Homework 8

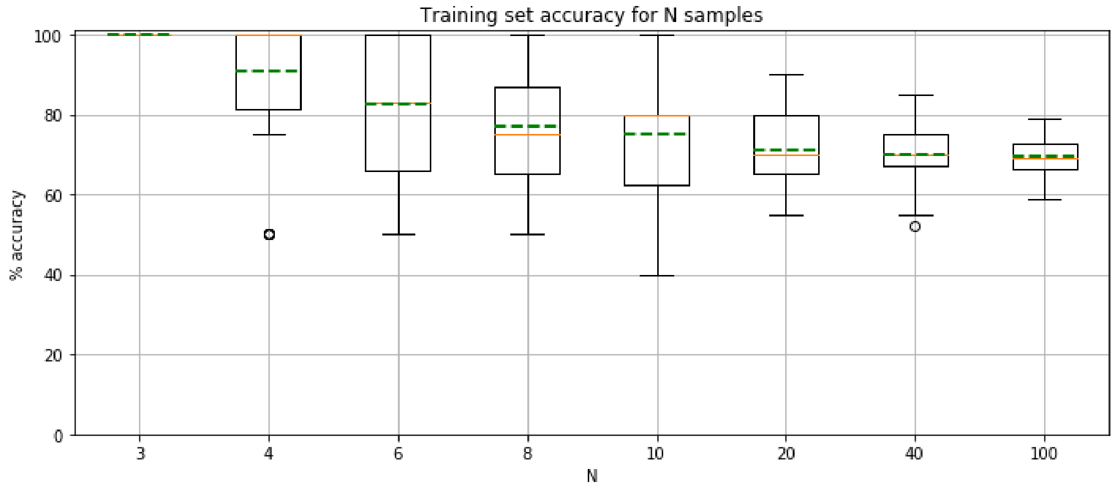
January 10, 2018

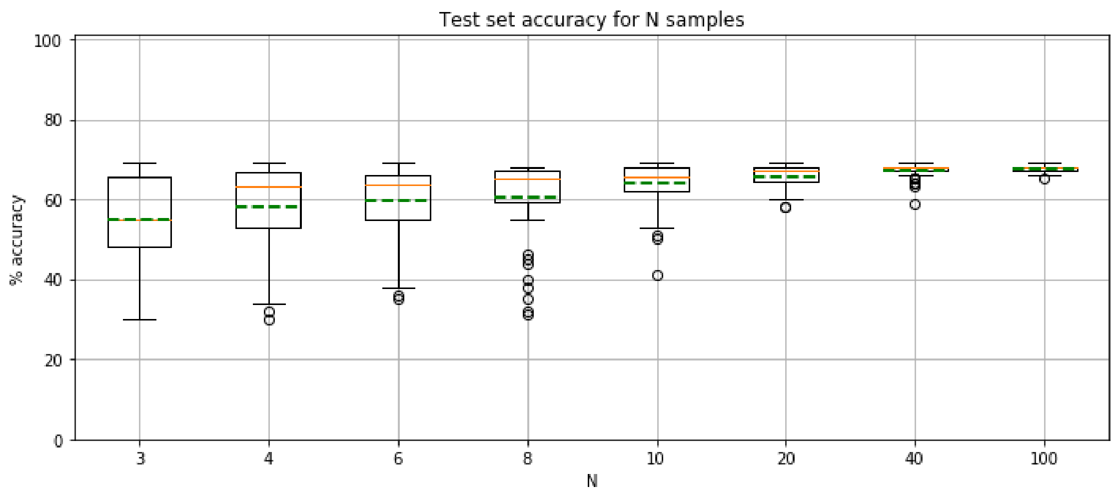
In [4]:
```
# Exercise H8.2: Variability of classification
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import mpl_toolkits.mplot3d
import mpl_toolkits.axes_grid1 as plt_ax
import scipy.stats as sp
import itertools
%matplotlib inline
```

