MI - H14

February 17, 2017

In [656]:
```
import numpy as np
from scipy.stats import multivariate_normal
import matplotlib.pyplot as plt
from matplotlib import colors, cm
from functools import reduce
import itertools
import sklearn.datasets
import os
import re
%matplotlib inline
```

In [661]:
```
def plot(data, ax=None, enum=False, title='', labels=None, legend=False,
    axes_defined = ax != None
    if not axes_defined:
        fig, ax = plt.subplots(1, 1, figsize=(13, 4))
    plotted = None
    if enum:
        plotted = ax.plot(data, **kwargs)
    else:
        mapping = np.array(data).T
        plotted = ax.plot(mapping[0], mapping[1], **kwargs)
    if labels:
        ax.set_xlabel(labels[0])
        if (len(labels) > 1):
            ax.set_ylabel(labels[1])
    if legend:
        ax.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
    ax.set_title(title)
    ax.grid(True)
    if not axes_defined:
        fig.tight_layout()
    return ax

def scatter(data, ax=None, enum=False, title='', labels=None, legend=Fals
    axes_defined = ax != None
    if not axes_defined:
        fig, ax = plt.subplots(1, 1, figsize=(13, 4))
    scattered = None
```

```
        if enum:
            scattered = ax.scatter(range(len(data)), data, **kwargs)
        else:
            mapping = np.array(data).T
            scattered = ax.scatter(mapping[0], mapping[1], **kwargs)
        if labels:
            ax.set_xlabel(labels[0])
            if (len(labels) > 1):
                ax.set_ylabel(labels[1])
        if legend:
            ax.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
        if xlim:
            ax.set_xlim(xlim)
        ax.set_title(title)
        ax.grid(True)
        if colorbar:
            cax = plt_ax.make_axes_locatable(ax).append_axes("right", size="5
            cbar = plt.colorbar(scattered, cax=cax)
            cbar.set_ticks([-1, 0, 1])
        if not axes_defined:
            fig.tight_layout()
        return ax

    def heatmap(ax, data, title, policy=False):
        if policy:
            cmap = cm.get_cmap('jet')
            bounds = [-1] + list(range(len(actions) + 1))
            norm = colors.BoundaryNorm(bounds, cmap.N)
            cax = ax.imshow(data, extent=[-np.pi/4, np.pi/4, -3, 3], aspect='
        else:
            cax = ax.imshow(data, extent=[-np.pi/4, np.pi/4, -3, 3], aspect='
        ax.set_xlabel('angle')
        ax.set_ylabel('velocity')
        ax.set_title(title)
        plt.colorbar(cax, ax=ax)
```

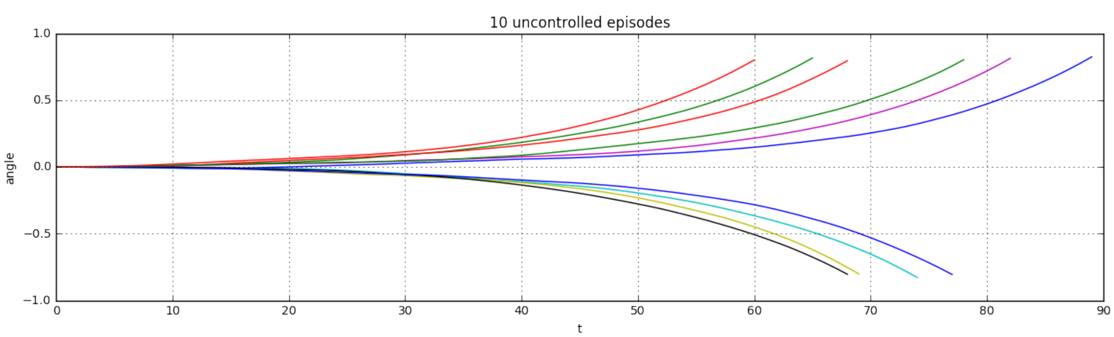
0.0.1 Global variables

```
In [19]: l = 1  # m
         m = 2  # kg
         g = 9.81  # m / s^2
         # Shape of state variable: (angle, velocity)

         ACTION_SUB = -4  # N
         ACTION_WAIT = 0  # N
         ACTION_ADD = 4  # N
         actions = (ACTION_SUB, ACTION_WAIT, ACTION_ADD)
```

```
In [53]: def reward(state):
             angle, _ = state
             if abs(angle) > np.pi/4:
                 return -1
             else:
                 return 0

         def is_failed(state):
             return reward(state) == -1

         # Calculations for dynamics in a small timestamp dt = 0.02 s
         def move(state, action, dt=0.02, sigma=3):
             angle, velocity = state
             epsilon = np.random.normal(0, sigma)
             next_velocity = velocity + (g / l) * np.sin(angle) * dt + (action / m)
             next_angle = angle + next_velocity * dt
             # TODO: Add reward, etc.
             return (next_angle, next_velocity)

         def simulate_uncontrolled():
             state = (0, 0)
             states = [state]
             while not is_failed(state) and len(states) < 1000:
                 state = move(state, ACTION_WAIT)
                 states.append(state)
             return state, states
```

