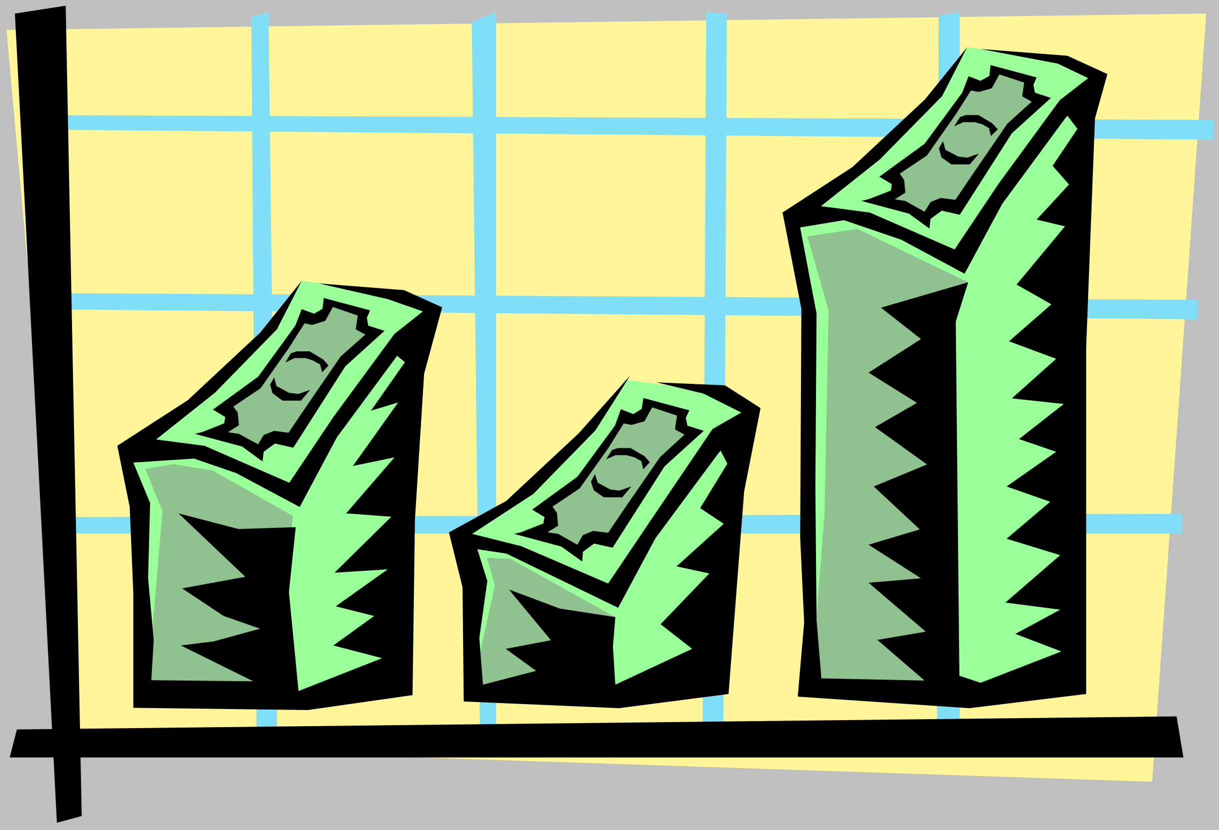


Introduction to Financial Analysis

What is the Time Value of Money?

Time value of money

The concept that a dollar in hand can be invested to earn a return so that more than one dollar will be available in the future.

1. Future Value of an Investment:

$$F = P(1 + r)^n$$

2. Present Value of Future Amount:

$$P = \frac{F}{(1 + r)^n}$$

3. Present Value Factors:

$$P = F\left[\frac{1}{(1 + r)^n}\right]$$

4. Present Value of an Annuity:

$$P = A\,(\text{af})$$

5. Straight-Line Depreciation:

$$D = \frac{I - S}{n}$$

Time Value of Money

- **Future value of an investment**
  - **The value of an investment at the end of the period over which interest is compounded.**

- **Compounding interest**
  - **The process by which interest on an investment accumulates and then earns interest itself for the remainder of the investment period.**

Future Value of an Investment

The value of a \$5,000 investment at 12 percent per year, 1 year from now is:

$$\$5{,}000(1.12) = \$5{,}600$$

If the entire amount remains invested, at the end of 2 years you would have:

$$\$5{,}600(1.12) = \$5{,}000(1.12)^2 = \$6{,}272$$

Time Value of Money

- **In general:**

$$F = P(1 + r)^n$$

where

F = future value of the investment at the end of n periods

P = amount invested at the beginning, called the principal

r = periodic interest rate

n = number of time periods for which the interest compounds

Application F.1

Future Value of a $500 Investment in 5 Years

$$F = P(1 + r)^n$$

$$P = \$500$$

$$r = 6\%$$

$$n = 5$$

$$500(1 + .06)^5 = 500(1.338) = \$669.11$$

Present Value of a Future Amount

- Present value of an investment
  - The amount that must be invested now to accumulate to a certain amount in the future at a specific interest rate.
- What is the present value of an investment worth \$10,000 at the end of year 1 if the interest rate is 12 percent?