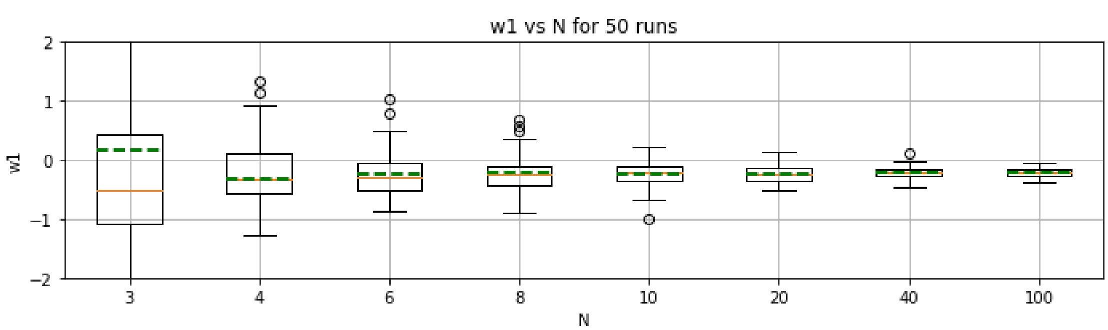
In [5]:
```
m1 = [0,1]
m2 = [1,0]
sigma = 2
testSampleSize = 1000
repeatTimes = 50

def calc_weights(train_set):
    y = train_set[:, 2]
    A = np.column_stack((np.ones(train_set.shape[0]), train_set[:, 0], train_set[:, 1]))
    wb = np.linalg.lstsq(A, y)[0]
    return wb[0], wb[1:]

def generateSample(N):
    if(N%2 == 0):
        numSamples1 = int(N/2)
        numSamples2 = numSamples1
    else:
        numSamples1 = int(N/2)
        numSamples2 = numSamples1 + 1
    p_c1 = np.zeros([numSamples1, 2])
    p_c2 = np.zeros([numSamples2, 2])
    p_c1 = np.random.normal(m1,np.sqrt(sigma),[numSamples1,2])
    p_c2 = np.random.normal(m2,np.sqrt(sigma),[numSamples2,2])
    p1_c1 = np.concatenate([p_c1.T, np.ones((numSamples1,1)).T]).T
    p2_c2 = np.concatenate([p_c2.T, -np.ones((numSamples2,1)).T]).T
    return np.concatenate([p1_c1, p2_c2])

test_set = generateSample(testSampleSize)
```

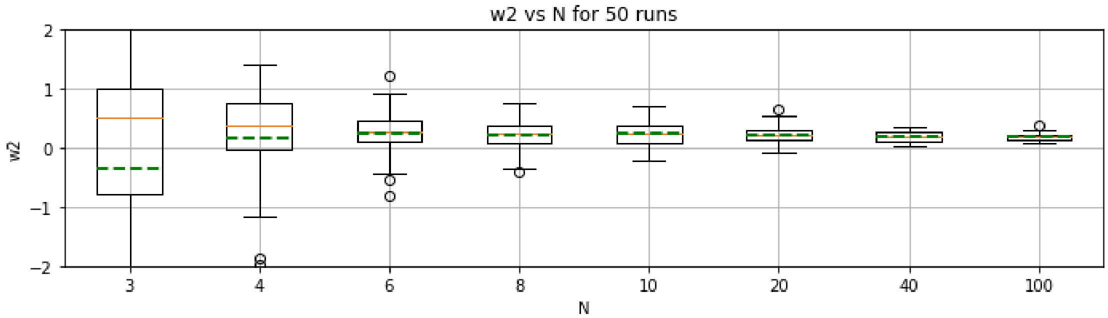
```
    N = [3, 4, 6, 8, 10, 20, 40, 100]
    train_w1s, train_w2s, train_bs, train_percentages, test_percentages = np.zeros((5, len(N
    for i, n in enumerate(N):
        train_w1s_runs, train_w2s_runs, train_bs_runs, train_percentages_runs, test_percenta
        for j in range(repeatTimes):
            train_set = generateSample(n)
            b, w = calc_weights(train_set)
            train_yT = np.sign(w.T.dot(train_set.T[:2]) + b)
            train_percentages_runs[j] = 100 * np.sum(train_yT == train_set[:, 2]) / train_se
            train_w1s_runs[j], train_w2s_runs[j] = w
            train_bs_runs[j] = b
            test_yT = np.sign(w.T.dot(test_set.T[:2]) + b)
            test_percentages_runs[j] = 100 * np.sum(test_yT == test_set[:, 2]) / test_set.sh
        train_w1s[i], train_w2s[i], train_bs[i] = train_w1s_runs, train_w2s_runs, train_bs_r
        train_percentages[i], test_percentages[i] = train_percentages_runs, test_percentages
```

