P2.

a).
$$M \doteq 2\frac{d}{\lambda}\left[1-\left(\frac{n_2}{n_1}\right)^2\right]^{1/2}$$

i.e. $M \doteq 2\frac{d}{\lambda}\left[1-\left(\frac{n}{n_1}\right)^2\right]^{\frac{1}{2}}$

$$= 2\frac{d}{\lambda}(n_1^2-n_2^2)^{1/2}$$

$$= 2\frac{d}{\lambda} NA$$

i.e. $M \doteq 2 \cdot \frac{10}{0.63} \cdot 1$

$$\doteq 31.74 = 32$$

b). Since $\sin\theta_m = m\frac{\lambda}{2d}$

$$V_m = c\cos\theta_m = c\sqrt{1-\sin^2\theta_m}$$

$$= c\sqrt{1-m^2\frac{\lambda^2}{4d^2}}$$

$$V_{max} = V_{31} = c\sqrt{1-31^2 \cdot \frac{0.63^2}{4\times10^2}}$$

$$= 0.216c$$

$$V_{min} = V_1 = c\sqrt{1-1\cdot\frac{0.63^2}{4\times10^2}}$$

$$= 0.9995c$$

c) Suppose that the original pulse of light is T

it turns into $\frac{2}{T}$.

P3.

a).

$\theta_c = \arcsin \frac{n_2}{n_1} \approx 61.04^\circ$

$\bar{\theta}_0 = 90^\circ - \theta_c = 28.96^\circ$

$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = (1.6^2 - 1.4^2)^{\frac{1}{2}} = 0.7746$

b) $M \doteq 2 \frac{d}{\lambda} NA$

$= 2 \times \frac{2}{0.87} \times 0.7746$

$= 3.56$

$= 4$

c) ~~$\sin\theta_m = m\frac{\lambda}{2d}$~~

~~i.e. The bounce angle $\theta_m = \arcsin(m\frac{\lambda}{2d})$~~

~~$V_m = c\cos\theta_m$~~

~~Since $m = 0$, $\theta_0 = \arcsin 0 = 0^\circ$~~

~~$V_0 = c\cos\theta_0 = c$.~~

For TE0 node

$\tan(\frac{hd}{2}) = \frac{1}{1.6\cos\theta_0}\sqrt{1.6^2\sin^2\theta_0 - 1.4^2}$

$\tan(\frac{\frac{2\pi \cdot 1.6}{\lambda}\cos\theta_0}{2}\cdot 2) = \frac{1}{1.6\cos\theta_0}\sqrt{1.6^2\sin^2\theta_0 - 1.4^2}$

$1.6\cos\theta_0 \approx \cancel{}\ \cancel{}\ 0.544$

$\cos\theta_0 \approx \cancel{}\ 0.034$

i.e. $V_0 = c\cos\theta_0 \approx \cancel{}$

$0.034c$.

P4.

$$M \doteq 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{\frac{1}{2}}$$

$$= \frac{2d}{1.3}(1.5^2 - 1.46^2)^{\frac{1}{2}} = 1$$

$d_{max} = 1.89\,\mu m$

$$M' \doteq 2\frac{d_{max}}{0.85}(1.5^2 - 1.46^2)^{\frac{1}{2}} = 1.53 = 2$$

P5. $M = 2\frac{d}{\lambda}(n_1^2 - n_2^2)^{\frac{1}{2}}$

For $TE_m > 0$

$$\tan\left(\frac{hd}{2}\right) = \frac{1}{n_1\cos\theta_0}\sqrt{n_1^2\sin^2\theta_0 - n_2^2}$$ have solution.

$$h = \frac{2\pi}{\lambda}n_1\cos\theta_0$$

$$\tan\left(\frac{\pi d n_1}{\lambda}\cos\theta_0\right) = \frac{1}{n_1\cos\theta_0}\sqrt{n_1^2 - n_2^2 - n_1\cos\theta_0}$$

Let $n_1\cos\theta_0 = x$

$$\tan\left(\frac{\pi d}{\lambda}x\right) = \frac{1}{x}\sqrt{n_1^2 - x^2 - n_2^2}$$

$$\frac{\sin^2\left(\frac{\pi d}{\lambda_0}x^2\right)}{\cos^2\left(\frac{\pi d}{\lambda_0}x^2\right)} = \frac{n_1^2 - n_2^2 - x^2}{x^2} = \frac{n_1^2 - n_2^2}{x^2} - 1$$

