Time Series Analysis

Axel Werwatz
Franziska Plitzko

Technische Universität Berlin
Institut für Volkswirtschaftslehre und Wirtschaftsrecht
FG Ökonometrie und Wirtschaftsstatistik

Organizational Issues

Axel Werwatz **`axel.werwatz@tu-berlin.de`**

Office hours: Tuesday 12:15 - 13:30 H 5103C

Lecture: Friday 12:15 - 13:45 H 0106

Franziska Plitzko **`franziska.plitzko@tu-berlin.de`**

Office hours: appointments via Mail

Tutorial (Ü): Thursday 8:30 - 10:00 **TEL 206_rechts**

occasionally EW 202

Next week (26.10.): Introduction to STATA

Check the homepage and the FAQ-Site!

www.statistik.tu-berlin.de/menue/home/

TEL TU Hochhaus, Ernst-Reuter-Platz 7

Course web page:

https://www.isis.tu-berlin.de/

- Fakultät VII

- Institut für Volkswirtschaftslehre und Wirtschaftsrecht

- Time Series Analysis WS17/18

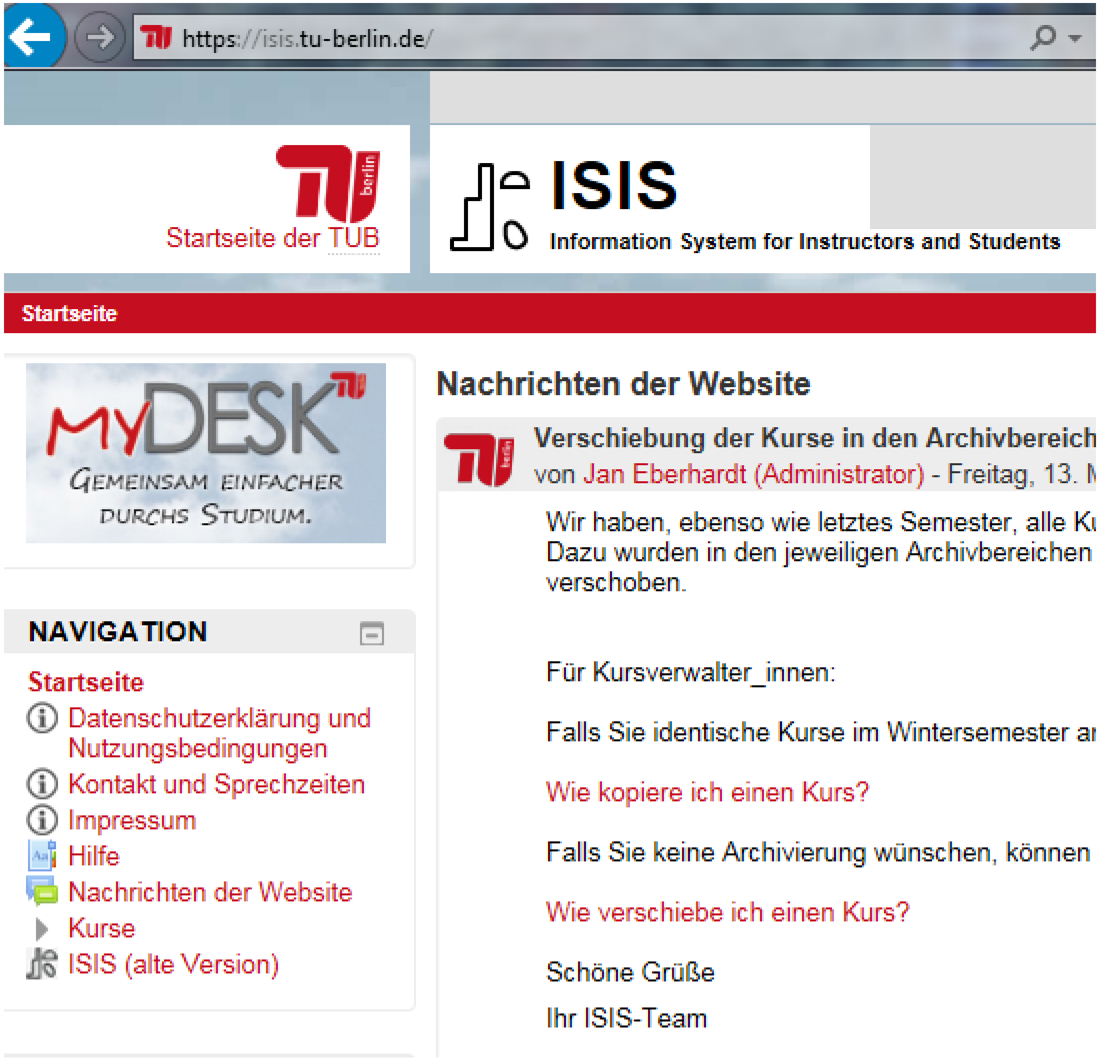
Password: Zeit1718

Proposed examination date:

23.2.2018 **A151** (first week of semester break)

12:00-14:00

ISIS screen shot



Registration for the exam:

TU STUDENTS:

1. Write down the asked information into the JOURNAL ("Registration for exam at our institute") on the ISIS-System

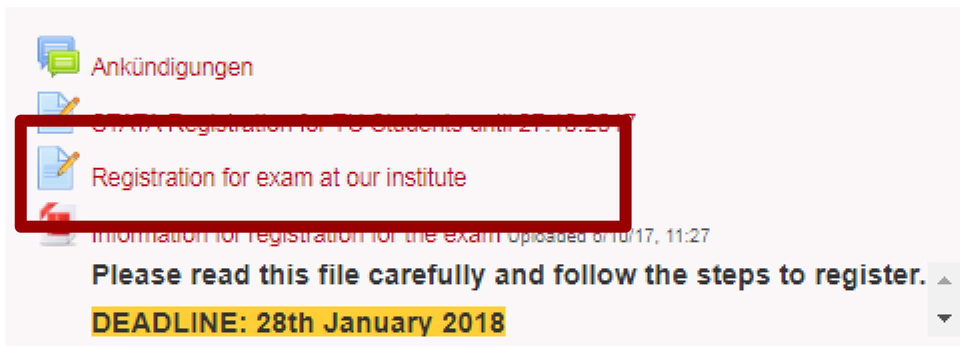
2. Register via QISPOS or "Prüfungsamt". QISPOS registration is possible from **16th October 2017 till 28th January 2018**. You can withdraw from the exam until the day before the actual exam via QISPOS.

   Yellow Sheets ("Gelbe Zettel") from the Prüfungsamt have to be given to our secretary Carola Haring (or the mailbox in front of our office H5103) until **28th January 2018**. We won´t accept those later on!

Organizational Issues

Registration for the exam:

Follow the instruction in the Journal on ISIS!



Organizational Issues

Make-up exam:

There is (potentially) a make-upexam at the end of the semester break. It is **only for those**

- who failed the exam in February
- or who were sick at the first and have a **medical certificate** (Attest) – please sent the original to the Prüfungsamt within the given deadline of your field of study and a scan or copy to our secretary (carola.haring@tu-berlin.de).

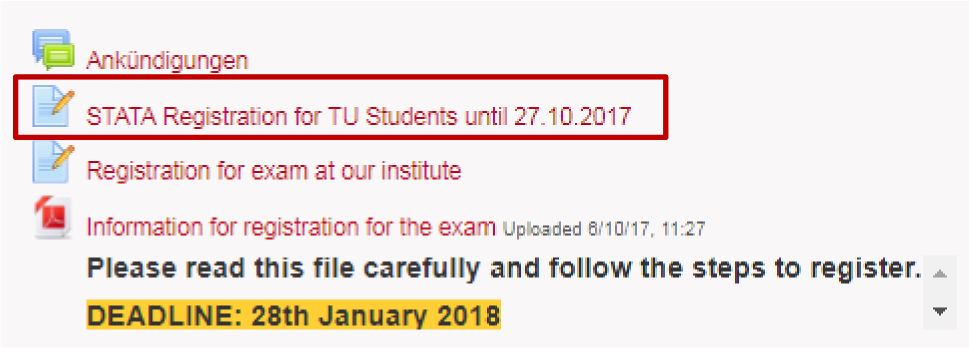
People who simply didn´t show up at the first exam are not eligible for the make-up exam.

Registration for the exam for guests:

E-Mail to **Franziska Plitzko** (franziska.plitzko@tu-berlin.de) **not later than 28.01.2018** including:

- course: Time Series Analysis
- last name,
- given name,
- student ID number (Msc. Statistics students **HU** and (if existent) TU no.),
- name of degree program (e.g. Wirtschaftsingenieurwesen, ...),
- aspired degree (e.g. Bachelor, Master, Diplom, PhD, ...),
- university

Please sign up for STATA!



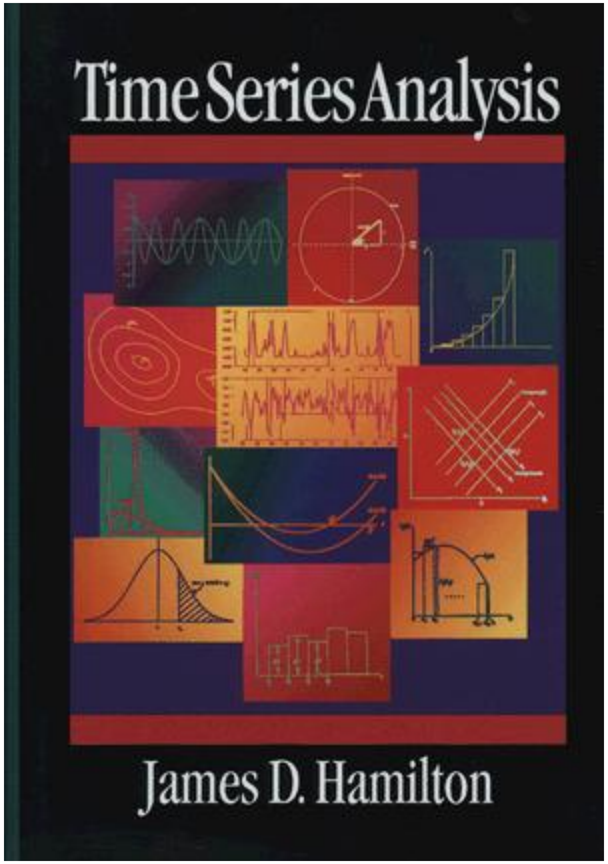
Lehrveranstaltungen - Übersicht

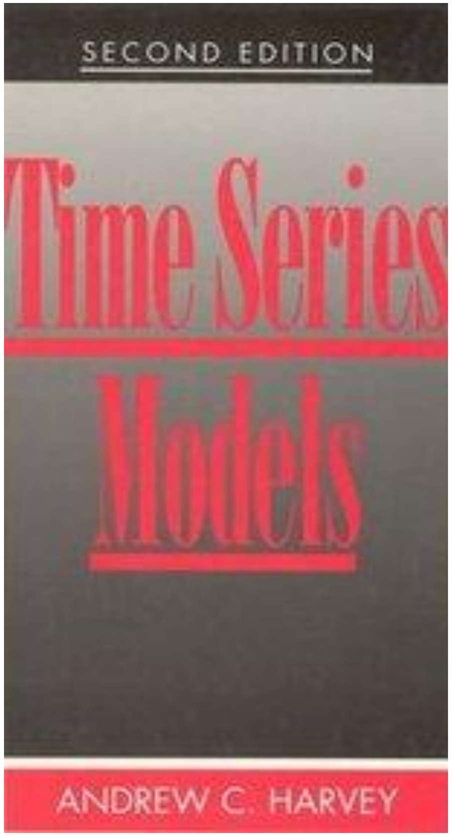
	Lehrveranstaltung	SWS	ECTS	WS/SS	Bachelor BE	Bachelor Wi-Ing	Master MINE	Master Wi-Ing	Ph.D.
Grundlagen Bachelor	Statistik I (IV +TUT)	6	6	WS/SS	Pflicht				
	Statistik II (IV +TUT)	6	6	WS/SS	Pflicht	Wahl-pflicht			
	Einführung in die Statistik (IV +TUT)	6	6	WS/SS		Pflicht			
	Ökonometrie (VL+Ü)	4	6	WS	Pflicht	Wahl-pflicht			
	Seminar „Angewandte Ökonometrie“	2	6	SS	Wahl-pflicht	Wahl-pflicht			
Vertiefungen Master / Ph.D.	Microeconometrics (VL+Ü)	4	6	WS			Pflicht	Wahl-pflicht	
	Treatment Effect Analysis (VL+Ü)	4	6	SS			Wahl-pflicht	Wahl-pflicht	Wahl
	Longitudinal- and Panel Data (VL+Ü)	4	6	WS			Wahl-pflicht	Wahl-pflicht	Wahl
	Seminar „Produktivität, Innovation und Firmenerfolg“	2	6	SS			Wahl-pflicht	Wahl-pflicht	Wahl
	Time Series Analysis (VL+Ü)	4	6	WS			Wahl-pflicht	Wahl-pflicht	
	Financial Econometrics (VL+Ü)	4	6	SS			Wahl-pflicht	Wahl-pflicht	
	Multivariate Analysis / Business Statistics (VL+Ü)	4	6	SS			Wahl-pflicht	Wahl-pflicht	
	Engineering Statistics (VL+Ü)	4	6	WS			Wahl-pflicht	Wahl-pflicht	
	Studienprojekt	4	12	WS			Wahl-pflicht		

