

## Experiment No. 1

### Name of the Experiment: Complexity of Algorithms

Duration: 1 cycle

**Background Study:** Chapter 1-2 (Theory and Problems of Data Structures Written by Seymour Lipschutz)

**Definition of Big-O Notation:**  $f(n)$  is  $O(g(n))$  if and only if there exist two constants  $c$  and  $n_0$  such that  $|f(n)| < c|g(n)|$  for all  $n > n_0$ .

$f(n)$  will normally represent the computing time of some algorithm. When we say that the computing time of an algorithm is  $O(g(n))$  we mean that its execution takes no more than a constant times  $g(n)$ .  $n$  is a parameter which characterizes the inputs and/or outputs. For example  $n$  might be the number of inputs or the number of outputs or their sum or the magnitude of one of them.

#### Example

Let  $f(n) = 3n+2$

Question: Find the Big-O notation of  $f(n)$ .

We have,  $f(n) = 3n+2$

We can write,  $f(n) \leq 3n + n$  if  $n \geq 2$

$f(n) \leq 4n$ ,

From definition,  $n_0 = 2$  and  $c = 4$ ,  $g(n) = n$

$f(n) \leq cg(n)$

$f(n)$  is  $O(g(n))$  or  $f(n)$  is  $O(n)$

**Problem I:** Find the Complexity of a Loop.

#### Algorithm 1.1:

1. Repeat for  $K = 1$  to  $n$  by 1
2. Write: K  
[End of Step 1 loop]
3. Exit

**Complexity:** the computing time of the loop,  $f(n) = n$ . So the complexity of the above algorithm is  $O(n)$ . Here  $f(n) \leq cg(n)$ ,  $c = 2$ ,  $n_0 = 0$ ,  $g(n) = n$ .

**Complexity Table:**

| n  | f(n) [from Program, Count Statement] | cg(n) [Theoretical] |
|----|--------------------------------------|---------------------|
| 10 |                                      |                     |
| 20 |                                      |                     |

**Graph:** Draw a Graph.

**Problem II:** Find the Complexity of the following Program.

#### Algorithm 1.2:

1. Repeat for  $K = 1$  to  $n$  by 1
2. Repeat for  $L = 1$  to  $n$  by 1
3. Write: L  
[End of Step 2 loop]
4. Write: K  
[End of Step 1 loop]
5. Exit

**Complexity:** the computing time of the loop,  $f(n) = n^2$ . So the complexity of the above algorithm is  $O(n^2)$ . (Let  $c = 2$ )

**Complexity Table:**

| n  | f(n) [from Program, Count Statement] | cg(n) [Theoretical] |
|----|--------------------------------------|---------------------|
| 10 |                                      |                     |
| 20 |                                      |                     |

**Graph:** Draw a Graph.

**Problem III:** Find the Complexity of the elementary Sort algorithm.

**Algorithm1.3:** (Given a nonempty array A with n numerical values. This algorithm sorts the values)

1. Repeat for i = 2 to n by 1
2. Repeat for k = i to 1 by -1
3. If  $A[k] < A[k-1]$  then:  
    Swap ( $A[k]$ ,  $A[k-1]$ )  
    [End of If Structure]  
    [End of Step 2 loop]  
    [End of Step 1 loop]
4. Exit

**Complexity:** The complexity of the above algorithm is  $O(n^2)$ . (Let  $c = 2$ )

**Complexity Table:**

| n  | f(n) [from Program, Count Statement] | cg(n) [Theoretical] |
|----|--------------------------------------|---------------------|
| 10 |                                      |                     |
| 20 |                                      |                     |

**Graph:** Draw a Graph.

**Problem IV:** Find the largest element in Array.

**Algorithm1.4:** (Given a nonempty array A with n numerical values. This algorithm finds the location LOC and the value MAX of the largest element of A)

1. Set  $K := 1$ ,  $LOC := 1$  and  $MAX := A[1]$
2. Repeat steps 3 and 4 while  $K \leq n$
3. IF  $MAX < A[K]$  then:  
    Set  $LOC := K$  and  $MAX := A[K]$ .  
    [End of If structure]
4.  $K := K + 1$ .  
    [End of step 2 loop]
5. Write: LOC, MAX.
6. Exit

**Complexity:** Not Required

**Problem V:** Linear Search.

**Algorithm1.5:** (Given a nonempty array A with n numerical values and a specific x of information is given. This algorithm finds the location LOC of x in the array A or Sets  $LOC = -1$ )

1. Set  $K := 1$ ,  $LOC := -1$
2. Repeat steps 3 and 4 while  $LOC = -1$  and  $K \leq n$
3. IF  $x = A[K]$  then:  
    Set  $LOC := K$ .  
    [End of If structure]
4.  $K := K + 1$ .  
    [End of step 2 loop]
5. If  $LOC = -1$  then: Write: x is not in the array A.  
    Else: Write: LOC is the location of x  
    [End of If structure]
6. Exit

**Complexity:** The worst case complexity is  $C(n) = n$  and the average case complexity is  $C(n) = (n+1)/2$ .

### MORE PROBLEMS

1. Programming Problems of Chapter 1 and 2 of "Data Structures" by Seymour Lipschutz.

**LAB REPORT:** You have to submit all assigned problems in next lab.