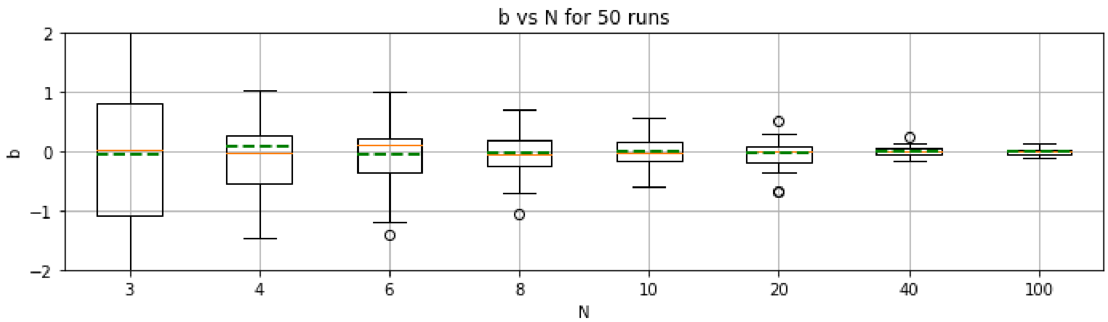
```
In [6]: def  errorbar_plot(percentage_matrix, ax, xticks, title='', labels=None, ylim=[0, 101],
            meanlineprops = dict(linewidth=2, color='green')
            ax.boxplot(percentage_matrix.T, meanprops=meanlineprops, meanline=True, **kwargs)
            ax.grid(True)
            ax.set_xticklabels(xticks)
            ax.set_title(title)
            ax.set_ylim(*ylim)
            if labels:
                ax.set_xlabel(labels[0])
                if (len(labels) > 1):
                    ax.set_ylabel(labels[1])
            return ax

        fig, ax1 = plt.subplots(1, 1, figsize=(10, 4.5))
        errorbar_plot(train_percentages, ax1, N, title='Training set accuracy for N samples', la
        fig.tight_layout()

        fig, ax2 = plt.subplots(1, 1, figsize=(10, 4.5))
        errorbar_plot(test_percentages, ax2, N, title='Test set accuracy for N samples', labels=
        fig.tight_layout()

        fig, ax3 = plt.subplots(1, 1, figsize=(10, 3))
        errorbar_plot(train_w1s, ax3, N, title='w1 vs N for 50 runs', labels=['N', 'w1'], ylim=[
        fig.tight_layout()

        fig, ax4 = plt.subplots(1, 1, figsize=(10, 3))
        errorbar_plot(train_w2s, ax4, N, title='w2 vs N for 50 runs', labels=['N', 'w2'], ylim=[
        fig.tight_layout()

        fig, ax5 = plt.subplots(1, 1, figsize=(10, 3))
        errorbar_plot(train_bs, ax5, N, title='b vs N for 50 runs', labels=['N', 'b'], ylim=[-2,
        fig.tight_layout()
```


Training set accuracy for N samples
% accuracy
N
0
20
40
60
80
100
3
4
6
8
10
20
40
100


Test set accuracy for N samples
% accuracy
N
0
20
40
60
80
100
3
4
6
8
10
20
40
100

