

# Time Series Analysis

Discussion Section 01

Please log in to **ISIS** (password: Zeit1718) and **download** the following files:

- USpop.dta

## Deterministic Models

- **Components of a Time Series**
- **Additive and Multiplicative Models**
- **Simple Trend Models**
- Smoothing Techniques
- Seasonal Adjustment

## Four components

$L_t$  long-term trend

$C_t$  cyclical component

$S_t$  'seasonal' component


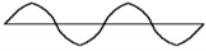
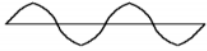
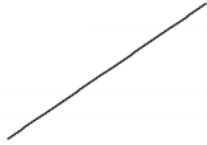








$I_t$  irregular component

### Additive Model

$$y_t = L_t + C_t + S_t + I_t \quad t = 1, \dots, T$$

### Multiplicative Model

$$y_t = L_t \cdot C_t \cdot S_t \cdot I_t \quad t = 1, \dots, T$$

	Nonseasonal	Additive Seasonal	Multiplicative Seasonal
Constant Level			
Linear Trend			
Damped Trend			
Exponential Trend			

## Exercise 1.1:

- Generate a time variable (named “time”) for the “USpop.dta” dataset.

### Recall:

- Creating New Variables: `generate newvar = exp`

- **System variables** (`_variables`)

`_n` contains the number of the current observation. It is useful for indexing observations or generating sequences of numbers and can be used with mathematical operators.

- Time series dates:
 

Format (%fmt)	Description	Coding
%td	daily	0 = 01jan1960, 1 = 02jan1960
%tw	weekly	0 = 1960w1, 1 = 1960w2
%tm	monthly	0 = 1960m1, 1 = 1960m2
%tq	quarterly	0 = 1960q1, 1 = 1960q2
%th	halfyearly	0 = 1960h1, 1 = 1960h2
%ty	yearly	1960 = 1960, 1961 = 1961

`format varlist %ty`

- Label the variable time “Time”.
- Plot the time series and describe its pattern. Which of the simple trend models do you think is appropriate?

## Exercise 1.2:

- Fit a linear trend model to the “USpop.dta” dataset. Calculate  $c_1$  and  $c_2$  and write down the estimated equation.
- Plot the original series and the fitted values. Do you think the linear model is appropriate?
- Predict the residuals. If the linear trend model would be the right one how should the residuals behave? Plot the residuals. What can you conclude?

**Note:**  $L_t = c_1 + c_2 t$

`regress depvar [indepvars]` fits a model of *depvar* on *indepvars* using linear regression

`predict newvar , xb` predicts the fitted values from the last estimation

`predict newvar , residuals` predicts the residuals from the last estimation

## Exercise 1.3:

- Plot “log(uspop)” against time.
- Fit a logarithmic linear trend model to the “USpop.dta” dataset and write down the estimated equation for  $L_t$ .

**Note:**  $L_t = f(t) = A \cdot e^{rt}$

Estimation by taking the logarithms of both sides and fitting the log-linear regression equation:

$$\log(L_t) = \log(A) + rt$$

- Plot the original series together with the fitted values against time, and the residuals against time. What can you conclude?

## Exercise 1.4:

- Calculate the US population growth rate between 1790 and 1800.
- Calculate the same growth rate using the log operator.

$$\log(x_1) - \log(x_0) \approx \frac{x_1 - x_0}{x_0} = \frac{\Delta x}{x_0} \quad x_1, x_0 > 0 \quad \text{for small changes in } x$$

**Note:** With the command `display` you can use Stata as a substitute for a hand calculator.  
Example: `display uspop[1]` displays the first observation of the variable “uspop”



## Exercise 1.5:

- Split the time series into two appropriate periods and fit a logarithmic linear trend model to each of them.
- Plot the original time series together with the fitted values. What can you conclude?
- If necessary try to split into two different periods.

## Exercise 1.6:

- Fit a quadratic trend model to the “USpop.dta” dataset. Write down the estimated equation.