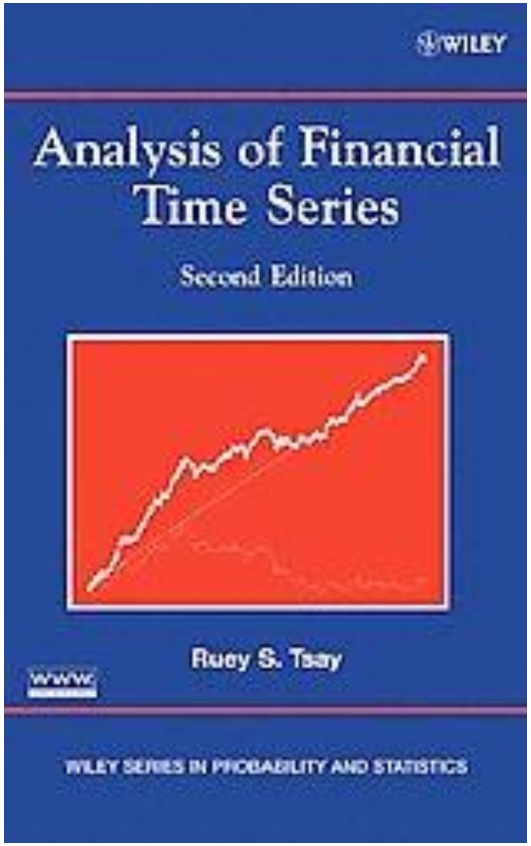
References

- *Schlittgen, Streitberg*: Zeitreihenanalyse (5. Aufl.)
- *Harvey*: Time Series Models
- *Harvey*: Forecasting, Structural Time Series Models and the Kalman Filter
- *Pindyck, Rubinfeld*: Econometric Models and Economic Forecasts (2. Aufl.)
- *Kirchgässner, Wolters*: Einführung in die moderne Zeitreihenanalyse

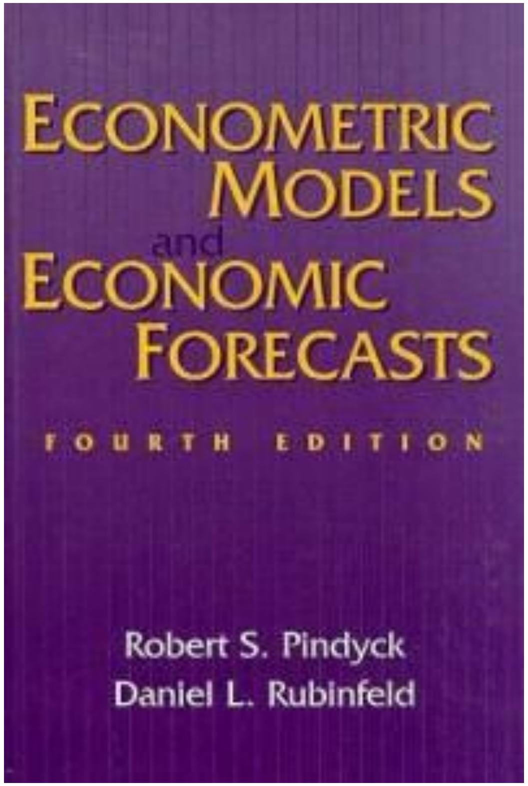
Literature

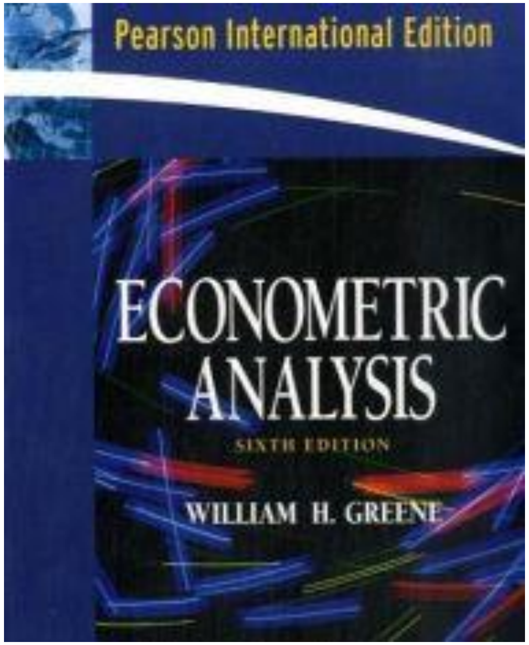

Time Series Analysis
James D. Hamilton

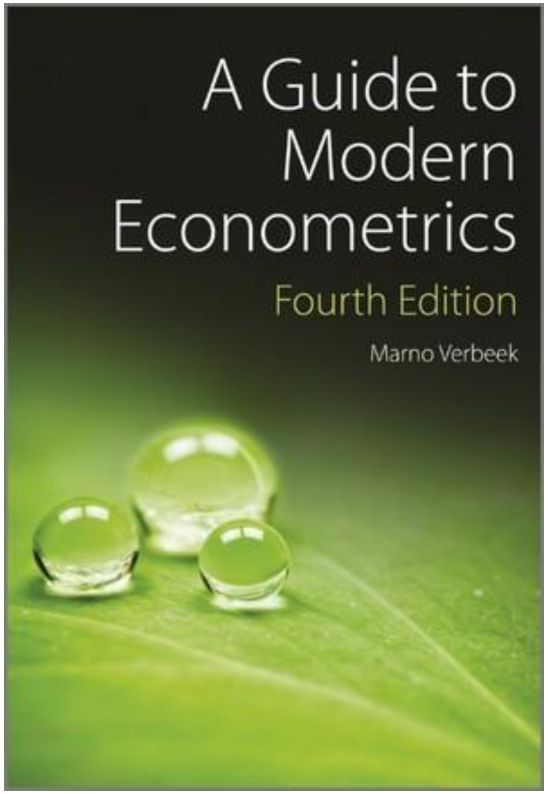

SECOND EDITION
Time Series
Models
ANDREW C. HARVEY


WILEY
Analysis of Financial
Time Series
Second Edition
Ruey S. Tsay
WILEY SERIES IN PROBABILITY AND STATISTICS

Literature


ECONOMETRIC MODELS and ECONOMIC FORECASTS
FOURTH EDITION
Robert S. Pindyck
Daniel L. Rubinfeld


Pearson International Edition
ECONOMETRIC ANALYSIS
SIXTH EDITION
WILLIAM H. GREENE


A Guide to Modern Econometrics
Fourth Edition
Marno Verbeek

Literature


Schlittgen · Streitberg
Zeitreihenanalyse
9. Auflage
Lehr- und Handbücher der Statistik
Oldenbourg


WiSo
Kurzlehrbücher
Gebhard Kirchgässner
Jürgen Wolters
Einführung in
die moderne
Zeitreihenanalyse
Verlag Vahlen

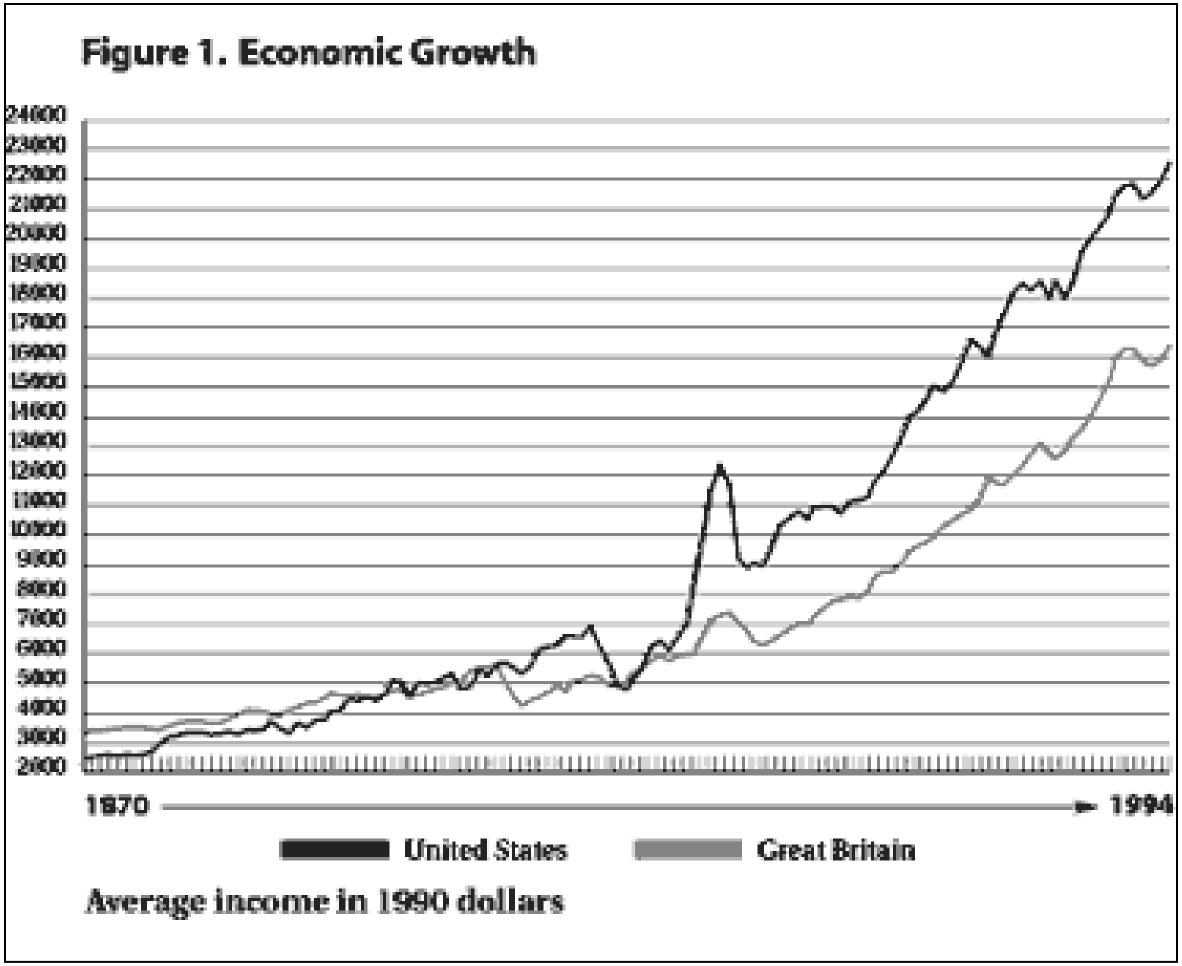
Schedule

Deter-ministic Models	• Components of a Time Series • Additive and Multiplicative Models • Simple Trend Models • Smoothing Techniques • Seasonal Adjustment

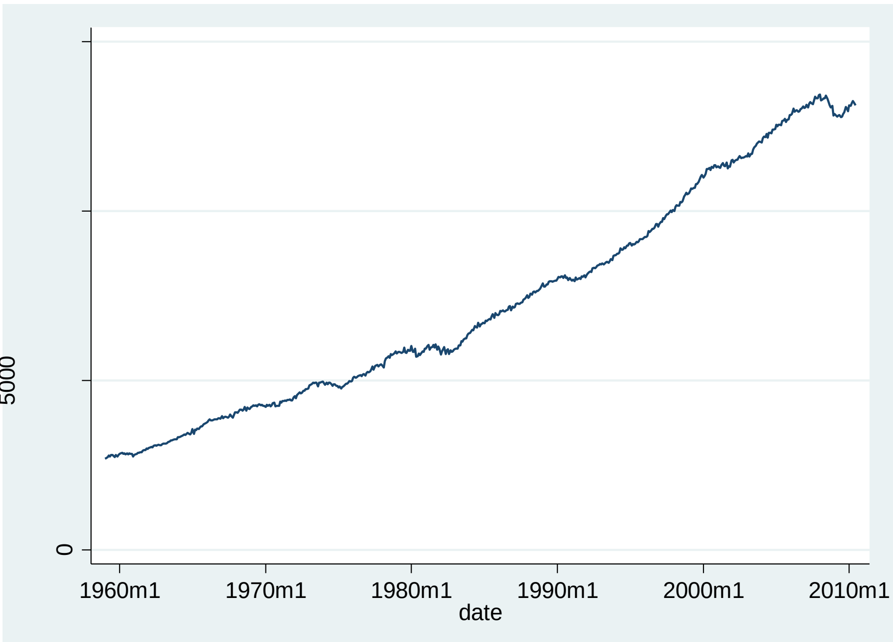
Stationary Stochastic Processes	• Introduction
	• Identification
	    • Autocorrelation Function
	    • Moving Average and Autoregressive Models
	    • Partial Autocorrelation Function
	    • ARMA Models
	• Estimation
	• Diagnostic Checking
	• Forecasting

Non-stationary Stochastic Processes	• Introduction • Nonstationarity and Trends • ARIMA Models • Unit Root Tests • Seasonal ARIMA

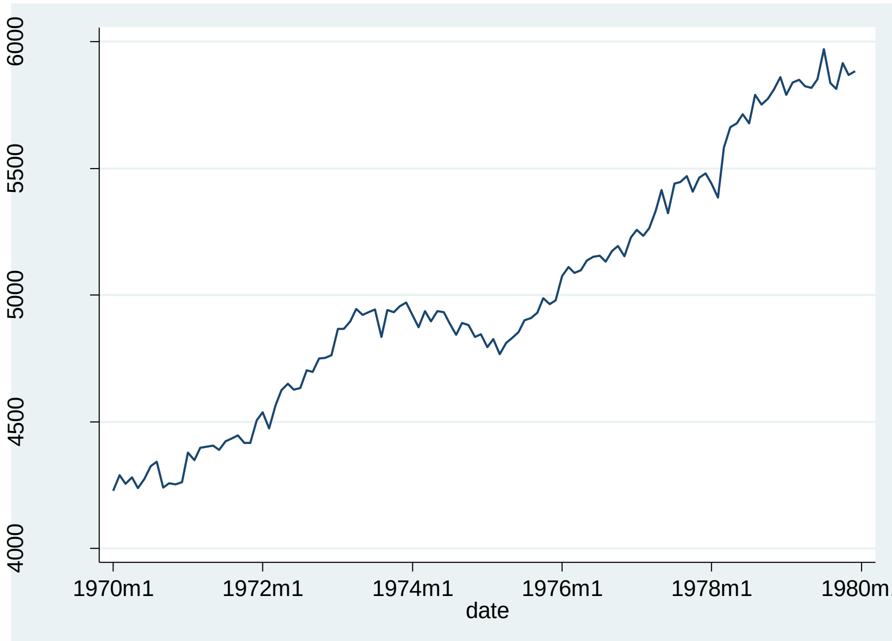
Time Series Analysis: **Dynamics!!**



Dynamics has several aspects!



