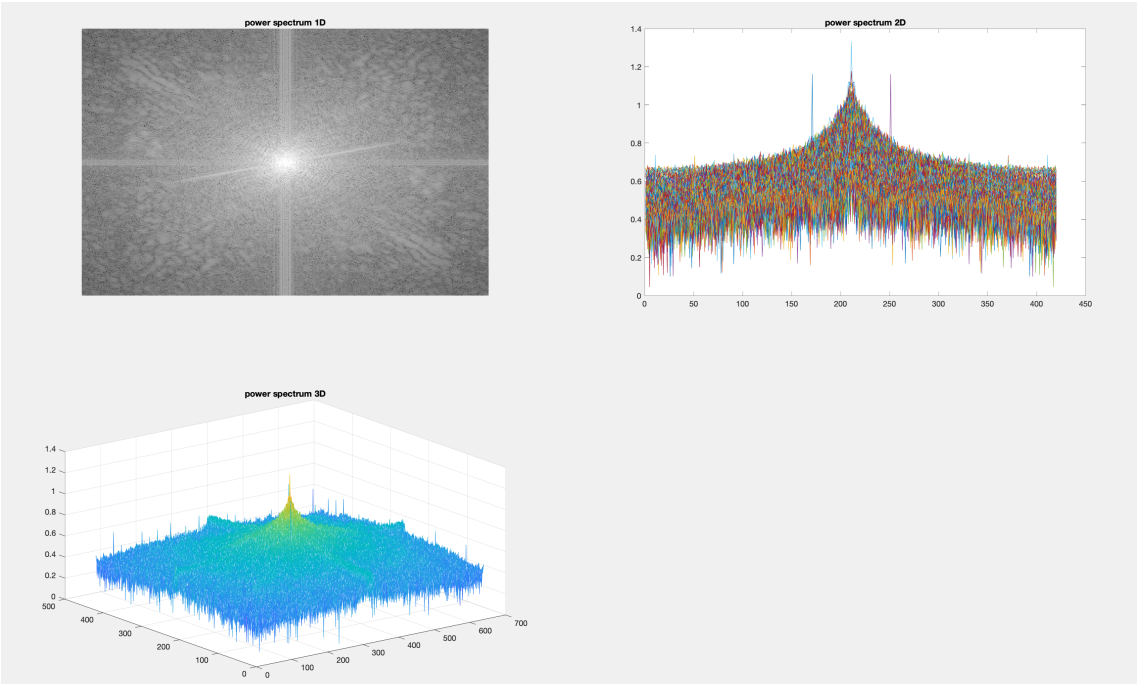

Figure 8: Power spectrums of noisy image

- Find a way to remove the periodic noise in the frequency domain. Then show the de-noised

image (in space) and its power spectrum. Discuss your approach and results.

I chose to use a notch filter to remove the noise because this type of filters removes frequencies in predefined neighborhoods about the center of frequency and from the analysis above, I know the this frequency. Due to the symmetry of the FT, the notch filters appears in pairs and that's good for this case because it will remove the 2 peaks representing the noise from the power spectrum. I used a Butterworth notch reject filter in order to remove this noise because I read in the book Gonzalez that the Butterworth was a good deal between an ideal and a gaussian filter due to the effects on its neighbours. So I removed those frequencies with this filter. The results are shown below. Obviously, we apply this filter in the frequency domain.

