Technische Universität Berlin

Distributed Algorithms

Hypercubes

Hypercube

- In general a hypercube is an n-dimensional representation of a square and a cube.
- Hypercubes are defined by their dimension
- You can add Dimensions to a shape, and thus creating a hypercube
- This is pretty intuitive up to the third dimension

0 Dimension

A shape with 0 Dimension is a point, it has no length, no heights, no depth

If you move the point over a length, you create a 1 dimensional Hypercube

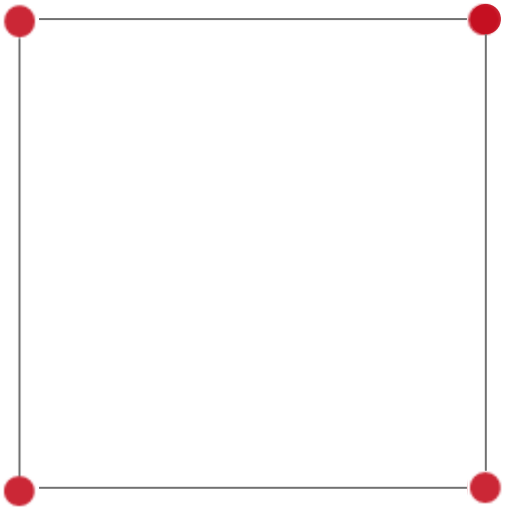
1 Dimension

A shape with 1 Dimension is a line, it has length.

But no heights and no depth

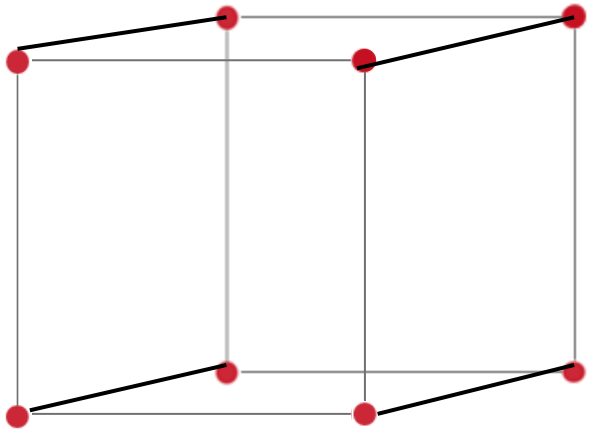
If you move the line at right angle to its length, you create a 2 Dimensional Hypercube

2 Dimension



A shape with 2 Dimension is a square, it has length and heights
But no depth
If you move this square in right angle to its lines you create a 3 Dimensional Hypercube

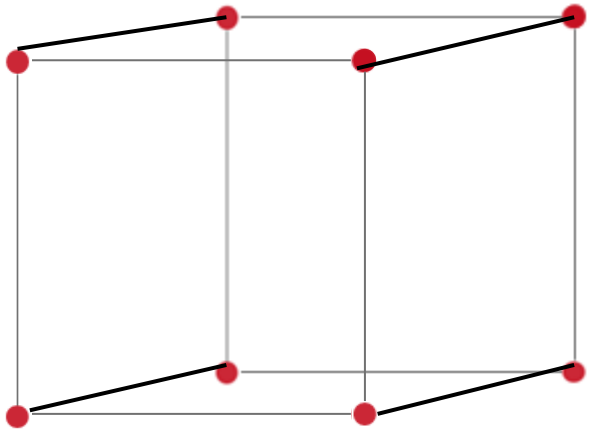
3 Dimension



A shape with 2 Dimension is a cube, it has length and heights and depth

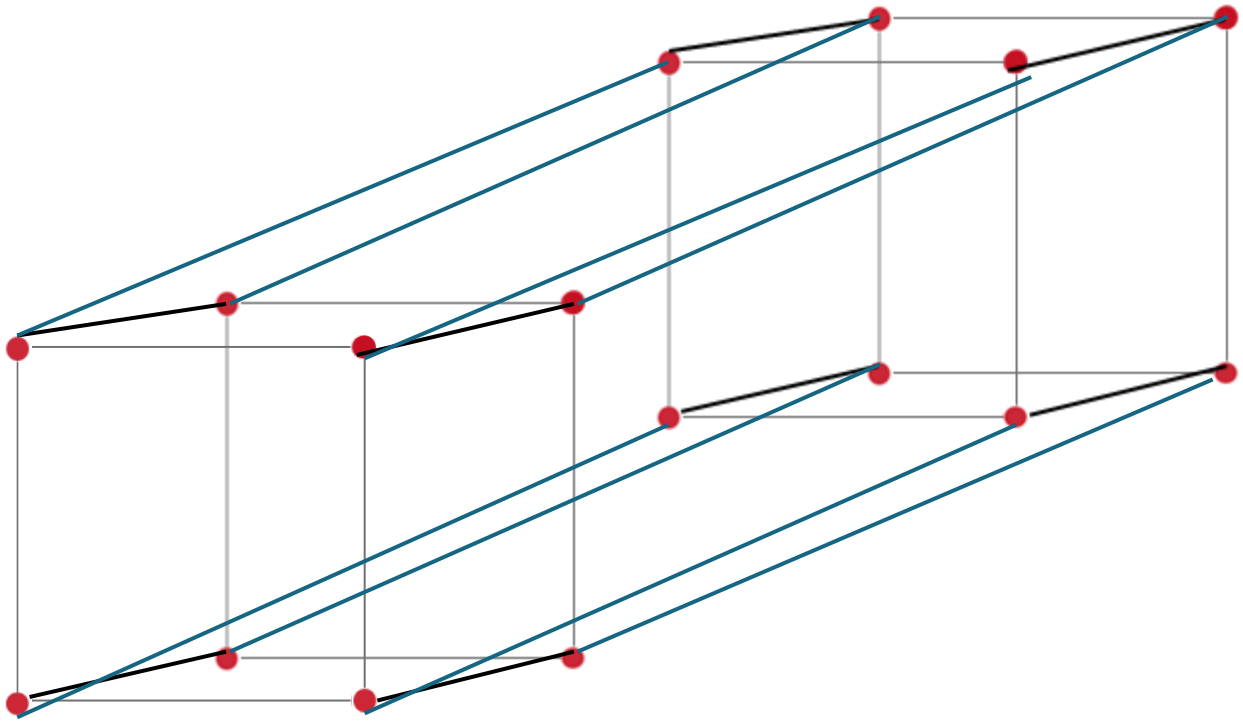
So far, so good. Now it gets al little tricky.

4 Dimensions ?



We now have to move the cube along the 4th dimension…

4 Dimensions



That is one possible graphical representation of a 4 dimension Hypercube

N – Dimension

We could go on, like that. Moving the given Hypercube along the 5^{th}, 6^{th} 7^{th} … n Dimension.

Some general rules and tips for understanding Hypercubes, considering a dimension n:

- Has $\mathbf{2^{(n-1)} * n}$ edges
- Has $\mathbf{2^n}$ nodes
- You can identify nodes by a binary representation with **n bits**.
- Two nodes would then be directly connected if exactly one bit differs in the binary representation.
- A shortest path in a hypercube is represented as a sequence of connected nodes.

Some sources

- A survey of the theory of hypercube graphs
  http://www.sciencedirect.com/science/article/pii/0898122188902131
- http://mathworld.wolfram.com/HypercubeGraph.html
- https://www.youtube.com/watch?v=G3NHtv6LCGQ
- http://demonstrations.wolfram.com/RotatingAHypercube/