Time Series Analysis

Discussion Section 02

Franziska Plitzko

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Organizational Issues

Please log in to **ISIS** (password: Zeit1718) and **download** the following file:

- coal_production.dta

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Exercise 2.15: Coal Production – Additional Models

Follow the **Univariate Box-Jenkins models for stationary time series** to estimate an **appropriate** model (if necessary consider different candidate models).

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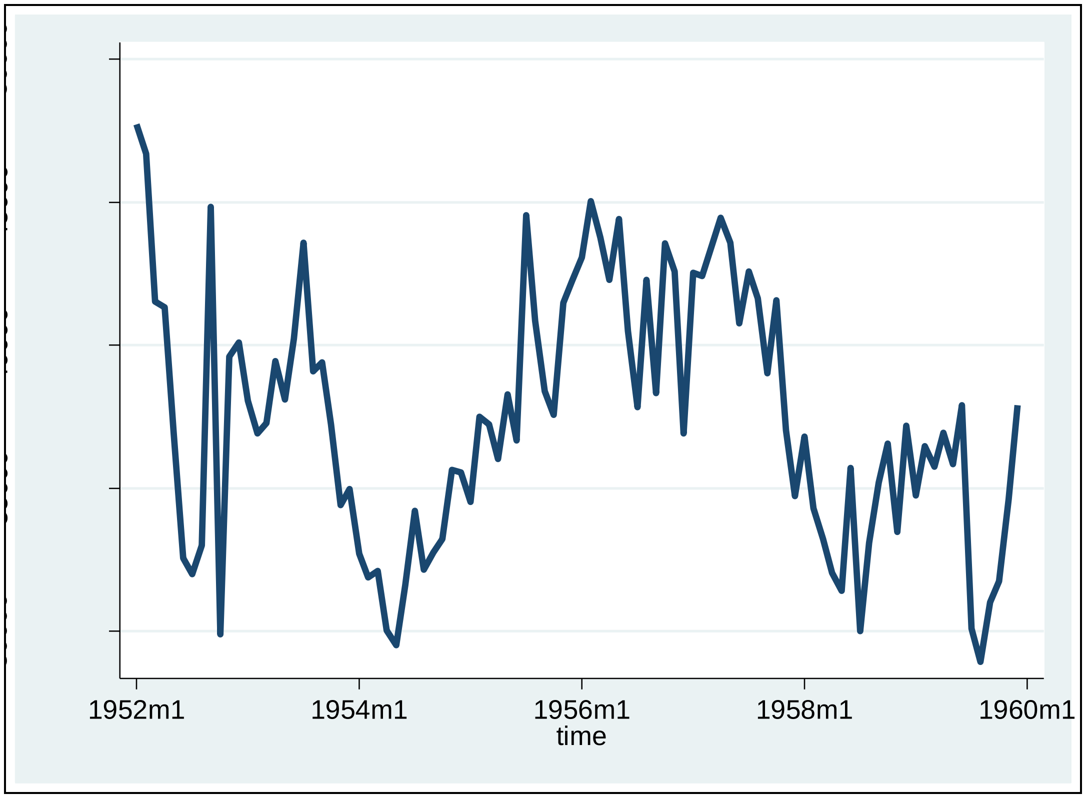
Exercise (coal_production.dta)

Coal Production – Original Series

```
. tsline coal_production, lwidth(thick)
```



Pankratz (1983) “Forecasting with univariate Box-Jenkins models”