$$F = \$10,000 = P(1 + 0.12)$$

$$P = \frac{F}{(1+r)^n} = \frac{10,000}{(1+0.12)^1} = \$8,929$$

Present Value of a Future Amount

- **In general:**

$$P = \frac{F}{(1 + r)^n}$$

where

F = future value of the investment at the end of n periods

P = amount invested at the beginning, called the principal

r = periodic interest rate (discount rate)

n = number of time periods for which the interest compounds

Application F.2

Present Value of $500 Received Five Years in the Future:

$$P = \frac{F}{(1 + r)^n}$$

F = $500

r = 6%

n = 5

$500/1.338 = $373.63

Present Value Factors

The present value of a future amount:

$$P = \frac{F}{(1+r)^n} = F\left[\frac{1}{(1+r)^n}\right]$$

where:

$[1/(1 + r)^n]$ is the present value factor (pf)

Present Value Factors

An investment will generate $15,000 in 10 years

If the interest rate is 12 percent, the following table shows that pf = 0.3220

The present value is:

$$P = F(\text{pf}) = \$15{,}000(0.3220)$$

$$= \$4{,}830$$

Present Value Factors

PRESENT VALUE FACTORS FOR A SINGLE PAYMENT (Partial)										
Number of Periods (n)	Interest Rate (r)									
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.12	0.14
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9259	0.9091	0.8929	0.8772
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8573	0.8264	0.7972	0.7695
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.7938	0.7513	0.7118	0.6750
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7350	0.6830	0.6355	0.5921
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.6806	0.6209	0.5674	0.4194
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6302	0.5645	0.5066	0.4556
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.5835	0.5132	0.4523	0.3996
8	0.9235	0.8635	0.7894	0.7307	0.6768	0.6274	0.5403	0.4665	0.4039	0.3506
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5002	0.4241	0.3606	0.3075
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.4632	0.3855	0.3220	0.2697

Application F.2

Present Value of \$500 Received Five Years in the Future

$$P = \frac{F}{(1 + r)^n}$$

F = \$500

r = 6%

n = 5

pf = .7473

\$500(.7473) = **\$373.65**

Present Value Factors

PRESENT VALUE FACTORS FOR A SINGLE PAYMENT (Partial)										
Number of Periods (n)	Interest Rate (r)									
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.12	0.14
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9259	0.9091	0.8929	0.8772
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8573	0.8264	0.7972	0.7695
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.7938	0.7513	0.7118	0.6750
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7350	0.6830	0.6355	0.5921
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.6806	0.6209	0.5674	0.4194
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6302	0.5645	0.5066	0.4556
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.5835	0.5132	0.4523	0.3996
8	0.9235	0.8635	0.7894	0.7307	0.6768	0.6274	0.5403	0.4665	0.4039	0.3506
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5002	0.4241	0.3606	0.3075
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.4632	0.3855	0.3220	0.2697

Annuities

- **Annuity**
  - **A series of payments of a fixed amount for a specified number of years**

- **At a 10% interest rate, how much needs to be invested so that you may draw out $5,000 per year for each of the next 4 years?**

$$P = \frac{\$5{,}000}{1+0.10} + \frac{\$5{,}000}{(1+0.10)^2} + \frac{\$5{,}000}{(1+0.10)^3} + \frac{\$5{,}000}{(1+0.10)^4}$$

$$= \$4{,}545 + \$4{,}132 + \$3{,}757 + \$3{,}415$$

$$= \$15{,}849$$

Annuities

- Find the present value of an annuity (af) from the following table
- Multiply the amount received each year (A) by the present value factor

$$P = A(\text{af})$$

where

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad i \neq 0$$

P = present value of an investment

A = amount of the annuity received each year

af = present value factor for an annuity

$P = A(\text{af}) = \$5{,}000(3.1699) = \$15{,}849$

Present Value Factors

PRESENT VALUE FACTORS OF AN ANNUITY (Partial)										
Number of Periods (*n*)	Interest Rate (*r*)									
	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.10	0.12	0.14
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9259	0.9091	0.8929	0.8772
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.7833	1.7355	1.6901	1.6467
3	2.9410	2.8839	2.8286	2.7751	2.7732	2.6730	2.5771	2.4869	2.4018	2.3216
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3121	3.1699	3.0373	2.9137
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	3.9927	3.7908	3.6048	3.4331
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.6229	4.3553	4.1114	3.8887
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.2064	4.8684	4.5638	4.2883
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.7466	5.3349	4.9676	4.6389
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.2469	5.7590	5.3282	4.9464
10	9.4713	8.9826	8.3302	8.1109	7.7217	7.3601	6.7201	6.1446	5.6502	5.2161