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

Figure 9: Formula of the Butterworth notch reject filter

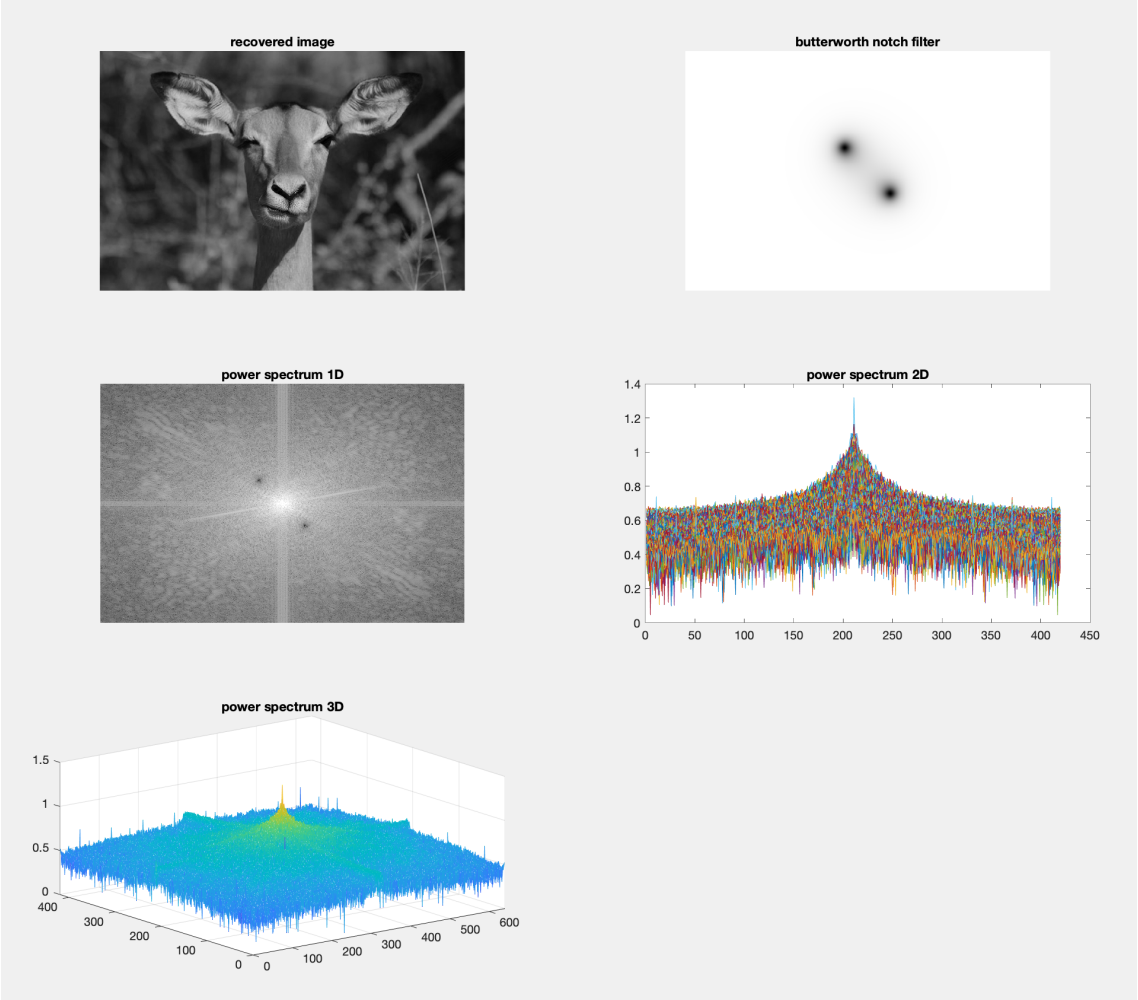

Figure 10: Power spectrums and result of Buttwerworth notch filter on the noisy image

Above, we can see the notch filter used to filter out the noise on the noisy image. The filter sets to 0 the frequencies that we want to get rid of. It also influences a little bit their neighbour since the function is not 0 or 1 but is more complex (see above) and is based on the distances between a frequency and the center frequency. I chose $n = 2$ to not influence too much the original frequencies (not the noisy ones). This is why the butterworth notch filter plot above is blurred. To my eyes, the result I got is pretty similar to the original image. We can see on the power spectrum that the peaks have been removed so the noise has been cancelled as wanted while the other frequencies seems to not have been much affected by this filter. I actually compared this power spectrum to the one from the original image and they look pretty similar. I don't include it here because it is not asked and probably not necessary.

3 Image restoration

- Choose an image of your liking and blur it with a spatial kernel h of your choice. Add random noise n following a specific noise distribution.

  I chose the same image. I applied a gaussian spatial kernel on it (size 5). I then added a gaussian noise on it. See the code for more explanations.

