In [7]:

```
import numpy

def convolution(x1, x2):
    return numpy.convolve(x1, x2)

def kernel(x1, x2):
    convo = convolution(x1, x2)
    normed = numpy.linalg.norm(convo)
    return normed*normed

def checkKernel(x1, x2):
    kern = kernel(x1, x2)
    return kern >= 0

print(checkKernel([1, 2, 0, 9], [0, 1, 0.5]))
```

```
True
```

In [ ]:

Exercise 1

b) Explicit feature map for $k(x, x') = \|x * x'\|^2$: $\sum_{-\infty}^{\infty} x(t-\tau)\, x'(\tau)$

$k(x, x')$ is PSD:

$$\sum_{i,j=1} c_i c_j\, k(x, x') \Leftrightarrow \sum_{i,j} c_i c_j \|x * x'\|^2 \Leftrightarrow \sum_{i,j} c_i \|x * x'\| c_j \|x * x'\| \Leftrightarrow$$

$$\Leftrightarrow \left(\sum_i c_i \|x * x'\|\right)^2 \geq 0 \quad \square$$

Exercise 2:

① $y_t = [w \star x]_t = \sum_{s=-\infty}^{\infty} w_s \cdot x_{t+s}$

② $y_t = [w * x]_t = \sum_{s=-\infty}^{\infty} w_s \cdot x_{t-s}$

we can apply the chain-rule

(a) ⇒ $$\frac{\partial E}{\partial x_u} = \sum_{-\infty}^{\infty} \frac{\partial E}{\partial y_t} \frac{\partial y_t}{\partial x_u} = \sum_{t=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot w_s \cdot \mathbb{1}_{\underline{\{t-s=u\}}}$$

$$\Downarrow$$

$$s = t-u$$

$$= \sum_{t=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot w_{t-u}$$

$$= \sum_{t=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot w_{(-u)+t}$$

① ⇒ $$= \left[\frac{\partial E}{\partial y} \star w\right]_{-u}$$

(b) $$\frac{\partial E}{\partial w_u} = \sum_{t=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot \frac{\partial y_t}{\partial w_u} = \sum_{t=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot x_{t-s}$$