Note:  $L_t = c_1 + c_2 t + c_3 t^2$

- Plot the residuals. What can you conclude?
- Fit a cubic trend model to the “USpop.dta” dataset. Write down the estimated equation.

Note:  $L_t = c_1 + c_2 t + c_3 t^2 + c_4 t^3$

- Plot the residuals. What can you conclude?

## Exercise 1.7:

- Compare the (at least) five models you have fitted to the dataset. Which of these models fits best to the series?
- Calculate for each model the forecast for the year 2000 and compare them with the US population in 2000 (281.55 million) and in 2010 (310.3 million).

Please log in to **ISIS** (password: Zeit1718) and **download** the following files:

- oil.dta
- Siemens.dta
- prod.dta

## Deterministic Models

- Components of a Time Series
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- Simple Trend Models
- **Smoothing Techniques**
- Seasonal Adjustment

## Smoothing Techniques...

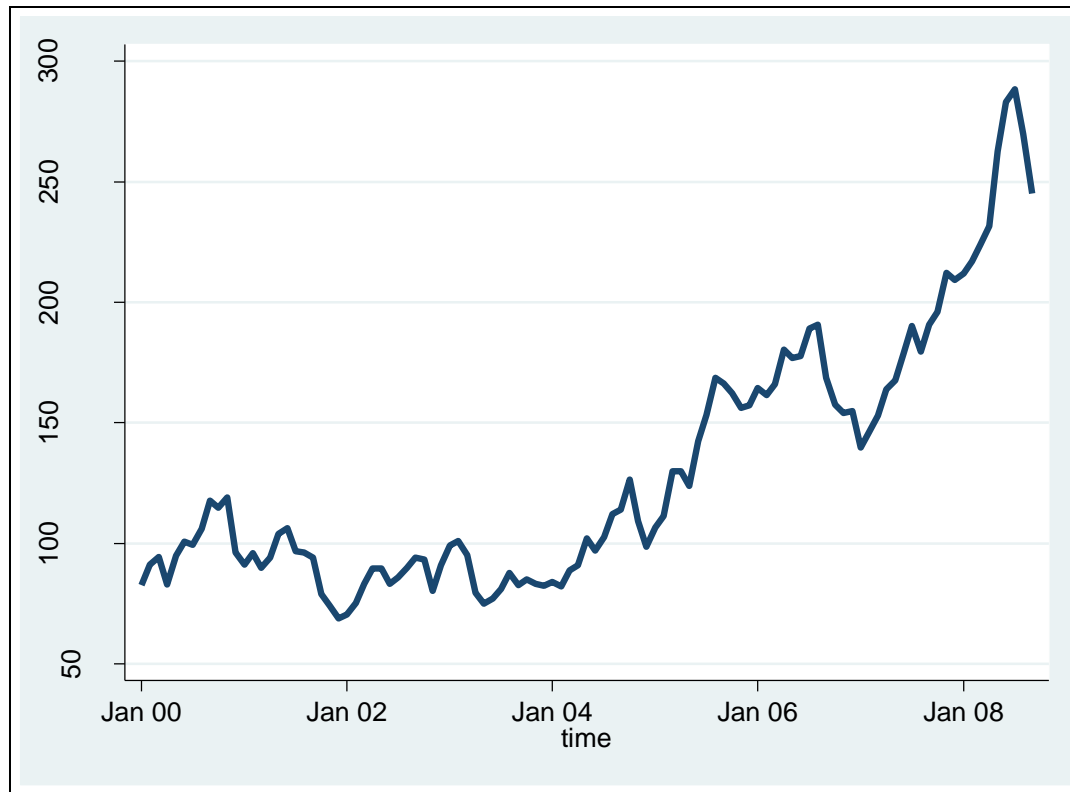
- Simple Moving Average
- Centered Moving Average
- Exponentially Weighted Moving Average
- Holt-Winter's two parameter exponential smoothing
- Hodrick-Prescott filter

... are useful for

- filtering/smoothing
- forecasting

## Crude oil import price index

Jan. 2000 - Sep. 2008



## Exercise 1.8:

Calculate the exponentially weighted moving average for the first four observations of the oil variable (import price index 2000 = 100). Use  $\bar{y}_0 = 82.6$  as the initial value and  $\alpha = .3$  as the smoothing parameter.