0.0.2 Exercise 14.1 (a) Uncontrolled Episodes

```
In [54]: ax = None
         for _ in range(10):
             final_state, all_states = simulate_uncontrolled()
             angles = [angle for angle, _ in all_states]
             ax = plot(angles, ax, enum=True, labels=['t', 'angle'], title='10 unco
```

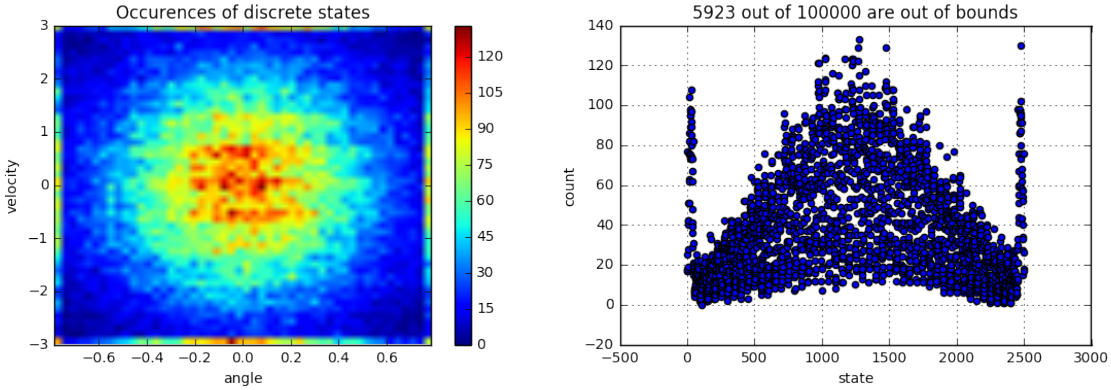


0.0.3 Exercise 14.1 (b) Discrete Spaces

```
In [481]: def discretize(value, value_space):
              idx = (np.abs(value_space - value)).argmin()
              return value_space[idx], idx

          def discrete_state(x, flatten=False, angle_range=angle_values, velocity_r
              angle, velocity = x
              angle, angle_index = discretize(angle, angle_range)
              velocity, velocity_index = discretize(velocity, velocity_range)
              if flatten:
                  return (angle, velocity), angle_index * len(velocity_range) + vel
              return (angle, velocity), (angle_index, velocity_index)
```

```
In [564]: D1 = 50
          D2 = 50
          angle_values = np.linspace(-np.pi/4, np.pi/4, D1).round(2)
          velocity_values = np.linspace(-3, 3, D2).round(2)
          discrete_space = list(itertools.product(angle_values, velocity_values))
          # Draw 100.000 continuous states from the multivariant normal dist and ma
          # FIXME: For some reason the following cov matrix doesn't work out: [[np.
          random_states = np.random.multivariate_normal([0, 0], [[np.pi/19, 0], [0,
          # 50 x 50 image plot for the colorcoded amount of each discrete state
          states = [discrete_state(x, angle_range=angle_values, velocity_range=velo
          # Count occurences for each discrete state
          counts = [states.count(disc_state) for disc_state in discrete_space]
```

```
In [565]: fig, axes = plt.subplots(1, 2, figsize=(13, 4))
          im = axes[0].imshow(np.reshape(counts, (50, 50)), extent=[-np.pi/4, np.pi
          axes[0].set_title('Occurences of discrete states')
          axes[0].set_xlabel('angle')
          axes[0].set_ylabel('velocity')
          plt.colorbar(im, ax=axes[0])
          ax = scatter(counts, ax=axes[1], enum=True, labels=['state', 'count'], ti
              len(states) - len([x for x in states if -np.pi/4 <= x[0] <= np.pi/4 a
              len(states)))
```

