Time Series Analysis

Excursus UNIT ROOT

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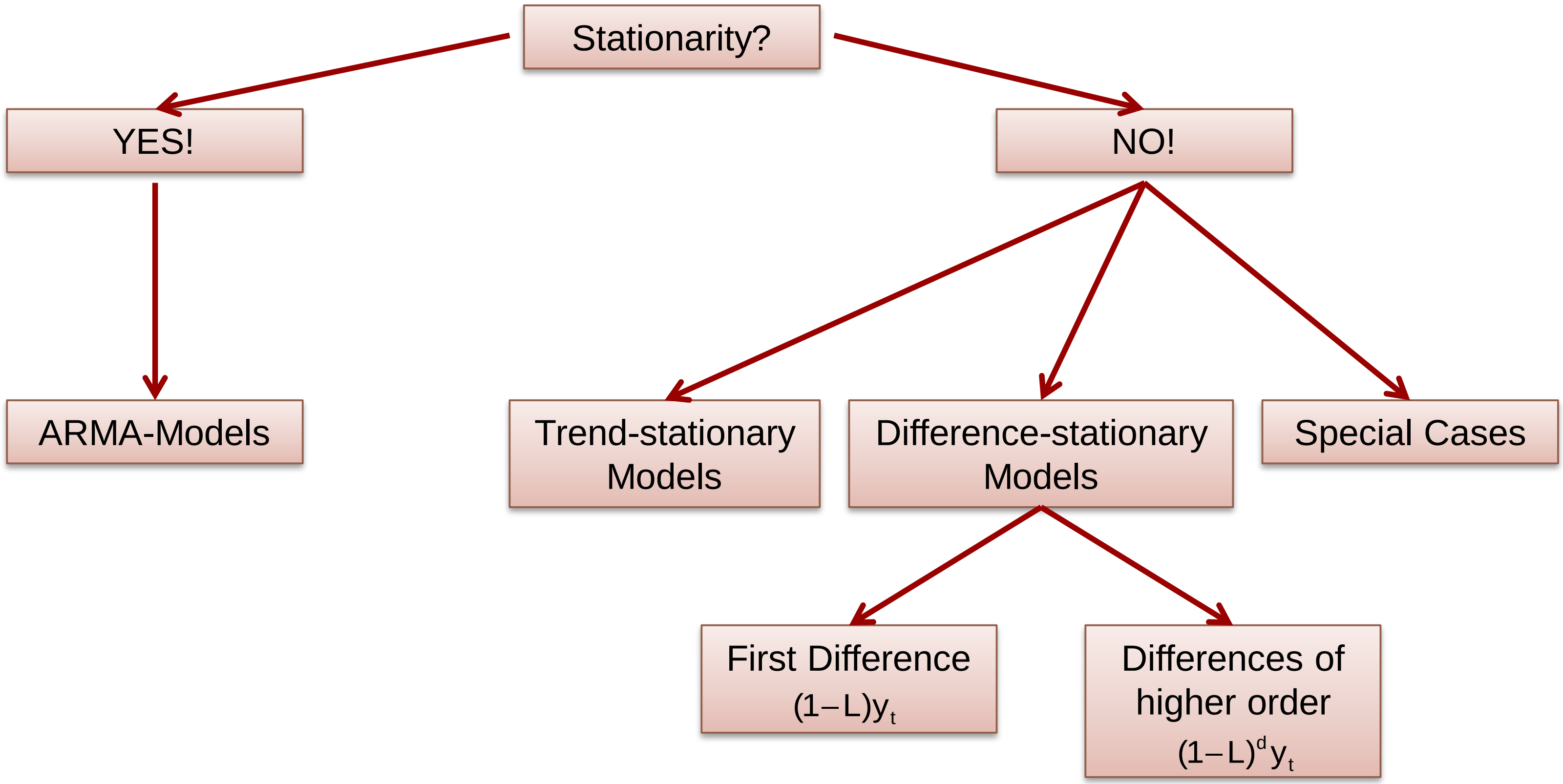
Overview



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Unit-Root Nonstationarity

The best known example of unit-root nonstationarity time series is the random walk model!

