Exercise #11.1 Directed Acyclic Graph

a) One possible topological sorting of the DAG:

F, E, B, A, H, D, G, C

or: E, B, F, A, D, H, G, C

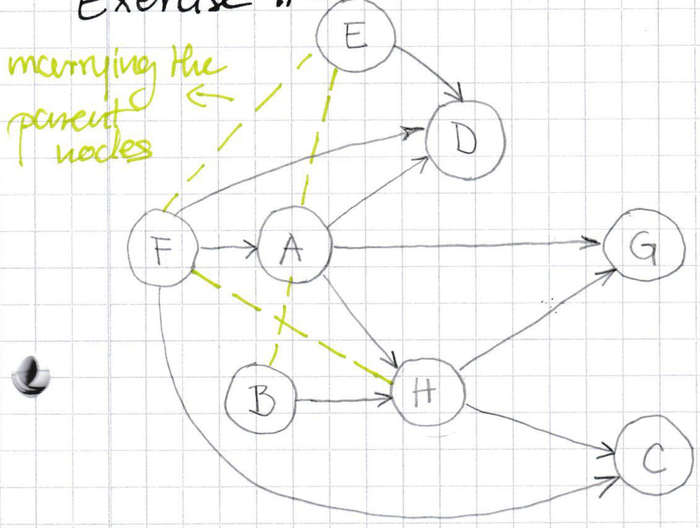
b) Factorization for the DAG:

$$P(A,B,C,D,E,F,G,H) = P(F) \cdot P(E) \cdot P(B) \cdot P(A|F) \cdot P(D|A,F,E) \cdot P(H|A,B) \cdot P(C|H,F) \cdot P(G|A,H)$$

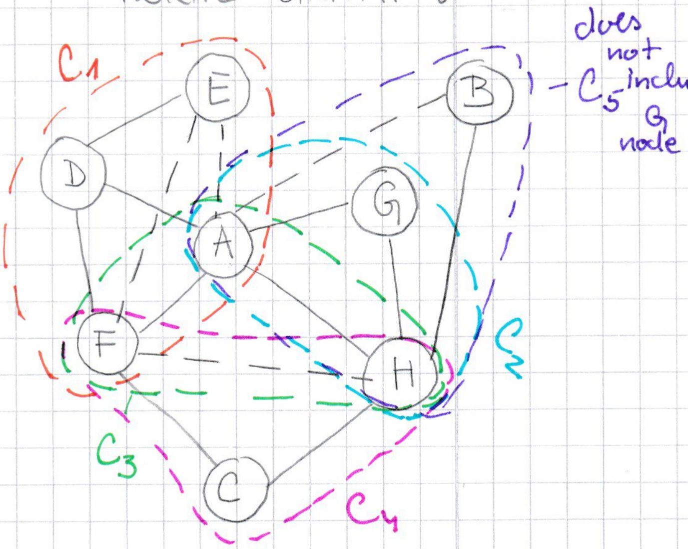
c) Nodes that belong to the Markov blanket of A:

