COUPLING OF ELECTROMAGNETIC FIELDS TO TRANSMISSION LINES

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Abstract

The prediction of terminal currents induced on a transmission line by an incident, electromagnetic field is investigated. The predictions of the distributed parameter, transmission line model are compared to those of two method of moments codes. The method of moments codes are used to provide baseline data in lieu of experimental data. It was found that the transmission line model provided very accurate predictions of the induced, terminal currents (differential mode currents) when the wire separation was electrically small. Various angles of incidence and polarization of the incident wave as well as values of the terminal impedances were investigated.

I. Introduction

The undesired interaction of wires in cable bundles with an incident electromagnetic field is often a major source of interference in electronic systems. Cables may be exposed to radar signals, ambient noise, EMP, lightning, and many other incident fields some of which may be essential signals for the system operation. These incident fields interact with the cables and induce currents along the wires in the bundles. The currents induced at the terminals of the cables may be of sufficient magnitude and/or spectral content to degrade the performance of the devices attached to the ends of the wires. It is the purpose of this paper to investigate the modeling and prediction of the coupling from an incident electromagnetic field onto a transmission line.

The basic problem is illustrated in Fig. 1. We will restrict our considerations to the simplest possible case: a two-wire line consisting of two parallel, perfectly conducting wires of radius r, separated by d and of length $\mathcal{L}$. The devices at the ends of the line will be represented by resistances R, both of which are assumed for this study to be identical. The incident field will be in the form of a uniform, plane wave. We are interested in predicting the induced, terminal, phasor currents I(0) and I($\mathcal{L}$).

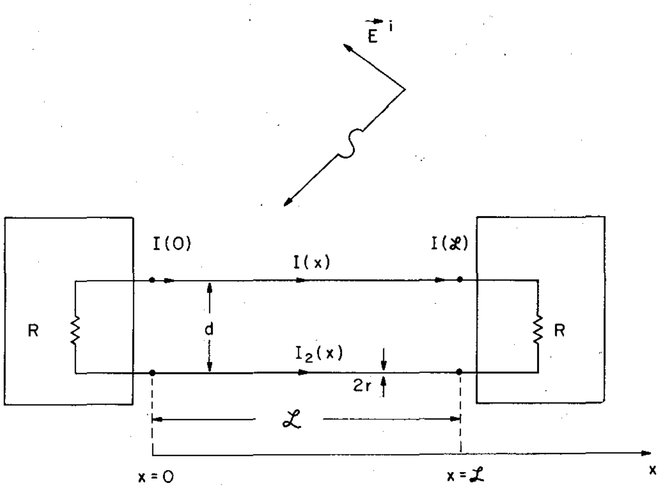

Fig. 1

Perhaps the most common model used to solve this problem is the transmission line (TL) model with distributed sources [1-4]. The advantages of this model are primarily its computational efficiency and its ability to provide qualitative insight into the coupling mechanism. Given the incident field magnitude, polarization, and angle of arrival and the parameters of the TL, it is a simple matter to compute the induced terminal currents at a multitude of frequencies [1-4]. In addition, the resulting equations for the terminal currents are simple enough to reveal the general behavior of these induced currents as the angle of arrival and/or polarization of the incident field are changed and as the values of the terminal impedances are changed. Thus one can deduce a considerable amount of information related to the vulnerability of the line for various conditions without the need for performing numerical computations.

An alternative prediction model would be to use the popular and highly effective method of moments (MOM) technique [5]. With this technique, the problem is modeled in a virtually exact fashion and the various assumptions inherent in the TL model are not employed. The disadvantages of the MOM technique are primarily in its computational inefficiency (compared to the TL model) and the inability to provide any qualitative details of the solution. The MOM technique requires the use of large, high-speed digital computers whereas the TL model can be implemented on hand calculators (at least for two-wire lines). Furthermore, in order to determine the interaction of the incident field with the line via the MOM techniques, one must obtain numerical results for a specific set of problem parameters, with little or no insight into the outcome of the solution.

Clearly the TL model is much more desirable than the MOM model in the above respects. However, certain assumptions and restrictions in the TL model limit its applicability. What is needed is to determine the range of problems for which the model provides adequate predictions. Some previous work has been done in the area of "validation" of this model [6,7], however much more work is needed before we have a full appreciation of its limitations. In this paper we intend to provide an extension of previous work. This work is not intended to "close the book" on the "validation" of the TL model. Considerably more work is needed in order to understand all of the limitations of this solution technique.

