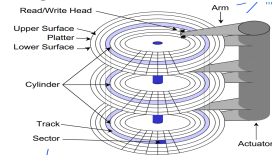# L1 - Data Storage

Magnetic Disks

- **Disk Access Time** Seek time + Rotational Latency + Transfer time
- **Response time** Queueing delay + Disk access time
- **Rotational Delay** $\frac{1}{2} \frac{60s}{RPM}$
- **Transfer Time** sectors on the same track *
$\frac{TimePerRevolution}{SectorsPerTrack}$

Buffer Manager

- **Buffer pool** Main memory allocated for DBMS
- **pin count** is incremented upon pinning
- **dirty bit** is updated when the page is unpinned (if modified)
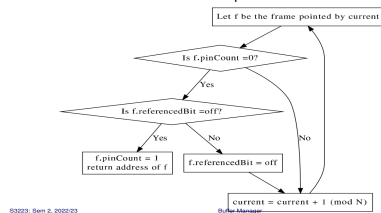- **R**eplacement is only possbile if pin count == 0
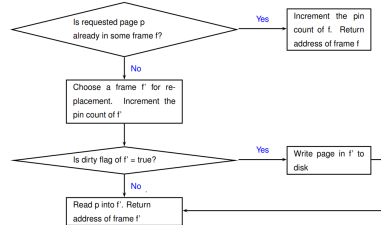
Replacement Policies LRU Policy

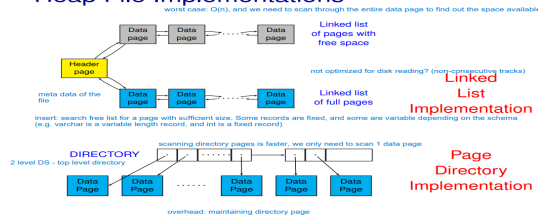- Maintains a queue of pointers to frames with pin count = 0

Clock Replacement Policy

N = number of frames in buffer pool

- Simplifies LRU with a second chance round robin system
- Each frame has a reference bit that is turned on when pin count reaches 0
- Repalces a page when referenced bit if off and pin count is 0

File Organisation

Heap File Implementations
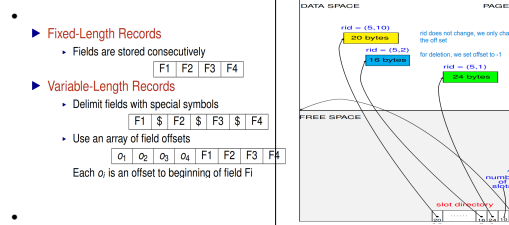

worst case: O(n), and we need to scan through the entire data page to find out the space available

Linked List Implementation

meta data of the file

insert: search free list for a page with sufficient size. Some records are fixed, and some are variable depending on the schema (e.g. varchar is a variable length record, and int is a fixed record)

scanning directory pages is faster, we only need to scan 1 data page

Page Directory Implementation

overhead: maintaining directory page

Page Formats: Fixed Length Records

- **Packed Organisation** Store records in contiguous slots
- **Unpacked Organisation** Uses a bit array to maintain free slots
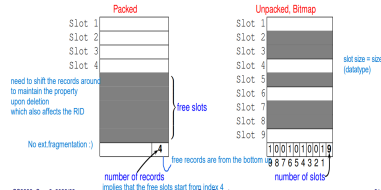
Page Formats: Slotted Page (variable length record)

- Store records in slots of *(record offset, record length)*
- Record Offset: Offset of the record from the start of the page

▶ Fixed-Length Records
  - Fields are stored consecutively
  | F1 | F2 | F3 | F4 |
▶ Variable-Length Records
  - Delimit fields with special symbols
  | F1 | $ | F2 | $ | F3 | $ | F4 |
  - Use an array of field offsets
  | o_1 | o_2 | o_3 | o_4 | F1 | F2 | F3 | F4 |
  Each o_i is an offset to beginning of field Fi

L2 And L3 - Indexing

- A search key is a sequence of k attributes. If k ¿ 1, composite key
- A search key is an unique index if it is a candidate key
- An index is stored as a file

