Theorem 1. *Let L be a modular lattice and let $a, b \in L$. Then*

$$\varphi_b : x \mapsto x \wedge b, x \in [a, a \vee b],$$

is an isomorphism between the intervals $[a, a \vee b]$ and $[a \wedge b, b]$. The inverse isomorphism is

$$\psi_a : y \mapsto x \vee a, y \in [a \vee b, b].$$

Proof. *It is sufficient to show that $\psi_a \varphi b(x) = x$ for all $x \in [a, a \vee b]$. Indeed, if this is true, then by duality, $\varphi_b \psi_a(y) = y$, for all $y \in [a \wedge b, b]$, is also true. The isotone maps φ_b and ψ_a, thus compose into the identity maps, hence they are isomorphisms, as claimed.*

So let $x \in [a, a \vee b]$. Then $\psi_a \varphi b(x) = (x \wedge b) \vee a$. Since $x \in [a, a \vee b]$, the inequality $a \leq x$ holds, and so modularity applies:

$$\psi_a \varphi_b(x) = (x \wedge b) \vee a = x \wedge (b\ veea) = x,$$

because $x \leq a \vee b$. $\square$