monthly U.S. real GDP
Jan 1960 – June 2010

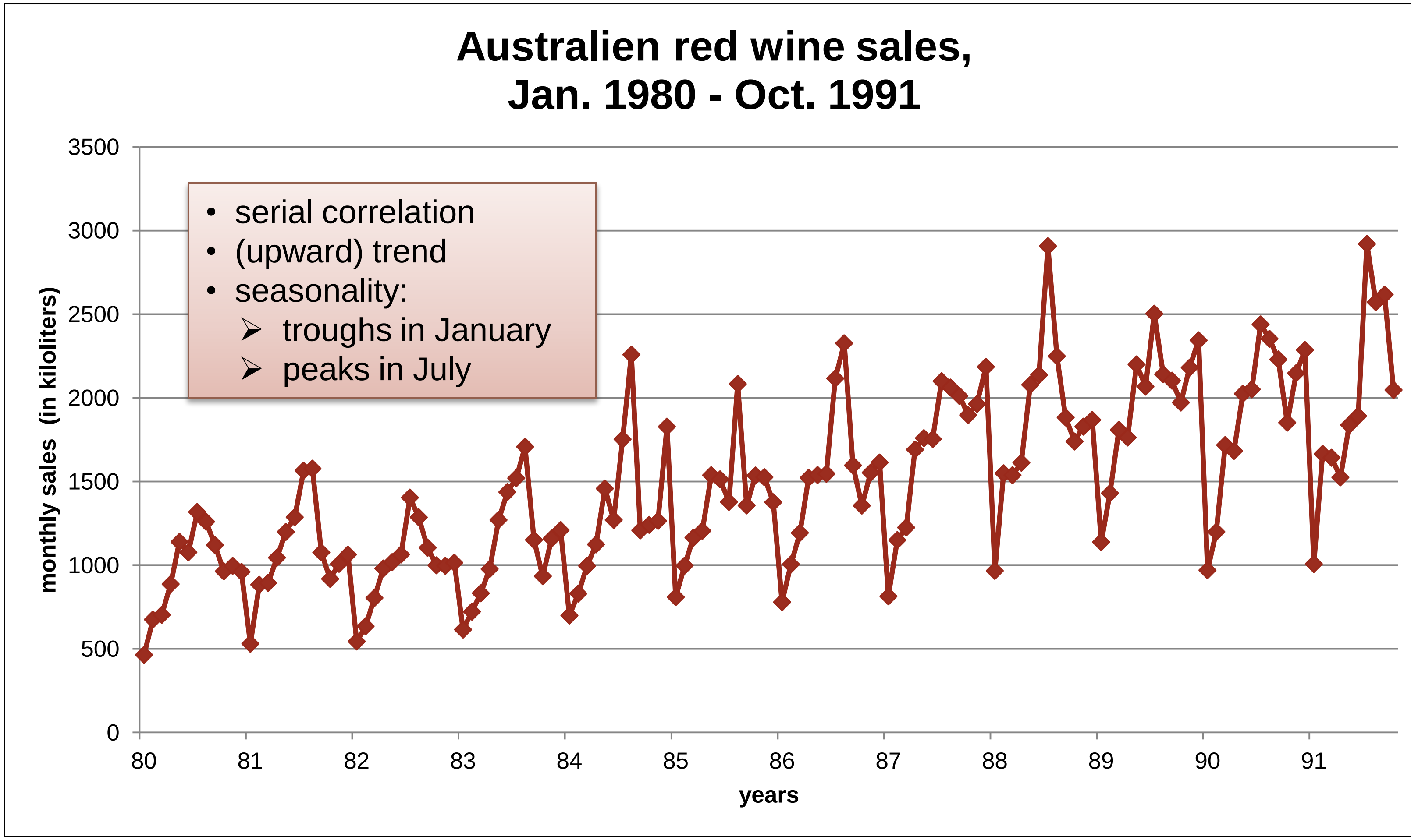


monthly U.S. real GDP
Jan 1970 – Dec 1979

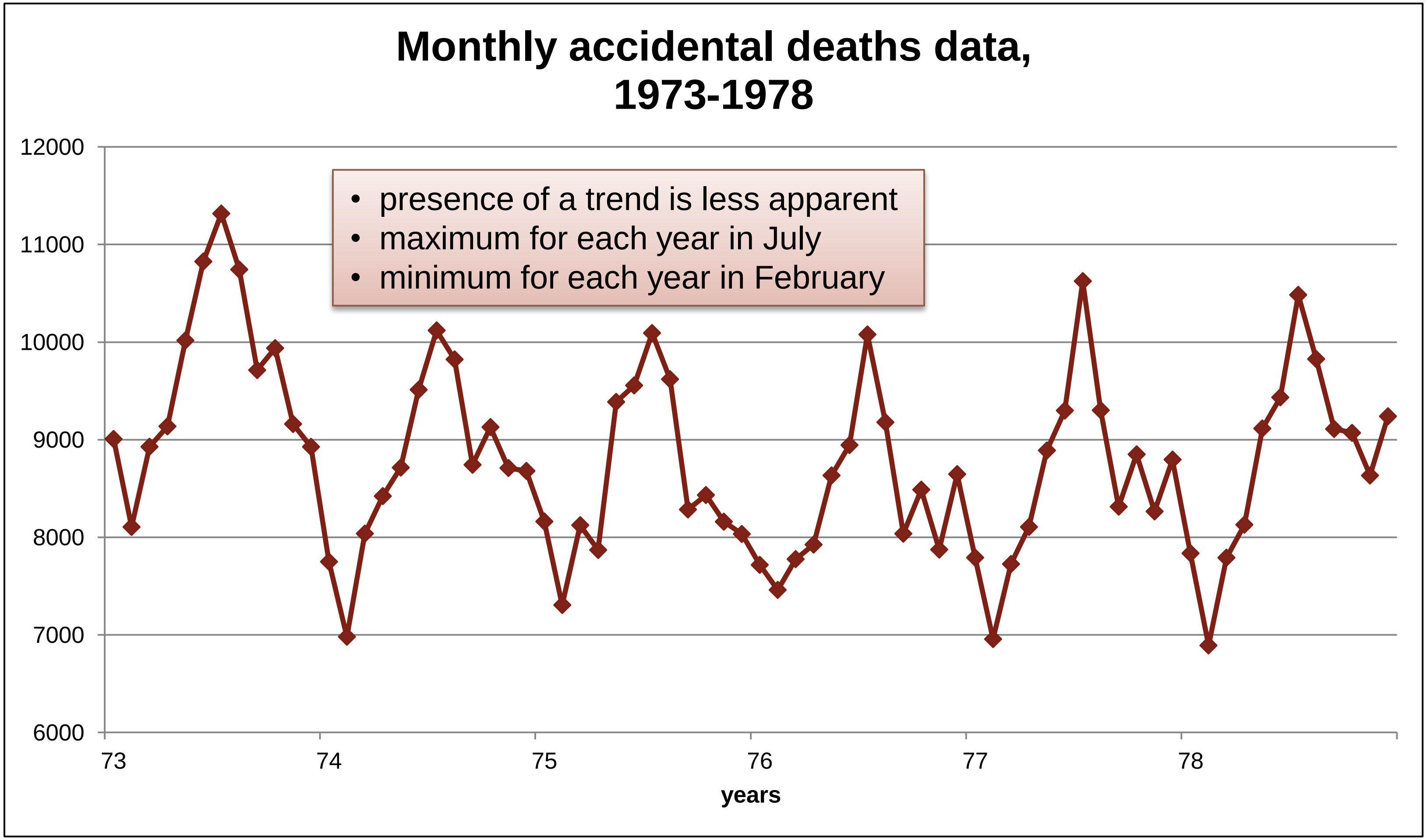
Motivation for Univariate Time Series Analysis

To obtain a forecast for a variable y_t from a regression equation may result in large forecast errors, when the future values of the explanatory are unknown and they have themselves large forecast errors.

In this situation it may be easier to forecast the variable y_t itself. Therefore <u>the prediction is solely based on the past behavior of the variable</u> by constructing an model for the time series which replicate its past behavior in a way that might help to forecast its future behavior.

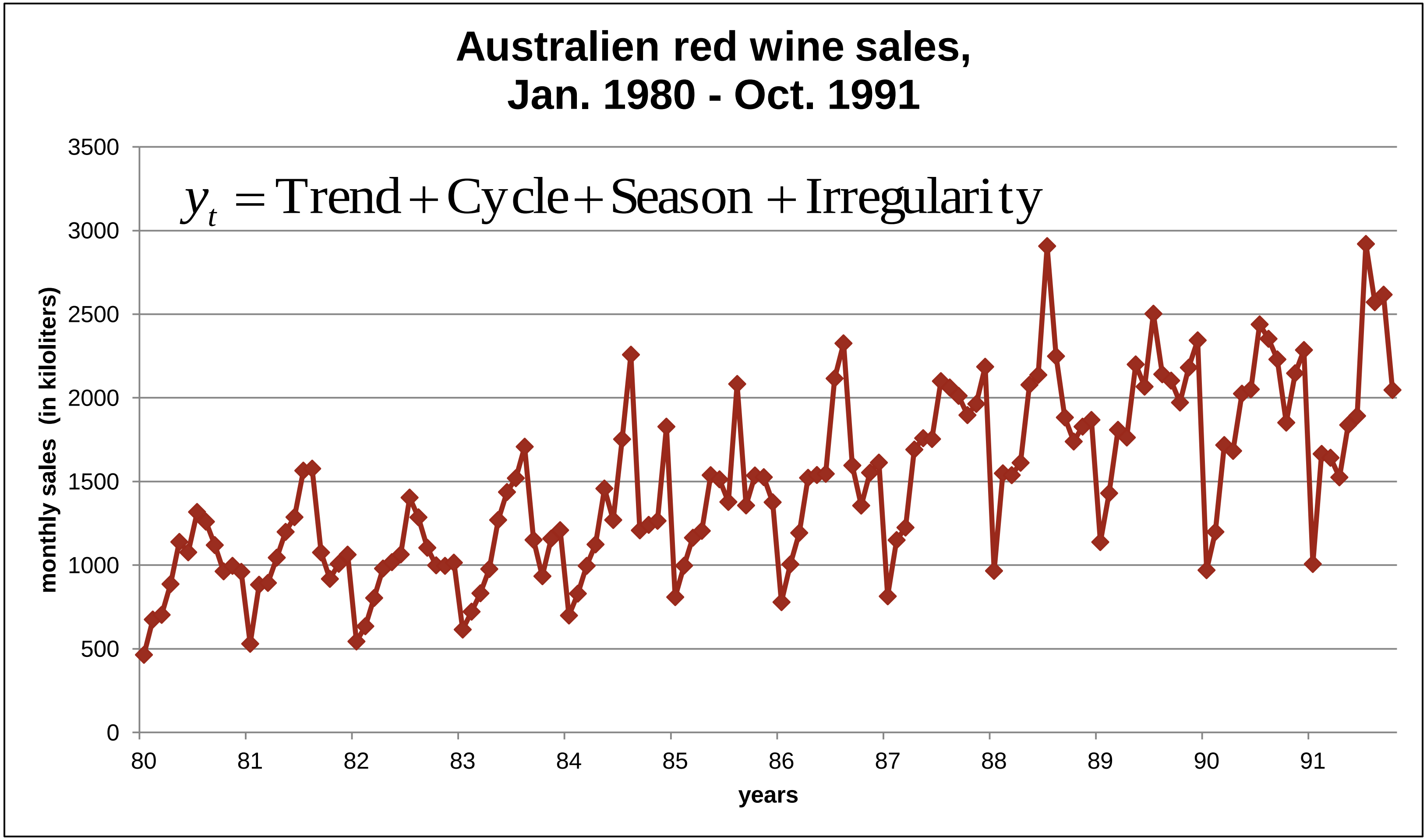


Brockwell/Davis (1996) “Introduction to Time Series and Forecasts”



Brockwell/Davis (1996) “Introduction to Time Series and Forecasts”

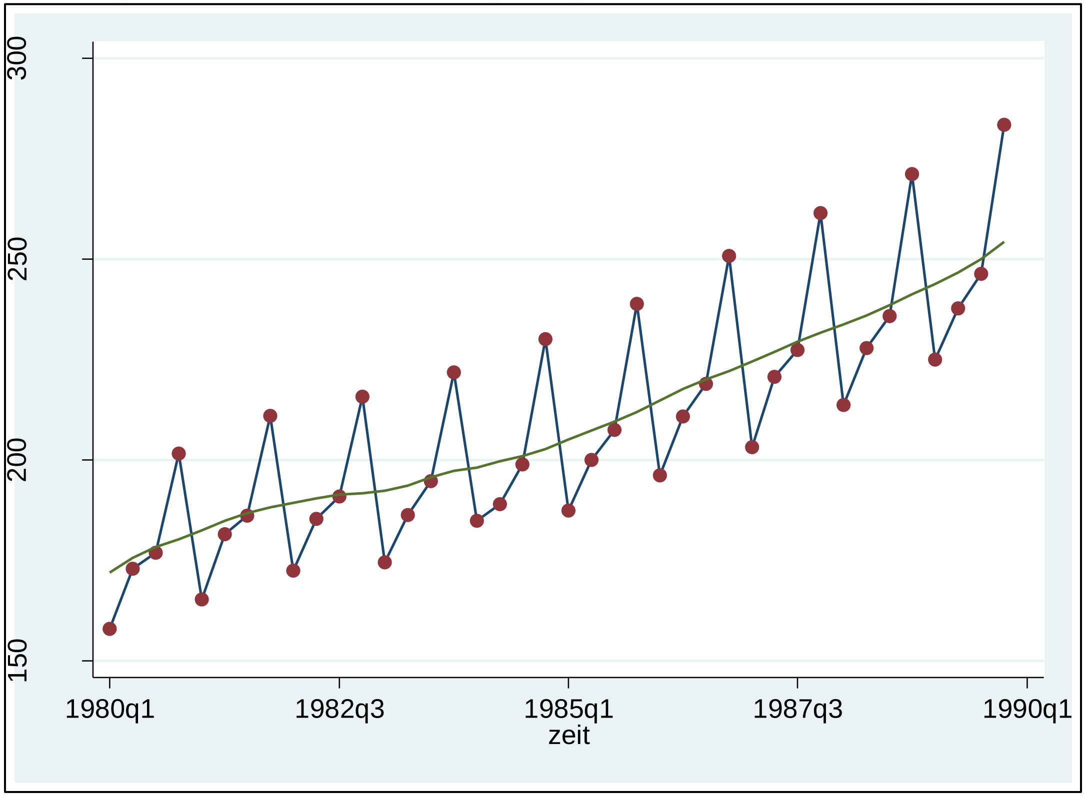
Introduction – Deterministic Time Series Analysis



