MI - H3

November 11, 2016

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In [11]: import numpy as np
         import matplotlib.pyplot as plt
         import matplotlib.cm as cm
         import itertools
         %matplotlib inline

In [ ]: # Exercise 1.a, b, c -> See notes

In [2]: # Array of {x_n, t_n} with n = 1, ... , 10
        # x_n: random from uniform distribution over [0, 1]
        # t_n: sin(2*pi*x_n) + Gaussian Noise (sigma=0.25)
        data = np.loadtxt('RegressionData.txt', skiprows=0, dtype=bytes, delimiter=

In [427]: def plot(ax, data, **kwargs):
              mapping = np.array(data).T
              ax.plot(mapping[0], mapping[1], **kwargs)
              ax.set_title(kwargs['label'])

          def scatter(ax, data, **kwargs):
              mapping = np.array(data).T
              ax.scatter(mapping[0], mapping[1], **kwargs)
              ax.set_title(kwargs['label'])

In [434]: yTs = data[:, 1]  # True value of attribute

          def quadratic_error(yT, y_x):
              return np.average(0.5 * (y_x - yT)**2)

          def mlp(epsilon=1e-5, t_max=3000, eta=0.5):
              # Initial MLP params
              weights_hidden = np.random.rand(3) - 0.5  # Weights for the single hi
              weights_output = np.random.rand(3) - 0.5  # Weights for output layer
              biases = np.random.rand(3) - 0.5  # Biases for all (1) hidden layers

              # Stop iteration if error value has converged (epsilon=1e-5) or max i
              errors = []
              iterations = 0
              y_xs = []
```

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    while iterations < t_max and (len(errors) < 2 or np.abs(errors[-1] -
        y_xs = []
        gradient_hidden = np.zeros(3)
        gradient_output = 0
        for x, yT in data:
            # Forward propagation (compute activation/transfer functions)
            y_x = (weights_output * np.tanh(weights_hidden * x - biases))
            y_xs.append(y_x)
            # Backward propagation:
            # Calculate local errors of the hidden layer: delta_v_i (laye
            # FIXME: "x - biases" ?
            local_errors = (weights_output * (1 - np.tanh(weights_hidden
            local_error_output = 1  # Because f_L (output layer) is the i
            # Calculate batch gradient using the local errors
            gradient_hidden += (y_x - yT) * local_errors * x
            gradient_output += (y_x - yT) * local_error_output * np.tanh(
        # Adopt gradient descent
        weights_hidden -= eta * gradient_hidden / len(data)
        weights_output -= eta * gradient_output / len(data)
        # Compute output error using the quadratic error cost function
        errors.append(quadratic_error(yTs, y_xs))
        iterations += 1
    return iterations, errors, weights_hidden, biases, y_xs

def mlp2(X, yTs, epsilon=1e-5, t_max=3000, eta=0.5):
    # Initial MLP params
    W = np.random.uniform(-0.5, 0.5, (2, 3))  # Weights between input and
    V = np.random.uniform(-0.5, 0.5, 4)  # Weights between hidden and out
    bias_output = np.random.uniform(-0.5, 0.5)

    # Stop iteration if error value has converged (epsilon=1e-5) or max i
    errors = []
    epochs = 0
    f = np.tanh
    df = lambda x: 1 - np.tanh(x)**2
    while epochs < t_max and (len(errors) < 100 or np.abs(errors[-1] - er
        yXs = []
        gVs = np.zeros(V.shape)
        gWs = np.zeros(W.shape)
        for x, yT in zip(X, yTs):
            # Forward propagation (compute activation/transfer functions)
            yX = V.T.dot(np.array([1, *f(W.T.dot(np.array([1, x])))]))
            yXs.append(yX)
            # Backward propagation: (comupte gradient decent using local
            gV = (yX - yT) * np.array([1, *f(W.T.dot(np.array([1, x])))])
            gW = np.zeros(W.shape)
            for i, j in list(itertools.product(range(gW.shape[1]), range(
                gW[j, i] = (yX - yT) * V[i] * df(W.T[i].dot([1, x])) * [1
```