Application F.3

Present Value of a $500 Annuity for 5 Years

P **= *A* (af)**

A **= $500 for 5 years at 6%**

af = 4.2124 (from table)

P **= $500(4.2124) = $2,106.20**

Nominal Versus Effective Interest Rates

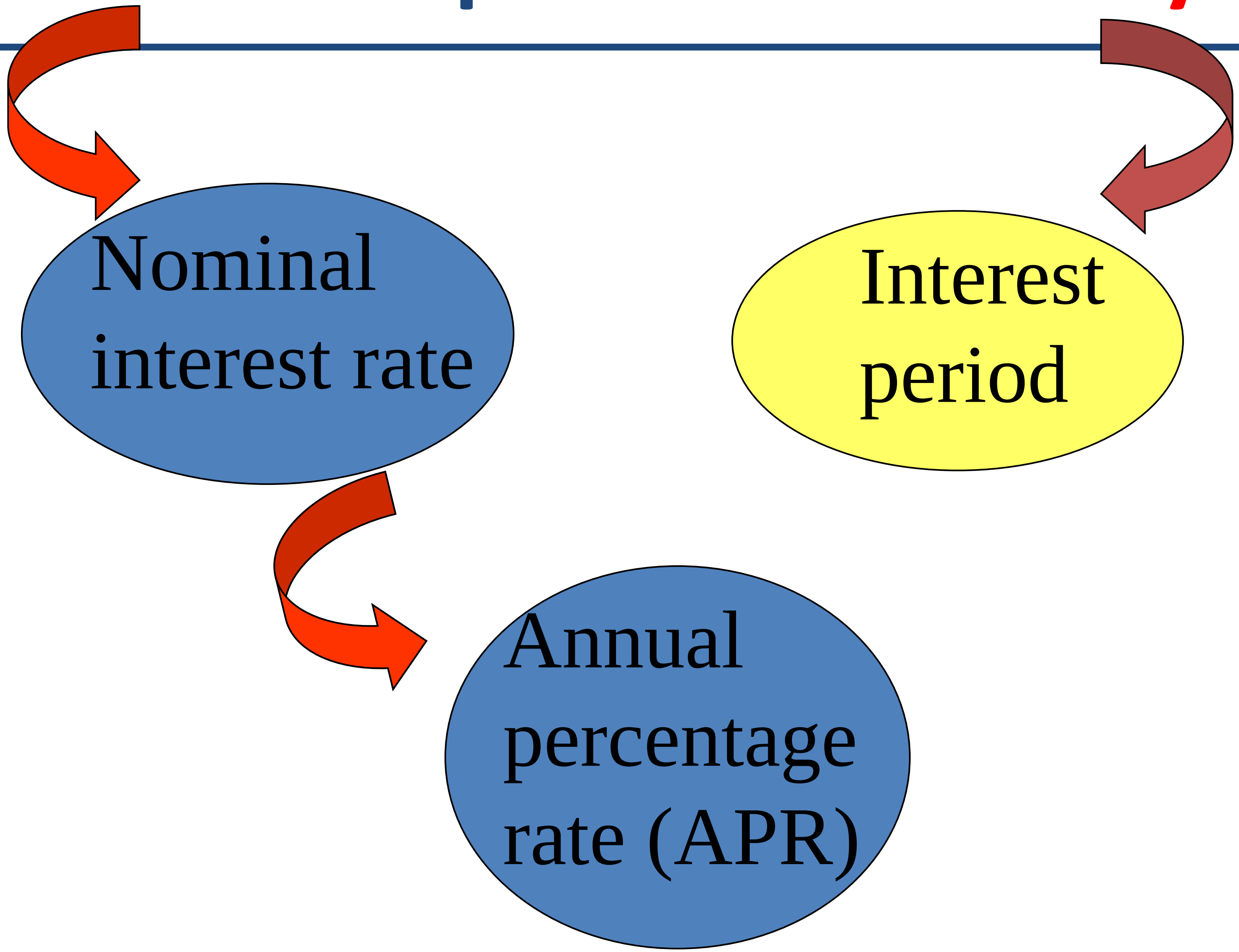
Nominal Interest Rate:

Interest rate quoted based on an annual period

Effective Interest Rate:

Actual interest earned or paid in a year or some other time period

18% Compounded Monthly



18% Compounded Monthly

- **What It Really Means?**
  - Interest rate per month (i) = 18% / 12 = 1.5%
  - Number of interest periods per year (N) = 12

- **In words,**
  - Bank will charge 1.5% interest each month on your unpaid balance, if you borrowed money
  - You will earn 1.5% interest each month on your remaining balance, if you deposited money

18% compounded monthly

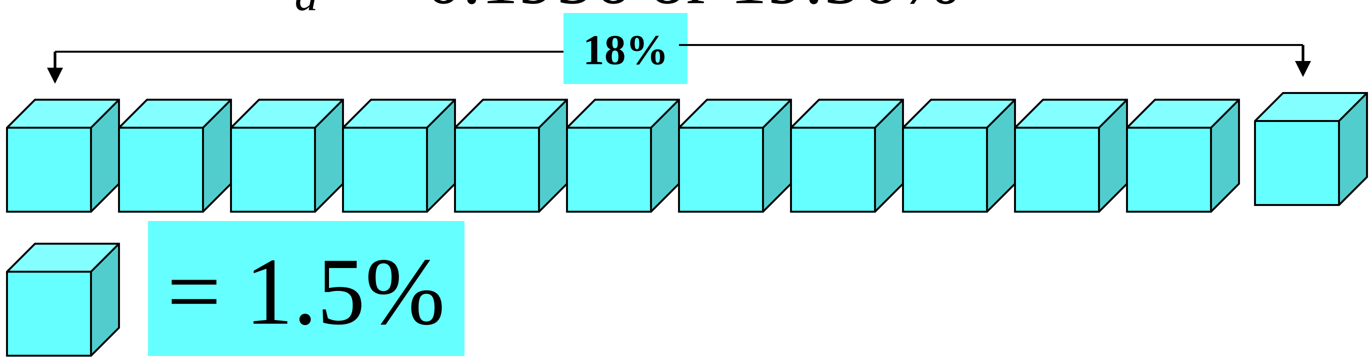
- **Question**: Suppose that you invest \$1 for 1 year at 18% compounded monthly. How much interest would you earn?

- **Solution**:

$$F = \$1(1+i)^{12} = \$1(1+0.015)^{12}$$

$$= \$1.1956$$

$$i_a = 0.1956 \text{ or } 19.56\%$$



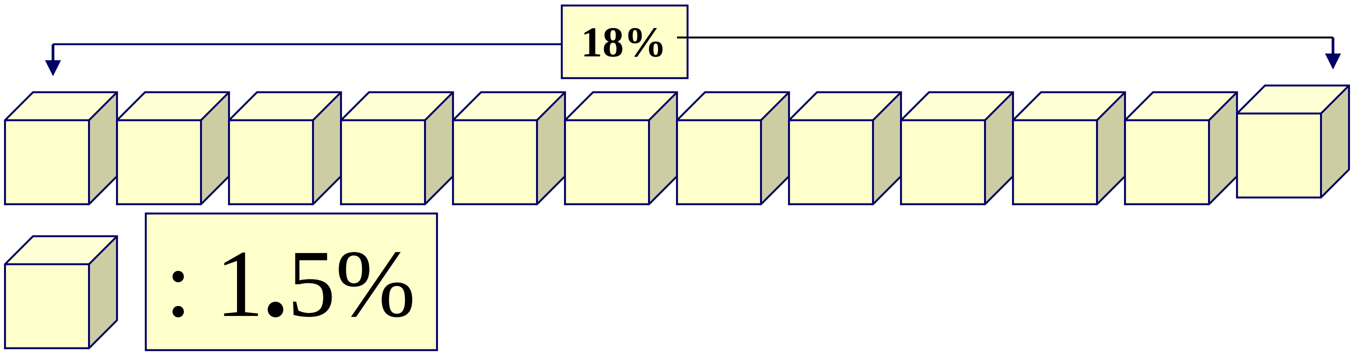
Effective Annual Interest Rate (Yield)

$$i_a = (1 + r/M)^M - 1$$

r = nominal interest rate per year

i_a = effective annual interest rate

M = number of interest periods per year



18% compounded **monthly**

or

1.5% per month for **12 months**

=

19.56 % compounded **annually**

Practice Problem

- If your credit card calculates the interest based on 12.5% APR, what is your monthly interest rate and annual effective interest rate, respectively?

- Your current outstanding balance is $2,000 and skips payments for 2 months. What would be the total balance 2 months from now?