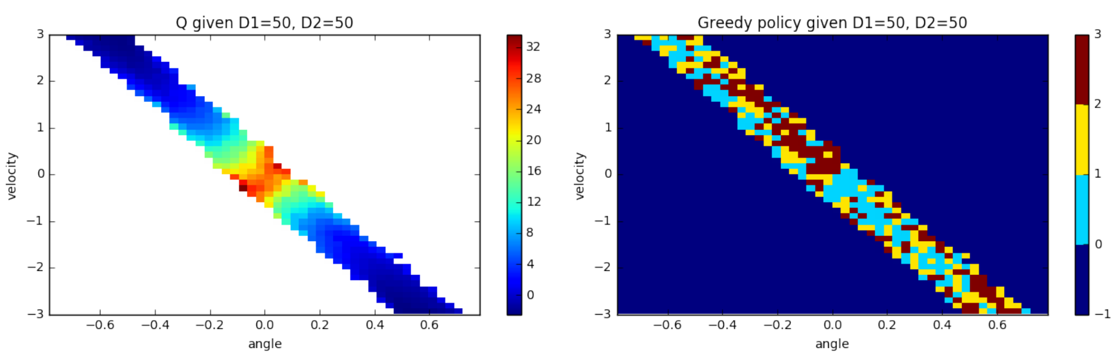


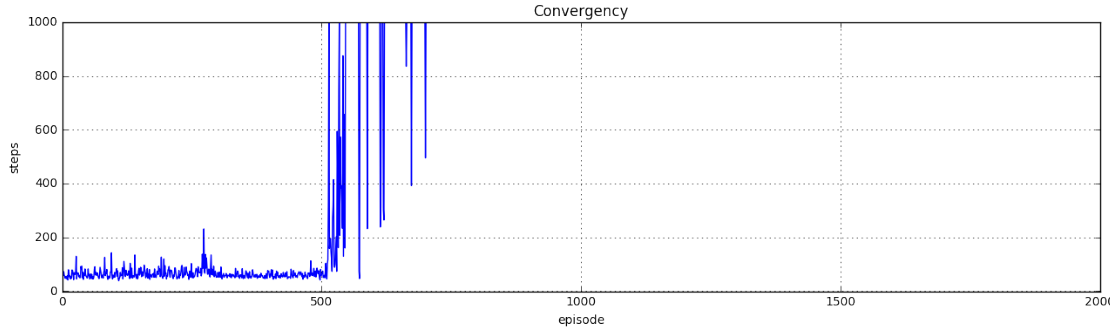
0.0.4 Exercise 14.2 (a) Q-Learning with e-greedy policy

