Cayley graphs of given degree and diameter on linear groups

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April 17, 2018

Overview

Motivation

- In it's simplest form, networks can be modeled by graphs with nodes as vertices and links between them as edges.
- In design of graphs we can take many restrictions into acount such degree, grith, diameter.
- Two important problems concerning degree and diameter and degree and grith of graph

The degree/diameter problem

Degree/diameter problem

Find graph with biggest possible number of vertices with given degree and diameter.

Degree/girth problem

Find graph with smallest possible number of vertices with given degree and diameter.

Moore bound

There is theoretical upper bound for largest order of graph with d-degree and k-diameter.

$$\begin{aligned} n_{d,k} \leq M_{d,k} &= 1 + d + d(d-1) + \cdots + d(d-1)^{k-1} \\ &= 1 + d(1 + (d-1) + \cdots + (d-1)^{k-1}) \\ &= \begin{cases} 1 + d\frac{(d-1)^k - 1}{d-2}, & \text{if } d > 2 \\ 2k+1, & \text{if } d = 2 \end{cases} \end{aligned} \tag{1}$$

Moore bound

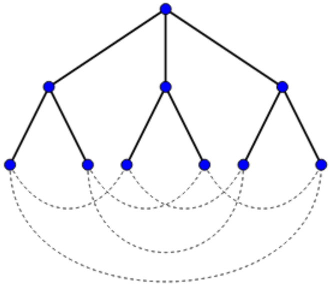

Figure: Peterssen graph is Moore graph with $d = 3$ and $k = 2$

Moore graphs

Graphs with order equal Moore bound are called Moore graphs and are reached only in few cases.

- If $d = 2$ for any $k \geq 1$
- If $k = 1$ for any $d \geq 2$
- For $k = 2$ for $d \in \{3, 7\}$, and possibly 57

For other cases we try to construct graphs with order as close to Moore bound as possible.

Moore graphs

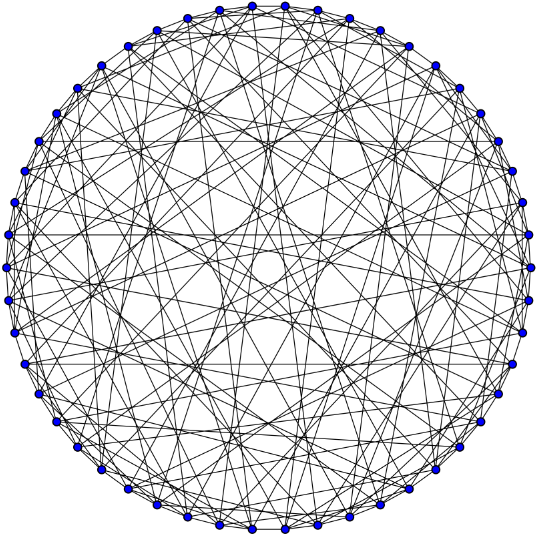

Figure: Hoffman-singleton graph is Moore graph with $d = 7$ and $k = 2$

Graph lifting

Let G be an undirected graph. We will assign direction to every edge of graph and making them *arcs*. *Arc* with *reversed* direction of e is denoted by e^{-1}.

Definition (Graph lifting)

Let G be a graph as above and let Γ be a finite group. The mapping

$$\alpha : D(G) \to \Gamma$$

will be called a *voltage assignment* if $\alpha(e^{-1}) = (\alpha(e))^{-1}$, for any arc $e \in D(G)$.

Graph lifting example

Obrazok zdvihu na petersenov graf.

Cayley graphs

General linear and Special linear groups

Generation of cayley graphs

Computer search of cayley graphs

The End