Brockwell/Davis (1996) “Introduction to Time Series and Forecasts”

Smoothing/filtering via moving average (of order 4)

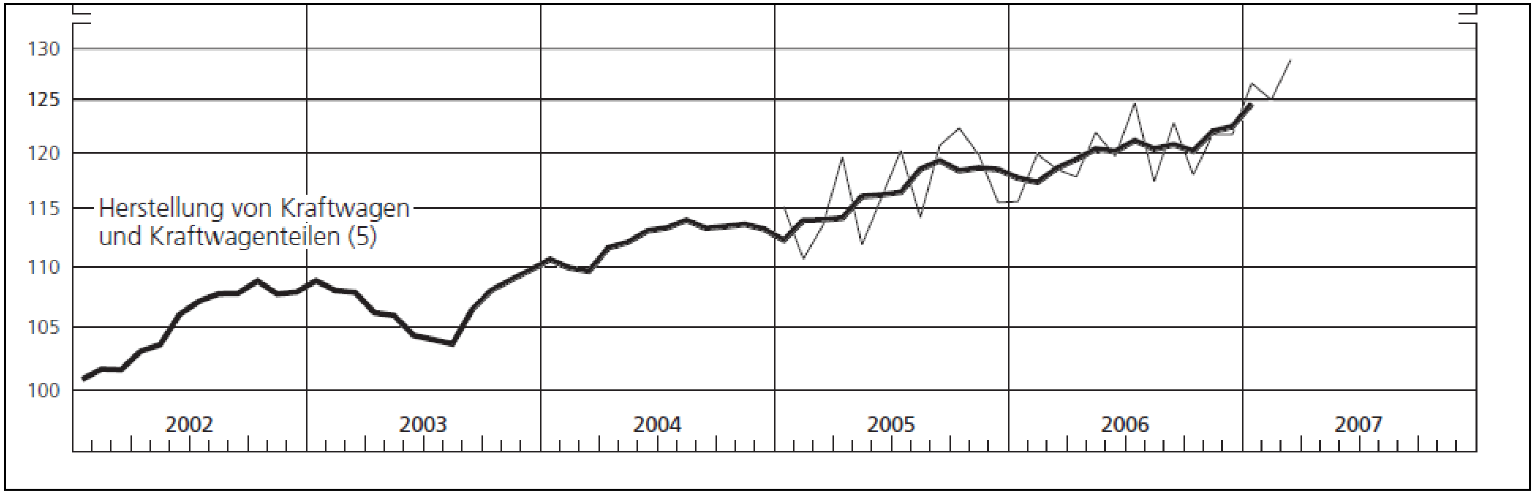
$$\tilde{y}_t = \frac{1}{2} \cdot \left[\frac{1}{4} \left(y_{t+1} + y_t + y_{t-1} + y_{t-2} \right) + \right] + \frac{1}{2} \cdot \left[\frac{1}{4} \left(y_{t+2} + y_{t+1} + y_t + y_{t-1} \right) \right]$$



Moving Averages

„Zur deutlicheren Kennzeichnung der **konjunkturellen** Entwicklung sind in den Schaubildern in der Regel neben saisonbereinigten Monatswerten daraus errechnete **gleitende Durchschnitte** dargestellt; die Zahl der in die Berechnung einbezogenen Werte ist an der jeweiligen Kurve (in Klammern) angegeben.“

Deutsche Bundesbank (Mai 1997), Saisonbereinigte Wirtschaftszahlen, S. 86



Deutsche Bundesbank (Mai 1997), Saisonbereinigte Wirtschaftszahlen, S. 35

Univariate **Box-Jenkins models** for stationary time series

Key: time series is a realization of a **stochastic process**

Which process? B+J: **ARMA model**

How can we find the right ARMA model?

General Procedure:

1. Identification
2. Estimation
3. Diagnostic Checking
4. Forecasting

Box-Jenkins models for **stationary** time series…

What is stationary?

Loosely speaking: “stable” stochastics

Looked at from t=0, process generating series has

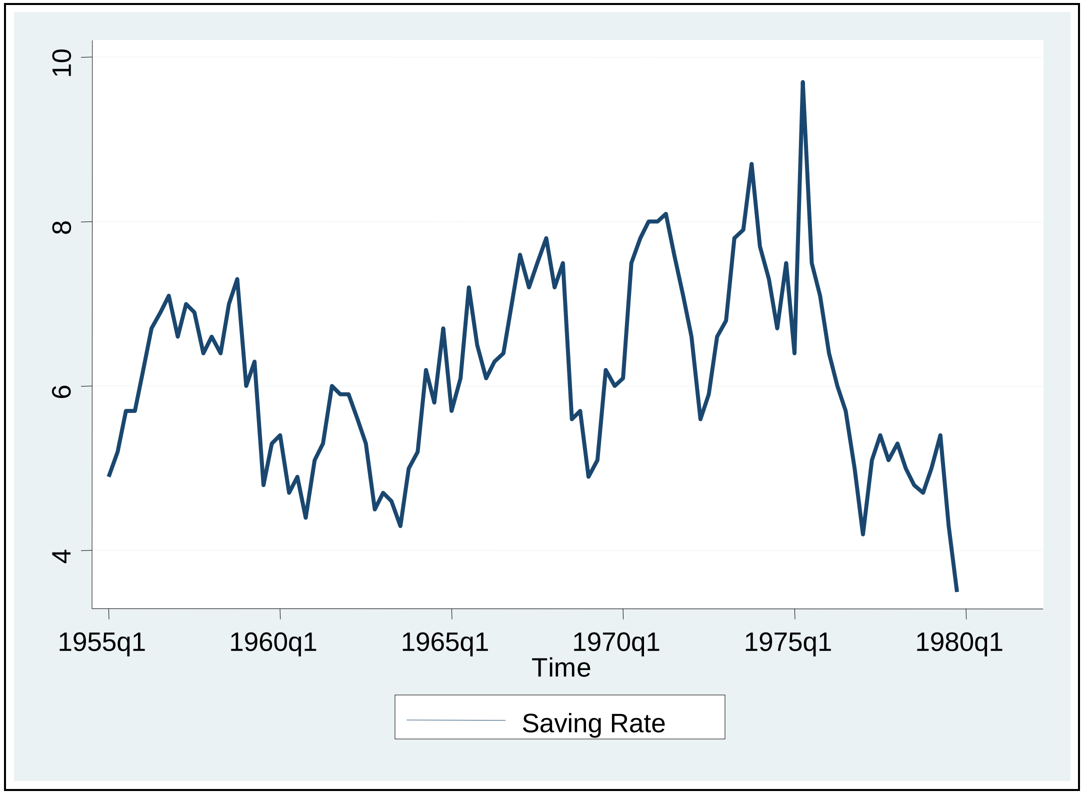
$$\mu_y = E(y_t) = E(y_{t+m})$$

$$\sigma_y^2 = E\left[(y_t - \mu_y)^2\right] = E\left[(y_{t+m} - \mu_y)^2\right]$$

$$\gamma_k = Cov(y_t, y_{t+k}) = E\left[(y_t - \mu_y)(y_{t+k} - \mu_y)\right] = Cov(y_{t+m}, y_{t+m+k})$$

for any t, k, and m

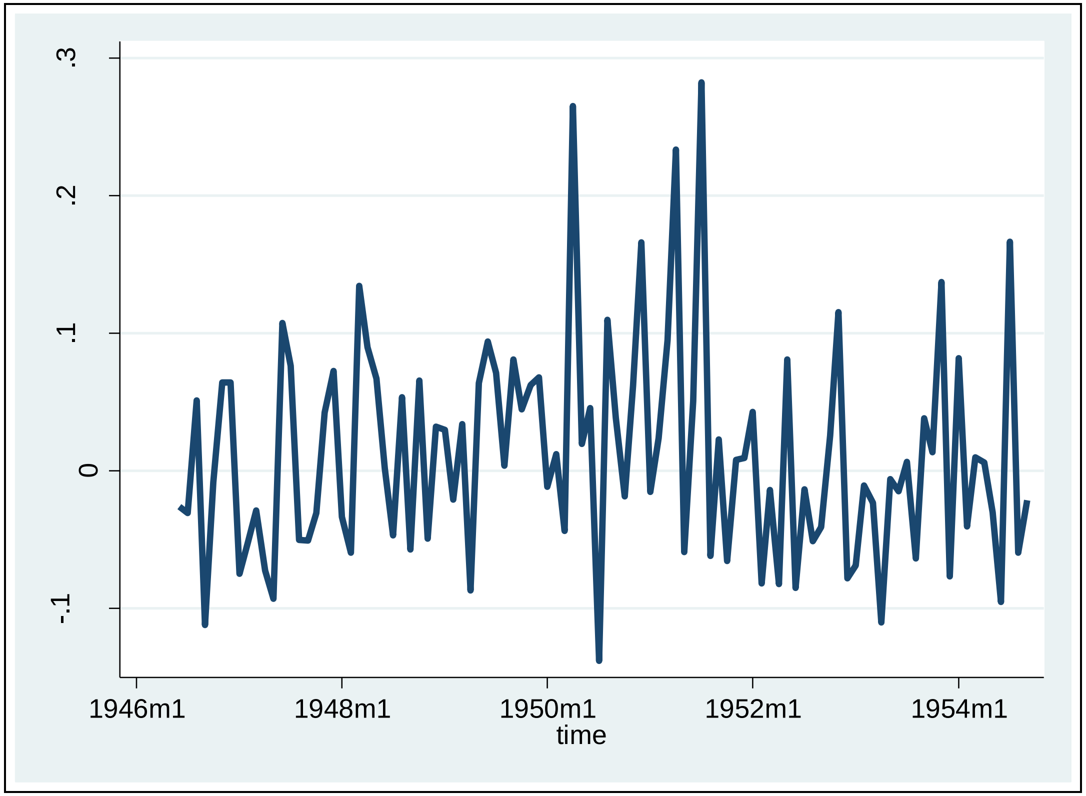
Example: 100 quarterly observations of the US saving rate for years 1955-1979



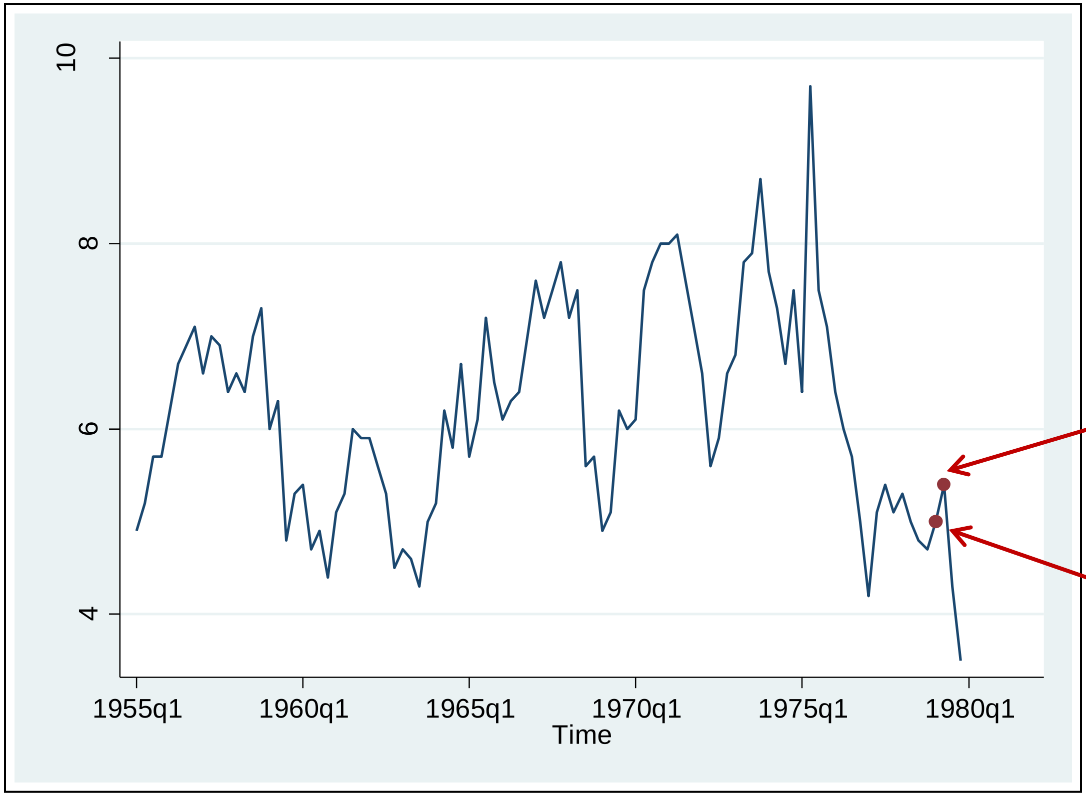
Note that the data are seasonally adjusted prior to publication by the U.S. Department of Commerce.

Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

Example: 100 monthly observations of simple returns of Merck stock for years 1946-1954



Example: quarterly US saving rate from 1955-1979



Are adjacent obs. related?

1979q2: $y_{98} = 5.4$

1979q1: $y_{97} = 5$

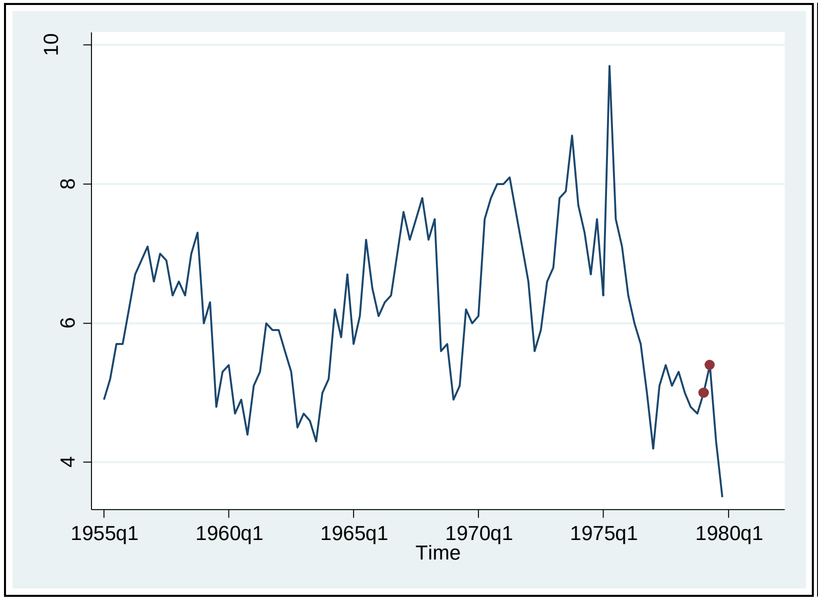
Dependency? → Correlation

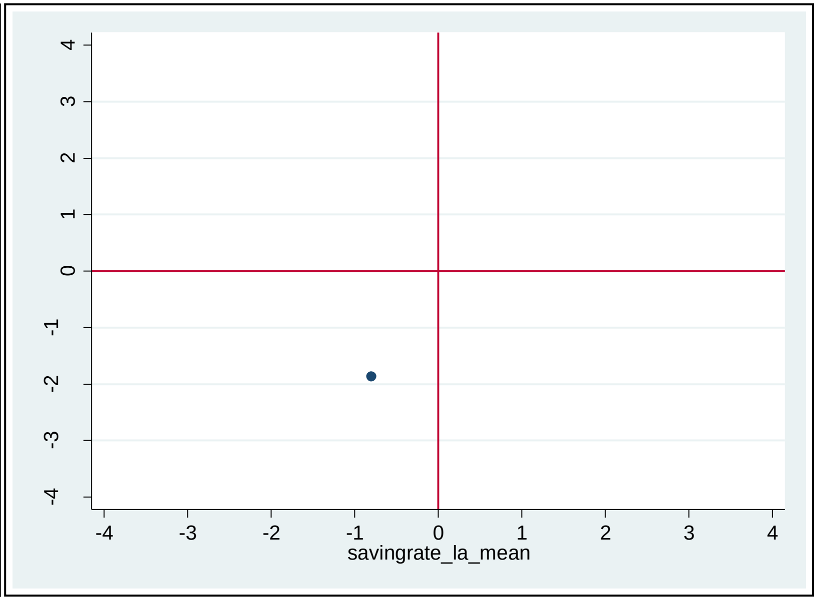
$$\hat{\rho} = \frac{\sum_{t=1}^{T-1} \left(y_t - \bar{y}\right)\left(y_{t+1} - \bar{y}\right)}{\sum_{t=1}^{T-1} \left(y_t - \bar{y}\right)^2}$$

Correlation of time series with its own past

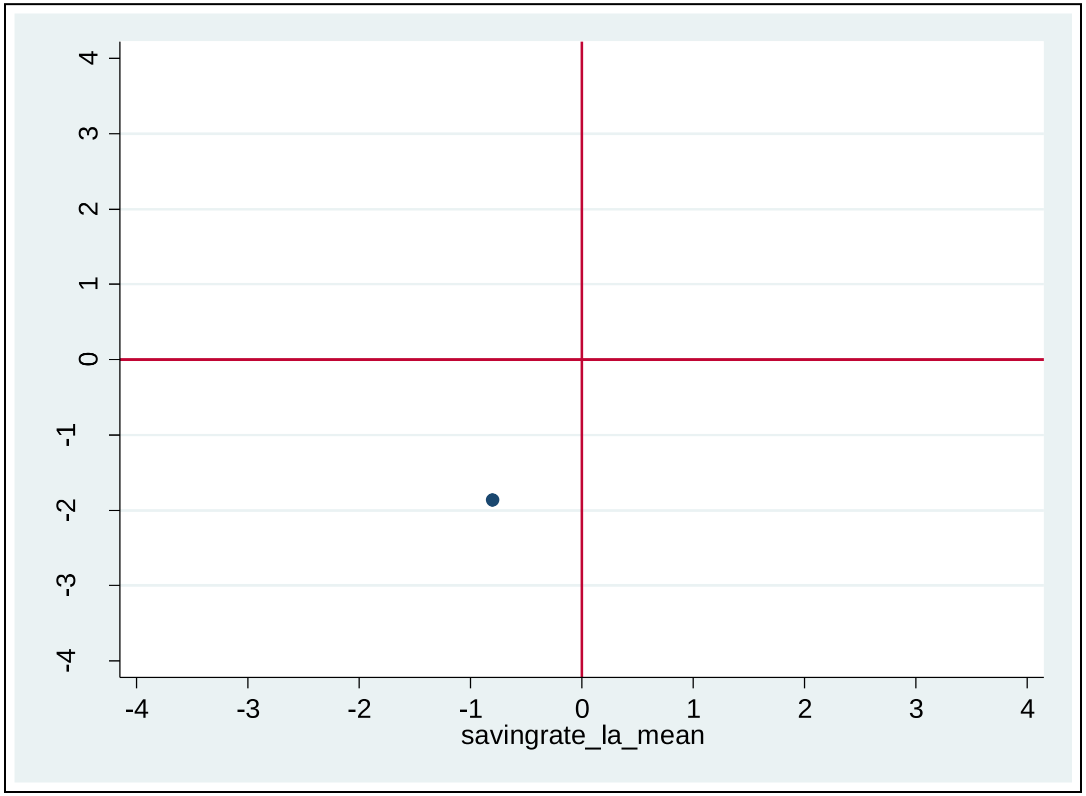
→ **Autocorrelation**

Example: quarterly US saving rate from 1955-1979

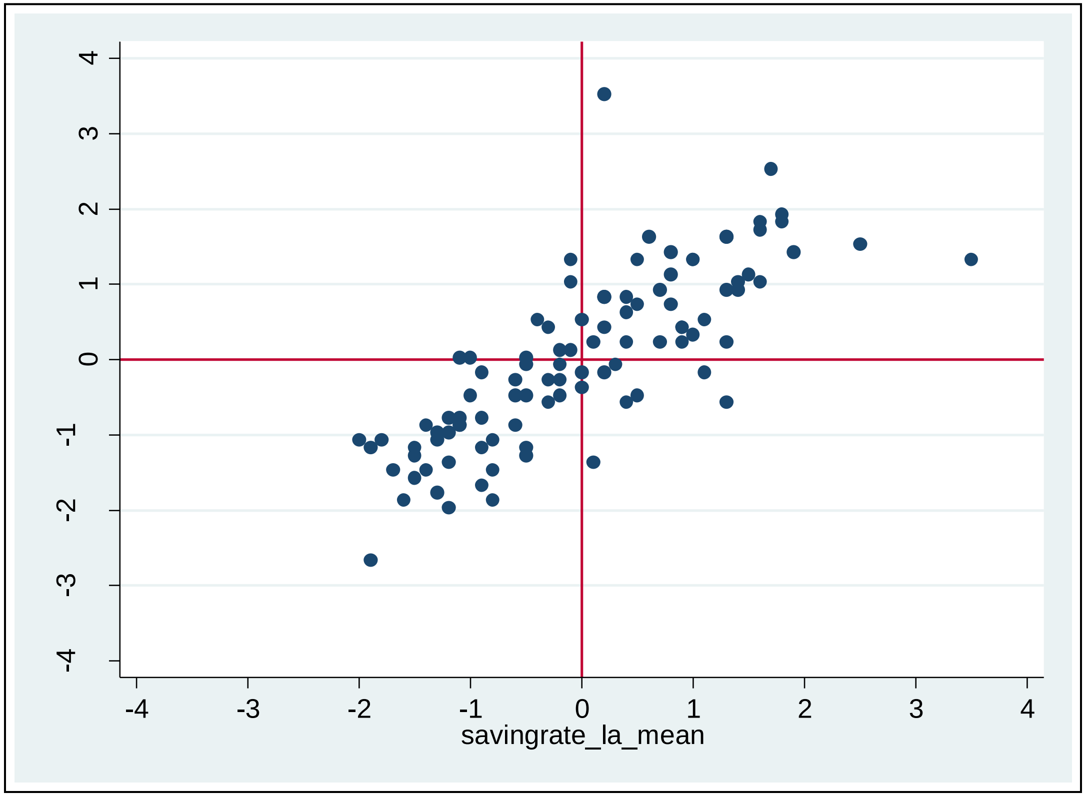




Example: quarterly US saving rate from 1955-1979



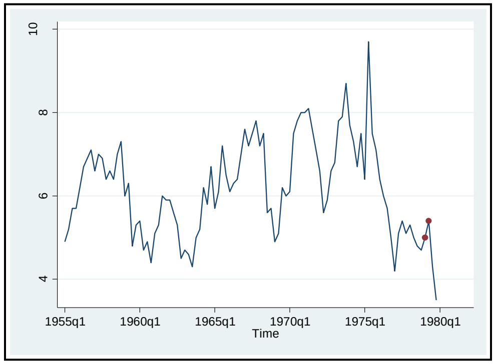
Example: quarterly US saving rate from 1955-1979

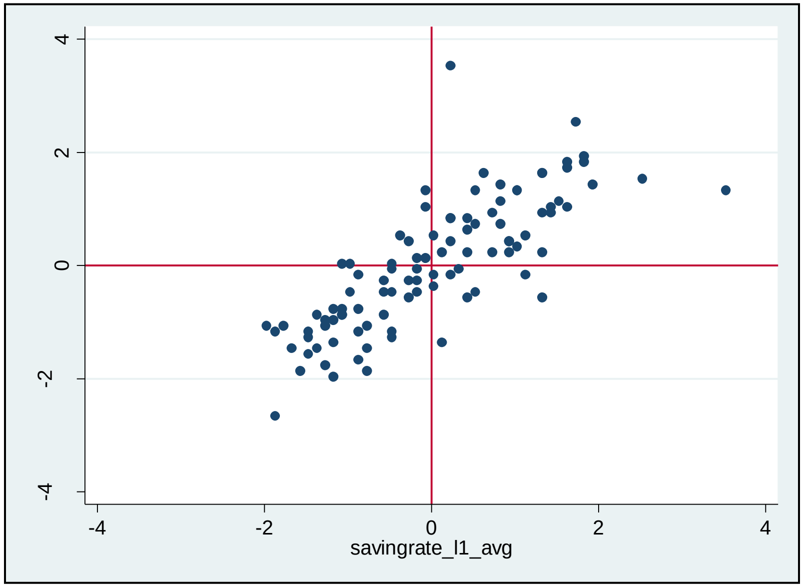


$$\hat{\rho} = \frac{\sum_{t=1}^{T-1} (y_t - \bar{y})(y_{t+1} - \bar{y})}{\sum_{t=1}^{T-1} (y_t - \bar{y})^2}$$

$$= 0.77$$

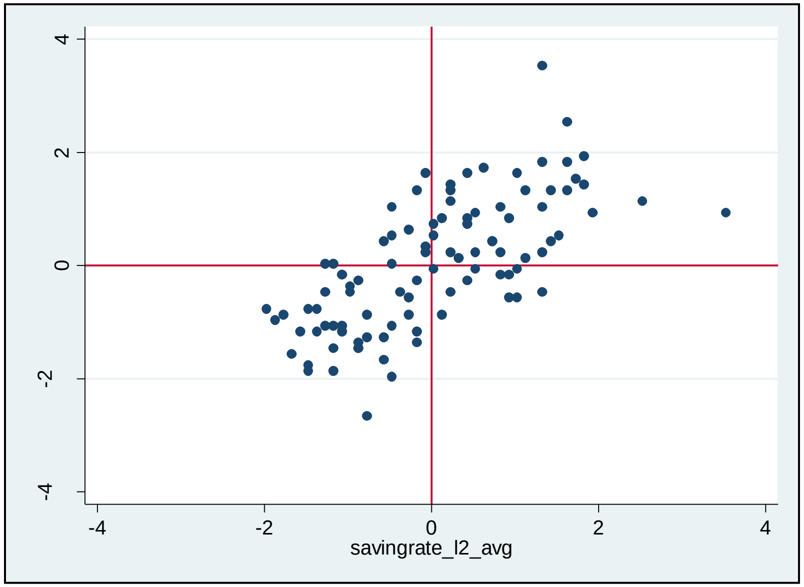
Example: Scatterplots





1 period apart

$\hat{\rho}_1 = 0.77$



2 periods apart

$\hat{\rho}_2 = 0.65$

Sample Autocorrelation Function with lag *k*:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{T-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{T-k}(y_t - \bar{y})^2}$$

A function of k

ACF: description of dependency structure in the series

ACF is key tool for finding a suitable time series model

Compare sample ACF with theoretical ACF of a model

Note: ACF is <u>symmetrical</u> and so ρ_k is plotted only for different <u>positive</u> values of k.