**Notice:**  $\bar{y}_t = \alpha y_t + (1 - \alpha)\bar{y}_{t-1}$

Data:	time	oil
	Jan. 2000	82.6
	Feb. 2000	91.2
	Mar. 2000	94.3
	Apr. 2000	82.9
	May 2000	94.9
	Jun. 2000	100.7
	...	...



## Exercise 1.9:

- Upload the “oil.dta” dataset.
- Generate the appropriate `time` variable (labeled “Time”).
- Plot the series.

## Exercise 1.10:

- Calculate the exponentially weighted moving average. Use  $\bar{y}_0 = 82.6$  as the initial value and  $\alpha = .3$  as the smoothing parameter.

### Stata commands:

#### Macro definition

```
. local lclname = exp
```

Example:

```
. local num = 5
. di `num'
5
```

#### Replace contents of existing variable

```
replace oldvar = exp
```

#### Loop over consecutive values

```
forvalues lname = range {
    commands referring to `lname'
}
```

where *range* is #1/#2 meaning #1 to #2 in steps of 1

## Exercise 1.11:

- Calculate the exponentially weighted moving average. Use  $\bar{y}_0 = 82.6$  as the initial value and  $\alpha = .3$  as the smoothing parameter.

### Stata command:

#### Exponential smoothing

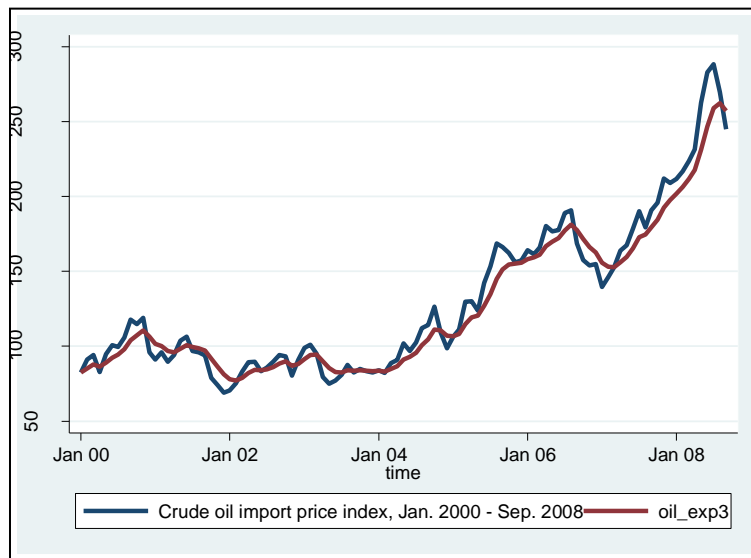
`tssmooth exponential`

`parms (#a)` specifies the parameter alpha for the exponential smoother;  $0 < \#a < 1$

`s0 (#)` specifies the initial value to be used

## Exercise 1.12:

- True or false and explain: The above EWMA model is appropriate for forecasting the oil price.
- For large values of  $\alpha$ , will the response to changes in the mean of the unfiltered series be slow or fast?



## Exercise 1.13:

- Upload the “Siemens.dta” dataset.
- Plot the series.
- Calculate the EWMA. Use the first observation as the initial value and  $\alpha=.3$  as the smoothing parameter.
- Plot the original series and the smoothed series.

## Exercise 1.14:

- Detrend the original series (assume a linear trend).  $L_t = c_1 + c_2 t$
- Smooth the detrended series using EWMA. Use the first observed value as the initial value and  $\alpha = .3$  as the smoothing parameter.
- Add back the trend.
- Plot the result together with the original series and the smoothed original series.