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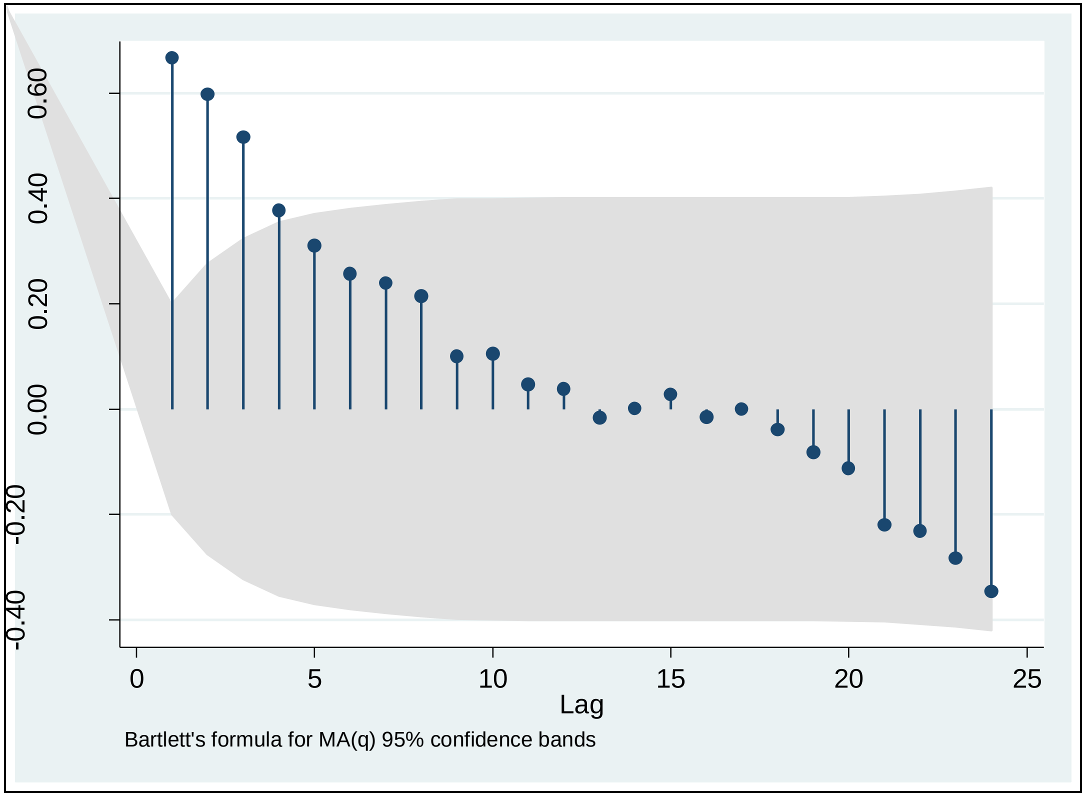
Identification

ACF: `. ac coal_production, lags(24)`



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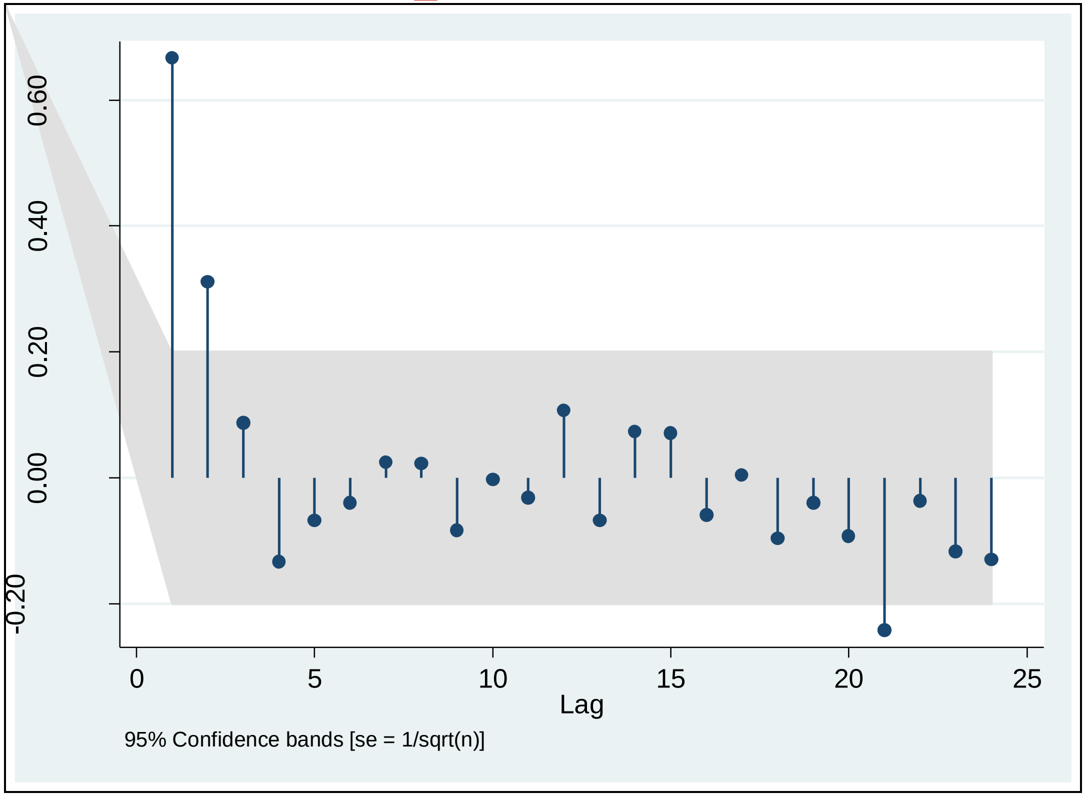
Identification

