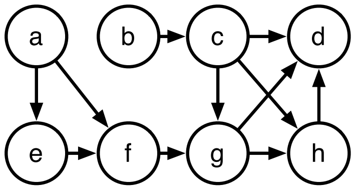
Exercises for the course
Machine Learning 1
Winter semester 2016/17

Abteilung Maschinelles Lernen
Institut für Softwaretechnik und theoretische Informatik
Fakultät IV, Technische Universität Berlin
Prof. Dr. Klaus-Robert Müller
Email: klaus-robert.mueller@tu-berlin.de

Exercise Sheet 8

Exercise 1: Graphical Models and Conditional Independence (5+5+5 P)



For the directed graphical model above, *show* by using the basic rules of probability theory **and** by using the rules of d-separation.

(a) whether a and b are conditionally independent given the empty set?

(b) whether a and b are conditionally independent given the variable h?

(c) *Draw* the directed graphical model for the following joint probability distribution:

$$P(a)P(b)P(c)P(d|c)P(e|a,d)P(g|e,a)P(h|b,g)$$

Exercise 2: Graphical Models and Discriminants (5+15 P)

Assume that we have an image $\boldsymbol{x}$ consisting d binary pixels x_i. Each image depicts a handwritten digit with class $y \in \{0, \ldots, 9\}$. Such an image can be modeled as follows. We make the assumption, that observed variables $x_1, \ldots, x_d$ are conditionally independent given y. That is, the joint probability distribution factorizes as

$$P(\boldsymbol{x}, y) = P(y) \cdot \prod_{i=1}^{d} P(x_i|y),$$

where each factor has its own parameters. In particular, we set $\beta_k = P(y = k)$, and $\theta_{ik} = P(x_i = 1|y = k)$. Please note that each pixel x_i is modeled independently.

(a) *Draw* the directed graphical model (i.e. the set of nodes, the node labels, the directed connections between nodes, and the parameters of the network to be defined).

(b) *Show* that the most likely class $k^\star = \arg\max_k P(y = k|\boldsymbol{x})$ for a given image $\boldsymbol{x}$ can be found by a set of linear discriminants $g_k(\boldsymbol{x}) = \boldsymbol{w}_k^\top \boldsymbol{x} + b_k$, where $k \in \{0, \ldots, 9\}$ is the class, and where the parameters of the discriminants $\{\boldsymbol{w}_k, b_k\}$ can be written as a function of $\{\beta_k, \theta_{ik}\}$.

Exercise 3: Finding the Most Likely Solution (15 P)

Last year for Christmas we received a robotic vacuum-cleaner that was programmed badly. Each day, we switch it on in the living room. Today, 7 minutes, after turning it on, we found the vacuum cleaner in the office. Unlike most vacuum cleaners, ours is rather fast and spends only exactly one minute in each room before moving onto and cleaning the next one (rather poorly unfortunately). What was the most likely sequence of rooms that our robot visited?

From almost a year of intense observation of robot-cleaning activities we have learned that when the robot is in the living room, it will be one minute later in the kitchen with probability 0.5, in the office with probability 0.3 and in the bedroom with probability 0.2. If the robot is in the office then it will get stuck under the desk for a minute with probability 0.1, with probability 0.662 it will end up in the kitchen. With a probability of just 0.008, it is rather unlikely that the robot goes from the office to the bedroom, which should come as no surprise since it has to climb a series of stairs to get to the bedroom. Finally, with probability 0.23 it will decide to clean the living room next. If the robot is in the bedroom, a minute later it will be in the living room with probability 0.3, with probability 0.6, it will have fallen down the stairs and into the office. Only in 10% of our observations, we found the robot in the kitchen the minute after cleaning the bedroom. Surprisingly, after spending a minute in the kitchen, the robot will always move to a different room, and each rooms is equally likely.

Exercise 4: Programming (50 P)

Download the programming files on ISIS and follow the instructions.