F, D, H, G, B, E

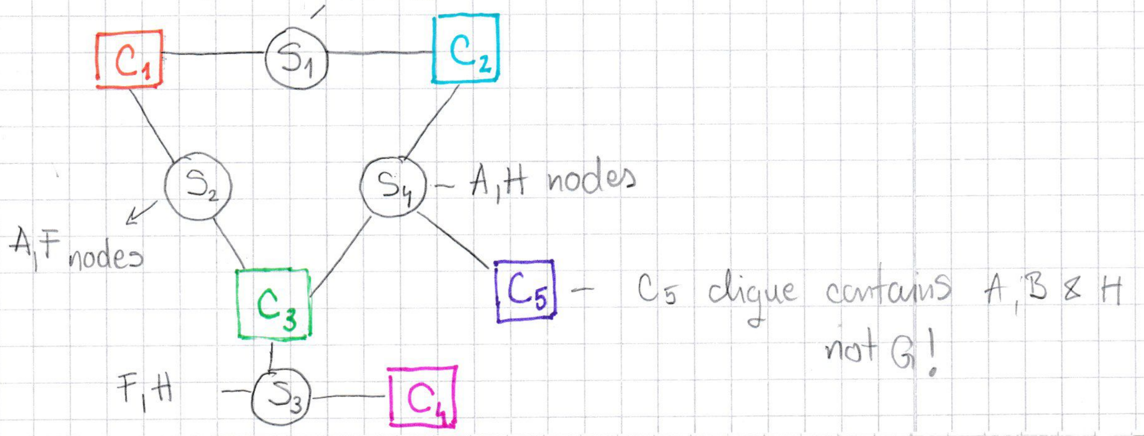
Exercise #11.2



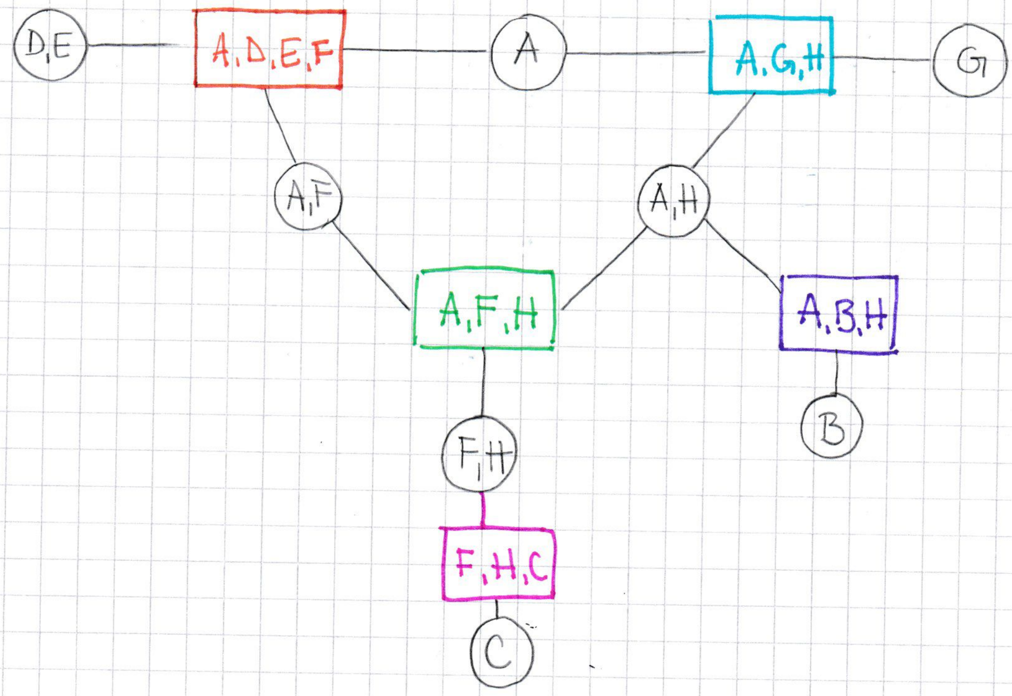
a) MORAL GRAPH:



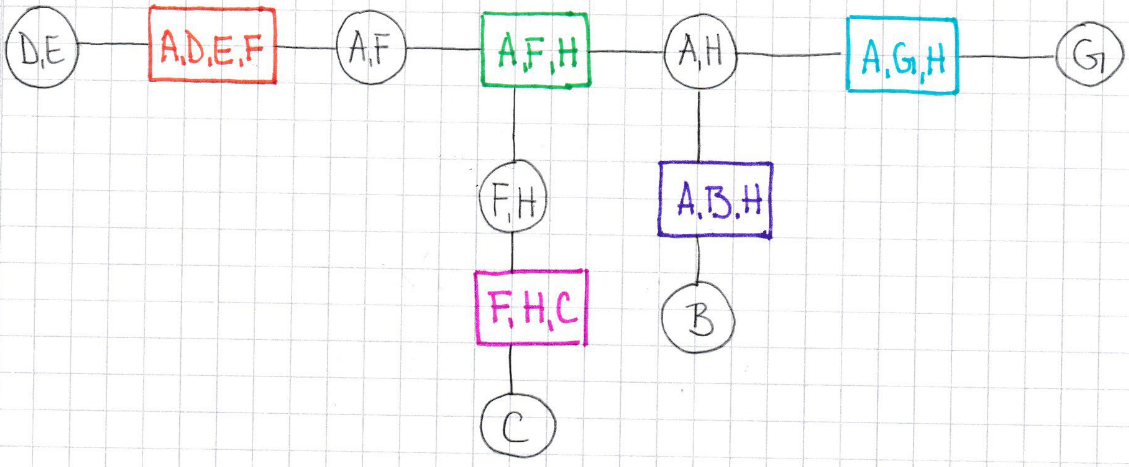
b) Bipartite graph:



Bipartite graph of cliques and separators



c) One possible junction tree

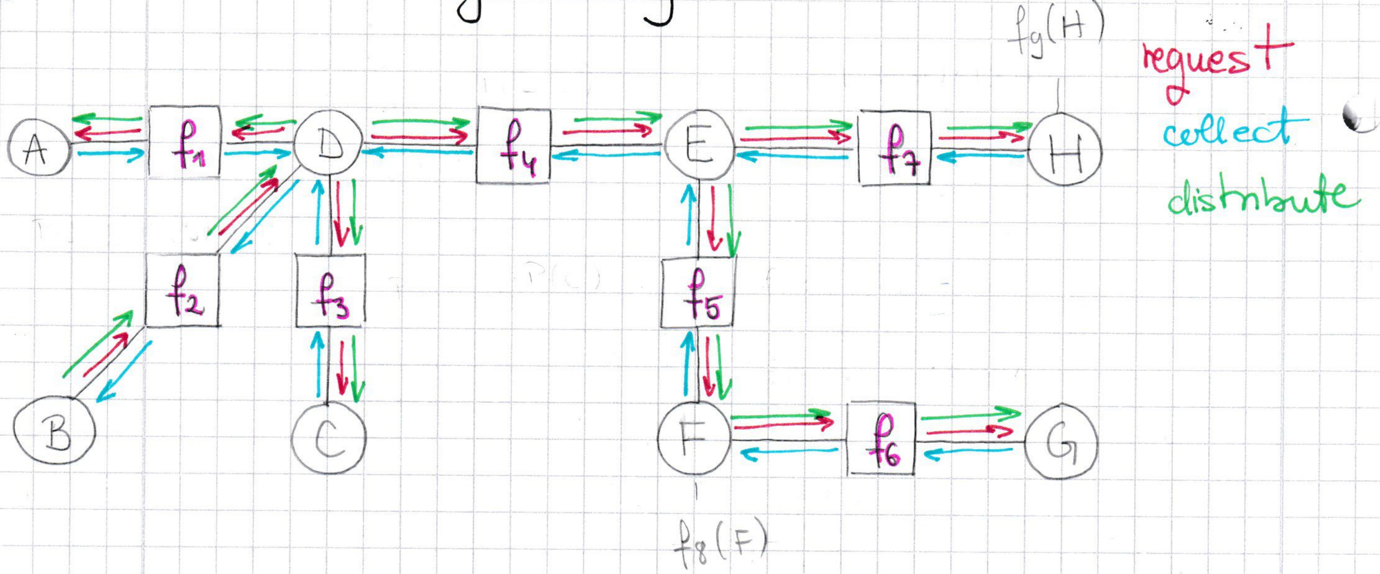


d) Running intersection property

- AF node contains A & F and the cliques that it connects ADEF & AFH also contain variables A & F.
- AH node contains variables A & H and the cliques that it connects AFH, AGH & ABH also contain variables A & H.

- Node FH contains variables F & H and the cliques that it connects AFH and FHC also contain variables F & H.

Exercise H11.3: Message Passing



If we assume that the tree has a root in node B we can also assume that the broadcast of the request goes from this node (B) to the leaves.

collect pass goes from leaves to the root (in this case node B)

distribute pass calculates all the other marginals

b) The computation performed for the message $\mu_{f_4 \to D}(D)$, e.g.

$$\mu_{f_3 \to D}(D) = \sum_C f_3(C,D)$$

$$\mu_{f_4 \to D}(D) = \sum_E f_4(D,E) \cdot \mu_{f_7 \to E}(E) \cdot \mu_{f_5 \to E}(E)$$

$$= \sum_E P(D|E)\left(\sum_H P(E|H) \cdot P(H)\right)\left(\sum_F P(E|F) \cdot \sum_G P(F|G) \cdot P(G)\right)$$

c) $\mu_{f_4 \to D}(D)$ after the evidence $F=f$ and $H=h$ has been observed

$$f_8(F) = \begin{cases} 1, & F=f \\ 0, & F \neq f \end{cases} \qquad f_9(H) = \begin{cases} 1, & H=h \\ 0, & H \neq h \end{cases}$$

$$\mu_{f_4 \to D}(D) = \sum_E P(D|E)\left(\sum_H P(E|H) \cdot f_9(H)\right)\left(\sum_F P(E|F) \cdot f_8(F) \sum_G P(F|G) \cdot P(G)\right)$$

d)

$$P(D \mid F=f, H=h) = \sum_E P(D \mid E) \cdot \left(\sum_H P(E \mid H) \cdot f_9(H) \right) \cdot \left(\sum_F P(E \mid F) \cdot f_8(F) \sum_G P(F \mid G) \cdot P(G) \right)$$

$$P(A \mid F=f, H=h) = \sum_D P(A \mid D) \cdot \sum_B P(D \mid B) \cdot P(B) \sum_C P(D \mid C) \cdot P(C)$$
$$\sum_E P(D \mid E) \cdot \sum_H P(E \mid H) \cdot f_9(H) \sum_F P(E \mid F) \cdot f_8(F) \cdot \sum_G P(F \mid G) \cdot P(G)$$

$$P(B \mid F=f, H=h) = \sum_D P(B \mid D) \cdot \sum_A P(D \mid A) \cdot P(A) \sum_C P(D \mid C) \cdot P(C)$$
$$\cdot \sum_E P(D \mid E) \cdot \sum_H P(E \mid H) \cdot f_9(H) \cdot$$
$$\cdot \sum_F P(E \mid F) \, f_8(F) \sum_G P(F \mid G) \cdot P(G)$$