ML2 – Group SSCCF

EX SHEET 6

EX 1

a) $\max_v v^T C v \quad \text{s.t. } \|v\|^2 = 1 \qquad (1)$

$\Leftrightarrow \; Cv = \lambda v \;,\; \lambda$ largest EV. $\qquad (2)$

Proof: Lagranges of (1):

$$\mathcal{L} = v^T C v - \alpha(\|v\|^2 - 1)$$

$$\frac{\nabla \mathcal{L}}{\partial v} = 2Cv - 2\alpha v = 0 \Leftrightarrow Cv = \alpha v$$

$\Rightarrow$ all Eigen~~values~~ vectors are extreme points.

But to choose the eigenvector that maximises

$$\max_{v \text{ EV}} v^T C v = \cancel{v^T \alpha v} \max v^T \alpha v$$
$$= \max \alpha v^T v \qquad (\|v\|^2 = 1)$$
$$= \max \alpha$$

$\Rightarrow$ So choose eigenvector associated to ~~big~~ largest eigenvalue.

b) Now let $v = \Phi^T \alpha$.

$Cv = \lambda v$

$\Leftrightarrow C\Phi^T\alpha = \lambda \Phi^T \alpha$

$\Leftrightarrow \Phi^T\Phi\Phi^T\alpha = \lambda\Phi^T\alpha$

$\Leftrightarrow \Phi^T \quad \Phi\Phi^T\alpha = \Phi^T \lambda\alpha$

$\Leftrightarrow \Phi^T \quad K\alpha = \Phi^T \lambda\alpha$

Observe that

$C = \Phi^T\Phi$ and

$K = \Phi\Phi^T$.

Solving this equation ~~is~~for α is equivalent to solving $K\alpha = \lambda\alpha$ for α.

<u>Ex2</u>

$\sum^N s_i^2 = \|y\|^2$.

Define $U := \begin{bmatrix} u_1 & - \\ \vdots & \\ u_N & \end{bmatrix}$

<u>Proof</u>:

$$\sum^N s_i^2 = \sum^N (u_i^T y)^2$$
$$= \langle Uy, Uy \rangle$$
$$= \langle U^T U y, y \rangle$$
$$= \langle I y, y \rangle$$
$$= \|y\|^2$$

Since u_i are the eigenvectors of <u>symmetric</u> K, U is orthogonal matrix. $\Rightarrow U^T U = I$