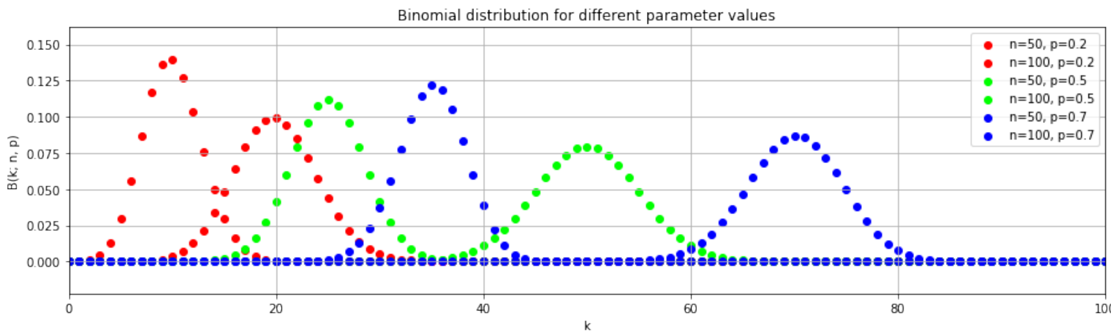

w1 vs N for 50 runs
w1
N
−2
−1
0
1
2
3
4
6
8
10
20
40
100



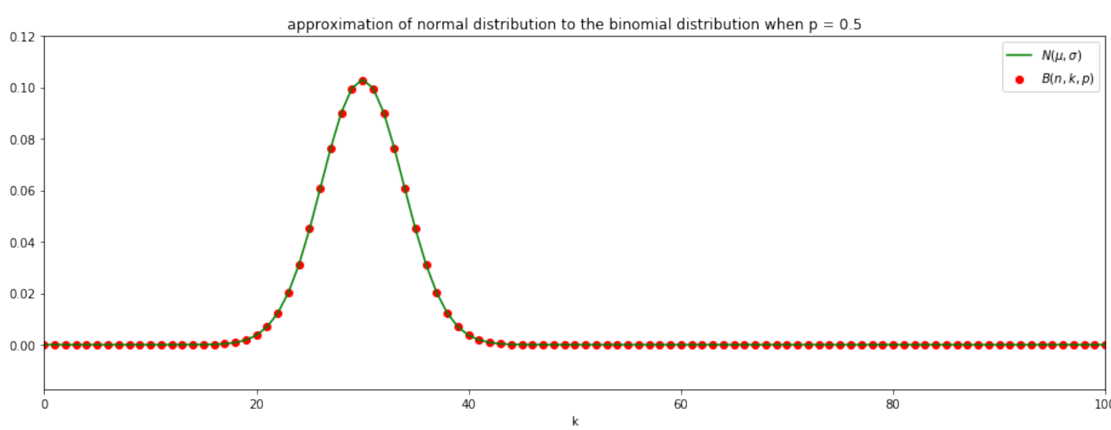


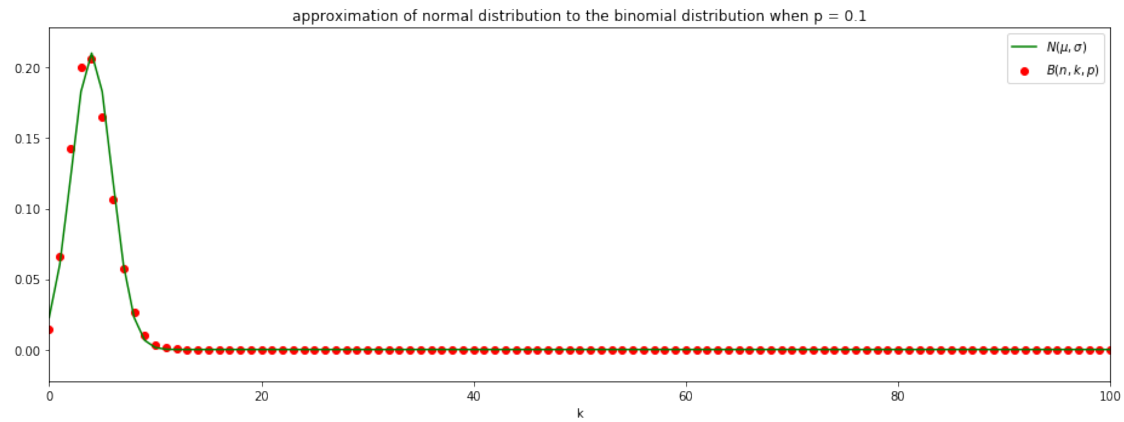
c) Increasing the sample size reduces the variance of our estimates. From the first two plots we see that the training set accuracy decreases when we increase the sample size, but the test set accuracy increases with increased sample size N. In both cases the variance is reduced when the sample size N increases.

```
In [7]: #  H8.3: The Binomial distribution
        ks = np.array(range(101))
        ns = np.array([50, 100])
        ps = np.array([0.2, 0.5, 0.7])
        fig, ax = plt.subplots(1, 1, figsize=(13, 4))
        for p, n in itertools.product(ps, ns):
            color = (ps == p) * 1.0
            color[color == 0] = (n - ns[0]) / ns[1]
            data = sp.binom(n, p).pmf(ks)
            scattered = ax.scatter(range(len(data)), data, color = color, label = 'n={}, p={}'.f
        ax.legend()
        ax.set_xlabel('k')
        ax.set_ylabel('B(k; n, p)')
        ax.set_xlim([ks[0], ks[-1]])
        ax.set_title('Binomial distribution for different parameter values')
        fig.tight_layout()
        ax.grid(True)
```