## Exercise 1.15:

Calculate the first three smoothed values of the Siemens time series using the Holt-Winter's two parameter exponential smoothing. Smoothing parameters and initial values:  $\alpha = 0.3$   $\gamma = 0.6$   $\tilde{y}_0 = 32.05$   $r_0 = 0$

time	SIE
08.10.2002	32.05
09.10.2002	32.42
10.10.2002	35
11.10.2002	37.53
...	...

**Notice:**  $\tilde{y}_t = \alpha y_t + (1 - \alpha)(\tilde{y}_{t-1} + r_{t-1})$  and  $r_t = \gamma(\tilde{y}_t - \tilde{y}_{t-1}) + (1 - \gamma)r_{t-1}$

Here  $r_t$  is a smoothed series representing the trend, for example the average rate of increase.

## Exercise 1.16:

- Smooth the Siemens.dta series using the Holt-Winter's two parameter exponential smoothing. Use the following smoothing parameters and initial values:  $\alpha = 0.3$   $\gamma = 0.6$   $\hat{y}_0 = 32.05$   $r_0 = 0$

**Notice:**  $\hat{y}_t = \alpha y_t + (1 - \alpha)(\hat{y}_{t-1} + r_{t-1})$

$$r_t = \gamma(\hat{y}_t - \hat{y}_{t-1}) + (1 - \gamma)r_{t-1}$$

- Plot the result together with the original series and the “simple” smoothed series.



## Exercise 1.17:

### Hodrick-Prescott filter

- Compare the "smoothness" of the filtered output series (prod.dta) for different values of  $\lambda$ .

**Notice:** 
$$\min \underbrace{\sum_{t=1}^T (y_t - \hat{G}_t)^2}_{\text{Fit}} + \lambda \underbrace{\sum_{t=2}^{T-1} [(\hat{G}_{t+1} - \hat{G}_t) - (\hat{G}_t - \hat{G}_{t-1})]^2}_{\text{Penalty for Non-Smoothness}}$$

Please log in to **ISIS** (password: Zeit1718) and **download** the following file:

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## Deterministic Models

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## Exercise 1.18:

- Show that a 12-month centered moving average is a weighted moving average over 13 periods.

## Seasonal Adjustment (Multiplicative model)

The objective is to eliminate the seasonal component  $S$ :

1. Isolate the combined long-term trend and cyclical components ( $G_t = L_t \cdot C_t$ ) by removing the combined seasonal and irregular components.
2. Divide the original data by the smoothed series to estimate the combined seasonal and irregular components:

$$y_t = L_t \cdot C_t \cdot S_t \cdot I_t \quad t = 1, \dots, T \quad \frac{y_t}{\hat{y}_t} = \frac{L_t \cdot C_t \cdot S_t \cdot I_t}{L_t \cdot C_t} = S_t \cdot I_t$$

3. Eliminate the irregular component as completely as possible. Average the values of the combined seasonal and irregular components corresponding to the same period. These averages will then be estimates of the seasonal indices.
4. Deseasonalize the original series by dividing each value by its corresponding seasonal index.

## Exercise 1.19:

- Load the “unemp.dta” dataset. Plot the series. Describe the seasonal pattern.
- Compute a series which is assumed to be relatively free of seasonal and irregular fluctuations.

**Recall:** `tssmooth ma newvar = var, weights([numlist_l]<#c>[numlist_f])`

The option is required for the weighted moving average and describes the span of the moving average, as well as the weights to be applied to each term in the average. The middle term literally is surrounded by < and >, so you might type `weights(1 2 <2> 2 1)`. `numlist_l` is optional and specifies the weights that are to be applied to the lagged terms when computing the moving average. `#_c` is required and specifies the weight to be applied to the current term. `numlist_f` is optional and specifies the weights to be applied to the forward terms when computing the moving average.

- Divide the original data by the smoothed series to estimate the combined seasonal and irregular components.

## Exercise 1.20:

- Average the values of the combined seasonal and irregular components corresponding to the same month. These averages will then be estimates of the seasonal indices.

**Note:** Final seasonal indices are computed by multiplying these averages by a factor that brings their sum to 12.

- Deseasonalize the original series by dividing each value by its corresponding seasonal index.