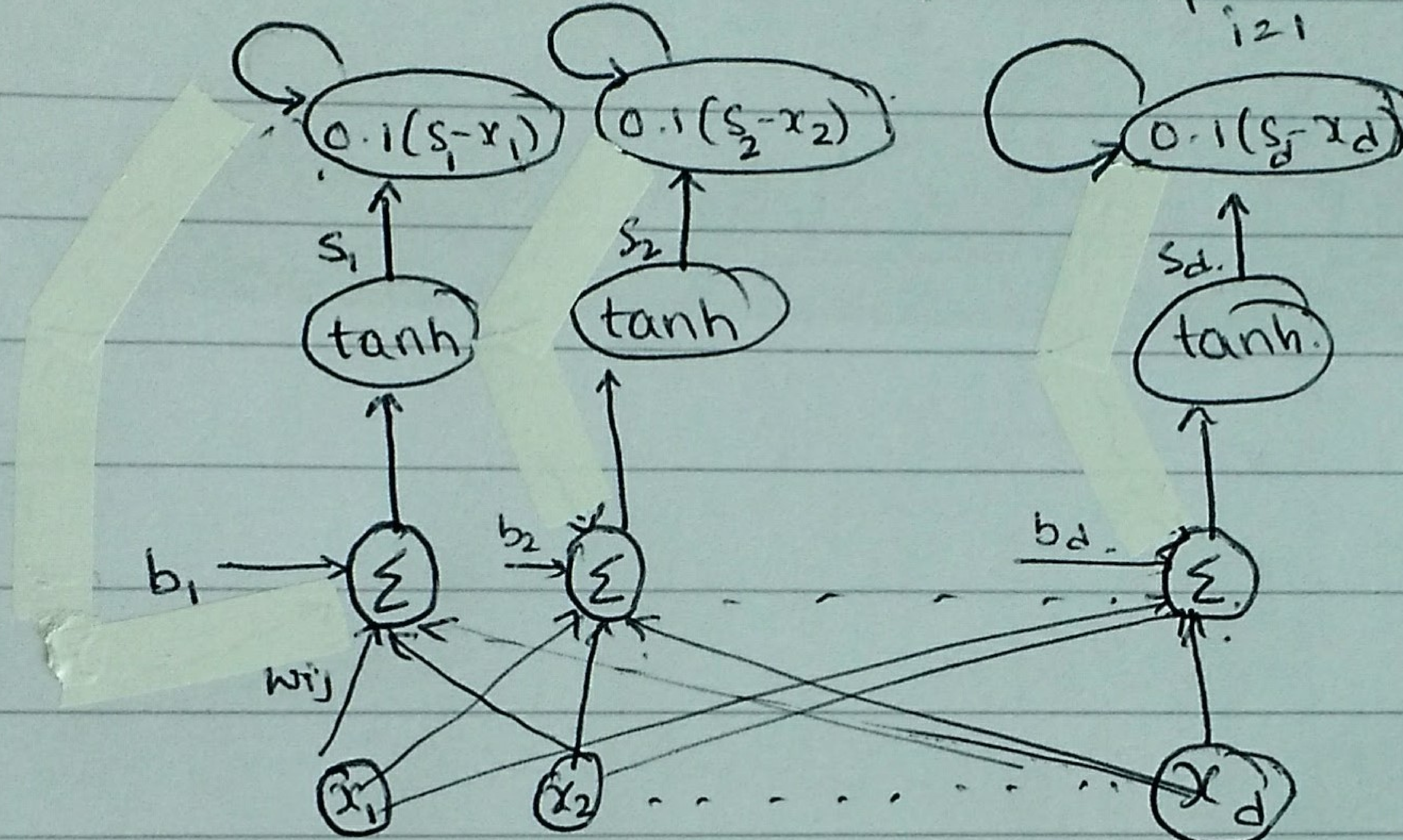
$$= \sum_{t=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot x_{t-u}$$

$$= \sum_{t=-\infty}^{\infty} \frac{\partial E}{\partial y_t} \cdot x_{(-u)+t}$$

① ⇒ $$= \left[\frac{\partial E}{\partial y} \star x\right]_{-u}$$

Exercise 3:

(a) $\forall_{j=i}^{d} : \dot{x}_j = 0.1\left(\tanh\left(\sum_{i=1}^{d} x_i w_{ij} + b_j\right) - x_j\right)$



Applying Euler discretization,

$$\frac{x_j^{(t)} - x_j^{(t-1)}}{t-(t-1)} = 0.1\, x_j^{(t-1)} + 0.1 \tanh\left(\sum_{i=1}^{d} x_i^{(t-1)} w_{ij} + b_j\right)$$

$$x_j^{(t)} = (1-0.1)\, x_j^{(t-1)} + 0.1 \tanh\left(\sum_{i=1}^{d} x_i^{(t-1)} w_{ij} + b_j\right)$$

$$= 0.9\, x_j^{(t-1)} + 0.1 \tanh\left(\sum_{i=1}^{d} x_i^{(t-1)} w_{ij} + b_j\right)$$

at time (t-1).

at time t.

at time t+1.

Exercise 3

(c) $$\frac{\partial x_j^t}{\partial x_j^{t-1}} = 0.9$$

(d) $$\frac{\partial x_j^t}{\partial w_{ij}} = 0.1\left(1 - \tanh^2\left(\sum_{i=1}^{d} x_i^{t-1} w_{ij} + b_j\right)\right) x_j^{t-1}$$

$$\frac{\partial x_j^t}{\partial b_j} = 0.1\left(1 - \tanh^2\left(\sum_{i=1}^{d} x_j^{t-1} w_{ij} + b_j\right)\right)$$