Format of data entries

- Format 1: k* is an actual data record with search value k
- Format 2: k* is the form (k, rid)
- Format 3: k* is the form (k, rid-list*)
- Note: Different formats affects the number of data entries stored in a page

Clustered Vs Unclustered

- Clustered: Order of data entries is the same as the oreder of data records. Can only be built on ordered field (e.g. primary key)
- Unclustered: Order of data entries does not correspond to the order of data records
- The implication is that we can read an entire clustered page with 1 I/O
- B+ Tree: Format 1 is clustered, Format 2 and 3 can be clustered if data records are sorted on the search key
- Hash: Only format 1 is clustered since hashing do not store data entries in search key order

Tree Based Index - B+ Tree

- Leaf nodes are doubly linked and store Data Entries
- Internal nodes sotre index entries (p0, k, p1 ... pk, k, pk+1)
- Internal nodes contains m entries, m $\in$ [d, 2d] $\rightarrow$ space utilisation $\geq$ 50%
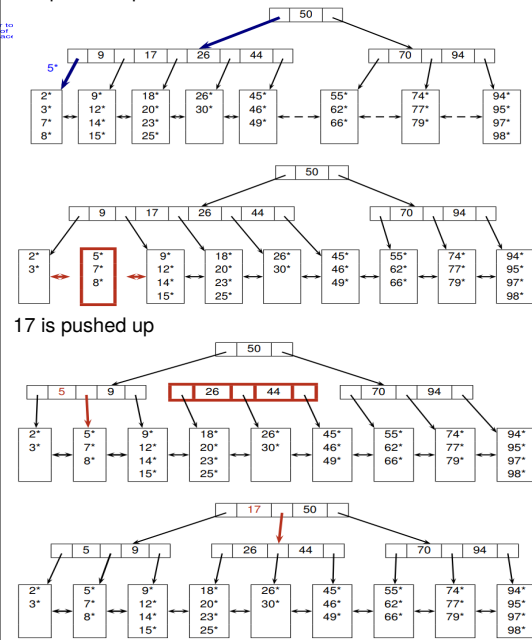- Root contains m entries, m $\in$ [1, 2d]

B+ Tree - Split Overflow Nodes

- Distribute d+1 entries to the new leaf node
- Create new entry index using smallest key in the new node (middle key)
- Insert new entry into parent node of overflowed node

The new node added is always the median value between the 2
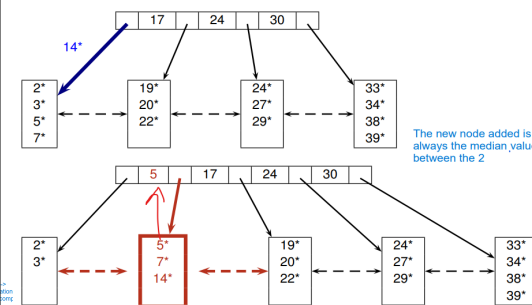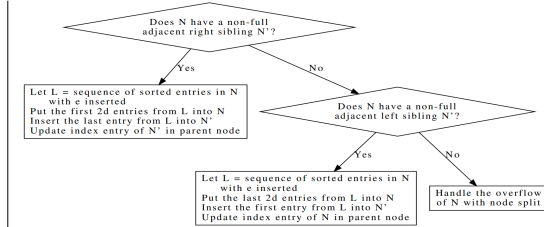
B+ Tree - Overflow Propagation

5 is pushed up

17 is pushed up

Excess middle node is pushed updated to parent node

B+ Tree - Redistribution of data entries

Two nodes are siblings if they have the same parent node

Let L = sequence of sorted entries in N with e inserted
Put the first 2d entries from L into N
Insert the last entry from L into N'
Update index entry of N' in parent node

Let L = sequence of sorted entries in N with e inserted
Put the last 2d entries from L into N
Insert the first entry from L into N'
Update index entry of N in parent node