PACF: `. pac coal_production, lags(24)`



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Estimation – Maximum Likelihood Estimation

AR(2) process

```
. arima coal_production, ar(1/2)
[...]
Sample:  1952m1 to 1959m12                         Number of obs      =         96
------------------------------------------------------------------------------
             |                 OPG
coal_produ~n |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
coal_produ~n |
       _cons |   37981.21   1409.507    26.95   0.000     35218.63    40743.79
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .4839235   .0815506     5.93   0.000     .3240873    .6437597
         L2. |   .3223401   .0719627     4.48   0.000     .1812958    .4633844
-------------+----------------------------------------------------------------
      /sigma |    3066.34   186.4115    16.45   0.000      2700.98      3431.7
------------------------------------------------------------------------------
```

$\hat{\varphi}_1 = 0.4839 \quad \hat{\varphi}_2 = 0.3223 \quad \hat{\mu} = 37981.21 \quad \hat{\sigma}_\varepsilon = 3066.34$

$\hat{\delta} = \hat{\mu}\left(1 - \hat{\varphi}_1 - \hat{\varphi}_2\right) = 7358.3429$

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Estimation – Maximum Likelihood Estimation

ARMA(1,1) process

```
. arima coal_production, ar(1) ma(1)
[...]
------------------------------------------------------------------------------
             |                 OPG
coal_produ~n |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
coal_produ~n |
       _cons |   37982.71   1463.531    25.95   0.000     35114.24    40851.17
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .8860966   .0591459    14.98   0.000     .7701727     1.00202
          ma |
         L1. |  -.3690676   .0956238    -3.86   0.000    -.5564868   -.1816484
-------------+----------------------------------------------------------------
      /sigma |   3084.761   196.4155    15.71   0.000     2699.794    3469.729
------------------------------------------------------------------------------
```

$$\hat{\varphi}_1 = 0.8861 \quad \hat{\theta}_1 = 0.3691 \quad \hat{\mu} = 37982.71 \quad \hat{\sigma}_\varepsilon = 3084.761$$

$$\hat{\delta} = \hat{\mu}\left(1 - \hat{\varphi}_1\right) = 4326.3598$$

Estimation – Maximum Likelihood Estimation

ARMA(1,2) process

```
. arima coal_production, ar(1) ma(1/2)
[...]
------------------------------------------------------------------------------
                |                 OPG
coal_production |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
----------------+-------------------------------------------------------------
coal_production |
          _cons |   38015.76    1427.53    26.63   0.000     35217.85    40813.67
----------------+-------------------------------------------------------------
ARMA            |
             ar |
            L1. |   .8413213   .0842891     9.98   0.000     .6761178    1.006525
                |
             ma |
            L1. |   -.407945   .0841274    -4.85   0.000    -.5728317   -.2430583
            L2. |   .2179235   .1293461     1.68   0.092    -.0355902    .4714372
----------------+-------------------------------------------------------------
         /sigma |   3037.224   182.7784    16.62   0.000     2678.985    3395.463
------------------------------------------------------------------------------
```

