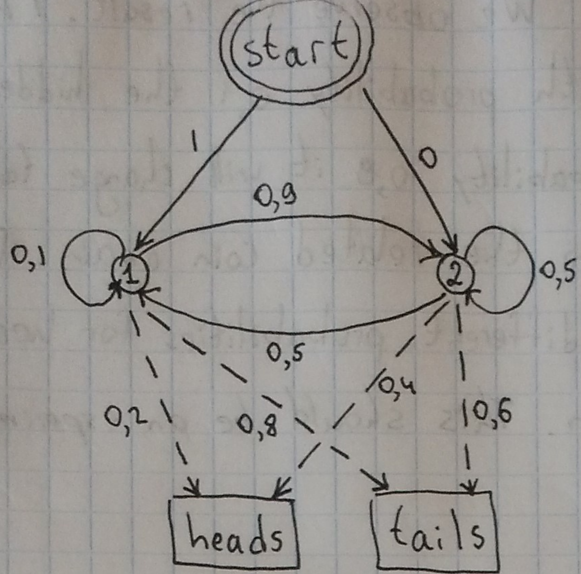
Exercise 1

(a)



(c)

$$P((q_1,q_2)\mid(Q_1,Q_2)=(tails,tails)) = \frac{\overbrace{P((Q_1,Q_2)=(tails,tails)\mid(q_1,q_2))}^{(*)}\,\overbrace{P((q_1,q_2))}^{(**)}}{\underbrace{P((Q_1,Q_2)=(tails,tails))}_{(***)}} =$$

$(***)$: $\sum P((Q_1,Q_2)=(tails,tails)\mid(q_1,q_2))\,P((q_1,q_2))$

$(**)$: $$P((q_1,q_2)) = \begin{pmatrix}0,1\\0,9\\0\\0\end{pmatrix} = \begin{pmatrix}P((s_1,s_1))\\P((s_1,s_2))\\P((s_2,s_1))\\P((s_2,s_2))\end{pmatrix}$$

$(*)$: $$\begin{pmatrix}0,8\cdot 0,8\\0,8\cdot 0,6\\0,8\cdot 0,6\\0,6\cdot 0,6\end{pmatrix} \begin{matrix}\leftarrow \text{given } (s_1,s_1)\\ \leftarrow \text{given } (s_1,s_2)\\ \vdots \\ \\ \end{matrix} = \begin{pmatrix}0,64\\0,48\\0,48\\0,\cancel{48}\,36\end{pmatrix}$$

$(*)\cdot(**)$: $$\begin{pmatrix}0,1\\0,9\\0\\0\end{pmatrix} \cdot \cancel{0,64} \begin{pmatrix}0,64\\0,48\\0,48\\0,36\end{pmatrix} = \begin{pmatrix}0,064\\0,432\\0\\0\end{pmatrix}$$

$\frac{(*)\cdot(**)}{(***)}$: $$\approx \begin{pmatrix}0,13\\0,87\\0\\0\end{pmatrix}$$

(b) We start in the first hidden state (as $\pi = (1,0)$) and throw a coin related to it. With probability 0,2 it will show heads and with 0,8 tails. We observe the result. Then a hidden transition takes place. With probability 0,1 the hidden state will remain the same, with probability 0,9 it will change to state 2. In that new hidden state we toss the related coin again. Depending on the hidden state there are different probabilities for heads and tails. We observe the result again. This should be an experiment for $M=2$ and $N=2$