Sample Autocorrelation Function with lag *k*:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{T-k} (y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{T-k} (y_t - \bar{y})^2}$$

A function of k

Note how ACF hinges on stationarity:

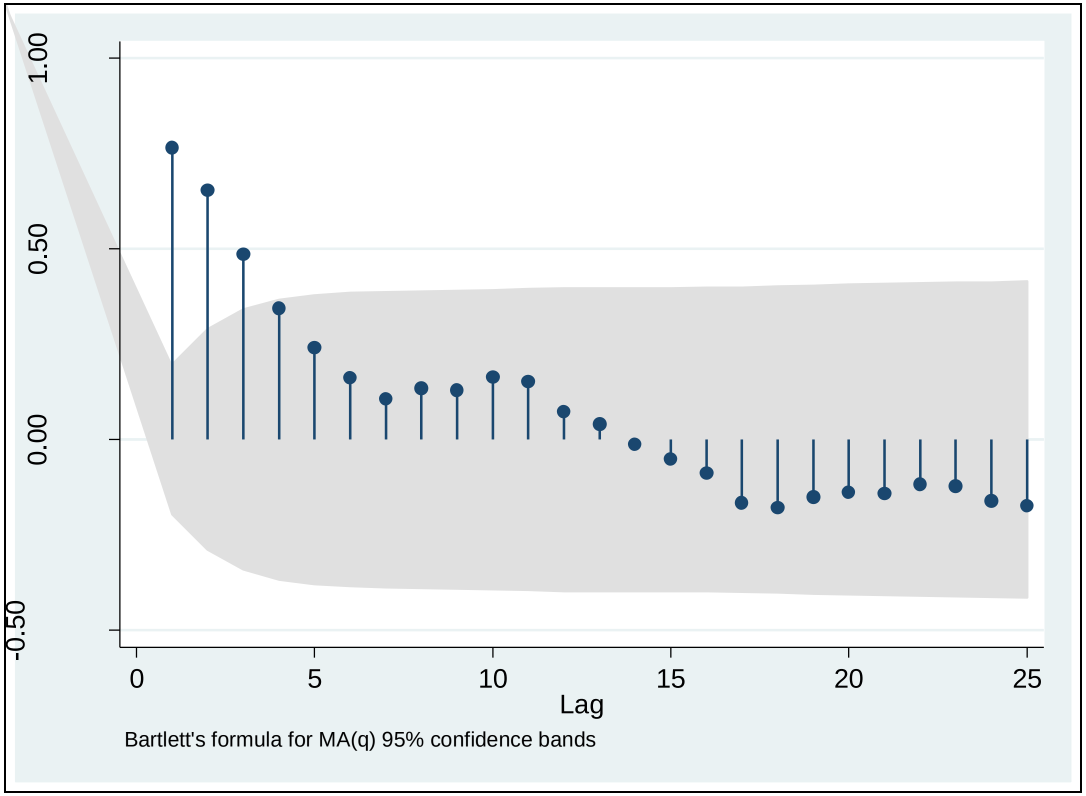
$$\rho_k = \frac{Cov(y_t, y_{t+k})}{\sigma_{y_t} \sigma_{y_{t+k}}}$$

autocorrelation in general

$$\rho_k = \frac{E[(y_t - \mu_y)(y_{t+k} - \mu_y)]}{\sigma_y^2}$$

autocorrelation under stationarity

Example: ACF of the saving rate

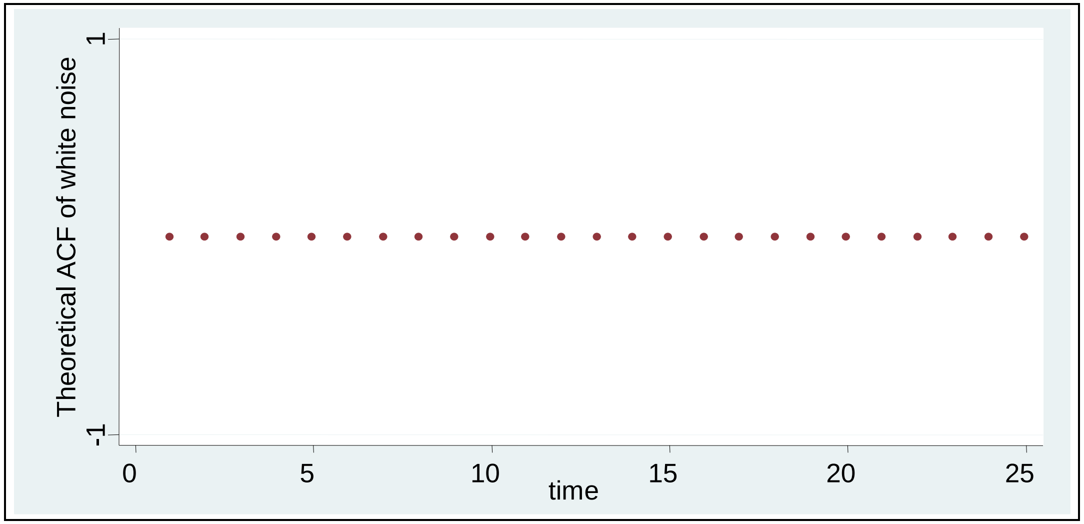


Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

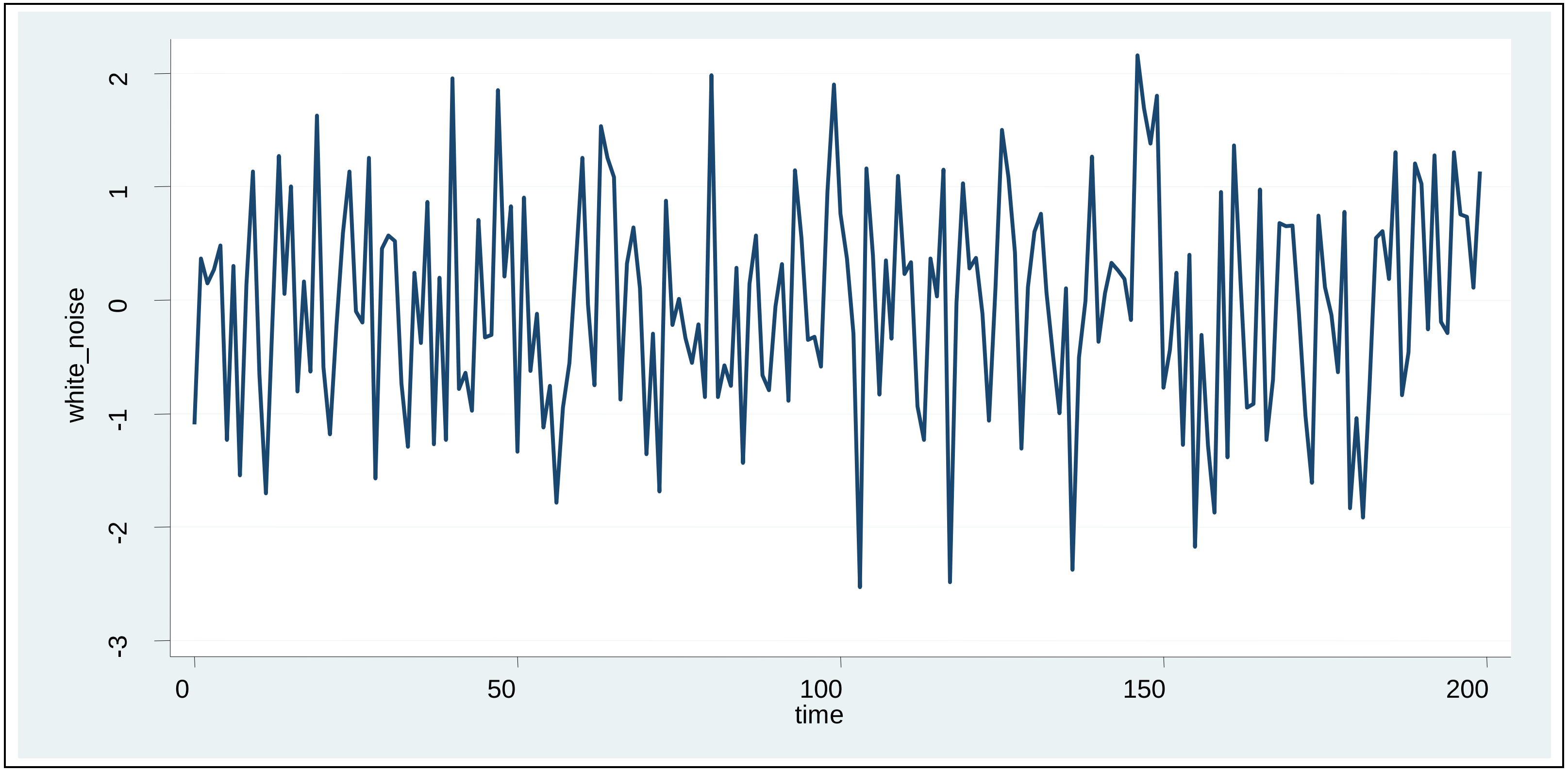
White Noise:

$y_t = \varepsilon_t$ with $\varepsilon_t \sim i.i.d.$ and $E(\varepsilon_t) = 0$

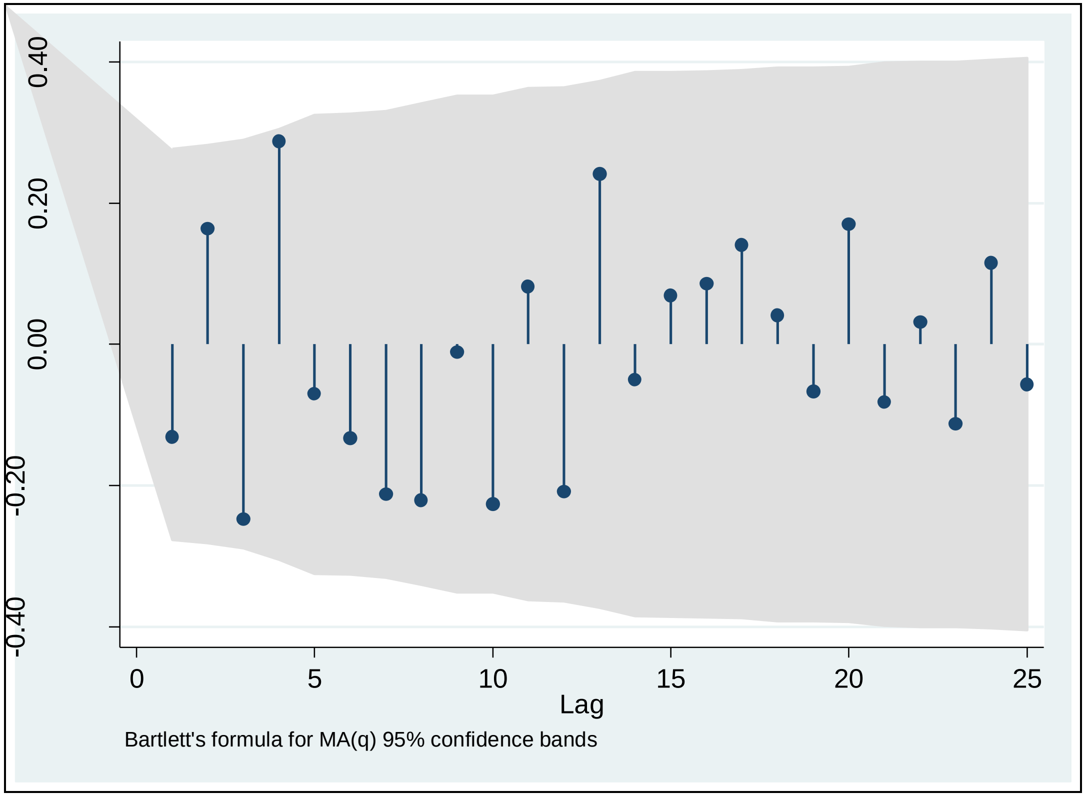
The autocorrelation function of white noise is given by $\rho_0 = 1$ and $\rho_k = 0$ for $k > 0$.