In [662]:
```
def egreedy_policy(Q, state_pos, action=None, e=0, seed=0):
    choice = -1
    randomize = np.random.RandomState(seed)
    if randomize.rand() < e or all(Q[state_pos] == 0):
        choice = randomize.randint(len(actions))
    else:
        # Find index of the best action optimizing Q[state, action]
        choice = Q[state_pos].argmax()
    if not action:
        return actions[choice], choice
    return int(action == choice)

def qlearning(Q, angle_values, velocity_values, eta=0.5, gamma=0.9, e=0):
    # SARSA algorithm for Q approximation (on policy)
    state, state_pos = discrete_state((0, 0), flatten=True,
                                      angle_range=angle_values, velocity_
    exact_state = state
    states = [state]
    seed = np.random.randint(100000)
    while not is_failed(state) and len(states) < 1000:
        r = reward(state)
        action, action_pos = egreedy_policy(Q, state_pos, e=e, seed=seed+
        exact_state = move(exact_state, action)
        next_state, next_state_pos = discrete_state(exact_state, flatten=
                                                    angle_range=angle_val
        Q[state_pos, action_pos] += eta * (r + gamma * Q[next_state_pos].
        state, state_pos = next_state, next_state_pos
        states.append(state)
    # Add the reward value (-1 if failed)
    r = reward(state)
    action, action_pos = egreedy_policy(Q, state_pos, e=e, seed=seed+len(
    exact_state = move(exact_state, action)
    next_state, next_state_pos = discrete_state(exact_state, flatten=True
                                                angle_range=angle_values,
    Q[state_pos, action_pos] += eta * (r + gamma * Q[next_state_pos].max(
    return Q, states

def train_qlearning(episodes=2000, D1=50, D2=50, eta=0.5, gamma=0.9, e=0,
    Q = np.zeros((D1 * D2, len(actions)))
    Qsteps = []
    angle_values = np.linspace(-np.pi/4, np.pi/4, D1).round(2)
    velocity_values = np.linspace(-3, 3, D2).round(2)
    D_space = np.array(list(itertools.product(angle_values, velocity_valu
```

```
        for i in range(episodes):
            Q, Qstates = qlearning(Q, angle_values, velocity_values, eta, gam
            Qsteps.append(len(Qstates))
            if i % 200 == 0 and not quiet:
                print('Episode {} (D1={}, D2={})'.format(i, D1, D2))
        return Q, D_space, Qsteps

    def plot_qlearning(Q, D_space, Qsteps, D1=50, D2=50, e=0):
        fig, axes = plt.subplots(1, 2, figsize=(13, 4))
        Q_values = Q.sum(axis=1)
        Q_values[Q_values != 0] = -np.log(-Q_values[Q_values != 0])
        Q_values[Q_values == 0] = np.nan
        heatmap(axes[0], Q_values.reshape(D1, D2), 'Q given D1={}, D2={}'.for
        # Plot policies
        policies = np.array([
                egreedy_policy(Q, i, e=e)[1] if any(Q[i] != 0) else -1
                for i in range(len(D_space))])
        cmap = colors.ListedColormap(['b','g','y','r'])
        bounds = [-1] + list(range(len(actions)))
        norm = colors.BoundaryNorm(bounds, cmap.N)
        heatmap(axes[1], policies.reshape(D1, D2), 'Greedy policy given D1={]
        fig.tight_layout()
        ax = plot(Qsteps, enum=True, title='Convergency', labels=['episode',
        ax.set_ylim([-1, 1001])
```

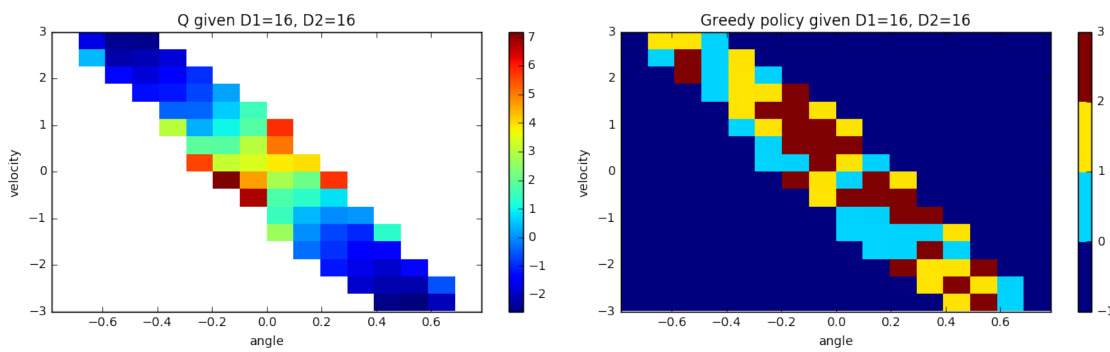
```
In [669]: Q50, Q50_D_space, Q50_steps = train_qlearning()
          plot_qlearning(Q50, Q50_D_space, Q50_steps)
```