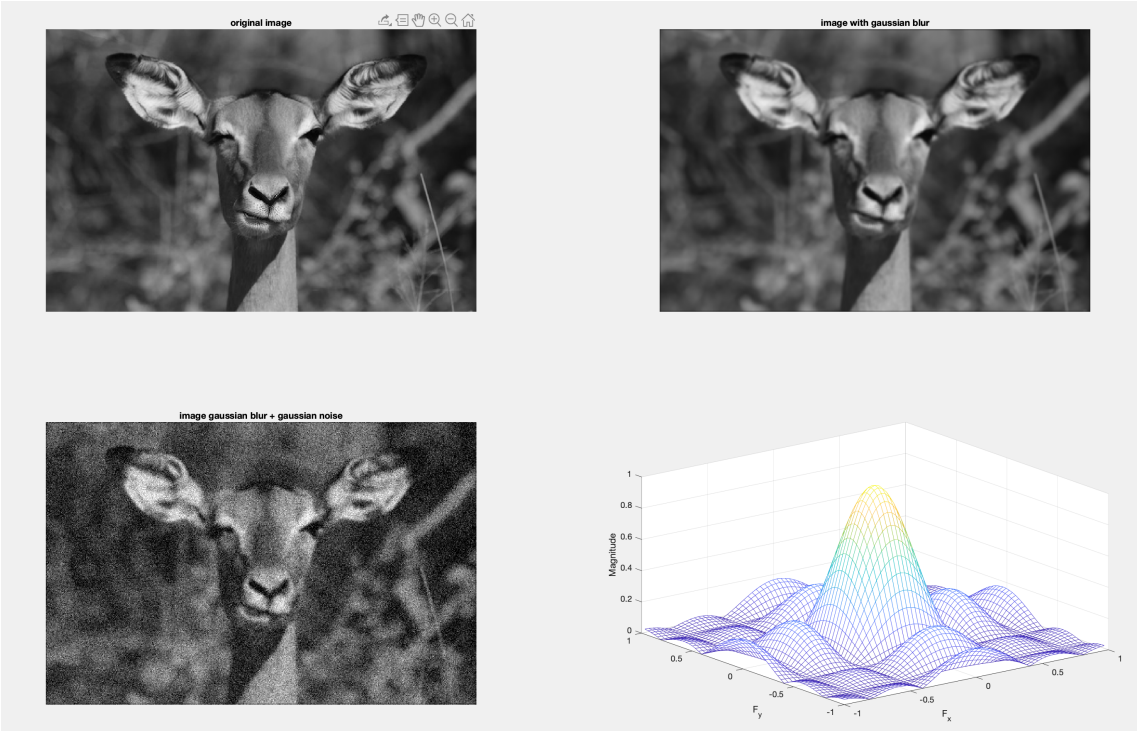

Figure 11:

- Calculate the frequency transform of your blurring function h.

  I calculated using the built in function fft2 and fft. I wasn't sure if I should implement it by hand in order to find the function itself or not.

- Calculate the 2D FFT of the degraded image(using an inbuilt function like fft2, numpy.fft.fft2). Display the degraded image's power spectrum in 1D, 2D, 3D and comment on it. What does it reveal about the blurring and noise?

  We can observe that the frequencies are concentrated in the center (on all power spectrums). We also observe the noise all around the center. We know that it is noise because there are a lot of low peaks/points (depending on the dimension of the figure) and it seems pretty

random. The different lines crossing the whole 1D spectrum are not present anymore, the frequencies of the original image have been attenuated by the blur and the noise and this is logic if you remember the formula:

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Figure 12:

However, we can see that the strongest frequencies (the ones in the center) are still pretty strong and dominant. Since the blur function multiplies the original image while the noise function gets added to it, the blur function is the most responsible one for decreasing the "weakest" frequencies. Blurring means remove the details on an image and keep the most important features on it so it is pretty straight forward that it attenuates the weakest ones.

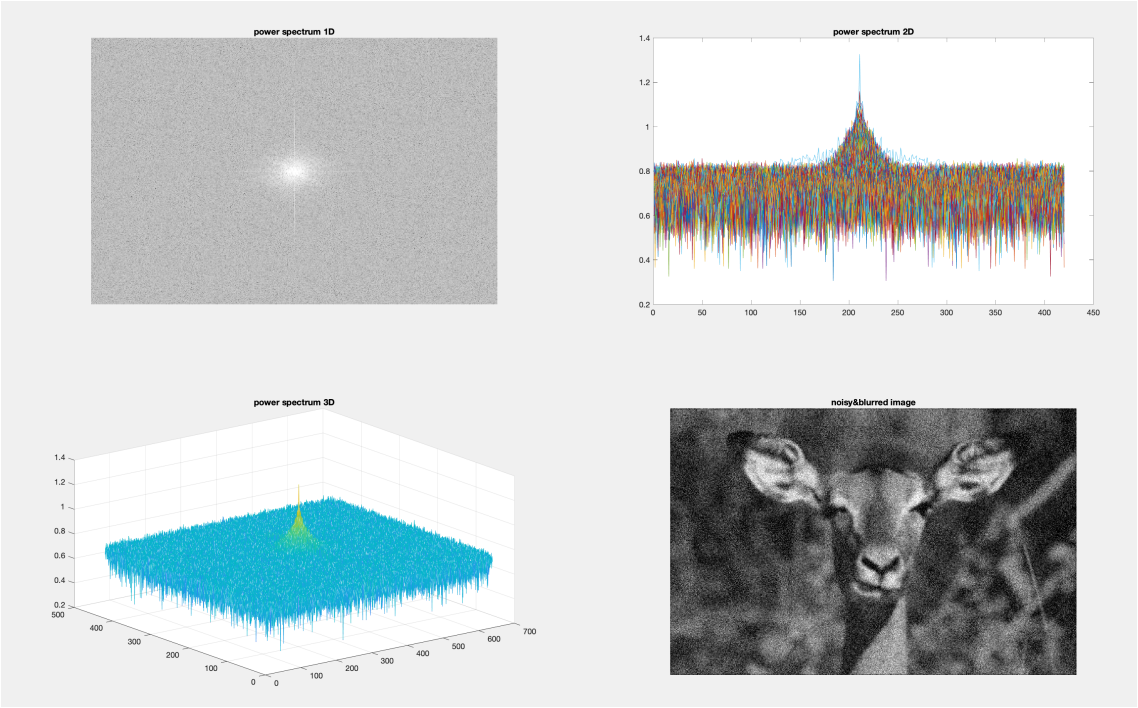

Figure 13:

- Find a way to remove the blurring in the frequency domain, using the inverse filtering methods we discussed in class. Then show the denoised image (in space) and its power spectrum. Discuss your approach and results.

  I implemented the inverse and the wiener but I don't have enough time to include the results on the report. Also, I think we didn't discuss much about those during class and we unfortunately can not ask you as many questions as we would like to so it is pretty difficult. I have read the book from Gonzalez and searched online but it still doesn't replace a teacher ...