Solution

Monthly Interest Rate:

$$i = \frac{12.5\%}{12} = 1.0417\%$$

Annual Effective Interest Rate:

$$i_a = (1 + 0.010417)^{12} = 13.24\%$$

Total Outstanding Balance:

$$F = B_2 = \$2,000(F/P, 1.0417\%, 2)$$

$$= \$2,041.88$$

Practice Problem

- **Suppose your savings account pays 9% interest compounded quarterly. If you deposit \$10,000 for one year, how much would you have?**

(a) Interest rate per quarter:

$$i = \frac{9\%}{4} = 2.25\%$$

(b) Annual effective interest rate:

$$i_a = (1+0.0225)^4 - 1 = 9.31\%$$

(c) Balance at the end of one year (after 4 quarters)

$$F = \$10,000(F/P, 2.25\%, 4)$$

$$= \$10,000(F/P, 9.31\%, 1)$$

$$= \$10,931$$

Effective Interest Rate per Payment Period (*i*)

$$i = [1 + r / CK]^{C} - 1$$

C = number of interest periods per payment period

K = number of payment periods per year

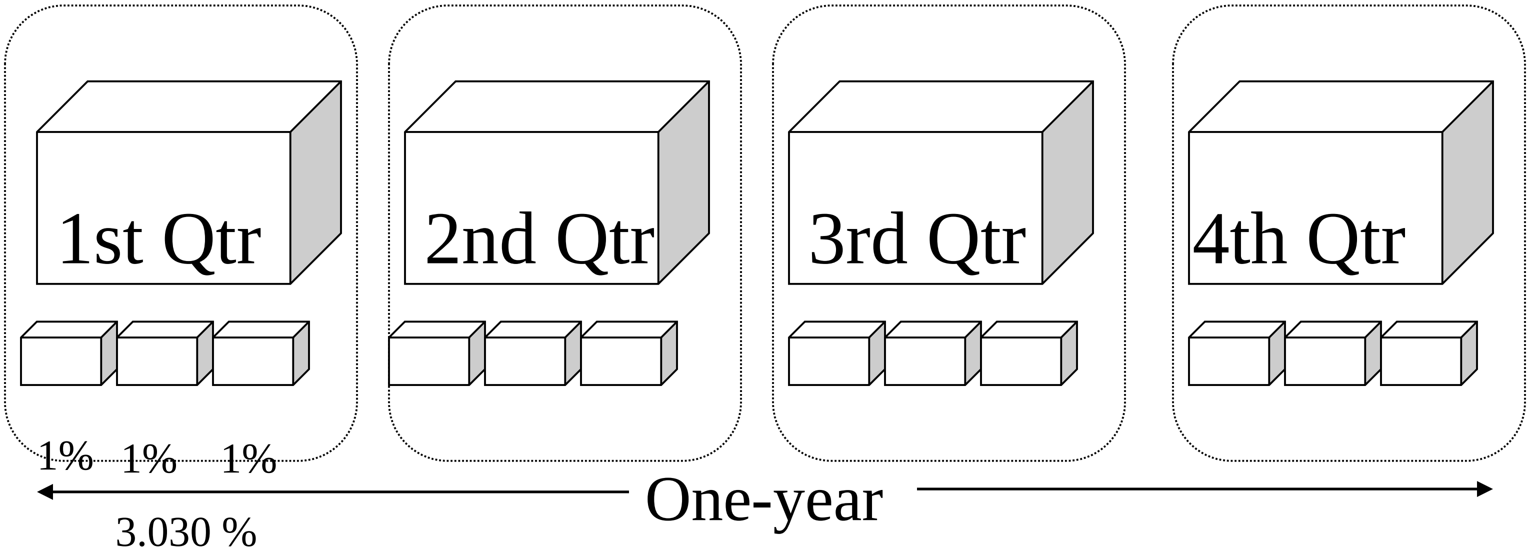
CK = total number of interest periods per year, or *M*

r /*K* = nominal interest rate per payment period

12% compounded monthly

Payment Period = Quarter

Compounding Period = Month



- **Effective interest rate per quarter**

$$i = (1+0.01)^3 - 1 = 3.030\%$$

- **Effective annual interest rate**

$$i_a = (1+0.01)^{12} - 1 = 12.68\%$$

$$i_a = (1+0.03030)^4 - 1 = 12.68\%$$

Effective Interest Rate per Payment Period with Continuous Compounding

$$i = [1 + r / CK]^C - 1$$

where CK = number of compounding periods per year

continuous compounding => $C \to \infty$

$$\begin{aligned} i &= \lim[(1 + r / CK)^C - 1] \\ &= (e^r)^{1/K} - 1 \end{aligned}$$