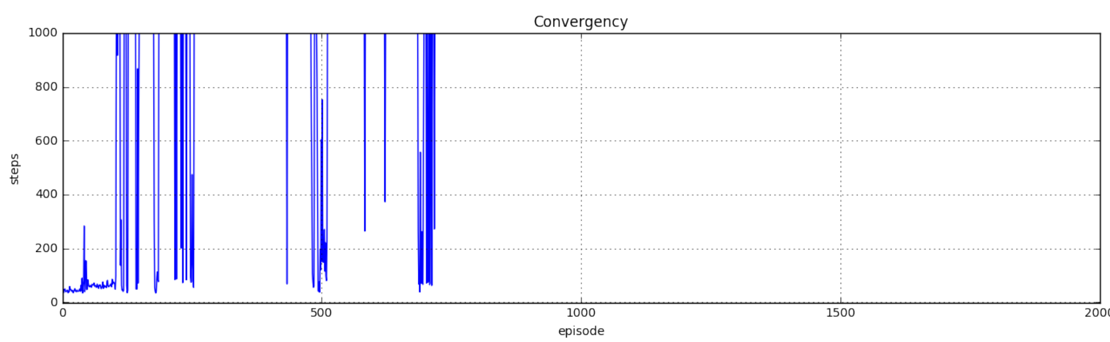


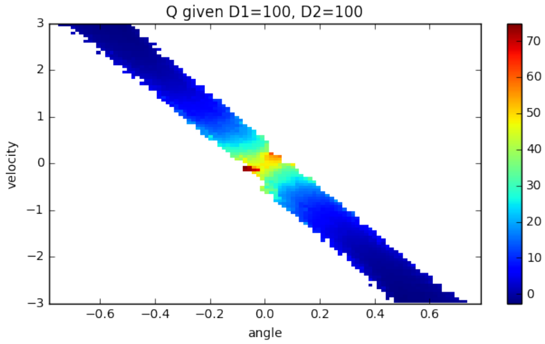


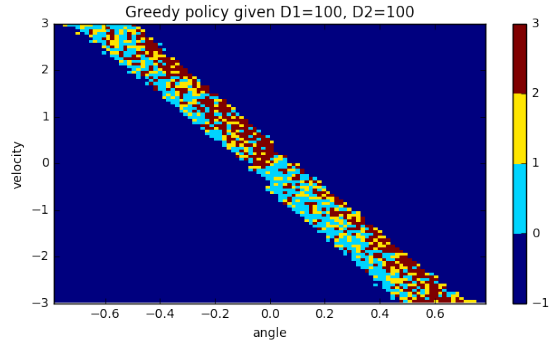
0.0.5 Exercise 14.2 (b) Changing D1 and D2

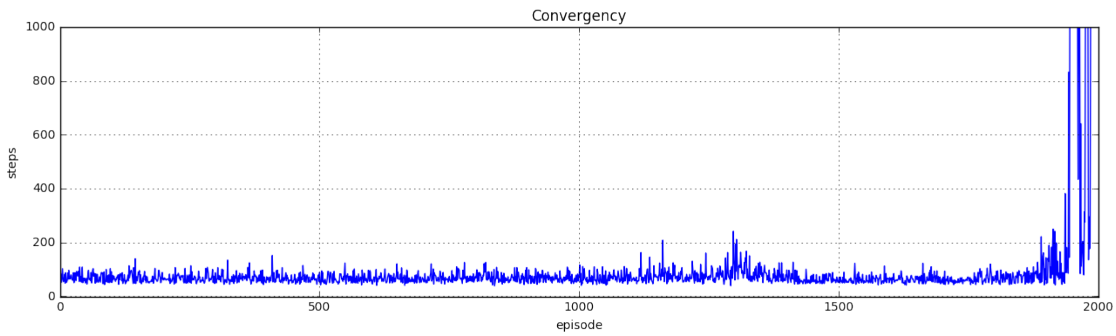
```
In [671]: for D in (16, 100, 200):
              Q, D_space, Q_steps = train_qlearning(D1=D, D2=D)
              plot_qlearning(Q, D_space, Q_steps, D1=D, D2=D)
```