$\hat{\varphi}_1 = 0.8413 \quad \hat{\theta}_1 = 0.4079 \quad \hat{\theta}_2 = -0.2179 \quad \hat{\sigma}_\varepsilon = 3037.224$

$\hat{\delta} = \hat{\mu}(1 - \hat{\varphi}_1) = 6032.2914$

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Estimation – Maximum Likelihood Estimation

MA(4) process

```
. arima coal_production, ma(1/4)
[...]
------------------------------------------------------------------------------
                 |                 OPG
coal_production  |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----------------+------------------------------------------------------------
coal_production  |
           _cons |   37645.08   847.4173    44.42   0.000     35984.17    39305.99
-----------------+------------------------------------------------------------
ARMA             |
              ma |
             L1. |    .456095   .0829638     5.50   0.000     .2934889    .6187011
             L2. |   .4808313   .1226394     3.92   0.000     .2404624    .7212001
             L3. |   .4652905   .1208325     3.85   0.000     .2284631    .7021178
             L4. |   .2279345   .1435482     1.59   0.112    -.0534149    .5092839
-----------------+------------------------------------------------------------
          /sigma |   3109.878    194.361    16.00   0.000     2728.937    3490.818
------------------------------------------------------------------------------
```

$$\hat{\theta}_1 = -0.4561 \quad \hat{\theta}_2 = -0.4808 \quad \hat{\theta}_3 = -0.4653 \quad \hat{\theta}_4 = -0.2279$$

$$\hat{\mu} = 37645.08 \quad \hat{\sigma}_\varepsilon = 3109.878$$

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Estimation – Summary

AR(2) process:

$$\hat{y}_t = \hat{\varphi}_1 y_{t-1} + \hat{\varphi}_2 y_{t-2} + \hat{\delta}$$

$$\hat{y}_t = 0.4839 y_{t-1} + 0.3223 y_{t-2} + 7358.3429$$

ARMA(1,1) process:

$$\hat{y}_t = \hat{\varphi}_1 y_{t-1} - \hat{\theta}_1 \varepsilon_{t-1} + \hat{\delta}$$

$$\hat{y}_t = 0.8861 y_{t-1} - 0.3691 \varepsilon_{t-1} + 4326.3598$$

ARMA(1,2) process:

$$\hat{y}_t = \hat{\varphi}_1 y_{t-1} - \hat{\theta}_1 \varepsilon_{t-1} - \hat{\theta}_2 \varepsilon_{t-2} + \hat{\delta}$$

$$\hat{y}_t = 0.8413 y_{t-1} - 0.4079 \varepsilon_{t-1} + 0.2179 \varepsilon_{t-1} + 6032.2914$$

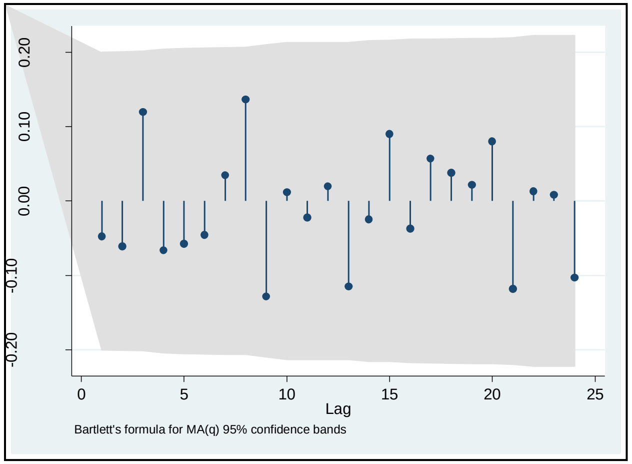
MA(4) process:

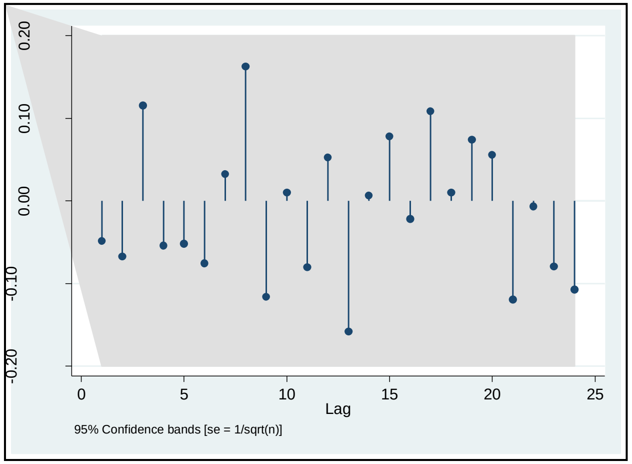
$$\hat{y}_t = \hat{\mu} - \hat{\theta}_1 \varepsilon_{t-1} - \hat{\theta}_2 \varepsilon_{t-2} - \hat{\theta}_3 \varepsilon_{t-3} - \hat{\theta}_4 \varepsilon_{t-4}$$

$$\hat{y}_t = 37645.08 + 0.4561 \varepsilon_{t-1} + 0.4808 \varepsilon_{t-2} + 0.4653 \varepsilon_{t-3} + 0.2279 \varepsilon_{t-4}$$

Diagnostic Checking

ACF and PACF of the AR(2) residuals





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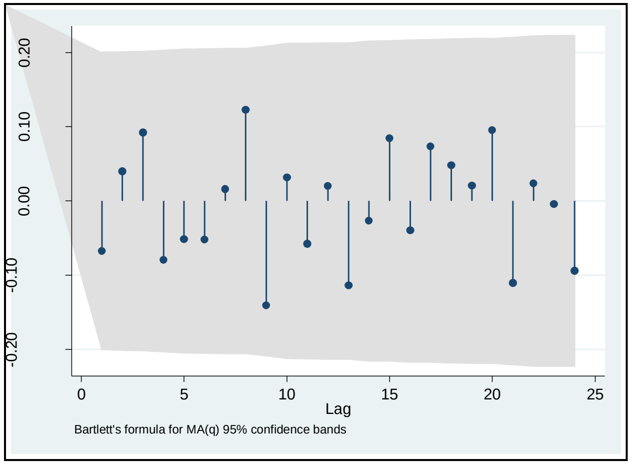
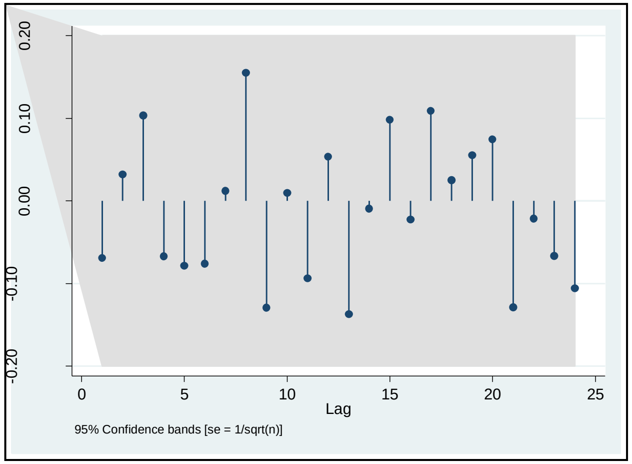
Diagnostic Checking