Case 0: 8% compounded quarterly

Payment Period = Quarter

Interest Period = Quarterly

1st Q	2nd Q	3rd Q	4th Q

1 interest period

Given $r = 8\%$,

$K = 4$ payments per year

$C = 1$ interest period per quarter

$M = 4$ interest periods per year

$$i = [1 + r / CK]^{C} - 1$$

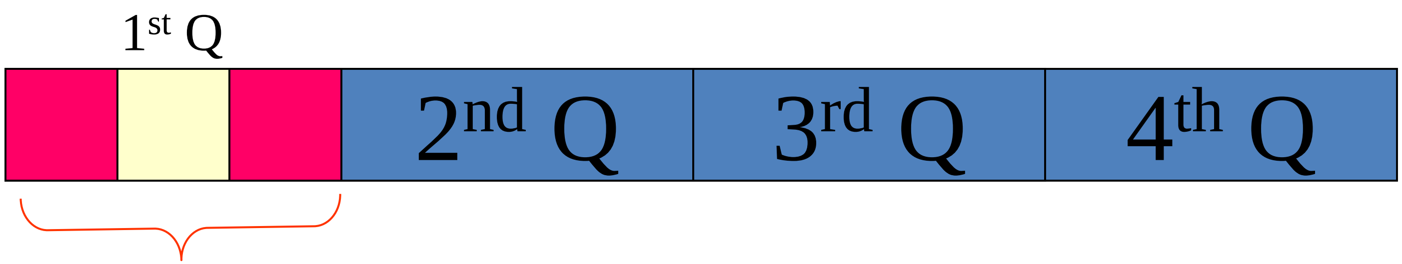
$$= [1 + 0.08 / (1)(4)]^{1} - 1$$

$$= 2.000\% \text{ per quarter}$$

Case 1: 8% compounded monthly

Payment Period = Quarter

Interest Period = Monthly



3 interest periods

Given r = 8%,

K = 4 payments per year

C = 3 interest periods per quarter

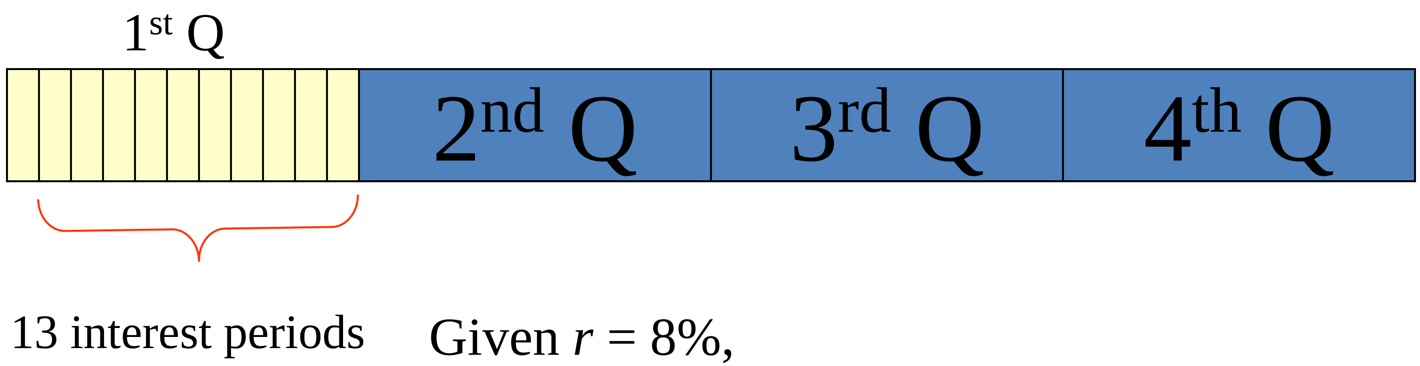
M = 12 interest periods per year

$$
\begin{aligned}
i &= [1 + r / CK]^{C} - 1 \\
&= [1 + 0.08 / (3)(4)]^{3} - 1 \\
&= 2.013\% \text{ per quarter}
\end{aligned}
$$