Handle the overflow of N with node split

B+ Tree - Underflow

- Underflow occurs when a node has less than d entries
- Underflow is resolved by redistributing entries between siblings
- An underflow node is merged if each of its adjacent siblings have exactly d entries
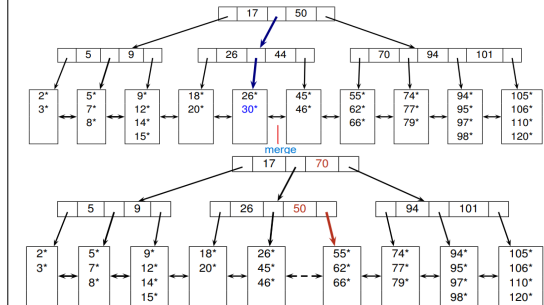
B+ Tree - Bulk Loading

- Initiazing a B+ tree by insertion is expensive (need to traverse tree n times)
- 1. Sort all data entries by search key
- 2. Initialise B+ tree with an empty root page
- 3. Load data entries into leaf pages
- 4. In asc order, insert the index entry of each leaf page into the rightmost parent node
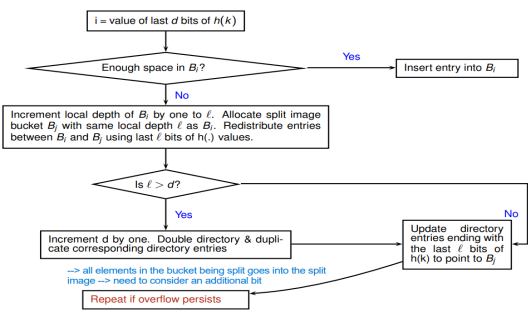
Hash Based Index

- Does not support range search, only equality queries

Static Hashing

- N buckets, each bucket has 1 primary page and $\geq$ 0 overflow pages
- To maintain performance, we need to routinely construct bigger hash tables and redistribute data entries

Dynamic Hashing - Extendible Hashing

- No overflow pages! A bucket can be thought of as a page
- At most 2 Disk I/Os for equality search (at most 1 if directory and bucket fits in memory)
- Instead of maintaining data entries, we maintain pointers to data entries in buckets
- Instead of maintaining buckets, maintain a directory of pointers to buckets
- The directory has 2^d buckets, where d is the global depth −¿ large overhead if hashing is uniform
- Each director entry diffets by a unique d-bit adddress
- Two directories are corresponding iff their addresses differ only in the dth bit
- All entries with the same local depth (l) have the same last l bits in h(k)

Column 1

i = value of last d bits of h(k)

Enough space in B_i? — Yes → Insert entry into B_i

No →

Increment local depth of B_i by one to ℓ. Allocate split image bucket B_j with same local depth ℓ as B_i. Redistribute entries between B_i and B_j using last ℓ bits of h(.) values.

Is $\ell > d$?

Yes → Increment d by one. Double directory & duplicate corresponding directory entries

No → Update directory entries ending with the last ℓ bits of h(k) to point to B_j

--> all elements in the bucket being split goes into the split image --> need to consider an additional bit
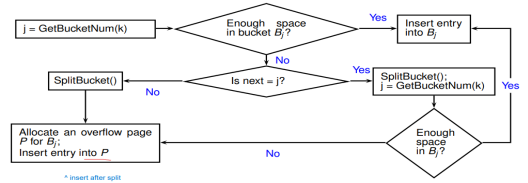
Repeat if overflow persists

Extendible Hashing - Split, Double
- Split and doubling is checked every time a bucket is full
- Doubling only happens if local depth = global depth
- The split image has the same depth as the split bucket
- Other than the split image of the split bucket, split image of other buckets points to the same corresponding bucket
- Each bucket is pointed by $2^{(d-l)}$ directories

Extendible Hashing - Deletion
- B_i is deallocated
- l decrement by 1
- Directory Entries that point to B_i points to its corresponding bucket

Dynamic Hashing - Linear Hashing

j = GetBucketNum(k)

Enough space in bucket B_j? — Yes → Insert entry into B_j

No → SplitBucket()

