Exercise 8

RCSKCK #

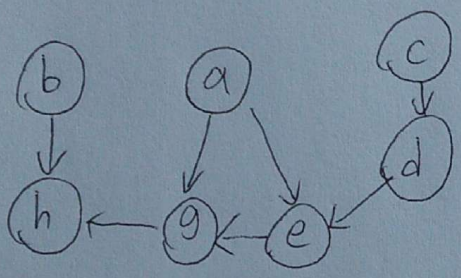
784506, 387470
387219
387215
384418
385402

1.

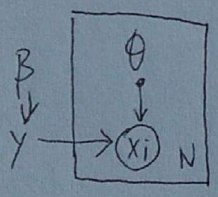
(a) a and b are conditionally independent given the empty set. Check all the path from a to b, they all pass through node g, then the path goes on to c, d, h. If it goes to c, then the arrows meet head-to-head at node g, it blocks the path. If it goes to h (or d), the arrows meet at node d (or h), which also blocks the path.
It means all paths between a and b are blocked, so a and b are conditionally independent given the empty set.

(b) a and b are not conditionally indepedent given h
path $a \to f \to g \to c \to b$, the arrows meet head-to-tail at f and c, neith f nor c is in set h. The arrows meet head-to-head at node g, but g has a descendant h, which is in the given set. The path is unblocked.
So a and b are not conditionally independent given h.

(c)



2. (a)



(b) $$P(y=K|X) = \frac{P(y=k, X)}{P(X)} = \frac{P(y=k)\prod_{i=1}^{d} P(x_i|y=k)}{P(X)}$$

$$= \frac{\beta_k \prod_{i=1}^{d} \theta_{ik}^{x_i}(1-\theta)^{1-x_i}}{P(X)}$$

To maximum $P(y=k|X)$, we have maximize $\beta_k \prod_{i=1}^{d} \theta_{ik}^{x_i}(1-\theta)^{1-x_i}$

3.

Define l : living room
k : kitchen
b : bedroom
o : office

The most likely path from l to l, k, b, o

step1
$l \to l$
$l \to k$
$l \to b$
$l \to o$

step2
$l \to k \to l$
$l \to o \to k$
$l \to k \to b$
$l \to k \to o$

step3
$l \to o \to k \to l$
$l \to k \to o \to k$
$l \to o \to k \to b$
$l \to k \to b \to o$

step4
$l \to k \to o \to k \to l$
$l \to k \to b \to o \to k$
$l \to k \to o \to k \to b$
$l \to o \to k \to b \to o$

step5
$l \to k \to b \to o \to k \to l$
$l \to o \to k \to b \to o \to k$
$l \to k \to b \to o \to k \to b$
$l \to k \to b \to o \to k \to o$

step6
$l \to o \to k \to b \to o \to k \to l$
$l \to k \to b \to o \to k \to o \to k$
$l \to o \to k \to b \to o \to k \to b$
$l \to k \to b \to o \to k \to b \to o$

optial path : $l \to k \to b \to o \to k \to b \to o$