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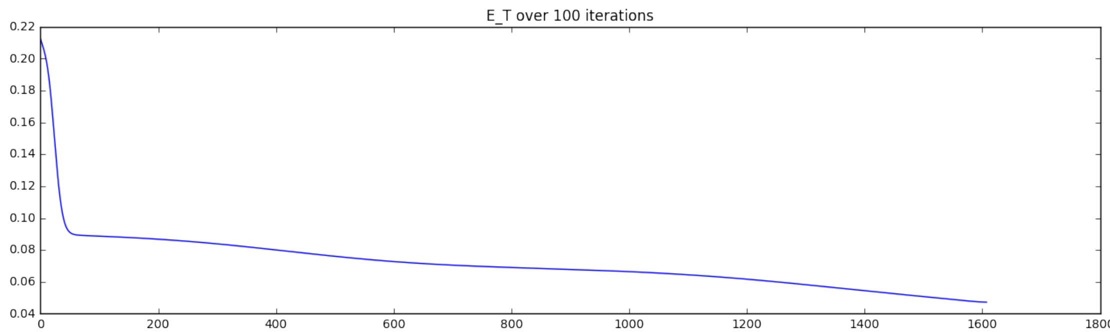
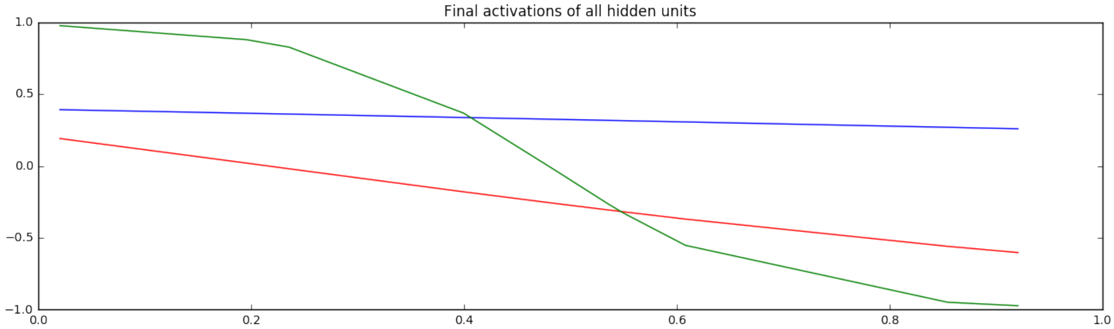
                # Calculate batch gradient using the local errors
                gVs += gV
                gWs += gW
            # Adopt gradient descent
            W -= eta * gWs / len(data)
            V -= eta * gVs / len(data)
            # Compute output error using the quadratic error cost function
            errors.append(quadratic_error(yTs, yXs))
            epochs += 1
        return epochs, errors, W, V, yXs
```

In [433]:
```
# Exercise 2.a: Plot E_T over iterations
np.random.seed(0)
epochs, errors, _, _, _ = mlp2(*data.T)
fig, ax = plt.subplots(1, 1, figsize=(13, 4))
plot(ax, list(enumerate(errors)), color='blue', label='E_T over {} iterat
fig.tight_layout()
```



In [432]:
```
# Exercise 2.b: Plot activation functions of hidden units from the final
np.random.seed(0)
_, _, W, V, _ = mlp2(*data.T)
input_space = data[:, 0]

fig, ax = plt.subplots(1, 1, figsize=(13, 4))
activators = [(x, np.tanh(W.T.dot(np.array([1, x])))) for x in input_spac
for i, color in enumerate(['red', 'blue', 'green']):
    activator = [(x, y[i]) for x, y in activators]
    activator.sort(key=lambda x: x[0])
    plot(ax, activator, color=color, label='Final activations of all hidc
fig.tight_layout()
```

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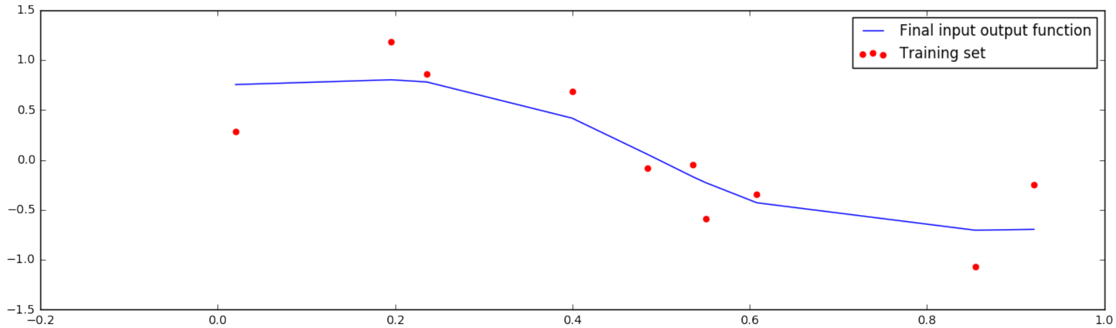
In [431]: # Exercise 2.c: Plot MLPs input-output function vs the original function
          np.random.seed(0)
          training_set = sorted(data, key=lambda x: x[0])
          input_space = data[:, 0]
          _, _, _, _, yXs = mlp2(*data.T)
          input_output_function = sorted(list(zip(input_space, yXs)), key=lambda x:

          fig, ax = plt.subplots(1, 1, figsize=(13, 4))
          scatter(ax, training_set, color='red', label='Training set')
          plot(ax, input_output_function, color='blue', label='Final input output f
          ax.set_title('')
          ax.legend()
          fig.tight_layout()
```

