How to Symbolic Traversal

Symbolic State Traversal for the state machine M based on the algorithm given lecture slides 5-10

Given M (assume M has no input):

$$statevariables: (s_0, s_1) \qquad \delta = \begin{cases} s_0' = \neg s_0 \\ s_1' = \neg s_1 \end{cases} \qquad S_{initial} = (0,0)$$

Step 1:

Create BDD manger class.

Step 2:

Create variables for current and next state bits (s_0, s_1, s_0', s_1')

Step 3:

Compute BDD for δ:

$$\delta_0 = BDD.\neg(s_0) \quad \delta_1 = BDD.\neg(s_1)$$

Step 4:

Compute BDD for transition relation $\tau = (s_0'.\delta_0 + \bar{s_0'}.\bar{\delta_0}).(s_1'.\delta_1 + \bar{s_1'}.\bar{\delta_1})$

$$\tau_0 = BDD.\vee 2\left(BDD.\wedge 2(s_0', \delta_0), BDD.\wedge 2\big(BDD.\neg(s_0'), BDD.\neg(\delta_0)\big)\right)$$

$$\tau_1 = BDD.\vee 2\left(BDD.\wedge 2(s_1', \delta_1), BDD.\wedge 2\big(BDD.\neg(s_1'), BDD.\neg(\delta_1)\big)\right)$$

$$\tau = BDD.\wedge 2(\tau_0, \tau_1)$$

Step 5:

Compute BDD for state characteristic function for initial state (0,0),

$$c_{s_0} = (s_0 = 0).(s_1 = 0)$$

Note: "==" is equivalent to XNOR

$$c_{s_0} = BDD.\wedge 2\big(BDD.xnor2(s_0, 0), BDD.xnor2(s_1, 0)\big)$$

Step 6:

$$c_{R_{it}} = c_{s_0}$$

Step 7:

$$c_R = c_{R_{it}}$$

Step 8:

Compute BDD for $img(s'_0, s'_1) = \exists_{s_0}\exists_{s_1} c_R.\tau$

$$temp1 = BDD.\wedge 2(c_R, \tau)$$

$$temp2 = BDD.\vee 2\big(BDD.coFactorTrue(temp1, s_1), BDD.coFactorFalse(temp1, s_1)\big)$$

$$img(s'_0, s'_1) = BDD.\vee 2\big(BDD.coFactorTrue(temp2, s_0), BDD.coFactorFalse(temp2, s_0)\big)$$

Step 9:

Form $img(s_0, s_1)$ from $img(s'_0, s'_1)$:

$$img(s_0, s_1) = \exists_{s'_0}\exists_{s'_1}(s_0 = s'_0).(s_1 = s'_1).img(s'_0, s'_1)$$

$$temp1 = BDD.\wedge 2\big(BDD.xnor2(s_0, s'_0), BDD.xnor2(s_1, s'_1)\big)$$

$$temp2 = BDD.\wedge 2\big(temp1, img(s'_0, s'_1)\big)$$

$$temp3 = BDD.\vee 2\big(BDD.coFactorTrue(temp2, s'_1), BDD.coFactorFalse(temp2, s'_1)\big)$$

$$img(s_0, s_1) = BDD.\vee 2\big(BDD.coFactorTrue(temp3, s'_0), BDD.coFactorFalse(temp3, s'_0)\big)$$

Step 10:

Compute BDD for new $c_{R_{it}}$:

$$c_{R_{it}} = BDD.\vee 2\big(c_R, img(s_0, s_1)\big)$$

Step 11:

Check $c_R = c_{R_{it}}$:

At this step, c_R consists of (0,0) and (1,1) and $c_{R_{it}}$ consists of (1,1), therefore it is not a fixed point and we have go back to step 7 and perform second iteration.

Step 12:

After second iteration both $c_R \wedge c_{R_{it}}$ consists of (0,0) and (1,1), therefore we reached a fixed point.

After this step, c_R is the symbolic representation of set of reachable states.