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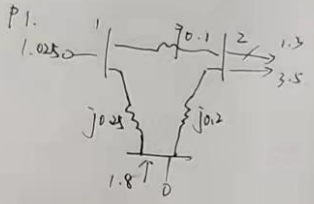
P1.



a).

Bus	Type	Input Data	Unknowns
1	Swing	$V_1 = 1.025$ $\delta_1 = 0°$	P_1, Q_1
2	Load	$P_2 = -3.5$ p.u. $Q_2 = -1.3$ p.u.	V_2, δ_2
3	Constant Voltage	$V_3 = 1.05$ p.u. $P_3 = 1.8$ p.u.	Q_3, δ_3

b). $f(x) = \begin{bmatrix} P(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} P_2(x) \\ \vdots \\ P_N(x) \\ Q_2(x) \\ Q_N(x) \end{bmatrix}$

$y = \begin{bmatrix} P_2 \\ P_3 \\ Q_2 \end{bmatrix} = \begin{bmatrix} -3.5 \\ 1.8 \\ -1.3 \end{bmatrix}$ p.u. $\quad x = \begin{bmatrix} \delta_2 \\ \delta_3 \\ V_2 \end{bmatrix}$

For Y_{bus}: $j\begin{bmatrix} -14 & 10 & 4 \\ 10 & -15 & 5 \\ 4 & 5 & -9 \end{bmatrix}$

i.e. $\begin{cases} P_2 = V_1 V_2 B_{21} \sin(\delta_2 - \delta_1) + V_2 V_3 B_{23} \sin(\delta_2 - \delta_3) + V_1 V_2 B_{12} \sin(\cancel{\delta_1} - \cancel{\delta_2}) \\ P_3 = V_1 V_2 B_{21} \cos(\cancel{\delta_2} - \delta_1) + V_2 V_3 B_{23} \cos(\cancel{\delta_2} - \cancel{\delta_3}) - V_1 V_2 B_{13} \cos(\cancel{\delta_2} - \cancel{\delta_3}) \\ Q_2 = V_1 V_3 B_{31} \sin(\cancel{\delta_3} - \cancel{\delta_1}) + V_2 V_3 B_{23} \sin(\delta_3 - \delta_2) + V_3 V_2 B_{23} \sin(\delta_2 - \delta_3) \end{cases}$

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→ For $\vec{f_0}(x)$ $\delta_2 = \delta_3 = 0,\ V_2 = 1$

i.e. $-3.5 = 1.025 V_2 \cdot 10 \cos(\delta_2 - 90^\circ) + 1.05 V_2 \cdot 5 \cos(\delta_2 - \delta_3 - 90^\circ)$

$\cancel{+11.3 = -1.025 \cdot 4 \cos(\delta_2 - 90^\circ) + 5 \cdot 1.05 V_2 \cos(\delta_3 - \delta_2 - 90^\circ)}$

$1.8 = V_2 [10 \sin(\delta_2 - 90^\circ)$

$1.8 = 1.05 \cdot 4 \sin(\delta_2) + 1.05 V_3 \cdot 5 \cos(\delta_3 - \delta_2 - 90^\circ)$

$-1.3 = -1.025 V_2 \cdot 10 \sin(\delta_2 - 90^\circ) - 1.05 V_2 \, 5 \sin(\delta_2 - \delta_3 - 90^\circ) + 15 V_2^2$

~~i.e.~~ $\cancel{\vec{f_0}}$

c).

$$J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

$$J_{122} = \frac{\partial P_2}{\partial \delta_2} = -V_2 [B_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + B_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23})] = 15.5$$

$$J_{123} = \frac{\partial P_2}{\partial \delta_3} = -1.05 \cdot 5 \sin(-90^\circ) = -5.25$$

$$J_{132} = \frac{\partial P_3}{\partial \delta_2} = 1.05 \cdot 5 \sin(-90^\circ) = -5.25$$

$$J_{133} = \frac{\partial P_3}{\partial \delta_3} = -1.025 [4 \cdot \sin(-90^\circ + 5 \cdot \sin(-90^\circ)) = 9.56$$

$$J_{222} = \frac{\partial P_2}{\partial V_2} = 0 \qquad J_{232} = \frac{\partial P_3}{\partial V_2} = 0$$

$$J_{322} = \frac{\partial Q}{\partial \delta_2} = 0 \qquad J_{333} = 0$$

i.e.
$$J = \begin{bmatrix} 15.5 & -5.25 & 0 \\ -5.25 & 9.56 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

P2.

a). Unknowns $X = \begin{bmatrix} Q_2 \\ Q_3 \\ V_2 \\ V_3 \end{bmatrix}$

b) Similarly to P1(b). we have.

$$\begin{cases} -3.5 = 1.025 V_2 \cdot 10 \cos(\delta_2 - 90^\circ) + V_2 V_3 \cdot 5 \cos(\delta_2 - \delta_3 - 90^\circ) \\ -1.3 = -1.025 V_2 \cdot 10 \cos \delta_3 - V_2 V_3 \cdot 5 \cos(\delta_2 - \delta_3) + 15 V_2^2 \\ 1.8 = 1.025 V_3 \cdot 4 \cos(\delta_3 - 90^\circ) + V_2 V_3 \cdot 5 \cos(\delta_3 - \delta_2 - 90^\circ) - 1.8 \\ 1.0 = 1.025 V_3 \cdot 4 \cos(\delta_3 - 90^\circ) - V_2 V_3 \cdot 5 \cos(\delta_3 - \delta_2) + 9 V_3^2. \end{cases}$$

c) Similarly to P1(c) $J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}$

$J_{122} = \frac{\partial P_2}{\partial \delta_2} = 15.25$

$J_{123} = \frac{\partial P_2}{\partial \delta_3} = -5$

$J_{132} = \frac{\partial P_3}{\partial \delta_2} = -5$

$J_{133} = \frac{\partial P_3}{\partial \delta_3} = 9.1$

$J_{222} = 0 \quad J_{322} = 0 \quad J_{223} = 0 \quad J_{322} = 0$

$J_{422} = \frac{\partial Q_2}{\partial V_2} = 14.75$

$J_{423} = \frac{\partial Q_2}{\partial V_3} = -5$

$J_{432} = \frac{\partial Q_3}{\partial V_2} = -5$

$J_{433} = \frac{\partial Q_3}{\partial V_3} = 8.9$

i.e $J = \begin{bmatrix} 15.25 & -5 & 0 & 0 \\ -5 & 9.1 & 0 & 0 \\ 0 & 0 & 14.75 & -5 \\ 0 & 0 & -5 & 8.9 \end{bmatrix}$