ACF and PACF of the ARMA(1,1) residuals





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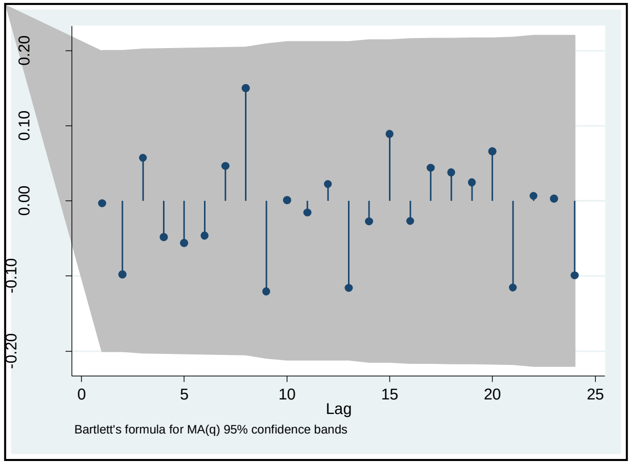
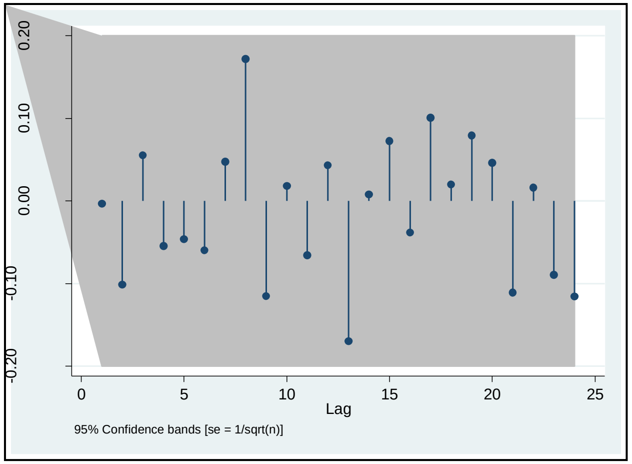
Diagnostic Checking

ACF and PACF of the ARMA(1,2) residuals





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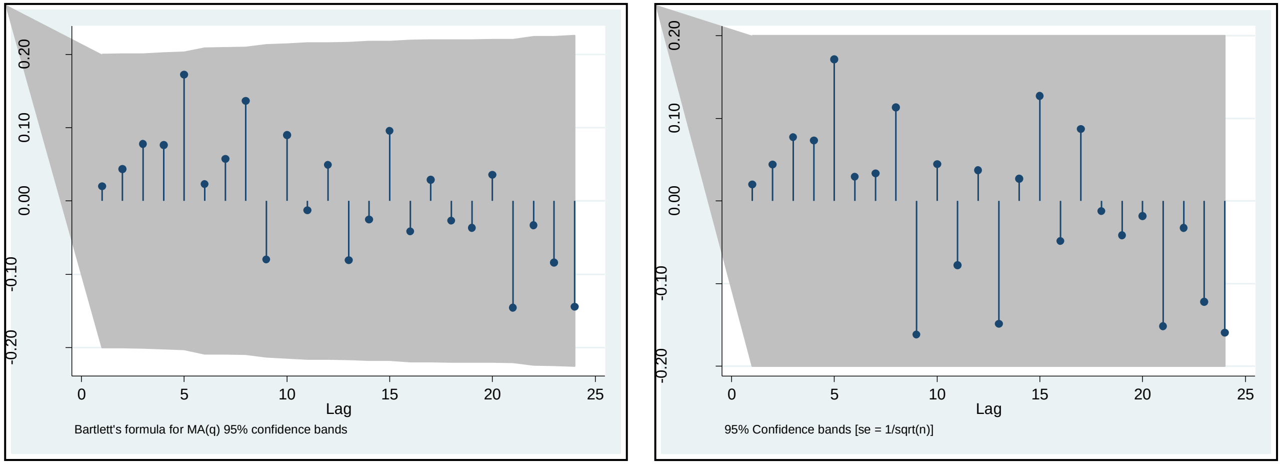
Diagnostic Checking

