MLZ Exercise sheet 11 20.07.2017

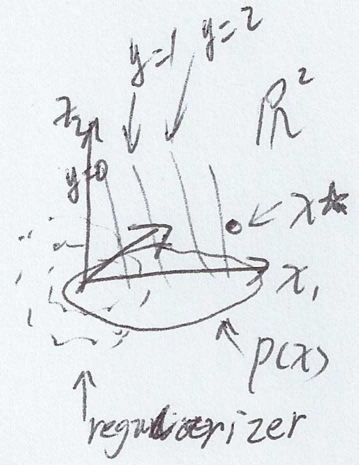
Exercise 1:

$y_i = W_i x + b_i$

$p_i = \frac{e^{y_i}}{\sum_{i'} e^{y_{i'}}}$ $p_i \geq 0 \sum_i p_i = 1$

$\log p_i = y_i - \log \sum \exp y_i$

1.(a) $\max_{x \in \mathbb{R}^d} \underbrace{w^T x + b - \lambda \|x^2\|}_{j(x)}$



$\frac{\partial J}{\partial x} = W - 2\lambda x = 0$

prototype: $x^* = \frac{w}{2\lambda}$

1.(b) $\max_{x \in \mathbb{R}^d} w^T x + b$ ~~flo~~ $p(x)$

$\underbrace{-\frac{1}{2}(y-u)^T \Sigma^{-1} (x-u)}_{J_2}$

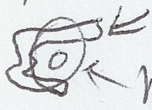
$\frac{\partial J_2}{\partial x} = W - \Sigma^{-1}(x-u) = 0$

$x^* = \Sigma \cdot W + u$

1.(c) $N(0,1)$ $p(x)$

$\boxed{z} \xrightarrow{g} \boxed{x} \rightarrow \boxed{y}$

$p(x)$



$$\max_z \underbrace{w^T(Az+c)+b \cancel{} - \gamma\|z\|^2}_{\cancel{j_3}\ j_3}$$

$$\frac{\partial J_3}{\partial z} = A^T w \cancel{+} 2\gamma z = 0$$

$$z = \frac{A^T w}{2\gamma}$$

$$x^* = \frac{AA^T w}{2\gamma} + c$$

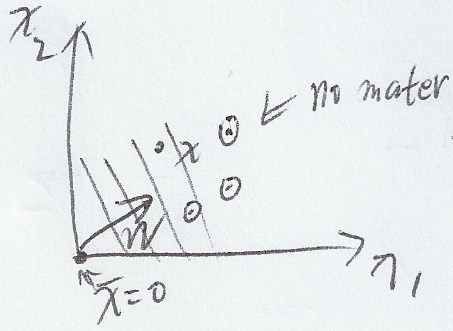
e.g. GAN can learn A, C

generative adversarial networks

Ex2 (a) $f(x) = w^T x$

$$R_i(x) = \left(\frac{\partial f}{\partial x}\Big|_x\right)^2 = \left(w_i|_x\right)^2$$

$$= w_i^2$$



$$\sum_i R_i(x) = \|\nabla_x f|_x\|^2$$

ⓐ→ⓑ

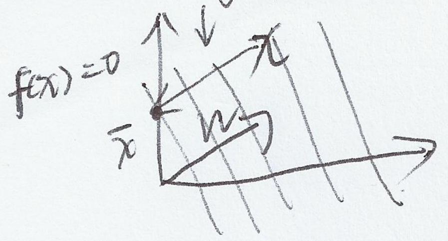
$$\sum_i R_i(x) = f(x)$$

(b) $$f(x) = \underset{\uparrow\atop 0}{\cancel{f(0)}} + \underbrace{\sum \frac{\partial f}{\partial x}\Big|_{x=0}(x_i - 0)}_{R_i(x)} + \underset{\uparrow\atop 0}{\cancel{\varepsilon}}$$

