2nd, SEM. /COMMON/ 2022(S)

TH-3 -ENGINEERING MATHEMATICS -II

Full Marks: 80

Time-3 Hrs

Answer any five Questions including Q No.1& 2 Figures in the right hand margin indicates marks

١. Answer All questions

 2×10

Define Modulus Function and represent it graphically.

Evaluate $\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

Solution Differentiate $\sec^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)$ with respect to x.

d. Define unit vector and find the unit vector of the given vector $2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}.$

Fivaluate the integral $\int (e^{3 \ln x} - e^{4 \ln x}) dx$.

Define Homogeneous Function and State Euler's Theorem.

Find the value of α so that $\vec{a} = \hat{i} + \hat{j} + \alpha \hat{k}$, $\vec{b} = 4\hat{i} - 3\hat{k}$ are perpendicular to each other.

Find the order and degree of the following differential equation

$$\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$$

Find the value of $\int_{-2}^{2} |x| dx$.

If $y = t^2$ and $x = t^3$ find $\frac{dy}{dx}$ at t = 1.

6 x 5

Answer Any six questions:

If $f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$ is continuous at x = 1, then find the

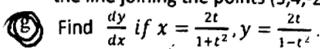
value of 'a' and 'b'.

Find $\frac{dy}{dx}$ if $y = (\ln x)^{tanx}$.

Determine the area within the curve $y^2 = 4ax$ and the x-axis, the ordinate x=4.

Evaluate $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx.$ Solve $(1 + x^2) dy + (1 + y^2) dx = 0.$

Find the scalar and vector projections of the vector 2i - 3j - 6k on the line joining the points (3,4,-2) and (5,6,-3).



i If $\sqrt{1-x^6} + \sqrt{1-y^6} = k(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$

Evaluate
$$\lim_{x\to 0} \frac{1-\cos^3 x}{x\sin 2x}$$
.

- i If $u = \tan^{-1}(x^2 + y^2 + z^2)$, show that $xu_x + yu_y + zu_z = \sin 2u$
 - If sum of two unit vectors is a unit vector, show that the magnitude of 3 their difference is $\sqrt{3}$.
- 5 i Evaluate $\int \frac{2x+11}{\sqrt{x^2+10x+29}} dx$. h if $y = \tan^{-1} z$, prove that $(1 + x^2)y_2 + 2xy_1 = 0$ 4
- Solve the following differential equation $(1+y^2)dx = (\tan^{-1} y - x)dy$ Find the derivative of $y = e^x$ by first principle. 3
- In a triangle AOB, angle $AOB=90^{\circ}$. If P, Q are the points of trisection of 7 \overline{AB} , prove that $OP^2 + OQ^2 = \frac{5}{9}AB^2$ by vector method. 4
 - Evaluate $\int e^x \left(\frac{1}{r^2} \frac{2}{r^3}\right) dx$.