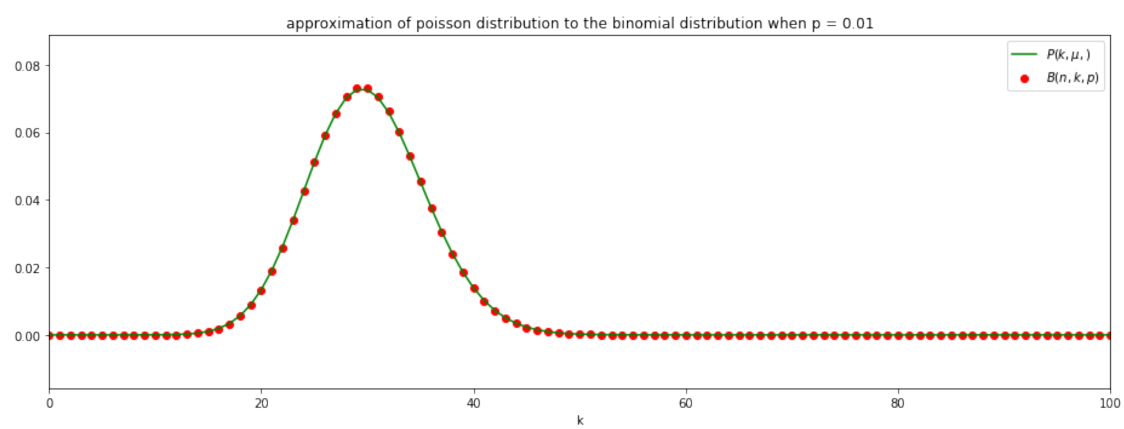


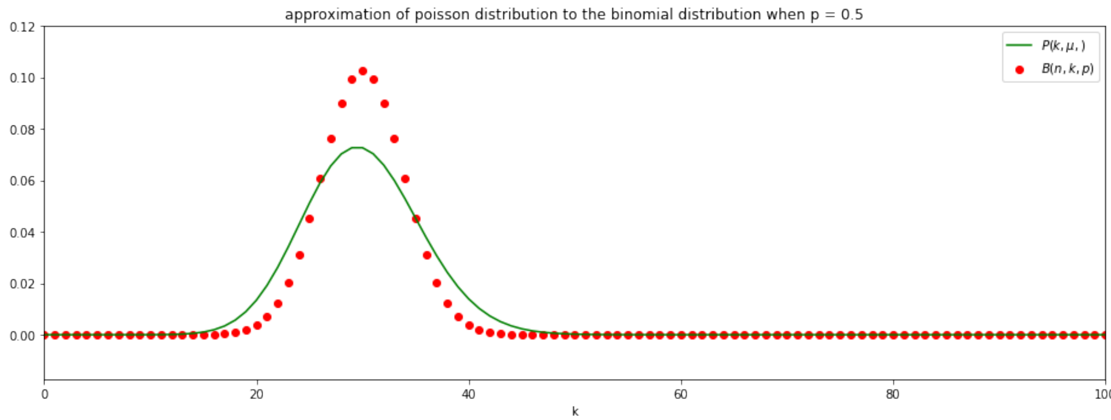

```
In [8]: def plot_bin_norm(n,p):
            fig, ax = plt.subplots(1, 1, figsize=(13, 5))
            k = np.array(range(int(101)))
            data_bin = sp.binom(n, p).pmf(k)
            ax.scatter(range(len(data_bin)), data_bin, color = 'red', label=r'$B(n,k,p)$')
            mean = n * p
            sigma = np.sqrt(n * p * (1-p))
            data_norm = sp.norm.pdf(k,mean, sigma)
            ax.plot(data_norm, color = 'green', label = r'$N(\mu, \sigma)$')
            ax.set_title('approximation of normal distribution to the binomial distribution when
            ax.set_xlabel('k')
            ax.set_xlim([0,100])
            ax.legend()
            fig.tight_layout()
        plot_bin_norm(60,0.5)
        plot_bin_norm(40,0.1)

        # Result Interpretaion:
        # - Approximation using normal distribution is good for p = 0.5
        #   and problematic when p is less around 0.1 as shown below
        # - the normal distribution being the distribution with maximum entropy for a given mean
```






```
In [9]: def plot_bin_poisson(n,p):
            fig, ax = plt.subplots(1, 1, figsize=(13, 5))
            k = np.array(range(int(101)))
            data_bin = sp.binom(n, p).pmf(k)
            ax.scatter(range(len(data_bin)), data_bin, color = 'red', label=r'$B(n,k,p)$')
            mean = n * p
            data_poisson = sp.poisson.pmf(k,mean)
            ax.plot(data_poisson, color = 'green', label = r'$P(k, \mu, )$')
            ax.set_title('approximation of poisson distribution to the binomial distribution whe
            ax.set_xlabel('k')
            ax.set_xlim([0,100])
            ax.legend()
            fig.tight_layout()

        plot_bin_poisson(3000,0.01)
        plot_bin_poisson(60,0.5)
        # Result Interpretaion:
        # - Poisson distribution is  good approximation for a high n and low p
```




approximation of poisson distribution to the binomial distribution when p = 0.5
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
100
k
$P(k, \mu,)$
$B(n, k, p)$