$$= \sum_i w_i x_i$$

$$R_i(x) = w_i x_i$$

Ex 2(c) gradient desent, chose x close to w



$$\begin{cases} w\tilde{x}=0 \\ \tilde{x}=x-tw \end{cases} \qquad \overbrace{w^T\tilde{x}}^{0} = w^Tx + tw^Tw \Rightarrow t = \frac{w^Tx}{w^Tw}$$

$$\tilde{x} = x - \frac{w^Tx}{w^Tw} w$$

$$f(x) = \sum_i \frac{\partial f}{\partial x_i}\Big|_{\tilde{x}} (x_i - \tilde{x}_i)$$

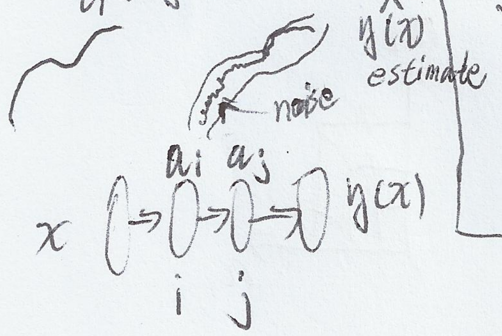
$$= \sum_i w_i \left(x_i - x_i + \frac{w^Tx}{w^Tw} x_i\right)$$

$$= \sum_i \underbrace{\frac{w_i^2}{w^Tw} \cdot w^Tx}_{R_i(x)}$$

$$\sum R_i(x) = \frac{\|w^2\|}{\|w^2\|} \cdot wx = wx$$

$=1$

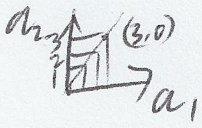
Ex3.



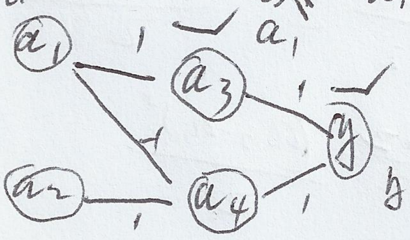
a)

$$R_i = \sum_j \frac{a_i w_{ij}^+}{\sum_i a_i w_{ij}^+} R_j$$

$w_{ij}^+ = \max(0, w_{ij})$



$R_1 = a_1$ $\quad R_3 = a_1$ $\quad$ a_1 is a_3, the same.



$y = \max(a_1, a_2)$

$R_2 = \max(0, a_2 - a_1)$

$\max(a, a_2 - a_4)$

$$R_3 = \frac{a_1 \cdot 1}{\cancel{\max(a_1,a_2)}} \cdot \max(a_1, a_2)$$

Ex3

(b) $R_1 = \max(a_1 - a_2)$?

$R_2 = a_2$

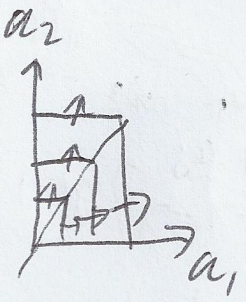
(c) $R_3 = 0.5 \quad a_3 = \frac{(a_1+a_2)}{2}$

$$R_4 = \frac{(a_1-a_2)^+}{2}$$

$$R_5 = \frac{(a_2-a_1)^+}{2}$$

$$R_1 = \frac{a_1 \cdot 1}{a_1 \cdot 1 + a_2 \cdot 1} R_3 + R_4$$

$$R_2 \quad \frac{a_2 \cdot 1}{a_1 \cdot 1 + a_2 \cdot 1} R_3 + R_5.$$



$$R_1 = \frac{a_1}{\cancel{(a_1+a_2)}} \cdot \frac{\cancel{(a_1+a_2)}}{2} + \frac{(a_1-a_2)^+}{2}$$

$$= \frac{a_1}{2} + \frac{(a_1-a_2)^+}{2}$$

$$R_2 = \frac{a_2}{2} + \frac{(a_2-a_1)^+}{2}$$