ACF and PACF of the MA(4) residuals



Diagnostic Checking Box-Ljung test for residuals

```
. corrgram res_AR2, lags(24)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
[...]
24      -0.1033  -0.1071   14.337  0.9387                            |
```

```
. di 1-chi2(22, 14.337)
.88908811
```

```
. corrgram res_ARMA11, lags(24)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
[...]
24      -0.0938  -0.1058   14.387  0.9374                            |
```

```
. di 1-chi2(22, 14.387)
.88717669
```

```
. corrgram res_ARMA12, lags(24)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
[...]
24      -0.0993  -0.1159   12.966  0.9667                            |
```

```
. di 1-chi2(21,12.966)
.90981018
```

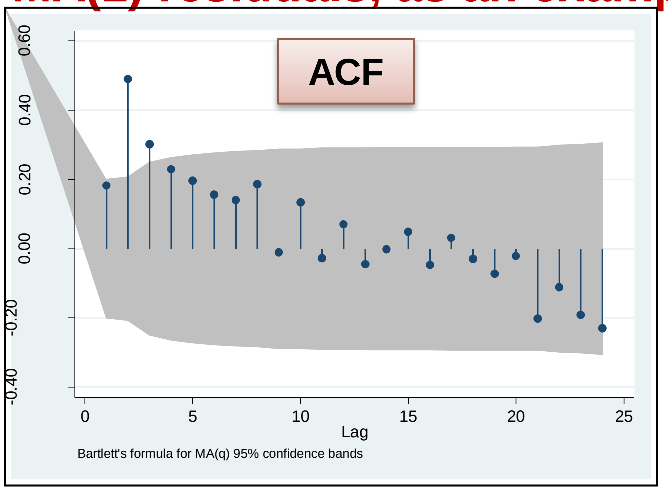
```
. corrgram res_MA4, lags(24)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
[...]
24      -0.1448  -0.1596   17.802  0.8125                           -|
```

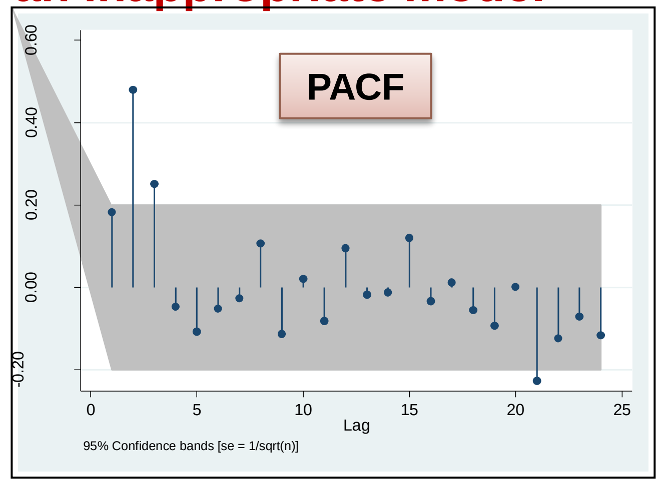
```
. di 1-chi2(20,17.802)
.60044967
```

Exercise (coal_production.dta)

Diagnostic Checking

MA(1) residuals, as an example for an inappropriate model






```
. corrgram res_MA1, lags(24)
                                                   -1       0       1 -1       0       1
 LAG       AC        PAC        Q      Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1         0.1822    0.1822    3.2866   0.0698           |-                 |-
2         0.4894    0.4795    27.261   0.0000           |---               |---
[…]
23       -0.1920   -0.0708    69.903   0.0000
24       -0.2299   -0.1163     76.81   0.0000
```

. di 1-chi2(23,76.81)

1.035e-07 **=> reject H_0**

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Diagnostic Checking

Comparison of the candidate models

	AR(2)	ARMA(1,1)	ARMA(1,2)	MA(4)	MA(1)
AIC	**16.098147**	16.110126	16.099898	16.168011	16.475249
BIC	**16.15157**	16.16355	16.180034	16.274859	16.501961

➔ARMA(1,2) and MA(4) have insignificant parameters and should not be considered, even though their residuals look fine

➔The residuals of the MA(1) model are not white noise, and so this model should not be considered

➔Only AR(2) and ARMA(1,1) seem to be appropriate (could test more)

➔AIC and BIC suggest AR(2) as the best model

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