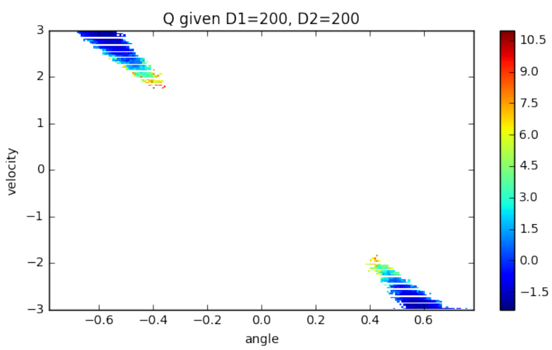


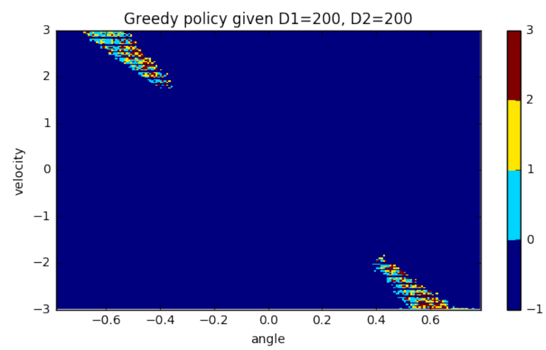


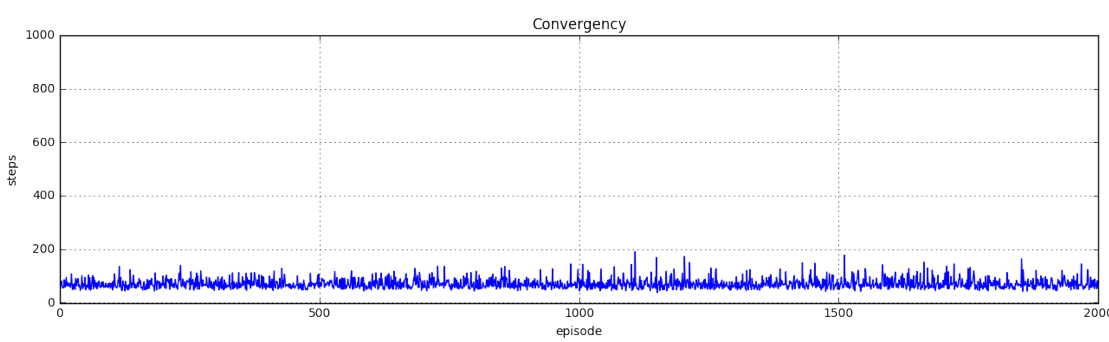

Q given D1=100, D2=100
velocity
angle


Greedy policy given D1=100, D2=100
velocity
angle


Convergency
steps
episode


Q given D1=200, D2=200
velocity
angle

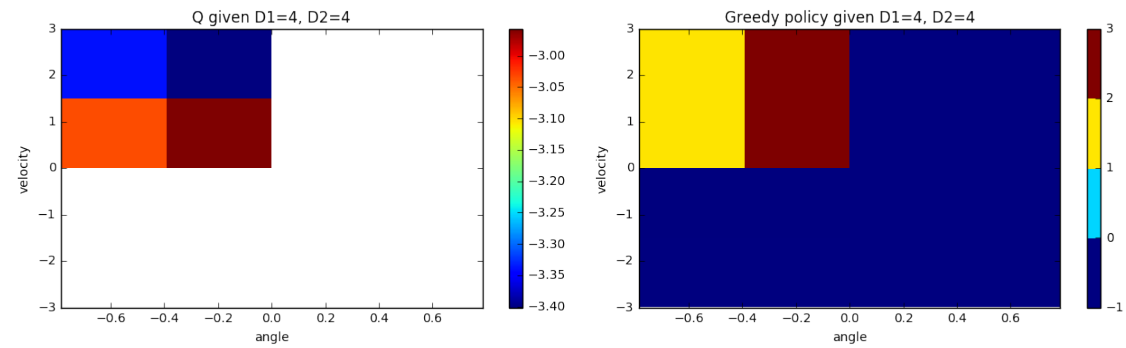

Greedy policy given D1=200, D2=200