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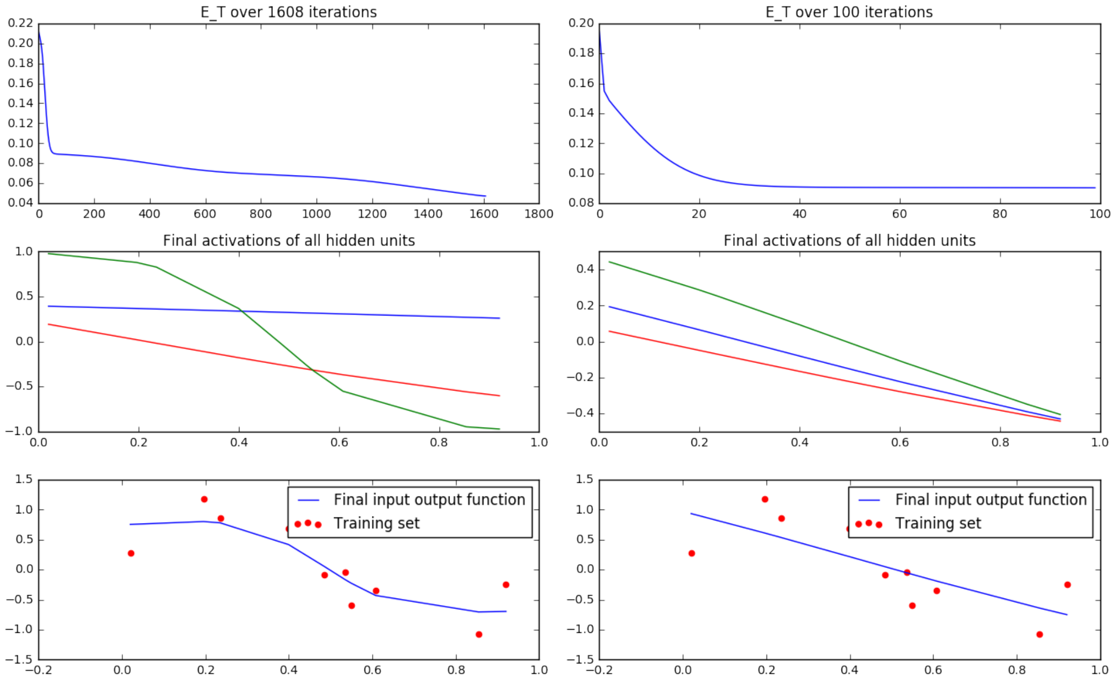
In [435]: # Exercise 2.d: Plot a-c twice next to each other - Discuss: Is there a c
          np.random.seed(0)
          fig, ax = plt.subplots(3, 2, figsize=(13, 8))
          training_set = sorted(data, key=lambda x: x[0])
          input_space = data[:, 0]

          for i in (0, 1):
```

```
        epochs, errors, W, V, yXs = mlp2(*data.T, eta=0.5)
        # 1.a
        plot(ax[0, i], list(enumerate(errors)), color='blue', label='E_T over
        # 1.b
        activators = [(x, np.tanh(W.T.dot(np.array([1, x])))) for x in input_
        for j, color in enumerate(['red', 'blue', 'green']):
            activator = [(x, y[j]) for x, y in activators]
            activator.sort(key=lambda x: x[0])
            plot(ax[1, i], activator, color=color, label='Final activations o
        # 1.c
        input_output_function = sorted(list(zip(input_space, yXs)), key=lambd
        scatter(ax[2, i], training_set, color='red', label='Training set')
        plot(ax[2, i], input_output_function, color='blue', label='Final inpu
        ax[2, i].set_title('')
        ax[2, i].legend()

    fig.tight_layout()
```



1 Discuss: Why are there differences?

Because the initial weights are generated randomly the gradient descent method most likely converges to another local minimum as before. That's why it finds a good solution but always depending on a different weighting matrix (See final activation functions).

2 Discuss: What is the motivation for a using a quadratic error function in this example?

The quadratic error cost functions is chosen because we know that the original function has a gaussian noise. This makes it also easier for the gradient descent method to compute dE/dY which equals yX-yT in this case.