For $n_1 \approx n_2$

We have $\tan\left(\frac{\pi d}{\lambda}x\right) = -1$

$$\frac{\pi d}{\lambda}x = -\frac{1}{4}\pi + \pi\cdot m + \Delta n \quad m = 1, 2, 3, 4$$

$$\frac{d}{\lambda_0}x = -\frac{1}{4} + m + \Delta n \quad m = 1, 2, 3, 4$$

i.e. $\lambda_0^2 \approx 8 n_1 \Delta n \cdot \frac{d^2}{m^2}$ For $(n_1 \to n_2)$.

P6.

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = 0.1$$

$$M_{TE} \approx \frac{\pi}{4}\left(\frac{2d}{\lambda_0}\right)^2 NA^2$$

$$M = \frac{2d}{\lambda} NA = \frac{\nu \cdot 2d}{c}$$

i.e. $\nu = \frac{c}{\lambda} NA = \frac{c}{\lambda} \cdot 0.1 = \frac{c}{2d} \cdot M = \frac{c}{2d} \cdot \sqrt{M_{TE} \cdot \frac{4}{\pi}}$

i.e. $\nu^2 = \frac{c^2}{4d^2} \cdot M_{TE} \cdot \frac{4}{\pi}$

$$= \frac{c^2}{\pi d^2} M_{TE}$$

i.e. $M_{TE} = \frac{\nu^2 \pi d^2}{c^2} = \frac{10^{-3} \times 10^{-9} \pi}{c^2} \nu^2$

P7.

$$NA = n_1 (2\Delta)^{\frac{1}{2}} = 0.0917$$

$$M = \frac{2d}{\lambda_0} NA = \frac{4a}{\lambda_0} NA$$

$$V = 2\pi \frac{a}{\lambda_0} NA$$

$$M \approx \frac{4}{\pi^2} V^2 = \frac{4\pi^2 a^2}{\lambda_0^2} NA^2 \cdot \frac{4}{\pi^2} = \frac{a^2}{\lambda_0^2} NA^2$$

$$\cancel{\frac{4a}{\lambda_0} NA} \quad \frac{a^2}{\lambda_0^2} NA^2$$

i.e. $\lambda_0 = \frac{a}{4} NA = \frac{5}{4} \cdot 0.0917 \approx 0.115 \mu m.$

P8.

$NA = n_1(2\Delta)^{\frac{1}{2}} = (n_1^2 - n_2^2)^{\frac{1}{2}} = 0.171 \qquad \Delta = \frac{n_1 - n_2}{n_1} = 0.0068$

$V = \frac{2\pi a}{\lambda} NA$

$= \frac{2\pi \cdot 20}{1.55} \times 0.17 = 13.78 \qquad M = \frac{4}{\pi^2} V^2 \approx 76.96$

For $\beta_{lm} = n_1 k_0 \left[1 - \frac{(l+2m)^2}{M} \Delta\right] \quad (l=1)$

$\beta_{1m} = n_1 k_0 \left[1 - \frac{(1+2m)^2}{M} \cdot 0.0068\right]$

$= 1.47 k_0 \left[1 - \frac{(1+2m)^2}{76.96} \times 0.0068\right]$

P9. $\cancel{V = \frac{2\pi a}{\lambda} NA}$. $V = \frac{2\pi a}{\lambda_0} NA$

$\cancel{= 0.1 \cdot 2\pi \frac{a}{0.87} = 5000}$ $\qquad M = \frac{4}{\pi^2} V^2 = 5000$

$\cancel{a =}$

a) i.e. $\frac{4\pi^2 a^2}{\lambda_0^2} NA^2 \cdot \frac{4}{\pi^2} = 5000$

i.e. $\frac{16}{\lambda_0^2} a^2 NA^2 = 5000$

$a^2 = \frac{5000 \lambda_0^2}{16 NA^2}$

$a = \frac{\sqrt{5000}}{4} \cdot \frac{\lambda_0}{NA}$

$= 153 \mu m$

b) $\sigma_\tau \approx \frac{L}{c_1} \cdot \left(\frac{\Delta}{2}\right)^2$