velocity
angle

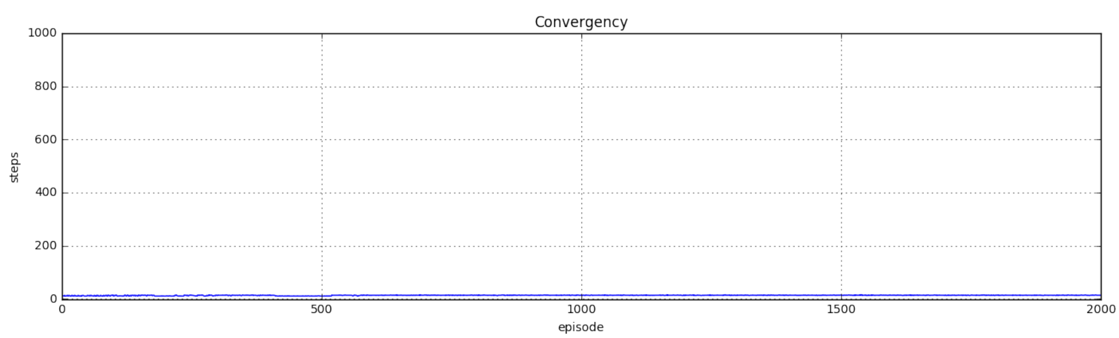

Convergency
steps
episode

0.0.6 14.2 (c)

Smallest number that earns a good policy: Should be D1=D2=2, for me it was around 16

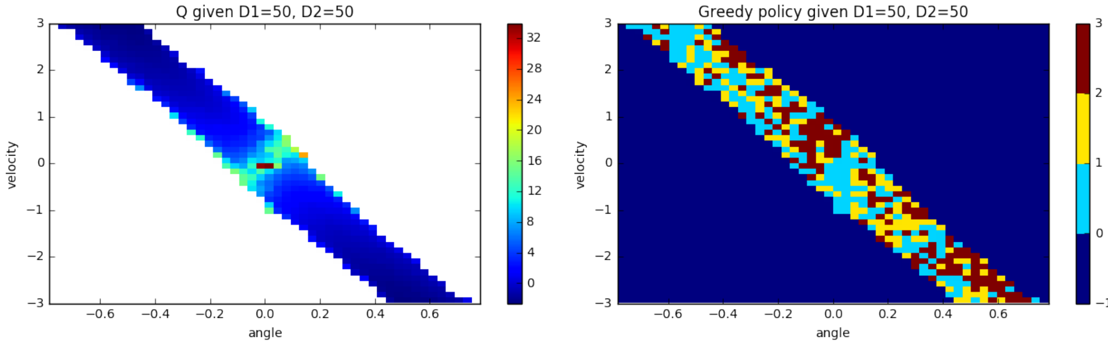
```
In [670]: Q, D_space, Q_steps = train_qlearning(D1=4, D2=4)
          plot_qlearning(Q, D_space, Q_steps, D1=4, D2=4)
```