Is next = j? — Yes → SplitBucket(); j = GetBucketNum(k) — Yes

No → Enough space in B_j? — Yes

No → Allocate an overflow page P for B_j; Insert entry into P

* insert after split

- **GetBucketNum(k)** returns bucket # where entry with search key k is located

$GetBucketNum(k) = \begin{cases} h_{level}(k) & \text{if } h_{level}(k) \geq next, \\ h_{level+1}(k) & \text{otherwise}. \end{cases}$ search key in unspilt
  search key in split

- **SplitBucket()** splits bucket B_{next}
  1. Redistribute the entries in B_{next} into $B_{next+N_{level}}$ using $h_{level+1}()$
  2. next = next + 1
  3. if (next = N_{level}) then { level = level + 1; next = 0 }

- One I/O for equality search (more per number of overflow pages in bucket)
- Performs worse than extendible hashing if distribution is skewed
- Does not require a directory
- Higher average space utilisation, but longer overflow chains
- Has a family of hash functions, with each having a range twice of its predecessor
- N_0: initial number of buckets
- $N_i = 2^i N_0$: number of buckets at start of round i
- $next$: the next bucket to be split, this is incremented every time split happnes
- $h_{i+1} = h(k) mod N_{i+1}$: hash function for round i, if the bucket > next (already split)
- $h_i = h(k) mod N_i$: hash function for round i+1, if the bucket > next
- Split Citeria: By default, split when a bucket overflows
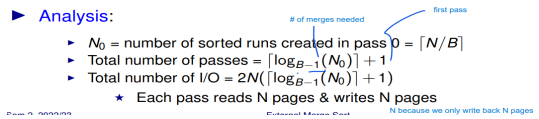
Linear Hashing - Deletion
- Essentially the inverse of insertion
- If the last bucket is empty → delete it, $next-$
- If $next$ is 0, set it to $M/2 - 1$, and we can decrement level by 1 (half of buckets have been deleted if $next$ is 0)
- Merging with corresponding bucket is optional

Column 2

L4: Query Evaluation - Sort, Select

Sorting - External Merge Sort
Projection, join, bulk loading etc all require sorting
- ▶ Uses B number of buffer pages
- ▶ Pass 0: Creation of sorted runs
  - ▶ Read in and sort B pages at a time
  - ▶ Number of sorted runs created = $\lceil N/B \rceil$
  - ▶ Size of each sorted run = B pages (except possibly for last run)
- ▶ Pass i, $i \geq 1$: Merging of sorted runs
  - ▶ Use $B-1$ buffer pages for input & one buffer page for output
  - ▶ Performs (B-1)-way merge
- ▶ Analysis: # of merges needed   first pass
  - ▶ N_0 = number of sorted runs created in pass 0 = $\lceil N/B \rceil$
  - ▶ Total number of passes = $\lceil log_{B-1}(N_0) \rceil + 1$
  - ▶ Total number of I/O = $2N(\lceil log_{B-1}(N_0) \rceil + 1)$
  - ★ Each pass reads N pages & writes N pages   N because we only write back N pages

External Merge Sort

External Merge Sort - Bocked I/O
- Read and write in blocks of **b** buffer pages (replace b with 1 for unoptimised)
- $\lfloor \frac{B-b}{b} \rfloor$ blocks for input, 1 block for output
- Can merge at most $\lfloor \frac{B-b}{b} \rfloor$ sorted runs in each merge pass
- $F = \lfloor \frac{B}{b} \rfloor - 1$ runs can be merged at each pass
- Num passes = $log_F N_0$

B+ tree sort
- B+ Tree is sorted by key
- Format 1 (clustered): Sequential Scan
- Format 2/3:Retrieve data using RID for each data entry
- Unclustered implies more I/Os

Access Path refers to the different ways to retrieve tuples from a relation. It is either a **file scan** or a **index plus matching selection condition**. The more **selective** the access paths, the fewer pages are read from the disk.
- Table scan: scan all data pages
- Index scan: scan all index pages
- Table intersection: combine results from multiple index scans (union, intersec). Find RIDs of each predicate and get the intersection

Query: Selection Covering Index
- I is a covering index of $query_Q$ if I contains all attributes of Q
- No RID lookup is needed, Index-only plan
- If data is unclustered, unsorted, no index -¿ best way is to collect all entries and sort by RID before doing I/O

