We know

$$C_{(N+1,N)} = 2\sum_{k=0}^{N}\binom{N}{k} \longrightarrow 1$$

From binomial theorem with $x = y = 1$

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k} x^n y^{n-k}$$

$$\Rightarrow (1+1)^n = \sum_{k=0}^{n}\binom{n}{k} 1^n 1^{n-k}$$

$$2^n = \sum_{k=0}^{n}\binom{n}{k} \longrightarrow 2$$

Plugging 2 in 1.

$$C_{(N+1,N)} = 2 \times 2^N = 2^{N+1}$$

$$C_{(N+2,N)} = 2 \sum_{k=0}^{N+1} \binom{N+1}{k}$$

$$C_{(N+2,N)} = C_{(N+1,N)} + C_{(N+1,N-1)}$$

$$= 2^{N+1} + 2 \sum_{k=0}^{N-1} \binom{N}{\substack{k \\ \cancel{N-1}}}$$

$$+ 2 \sum_{k=0}^{N} \binom{N}{k} - 2 \binom{N}{N}$$

$$+ 2 \cdot 2^{N} - 2$$

$$2^{N+1} + 2^{N+1} - 2$$

$$= 2 \cdot 2^{N+1} - 2$$

$$= 2^{N+2} - 2.$$

$$\therefore C_{(N+2,N)} < 2^{N+2}.$$