CS3231
AY23/24 S1
github.com/SeekSaveServe

Lectures

L1: Preliminaries, DFA

Types of Proof

- Deductive Proofs
- Modus Ponens, $A \rightarrow B$
- Proof by contradiction
- Counter-example
- Equivalence (iff)
- Converse, $A \rightarrow B$ and $B \rightarrow A$ shows iff and equivalence
- Inductive Proofs
- *Proof.* Prove base case
  Assume true for $n = k$
  Prove true for $n = k + 1$ $\square$
- Structural Induction: if a claim holds true for a tree of height k, then it holds true for a tree of height $k + 1$
- Mutual Induction: Showing several claims to be true simultaneously
- Diagonalisation: Showing that a set is uncountable by demonstrating a contradiction to the assumption that it is possible to enumerate all elements of the set (countable). This can be done by changing the i^{th} digit of the i^{th} element of the enumeration, ensuring that the new element differs from existing elements in the enumeration by at least 1 digit.

Central Concepts of Automata Theory

- **Alphabet** Σ: finite non-empty set of symbols (e.g. $\{0, 1\}$)
- **String**: finite sequence of symbols from Σ (e.g. 0101)
- **Empty String** ϵ: string with no symbols
- **Length of String** $|w|$: number of symbols in string w
- **Powers of n Alphabet** Σ^n: set of all strings of length n over Σ (e.g. $\{0, 1\}^2 = \{00, 01, 10, 11\}$)
- **Concatenation of Strings**
  $w_1 = 0101, w_2 = 1010, w_1w_2 = w_1.w_2 = 01011010$
- **Substring** ab is a substring of $babaa$ but bb is not
- **Subsequence** bba is a subsequence of $babaa$ but abb is not. Relative order matters, can skip.
- **Language** L A set of strings over Σ

Strict Definitiion of a Language

- A language is strictly defined. When a turing machine accepts a language, it accepts exactly that language, not a superset or subset
- $L = \{x : x$ is a binary representation of a prime number$\}$ is not clearly defined since 011 and 11 are both binary representations of 3
- $L_1.L_2 = \{xy : x \in L_1, , y \in L_2\}$
- $L^* = \{x_1x_2...x_n : x_1, x_2..., x_n \in L, n \in \mathbb{N}\}$ since n can be 0, $\epsilon \in L^*$
- $L^* = \{\epsilon\} \cup L \cup L^2 \cup L^3 \cup ...$
- $L^+\{x_1x_2...x_n : x_1, x_2..., x_n \in L, n \geq 1\}$
- $L^+ = L \cup L^2 \cup L^3 \cup ...$, note that it may or may not include ϵ since ϵ can be included in L
- Number of srings over any fixed fininte alphabet is countable
- Number of languages over any non-empty alphabet is uncountable

Finite Automata

- Regular language is accepted by a finite automata
- E.g. On-Off Switch. Inputs toggles the state

Deterministic Finite Automata (DFA)

- A DFA is a 5-tuple $A = (Q, \Sigma, \delta, q_0, F)$ where
- Q is a finite set of states
- Σ is a finite alphabet
- $\delta(q_i, x|q_i \in Q, x \in \Sigma) : Q \times \Sigma \rightarrow Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states