Machine Intelligence 1 **Exercise Sheet 12**
WS 2016/17, Obermayer/Böhmer due: 02.02.2017

Junction Trees

Exercise T12.1: Junction trees (tutorial)

(a) What is a *decomposable graph*?

(b) What are *cliques* and *separators*?

(c) What is the *running intersection property*?

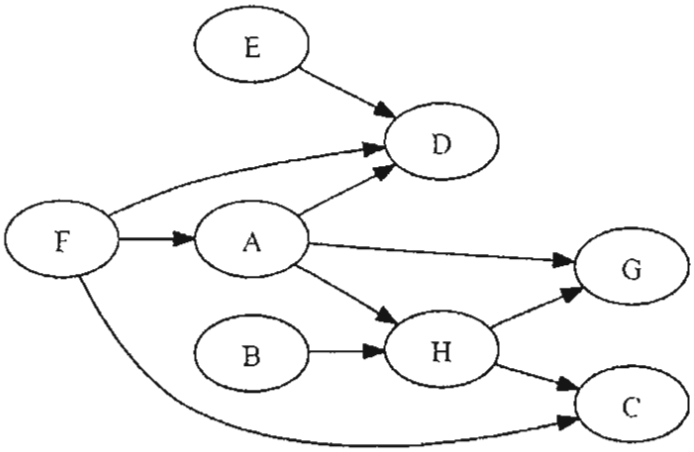
(d) How is a *junction tree* build from cliques and separators?

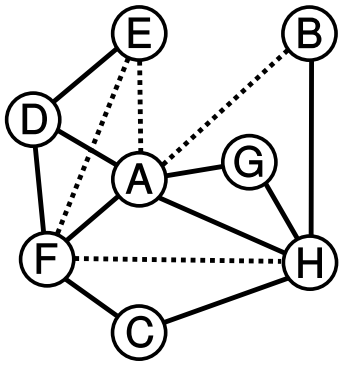
(e) Generate a junction tree from a DAG.

(f) How is evidence introduced to the junction tree?

Exercise H12.1: Construct a junction tree (homework, 4 points)

Given is a DAG (below left) and the corresponding moral graph (below right). The dotted edges in the latter are added during moralization.





(a) (1 point) Determine the cliques of the moral graph.

(b) (1 point) Draw the corresponding bipartite graph of cliques and separators.

(c) (1 point) Construct one possible junction tree from the bipartite graph.

(d) (1 point) Show that the *running intersection property* holds for your tree.

Exercise H12.2: Conditional dependence (homework, 4 points)

Consider three variables, $a, b, c \in \{0, 1\}$, with the following joint distribution:

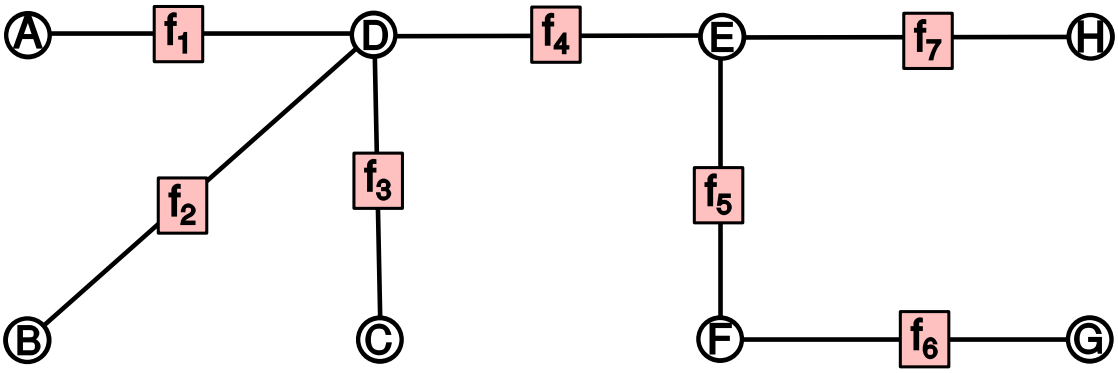
a	b	c	$p(a, b, c)$
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

(a) (2 point) Show by direct evaluation that a and b are *marginally dependent*, i.e. $p(a, b) \neq p(a)\, p(b)$. Describe how you computed this.

(b) (2 point) Show by direct evaluation that a and b are *conditionally independent* given c, i.e. $p(a, b|c) = p(a|c)\, p(b|c)$. Describe how you computed this.

Exercise H12.3: Message Passing (homework, 3 points)

Given is the following junction tree:



(a) (1 point) Describe the order in which messages are generated in the "request", the "collect" and the "distribute" pass, starting at node B.

(b) (1 point) Write out the computation performed for the message $\mu_{f_4 \to D}(D)$, e.g. $\mu_{f_3 \to D}(D) = \sum_C f_3(C, D)$.

(c) (1 point) Write out the message $\mu_{f_4 \to D}(D)$ as in (b), after the evidence $F = f$ and $H = h$ has been observed.

Total 11 points.

Note: This exercise sheet counts as 10 points, but allows you to earn up to 11 points.