Case 2: 8% compounded weekly

Payment Period = Quarter

Interest Period = Weekly



Given r = 8%,

K = 4 payments per year

C = 13 interest periods per quarter

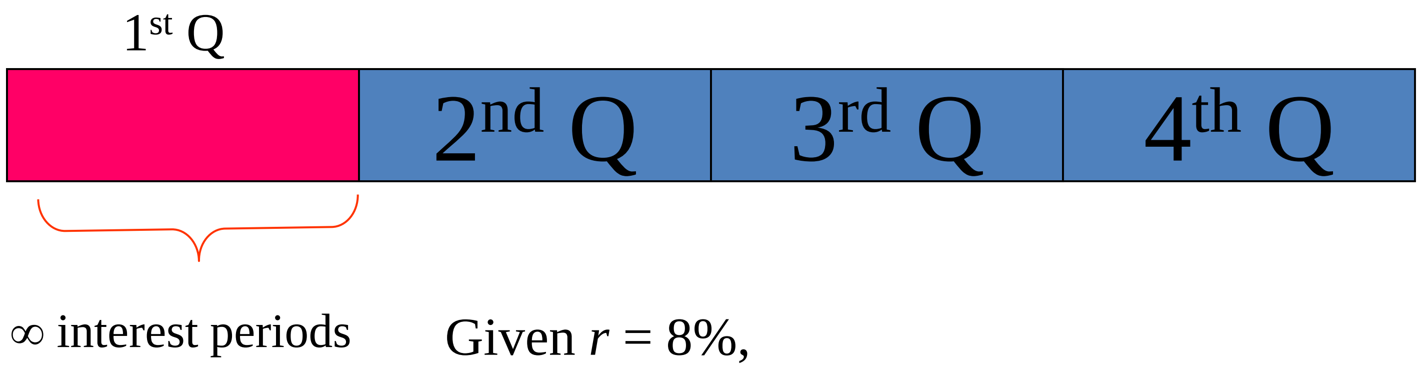
M = 52 interest periods per year

$$
\begin{aligned}
i &= [1 + r / CK]^{C} - 1 \\
&= [1 + 0.08 / (13)(4)]^{13} - 1 \\
&= 2.0186\% \text{ per quarter}
\end{aligned}
$$

Case 3: 8% compounded continuously

Payment Period = Quarter

Interest Period = Continuously



Given r = 8%,

K = 4 payments per year

$$
\begin{aligned}
i &= e^{r/K} - 1 \\
&= e^{0.02} - 1 \\
&= 2.0201\% \text{ per quarter}
\end{aligned}
$$

Summary: Effective interest rate per quarter

Case 0	Case 1	Case 2	Case 3
8% compounded quarterly	8% compounded monthly	8% compounded weekly	8% compounded continuously
Payments occur quarterly	Payments occur quarterly	Payments occur quarterly	Payments occur quarterly
2.000% per quarter	2.013% per quarter	2.0186% per quarter	2.0201% per quarter

Techniques of Analysis

- **Three basic financial analysis techniques that rely on cash flows:**
  - Net present value method
  - Internal rate of return method
  - Payback method
- **Two important points**
  - Consider only incremental cash flows
  - Convert cash flows to after-tax amounts

Depreciation and Taxes

- Depreciation
  - An allowance for the consumption of capital
  - Not a cash flow but it does affect net income

- Straight-line depreciation
  - Subtract the estimated salvage value of the asset to be depreciated
- Salvage value is the cash flow from disposal at the end useful life.

Depreciation and Taxes

- General expression for annual depreciation:

$$D = \frac{I - S}{n}$$

where

D = annual depreciation

I = amount of investment

S = salvage value

n = number of years of project life

Depreciation and Taxes

- Accelerated depreciation or Modified Accelerated Cost Recovery System (MACRS)
  - 3-year class
  - 5-year class
  - 7-year class
  - 10-year class
- Income-tax rate varies with location
- Include all relevant income taxes in analysis

Accelerated Depreciation

MACRS DEPRECIATION ALLOWANCES				
	Class of Investment			
Year	3-Year	5-Year	7-Year	10-Year
1	33.33	20.00	14.29	10.00
2	44.45	32.00	24.49	18.00
3	14.81	19.20	17.49	14.40
4	7.41	11.52	12.49	11.52
5		11.52	8.93	9.22
6		5.76	8.93	7.37
7			8.93	6.55
8			4.45	6.55
9				6.55
10				6.55
11				3.29
	100.0%	100.0%	100.0%	100.0%

Analysis of Cash Flows

- **Four steps:**
  1. **Subtract the new expenses attributed to the project from new revenues**
  2. **Subtract the depreciation to get pre-tax income**
  3. **Subtract taxes to get net operating income (NOI)**
  4. **Compute the total after-tax cash flow by adding back depreciation, i.e.,**
     NOI + D

Example F.1

A local restaurant is considering adding a salad bar. The investment required to remodel the dining area and add the salad bar will be $16,000. Other information about the project is as follows:

1. The price and variable cost are $3.50 and $2.00
2. Annual demand should be about 11,000 salads
3. Fixed costs, other than depreciation, will be $8,000
4. The assets go into the MACRS 5-year class for depreciation purposes with no salvage value
5. The tax rate is 40 percent
6. Management wants to earn a return of at least 14 percent

Determine the after-tax cash flows for the life of this project.