CNF Predicate
- Find RIDs of each predicate and get the intersection
- Conjuncts are in the form (R.A op c V R.a op R.b)
- CNF are conjuncts (or terms) connected by $\land$

Matching Predicates - B+ Tree
- Non-disjunctive CNF (no $\lor$)
- At most one non-equality comparison operator which must be on the **last attribute in the CNF**
- $(k_1 = c_1) \land (k_2 = c_2) \land ...k_i opc_i | I = (k_1, k_2...k_n)$
- The order of k matters, and there cannot be missing K_i in the middle of the CNF
- Having inequality operator before equality operator makes the query to be less selective

Matching Predicates - Hasing
- No inequality operators
  $(k_1 = c_1) \land ...k_i = c_n | I = (k_1, k_2...k_n)$
- Unlike B+ tree, **all predicates must match**

Column 3

I=(age, weight, height), p=($age \geq 20 \land age \geq 18 weight = 50 \land height = 150 \land level = 3$)

Primary Conjuncts : The subset of conjuncts in p that I matches
Primary Conjuncts: $age \geq 20 \land age \geq 18$

Covered Conjuncts : The subset of conjuncts in p that I covers (conjuncts that appear in I). Primary conjunct $\subseteq$ covered conjunct
Covered Conjuncts: $age \geq 20 \land age \geq 18 \land height = 150$

Cost Notation

Notation	Meaning
r	relational algebra expression
$\|r\|$	number of tuples in output of r — data records
$\|r\|$	number of pages in output of r
b_d	number of data records that can fit on a page
b_i	number of data entries that can fit on a page
F	average fanout of B+-tree index (i.e., number of pointers to child nodes)
h	height of B+-tree index (i.e., number of levels of internal nodes)
	$h = \lceil log_F(\lceil \frac{\|R\|}{b_i} \rceil) \rceil$ if format-2 index on table R
B	number of available buffer pages

Cost of B+-tree index evaluation of p
Let p'=primary conjuncts of p — p_c=covered conjuncts of p
1. Navigate internal nodes to locate the first leaf page

$$Cost_{internal} = \begin{cases} \lceil log_F(\lceil \frac{\|R\|}{b_d} \rceil) \rceil & ||Format1 \\ \lceil log_F(\lceil \frac{\|R\|}{b_i} \rceil) \rceil & ||Otherwise \end{cases}$$

1.1 This is traversing the height of B+ tree
2. Scan leaf pages to access all qualifying data entries

$$Cost_{leaf} = \begin{cases} \lceil \frac{\|\sigma_{p'}(R)\|}{b_d} \rceil & ||Format1 \\ \lceil \frac{\|\sigma_{p'}(R)\|}{b_i} \rceil & ||Otherwise \end{cases}$$

2.1 This is the cost of reading qualifying conjuncts
2.2 Using p_c would be wrong since covering conjuncts may be non-matching which results in more reads from the leaves
3 Retrieve qualified data records using RID lookups. 0 if I is covering OR format 1 index. $\|\sigma_{p_c}(R)\|$ otherwise

Cost of RID lookups could be reduced by first sorting the RIDs

assuming clustered/congiguous $\lceil \frac{\|\sigma_{p_c}(R)\|}{b_d} \rceil \leq Cost_{rid} \leq min\{\|\sigma_{p_c}(R)\|, |R|\}$ sequenced I/O
  unclustered
  ceiling because we have to read the additional page for the remainder RIDs

Cost of Hash index evaluation of p
- Format 1: cost to retrieve **data entries** is at least $\lceil \frac{\|\sigma_{p'}(R)\|}{b_d} \rceil$
- Format 2: cost to retrieve **data entries** is at least $\lceil \frac{\|\sigma_{p'}(R)\|}{b_i} \rceil$
- Format 2: Cost to retrieve **data records** is 0 if it is a covering index (all information in data entry) OR $\|\sigma_{p'}(R)\|$ otherwise