Simulated values of i.i.d. N(0, 1) noise:



Autocorrelation Function for simulated values of i.i.d. N(0, 1) noise:



Autoregressive Process of order 1, AR(1):

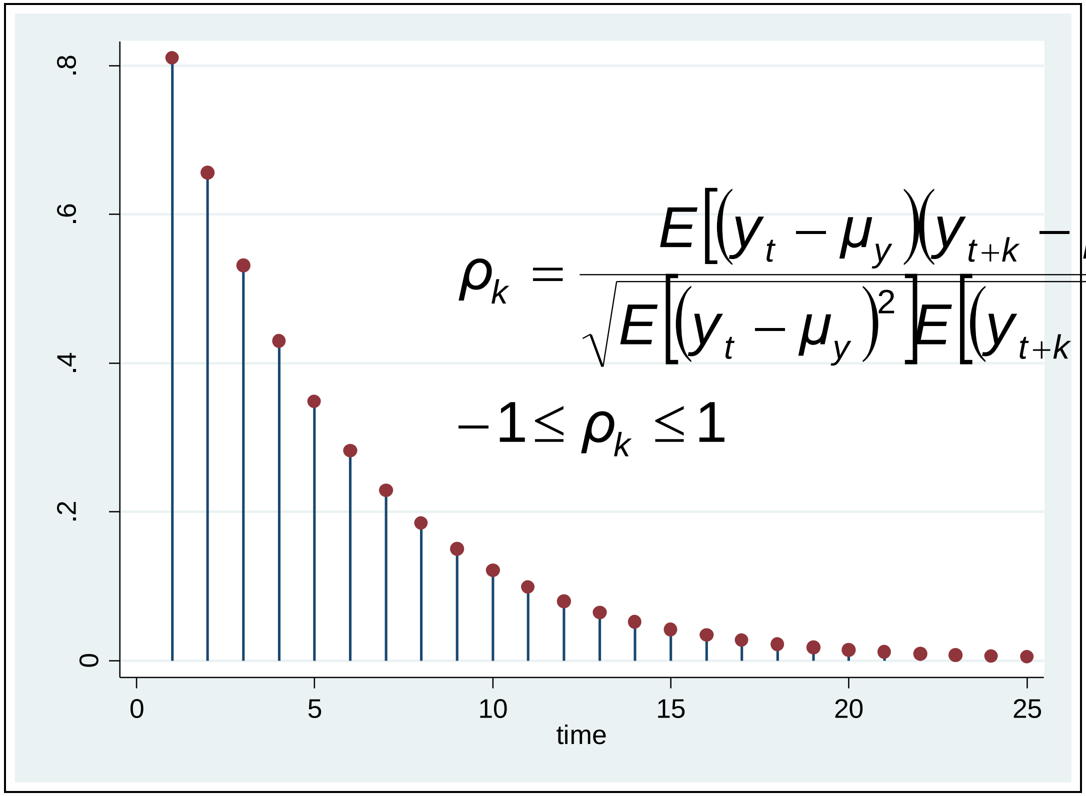
$$y_t = \varphi_1 y_{t-1} + \delta + \varepsilon_t \qquad \varepsilon_t \sim \text{i.i.d.}\left(0, \sigma_\varepsilon^2\right)$$

$$\text{with} \left|\varphi_1\right| < 1 \text{ (stationarity)}$$

$$E(y_t) = \mu = \frac{\delta}{1-\varphi_1} \qquad E\left(y_{T+1} \middle| \Omega_T\right) = \varphi_1 y_T + \delta$$

$$Var(y_t) = \frac{\sigma_\varepsilon^2}{1-\varphi_1^2} \qquad Var\left(y_{T+1} \middle| \Omega_T\right) = \sigma_\varepsilon^2$$

Theoretical ACF of an AR(1) process:



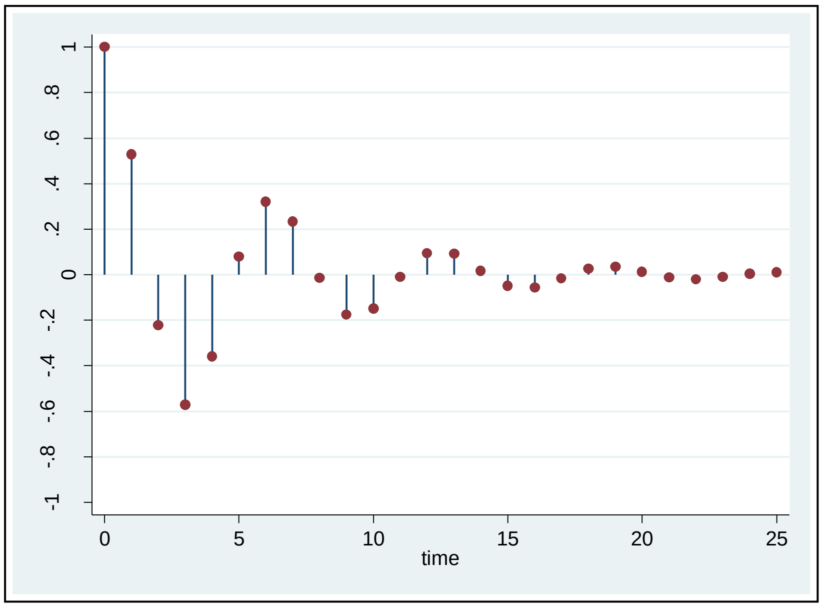
$$\rho_k = \frac{E\left[\left(y_t - \mu_y\right)\left(y_{t+k} - \mu_y\right)\right]}{\sqrt{E\left[\left(y_t - \mu_y\right)^2\right]E\left[\left(y_{t+k} - \mu_y\right)^2\right]}} = \frac{Cov\left(y_t, y_{t+k}\right)}{\sigma_{y_t}\sigma_{y_{t+k}}}$$

$$-1 \leq \rho_k \leq 1$$

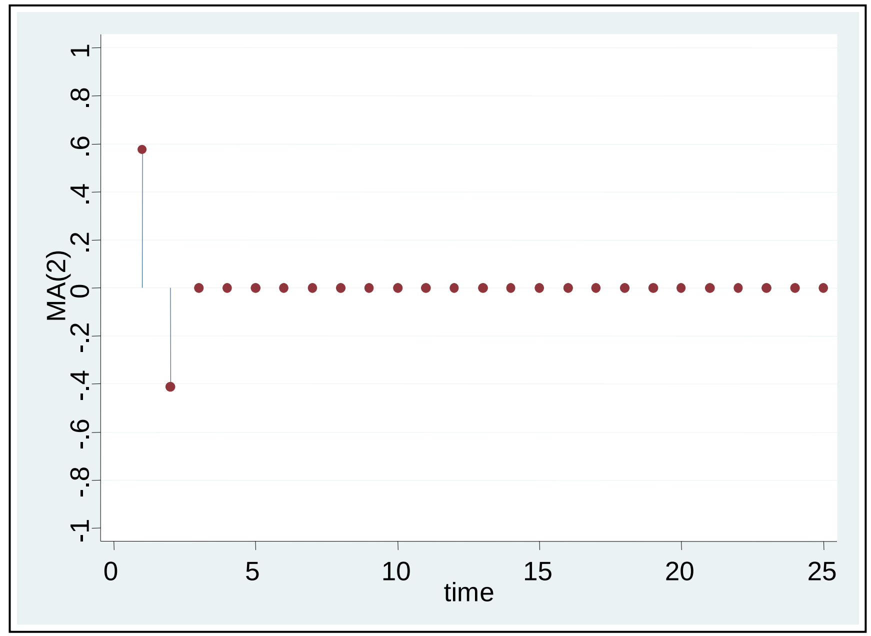
Examples: Models and their ACFs

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \delta + \varepsilon_t$$

$$y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$



Theoretical ACF for φ_1 = 0.9 and φ_2 = -0.7 :



Theoretical ACF for θ_1 = -0.6, θ_2 = 0.3

ARMA(p,q):

$$y_t = \underbrace{\varphi_1 y_{t-1} + \ldots + \varphi_p y_{t-p}}_{\text{AR(p)}} + \delta + \varepsilon_t \underbrace{- \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q}}_{\text{MA(q)}}$$

Examples:

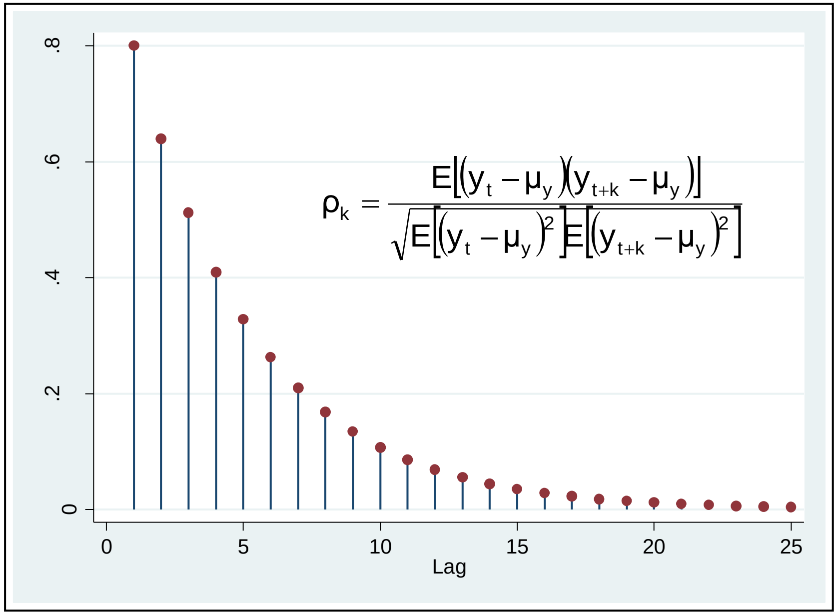
$$\text{AR}(1): y_t = \varphi_1 y_{t-1} + \delta + \varepsilon_t$$

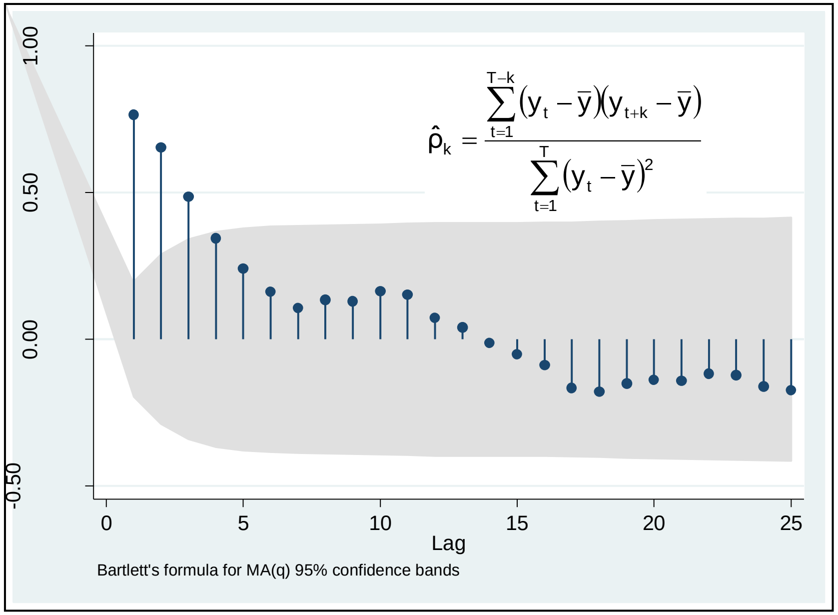
$$\text{MA}(2): y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

$$\text{ARMA}(1,1): y_t = \varphi_1 y_{t-1} + \delta + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Theoretical ACF of an AR(1) process:

Empirical ACF of the saving rate series





Example: AR(1) model

```
ARIMA regression

Sample:  1955q1 to 1979q4                        Number of obs      =       100
                                                 Wald chi2(1)       =    169.73
Log likelihood = -106.0871                       Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
  savingrate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
savingrate   |
       _cons |   6.013607   .4074939    14.76   0.000     5.214934    6.812281
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .8117232   .0623055    13.03   0.000     .6896067    .9338398
-------------+----------------------------------------------------------------
      /sigma |   .6952772   .0296021    23.49   0.000     .6372581    .7532963
------------------------------------------------------------------------------
```

Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

Autoregressive Process of order 1, AR(1):

$$y_t = \varphi_1 y_{t-1} + \delta + \varepsilon_t$$

Estimation:

$$\hat{y}_t = \hat{\varphi}_1 y_{t-1} + \hat{\delta}$$

$$\hat{\varphi}_1 = .8117232$$

$$\hat{\delta} = 6.013607 \cdot (1 - .8117232) = 1.1322227$$

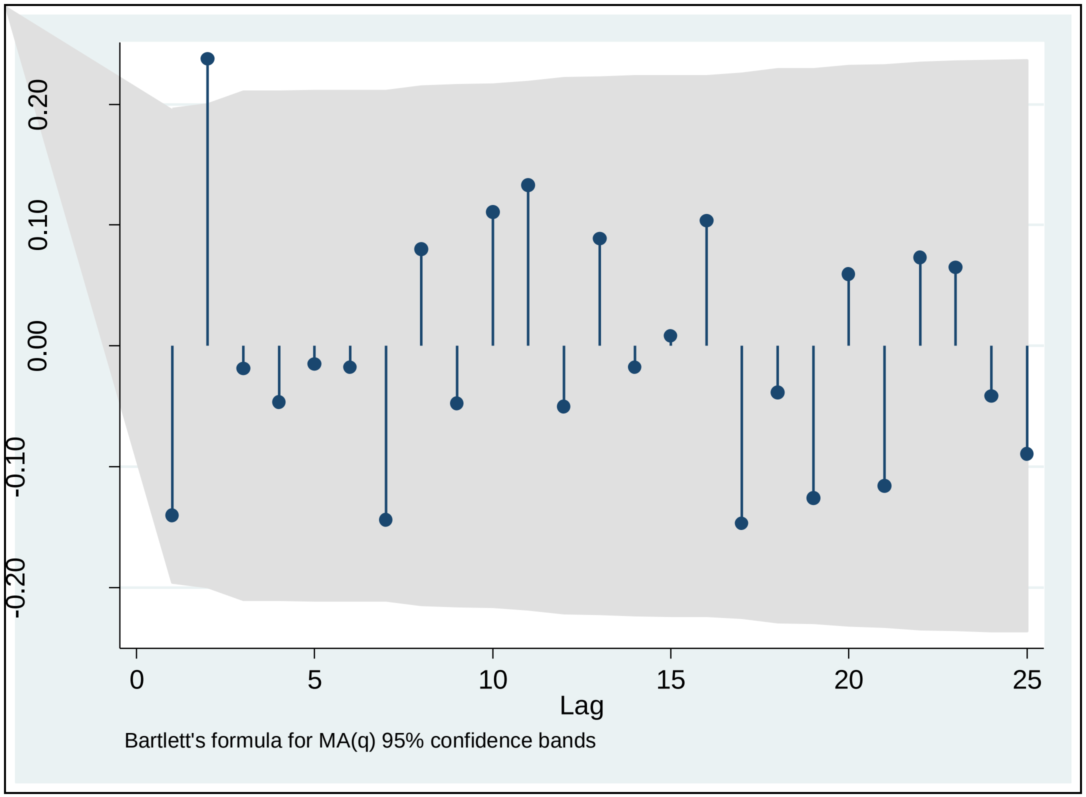
$$\hat{y}_t = .8117232 \cdot y_{t-1} + 1.1322227$$