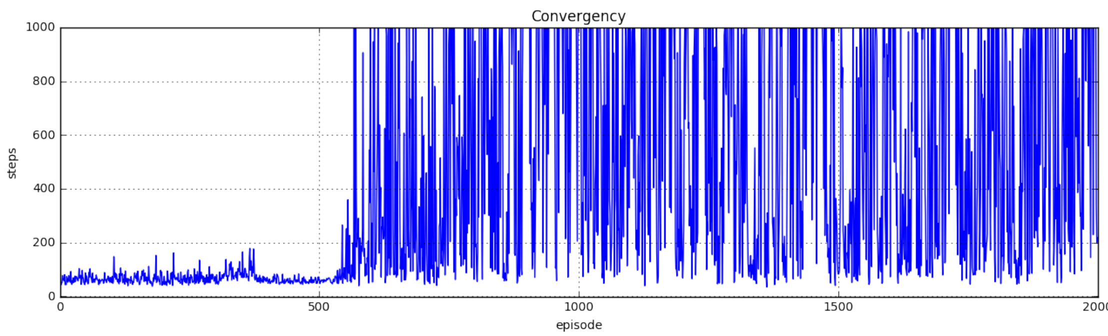




0.0.7 14.2 (d) e-greedy policy (e = 0.1)

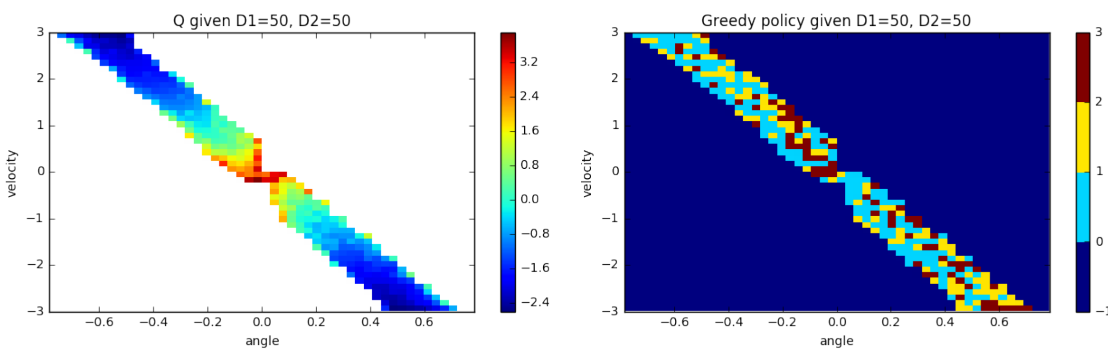
```
In [672]: Qe, Qe_D_space, Q_steps = train_qlearning(e=0.1)
          plot_qlearning(Qe, Qe_D_space, Q_steps, e=0.1)
```

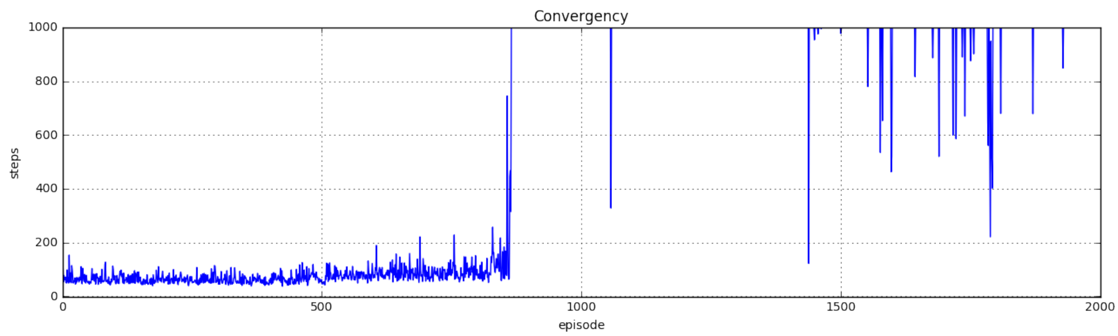




0.0.8 14.2 (e) fast learning rate (eta = 1)

```
In [673]: Qeta, Qeta_D_space, Qeta_steps = train_qlearning(eta=1)
          plot_qlearning(Qeta, Qeta_D_space, Qeta_steps)
```




Convergency
steps
episode
1000
800
600
400
200
0
0
500
1000
1500
2000