L5:Query Evaluation - Projection and Join
- $\pi^*(R)$ refers to projection without removing duplicates
- $\pi(R)$ involves 1.Removing unwanted attributes 2. Removing duplicates
- Sorting is better if we have many duplicates or if hte distribution is nonuniform(overflow more likely for hashing paritions)
- Sorting allows results to be sorted
- If $B > \sqrt{|\pi_L^*(R)|}$, then both sorting and hashing has similar I/O costs ($\lceil \frac{R}{B} \rceil \to |R| + 2 * |\pi_L^*(R)|$)

Column 4

Approach 1: project based on sorting
- **Naive**: Extract attributes L from records $\to \pi_L^*(R) \to$ Sort attributes $\to$ Remove duplicates
- Cost: Cost to scan records ($|R|$) + Cost to output to temporary result ($|\pi_L^*(R)|$) $\to$ cost to sort records ($2|\pi_L^*(R)| \log_m(N_0) + 1$) $\to$ Cost to scan data records $|\pi_L^*(R)|$
- **Optimisation**: Create Sorted runs with attributes L only (Pass 0) $\to$ Merge sorted runs and remove duplicates $\to \pi_L(R)$

Approach 2: project based on hashing
- Build a main-memory hash table to detect and remove duplicates. Insert to the hashtable if then entry is not already in it.
- 1. Partition R into $R_1, R_2...R_{B-1}$, hash on $\pi_L(t)$ for $t \in R \leftarrow (\pi_L^*(R_i)$ does not intersect $\pi_L^*(R_j), i! = j)$
- 1.1 Use 1 buffer for input and (B-1) for output
- 1.2 Read R 1 page at a time, and hash tuples into B-1 partitions
- 1.3 Flush output buffer to disk when full
- 2. Eliminate duplicates in each partition $\pi_L^*(R_i)$
- $\pi_L(R) = \cup_i^{B-1}(\pi_L(R_i))$
- 2.1 For each partition, Initialise an in-memory hash table and insert each tuple into B_j if $t \notin B_j$

Parition overflow: Hash table $\pi_L^*(R_i)$ is larger than available memory buffers.
Solution: Recursively apply hash-based partitioning to overflowed partitions.
Analysis: Effective (no overflow) when B $> \frac{|\pi_L^*(R)|}{B-1} * f \approx \sqrt{f * \pi_L^*(R)}$
If no partition overflow: (partition)$|R| + \pi_L^*(R)| +$ (duplicate elimination)$|\pi_L^*(R)|$

Index based projection: Do index scan if the wanted attribtues $\subseteq$ search key

Join $R \bowtie_\theta S$, where R is the outer relation and S is the inner relation

- **Tuple-based**
  - Cost: $|R| + \|R\| * |S|$
  - for each tuple r in R
  - for each tuple s in S
  - if (r matches s) then output (r, s)4 to result
- **Page-based**
  - Load P_R and P_S to main memory
  - Cost: $|R| + |R| * |S|$
  - for each page P_R in R
  - for each page P_S in S
  - for each tuple $r \in P_R$
  - for each tuple $s \in P_S$
  - if (r matches s) then output (r, s)4 to result
- **Block-based**
  - Allocate 1 page for S, 1 for output, B-2 for R
    $|R| \leq |S|$
  - Cost:
    $|R| + (\lceil \frac{|R|}{B-2} \rceil * |S|)$
  - while Scanning R

- read next (B-2) pages of R to buffer
- for P_S in S
- read P_S into Buffer
- for $r \in buffer \land s \in P_S$
- if (r matches s) then output (r, s)4 to result
- **Index Nested Loop Join**
  - There is an index on the join attributes of S
  - Uniform distribution: r joins $\lceil \frac{\|S\|}{\|\pi_{B_j}(S)\|} \rceil$ tuples in S
  - format 1 B+Tree:$|R| + \|R\| * J$
  - $J = \log_F(\lceil \frac{\|S\|}{b_d} \rceil)$ (tree traversal)+$\lceil \frac{\|S\|}{b_d\|\pi_{B_j}(S)\|} \rceil$ (search leaf nodes)
  - for $r \in R$
  - use r to probe S's index to find matching tuples