Residuals:

$$\hat{\varepsilon}_t = y_t - \hat{y}_t$$

Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

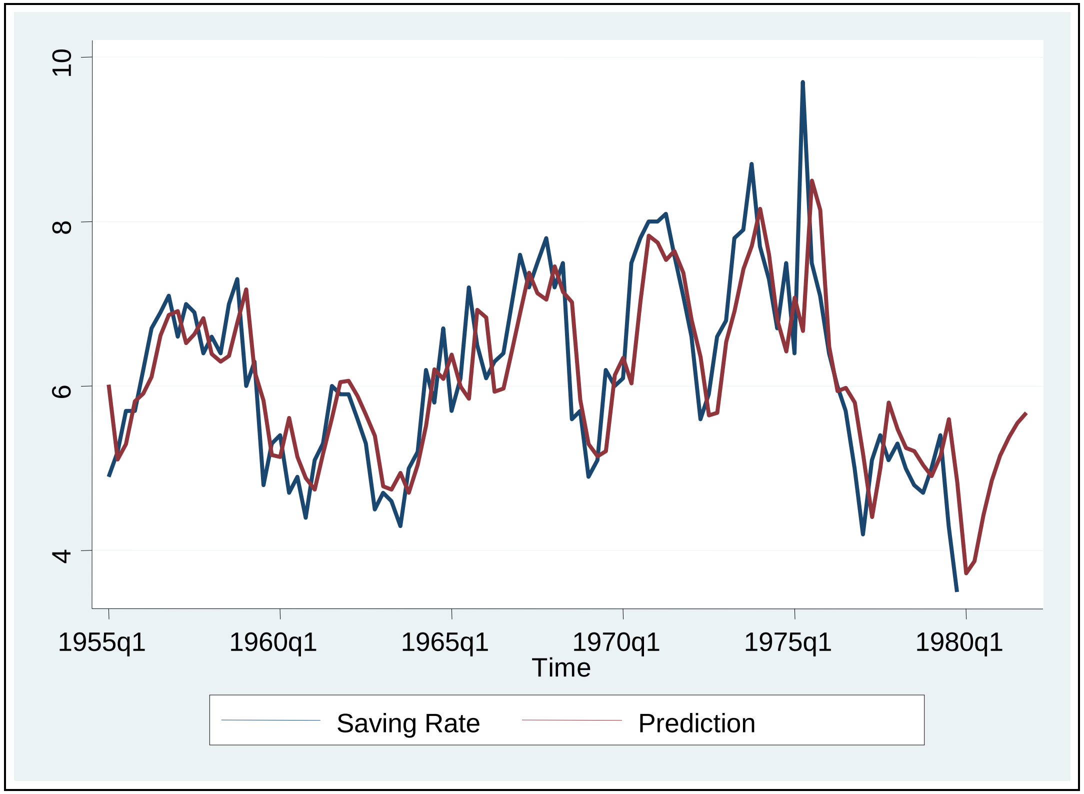
Example: ACF of the residuals



The **ACF** of the residuals suggeststhat the AR(1) model is not adequate because of the significant spike at lag 2.

Pankratz (1983) "Forecasting with univariate Box-Jenkins models"

Example: Saving Rate Prediction for years 1980-1981 with an ARMA(1,2) model



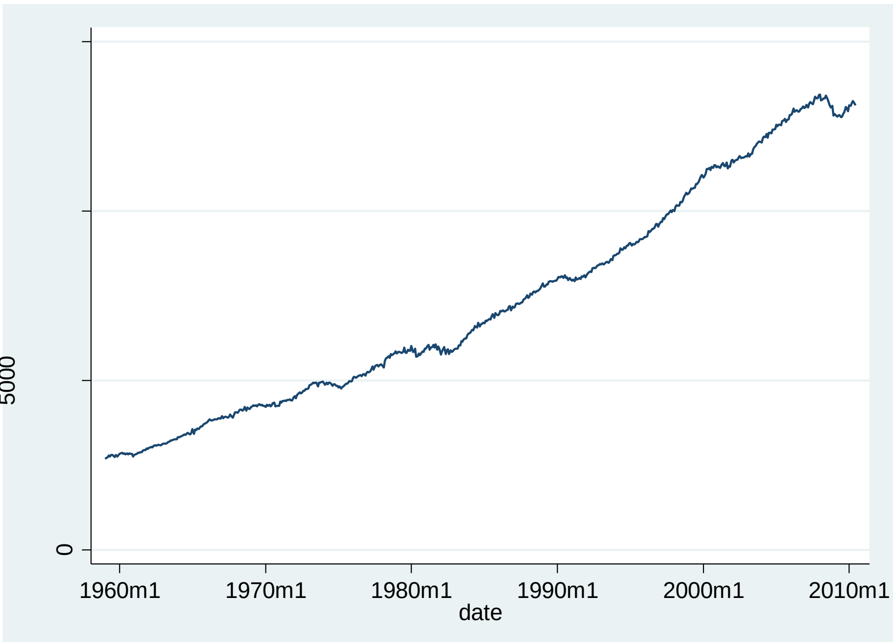
Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

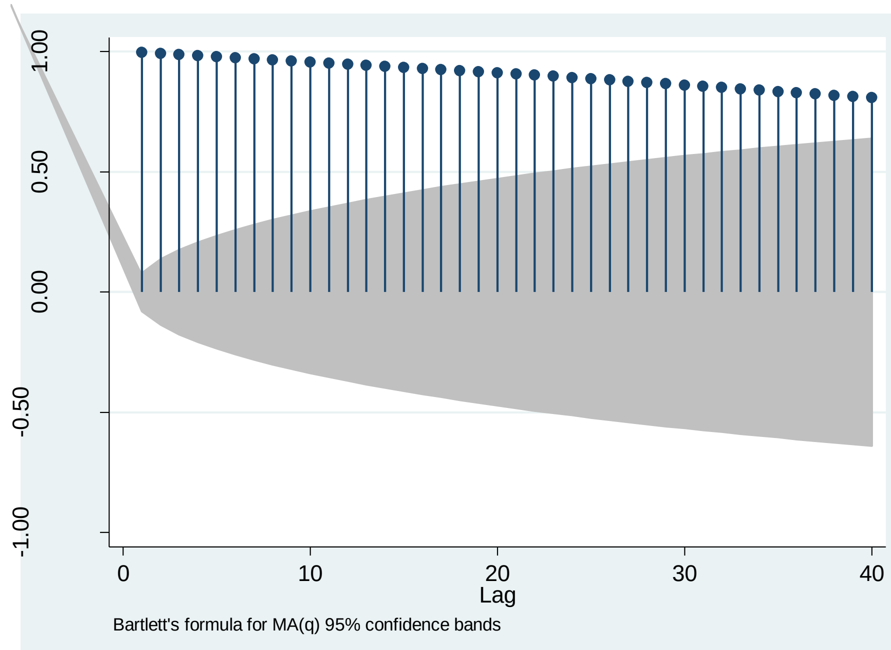
Example: Checking the ARMA(1,2) model for its ability to forecast

Time		Forecast Values	Observed Values	Forecast Errors (in percent)
1978	1	5.5948	5.3	-5.56
	2	5.4931	5.0	-9.86
	3	5.7277	4.8	-19.33
	4	5.8864	4.7	-25.24
1979	1	5.9937	5.0	-19.87
	2	6.0663	5.4	-12.34
	3	6.1153	4.3	-42.22
	4	6.1485	3.5	-75.67

Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

What about trends and other forms of nonstationarity?





monthly U.S. real GDP
Jan 1960 – June 2010

ACF of monthly U.S. real GDP

We can incorporate trends (in two ways)

Ericsson: Comment on "Economic Forecasting in a Changing World"

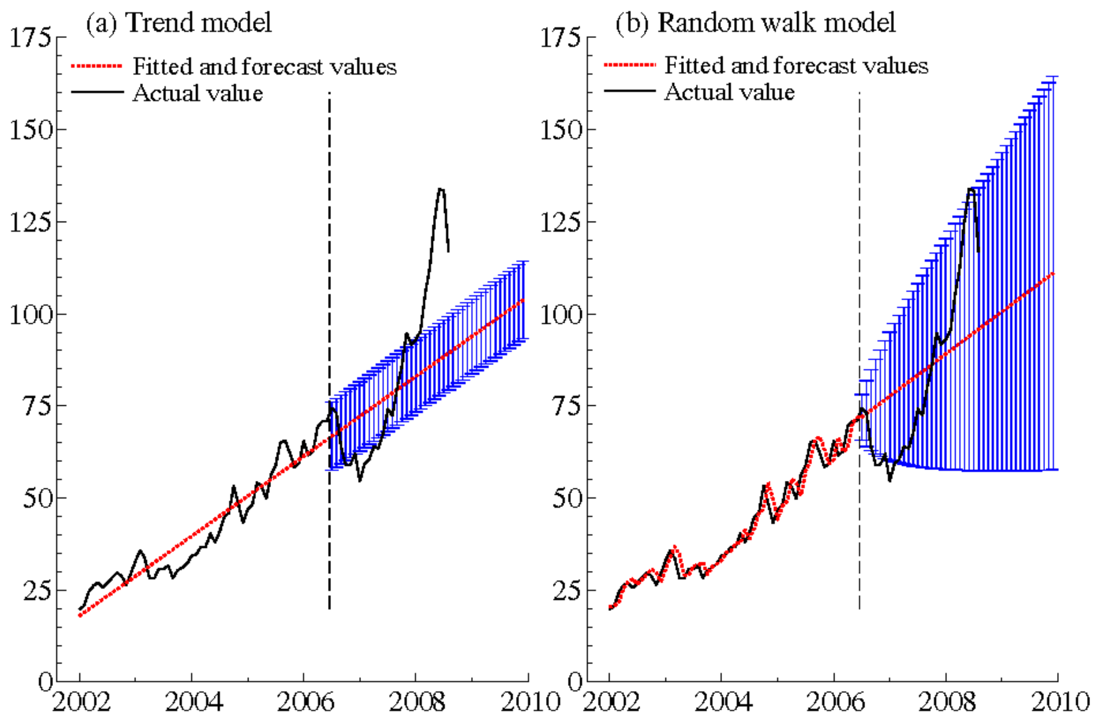


Figure 2: Actual, fitted, and forecast values from the trend and random walk models of the oil price, with 95% confidence intervals for the forecasts.

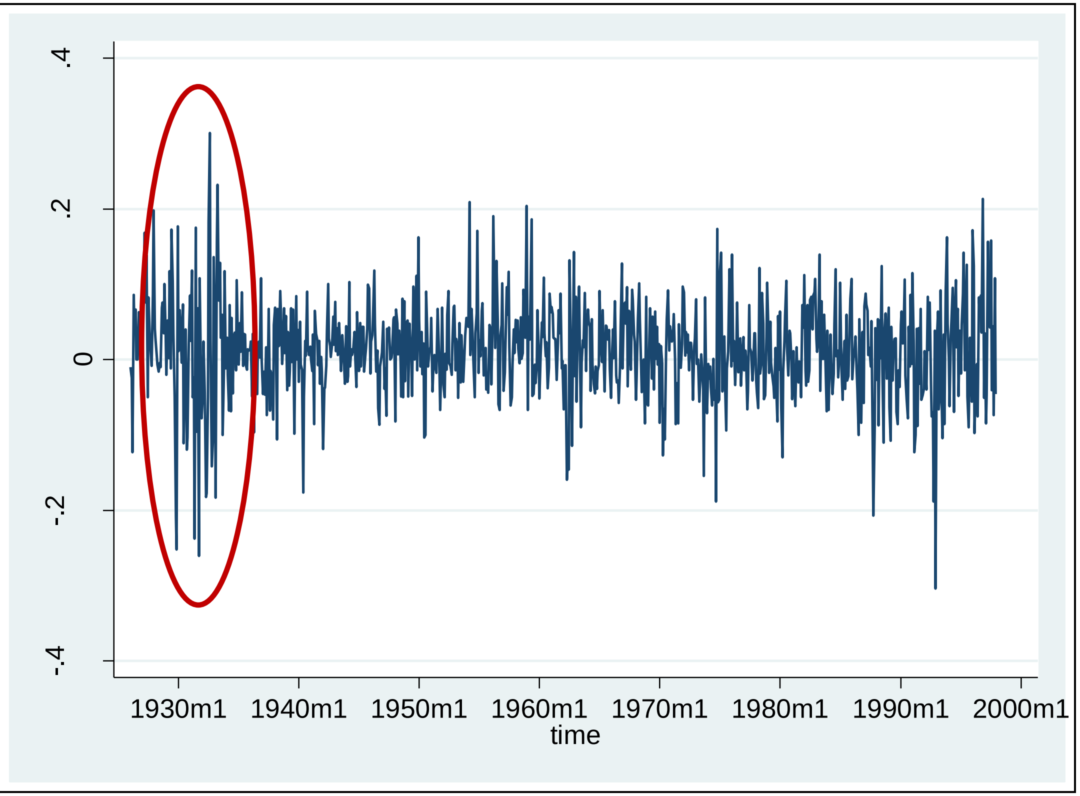
Schedule

Deter-ministic Models	• Components of a Time Series • Additive and Multiplicative Models • Smoothing Techniques • Seasonal Adjustment

Stationary Stochastic Processes	• Introduction
	• Identification
	    • Autocorrelation Function
	    • Moving Average and Autoregressive Models
	    • Partial Autocorrelation Function
	    • ARMA Models
	• Estimation
	• Diagnostic Checking
	• Forecasting

Non-stationary Stochastic Processes	• Introduction • Nonstationarity and Trends • ARIMA Models • Unit Root Tests • Seasonal ARIMA

What we don‘t cover in this class



changing conditional variance