Calculating After-Tax Cash Flows

	Year						
Item	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
Initial Information							
Annual demand (salads)		11,000	11,000	11,000	11,000	11,000	
Investment	$16,000						
Interest (discount) rate	0.14						
Cash Flows							
Revenue		$38,500	$38,500	$38,500	$38,500	$38,500	
Expenses: Variable costs		22,000	22,000	22,000	22,000	22,000	
Expenses: Fixed costs		8,000	8,000	8,000	8,000	8,000	
Depreciation (*D*)		3,200	5,120	3,072	1,843	1,843	922
Pretax income		$5,300	$3,380	$5,428	$6,657	$6,657	– $922
Taxes (40%)		2,120	1,352	2,171	2,663	2,663	– 369
Net operating income (NOI)		$3,180	$2,208	$3,257	$3,994	$3,994	– $533
Total cash flow (NOI + *D*)		$6,380	$7,148	$6,329	$5,837	$5,837	$369

Net Present Value Method

- **NPV**
  - The method that evaluates an investment by calculating the present values of all after-tax total cash flows and then subtracting the initial investment amount from their total.

- **Discount rate:** interest rate used in discounting the future value to its present value.

- **Hurdle rate:** interest rate that is the lowest desired return on investment.

Application F.4

Find the NPV for Example Project

Year 1: $500
Year 2: $650
Year 3: $900

The discount rate is 12%, and the initial investment is $1,550, so the project's NPV is:

Present value of investment (Year 0):	($1,550.00)
Present value of Year 1 cash flow:	446.45
Present value of Year 2 cash flow:	518.18
Present value of Year 3 cash flow:	640.62
Project NPV:	$ 55.25

Internal Rate of Return

- Internal rate of return (IRR)
  - The discount rate that makes the NPV of a project zero.
  - The IRR can be found by trial and error, beginning with a low discount rate and calculating the NPV until the result is near or at zero.
  - A project is successful only if the IRR exceeds the hurdle rate.

Application F.5

IRR for Example Project

Discount Rate	NPV		
10%	\$500(0.9091) + \$650(0.8264) + \$900(0.7513) - \$1550	=	\$117.88
12%	\$500(0.8929) + \$650(0.7972) + \$900(0.7118) - \$1550	=	\$55.25
14%	\$500(0.8772) + \$650(0.7695) + \$900(0.6750) - \$1550	=	(\$3.73)

IRR is slightly less than 14%

Payback Method

- **Payback method**
  - A method for evaluating projects that determines how much time will elapse before the total after-tax cash flows will equal, or pay back, the initial investment
  - Payback is widely used, but often criticized for encouraging a focus on the short run and for failing to consider the time value of money.

Example F.2

What are the NPV, IRR, and payback period for the salad bar project in Example F.1?

Management wants to earn a return of at least 14 percent on its investment, so we use that rate to find the pf values in Table F.1. The present value of each year's total cash flow and the NPV of the project are as follows:

2013:	**\$6,380(0.8772)=**	**\$5,597**
2014:	**\$7,148(0.7695)=**	**\$5,500**
2015:	**\$6,329(0.6750)=**	**\$4,272**
2016:	**\$5,837(0.5921)=**	**\$3,456**
2017:	**\$5,837(0.5194)=**	**\$3,032**
2018:	**\$ 369(0.4556) =**	**\$ 168**

Example F.2

NPV of project

= ($5,597 + $5,500 + $4,272 + $3,456 + $3,032 + $168) – $16,000

= $6,025

Because the NPV is positive, the recommendation would be to approve the project.

IRR of project

- Begin with the 14 percent discount rate
- Increment at 4 percent with each step to reach a negative NPV with a 30 percent discount rate.
- If we back up to 28 percent to “fine tune” our estimate, the NPV is $322.
- Therefore, the IRR is about 29 percent.

Example F.2

Discount Rate	NPV
14%	$6,025
18%	$4,092
22%	$2,425
26%	$ 977
30%	–$ 199

Example F.2

- **Payback for Project**
  - **To determine the payback period, add the after-tax cash flows at the bottom of the table in Example F.1 for each year until you get as close as possible to $16,000 without exceeding it.**
  - **For 2013 and 2014, cash flows are $6,380 + $7,148 = $13,528.**
  - **The payback method is based on the assumption that cash flows are evenly distributed throughout the year, so in 2015 only $2,472 must be received before the payback point is reached.**
  - **As $2,472/$6,329 = 0.39, the payback period is 2.39 years.**

Application F.6

What is the payback period for the project in Application F.4?

Payback for year 1	**$1550 - 446.45 = $1103.55**
Payback for years 1 and 2	**$1103.55 - $518.18 = $585.37**
Proportion of year 3	**$585.37 / $640.62 = .9138**
Payback period for project	**2.9138 years**