A time series p_t is a random walk if it satisfies:

$$p_t = p_{t-1} + a_t$$

Where p_0 is a real number denoting the starting value of the process and a_t is a white noise process. If a_t has a symmetric distribution around zero, then conditional on p_{t-1}, p_t has a 50-50 chance to go up or down, implying that p_t would go up or down at random. If we treat the random walk as a special AR(1)-Model, then the coefficient of p_{t-1} is unity (meaning $\varphi_1 = 1$), which does not satisfy the weak stationarity condition of an AR(1)-Model. A random walk series is, therefore, not weakly stationary, and we call it a unit-root nonstationary time series.

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Unit-Root Nonstationarity

Why is the weak stationarity condition not fulfilled?

Weakly stationarity needs time-invariant first and second moments for the given time series.

We know, given our example of the random walk:

$$\begin{aligned}
\mathrm{Var}(p_t) &= E\left[p_t^2\right] \\
&= E\left[\left(p_{t-1} + a_t\right)^2\right] \\
&= E\left[p_{t-1}^2\right] + \sigma_a^2 \\
&= E\left[\left(p_{t-2} + a_t\right)^2\right] + \sigma_a^2 \\
&= E\left[p_{t-2}^2\right] + 2\sigma_a^2 \\
&= \ldots \\
&= E\left[p_{t-n}^2\right] + n\sigma_a^2
\end{aligned}$$

Since $\sigma_a^2 > 0$ the variance of the random walk is infinite and hence undefined.

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Unit-Root Nonstationarity

From our example 2.23:

$$\left(1-1.34L-0.07L^2+0.41L^3\right)\tilde{y}_t=\varepsilon_t$$

The lag order polynomial has a unit root if

$$a(z)=\left(1-\varphi_1 z-\varphi_2 z^2-\varphi_3 z^3\right)=0 \text{ for } z=1$$

$$\left(1-1.34-0.07+0.41\right)=0$$

This holds in this example, so we have a unit root as a solution for this polynomial ➔ Our $\tilde{y}_t$ is not stationary! But maybe we can get rid of the nonstationarity if we use the first difference. Or do we need an higher order?

$$\left(1-0.34L-0.41L^2\right)\underbrace{\left(1-L\right)\tilde{y}_t}_{x_t}=\varepsilon_t$$

$$\left(1-0.34L-0.41L^2\right)x_t=\varepsilon_t$$

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Unit-Root Nonstationarity

From our example 2.23:

$$\left(1-0.34L-0.41L^2\right)x_t=\varepsilon_t$$

Test if x_t is stationary: $1-0.34L-0.41L^2\overset{!}{=}0$

$$\Leftrightarrow -2.44+0.83L+L^2=0$$

We know from the p-q-formula: $x^2+px+q=0$

$$x_{1/2}=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}$$

So we get:

$$L_{1/2}=-\frac{0.83}{2}\pm\sqrt{\left(\frac{0.83}{2}\right)^2+2.44}$$

$$=-0.415\pm\sqrt{2.612}$$

$$\left.\begin{aligned}L_1&=1.2012\\L_2&=-2.031\end{aligned}\right\}$$

L≠1, so there is no unit root anymore and our x_t seems to be stationary.

For references see:

Hamilton, James Douglas (1994), Time Series Analysis (Vol. 2), Princeton: Princeton university press, pp. 18, pp. 435.

Kirchgässner, Gebhard; Wolters, Jürgen (2006), Einführung in die moderne Zeitreihenanalyse, Vahlen, S. 147ff.

Tsay, Ruey S. (1951), Analysis of Financial Time Series, Second Edition, John Wiley Sons, pp. 64.

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