ML2
Exercise sheet 7

Ex1 (a) $$k(x,x') = \sum_{m=1}^{M} \beta_m \sum_{n=1}^{N-m+1} \mathbb{1}(u_{m,n}(x) = u_{m,n}(x')),$$

show $$\sum_{i=1}^{k}\sum_{j=1}^{k} \alpha_i \alpha_j k(x_i, x_j) \geq 0$$

① $$\sum_{i=1}^{k}\sum_{j=1}^{k} \alpha_i\alpha_j \sum_{m=1}^{M} \beta_m \sum_{n=1}^{N-m+1} \mathbb{1}(u_{m,n}(x) = u_{m,n}(x'))$$

② $$\sum_{n=1}^{N-m+1} \mathbb{1}(u_{m,n}(x) = u_{m,n}(x')) = \sum_{s_m} \left(\mathbb{1}(u_{m,n}(x) = s_m) \wedge \mathbb{1}(u_{m,n}(x') = s_m)\right)$$

let $s_m \in \{A,G,C,T\}^m$ is a ~~stri~~ sub string in x or x' with length m

$$= \sum_{s_m} \mathbb{1}(u_{m,n}(x) = s_m) \cdot \sum_{s_m} \mathbb{1}(u_{m,n}(x') = s_m)$$

①,② $\Rightarrow =$ $$\underbrace{\sum_{m=1}^{M} \beta_m}_{\geq 0} \underbrace{\sum_{i=1}^{k} \alpha_i \sum_{s_m} \mathbb{1}(u_{m,n}(x) = s_m)}_{\geq 0} \cdot \underbrace{\sum_{j=1}^{k} \alpha_j \sum_{s_m} \mathbb{1}(u_{m,n}(x) = s_m)}_{\geq 0}$$

$\Rightarrow \geq 0$

b) $$\varphi(x_i) = \sqrt{\beta \sum_{s_m} I(u_{1,n}(x) = s_m)}$$

$$\varphi(x_j) = \sqrt{\beta \sum_{s_m} I(u_{1,n}(x') = s_m)}$$

c) $$\varphi(x_i) = \sqrt{\sum_{s_m} I(u_{2,n}(x) = s_m)}$$

$$\varphi(x_j) = \sqrt{\sum_{s_m} I(u_{2,n}(x') = s_m)}$$