Ideally, in order to investigate the limitations of the TL model we need to compare its predictions to accurate, experimental results. At an early stage in this work it became clear that obtaining experimental results would be quite difficult. Therefore, in lieu of experimental results we have used two MOM codes to provide this baseline data against which we will compare the predictions of the TL model. These two codes will be discussed in the next section.

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II. Prediction Models

The transmission line model is derived in numerous places and that derivation will not be repeated here [1-4,8]. The notation is illustrated in Fig. 2(a). The line is directed along the x axis of a rectangular coordinate system and the phasor currents in the upper and lower wires at a particular x are assumed to be equal in magnitude and oppositely directed. The phasor voltage at a particular x is the line integral of the transverse electric field at that x. The transmission line equations are derived from the per-unit-length circuit in Fig. 2(b) in the limit as $\Delta x \to 0$ and become

$$\frac{dV(x)}{dx} + j\omega\ell\, I(x) = V_s(x) \tag{1a}$$

$$\frac{dI(x)}{dx} + j\omega c\, V(x) = I_s(x) \tag{1b}$$

The parameters ℓ and c are the per-unit-length inductance and capacitance, respectively, of the line. The distributed sources $V_s(x)$ and $I_s(x)$ are a result of the incident field. These are given by

$$V_s(x) = j\omega\mu_0 \int_0^d H_z^i(y,x)\, dy \tag{2a}$$

$$I_s(x) = -j\omega c \int_0^d E_y^i(y,x)\, dy \tag{2b}$$

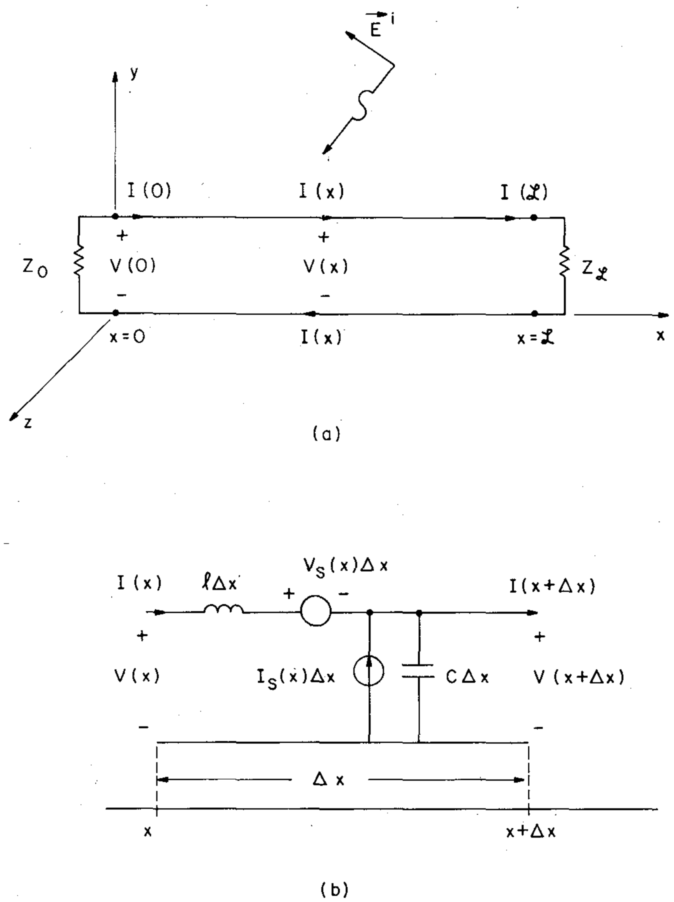


Fig. 2

where H_z^i is the z component of the incident magnetic field in the plane of the wires and E_y^i is the y component of the incident electric field between the two wires. Equations (1) may be solved in terms of the chain parameter matrix (CPM) relating voltages and currents at two positions on the line:

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} \phi_{11}(x) & \phi_{12}(x) \\ \phi_{21}(x) & \phi_{22}(x) \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + \begin{bmatrix} \hat{V}_s(x) \\ \hat{I}_s(x) \end{bmatrix} \tag{3}$$

where $\phi_{11}(x) = \phi_{22}(x) = \cos(\beta x)$, $\phi_{12}(x) = -jv\sin(\beta x)\ell$, $\phi_{21}(x) = -jv\sin(\beta x)c$, $v = 1/\sqrt{\ell c}$ and $\beta = \omega\sqrt{\ell c}$. The equivalent source terms in (3) are convolutions of $V_s(x)$ and $I_s(x)$ with the ϕ_{ij}'s [3]:

$$\hat{V}_s(x) = \int_0^x [\phi_{11}(x-\tau)V_s(\tau) + \phi_{12}(x-\tau)I_s(\tau)]d\tau \tag{4a}$$

$$\hat{I}_s(x) = \int_0^x [\phi_{21}(x-\tau)V_s(\tau) + \phi_{22}(x-\tau)I_s(\tau)]d\tau \tag{4b}$$

The CPM in (3) only relates the voltages and currents at two points on the line. In order to solve for the voltage and/or current at a specific position we incorporate the terminal conditions,

$$V(0) = -Z_0 I(0) \tag{5a}$$

$$V(\mathcal{L}) = Z_{\mathcal{L}} I(\mathcal{L}) \tag{5b}$$

into (3). A computer program was written [8] to obtain the induced, terminal currents via this method. Certain angles of incidence and polarization of the incident wave were investigated and simplified expressions for these cases are given in [8].

The two MOM codes used to provide baseline data are the OSMOM code described in [9] and the WRSMOM code described in [10]. The OSMOM code was written by J. H. Richmond of Ohio State University. It employs piecewise sinusoidal current expansion functions. The Galerkin method is used and thus the testing functions are also piecewise sinusoids. The WRSMOM code was written by D. E. Warren of the Rome Air Development Center, Griffiss AFB, NY and uses pulse expansion functions and point matching. This is essentially the same technique described by Harrington in chapter 4 of [5]. Thus the two MOM codes are considerably different, not only in choice of expansion functions but also in choice of testing functions. We found that these differences produced significant differences in predictions for our seemingly simply problem. Essentially, the OSMOM code provided remarkably good correlation with the TL predictions for the chosen problem dimensions whereas in general the WRSMOM code did not. This is not intended to imply a fault in the WRSMOM code. The pulse expansion, point matching technique (used in WRSMOM) is very difficult to successfully apply in certain cases, particularly those involving corners or wire junctions. The TL structure considered here heavily taxed the modeling capabilities of the technique.

The total currents on each wire are denoted as $I_1(x)$ and $I_2(x)$ and are directed in the positive x direction as shown in Fig. 1. We can decompose these into differential mode currents

$$I_D(x) = \frac{I_1(x) - I_2(x)}{2} \tag{6}$$

and common mode currents

$$I_C(x) = \frac{I_1(x) + I_2(x)}{2} \tag{7}$$

as shown in Fig. 3. The TL model is intended to predict only the differential mode currents, $I_D(x)$, whereas the MOM codes predict the total currents, $I_1(x)$ and $I_2(x)$. Therefore, in order to provide relevant comparisons, the total currents $I_1(x)$ and $I_2(x)$ obtained with the MOM codes are separated into differential mode and common mode currents, as in (6) and (7), and the differential mode currents computed via (6) are compared to the predictions of the TL model.

From the standpoint of using these models to predict the effect of an incident field on the devices which the wires interconnect, we are only interested in predicting the currents induced in the terminations. In this case, only the differential mode currents are of interest since the common mode currents tend to cancel out at the terminations [8]. This is particularly true if the wire separation is electrically small so that the loads may be considered as lumped parameters. Thus the TL model should be adequate for predicting vulnerability of the terminal equipment even though it does not (and is not intended to) predict the total current at various points on the line.

III. The Chosen Configuration

It is, of course, impossible to investigate all possibilities of wire length, wire separation, wire radius, frequency, values of terminal impedances, and wave angles of incidence and polarization. In this study we have chosen representative values for the dimensions. The line length was chosen to be one meter ($\mathcal{L}$ = 1m). Although lengths of typical cables have a wide variety of values, this chosen length facilitated the modeling with the MOM codes. In addition, so long as the line length is much greater than the wire separation we would not expect the general conclusions obtained for a 1 meter line to change if the line length is increased. The wire separation was chosen to be 1cm (.01m). This provided a line length much greater than the wire separation. It also seems to be a reasonable compromise for typical wire separations in cable bundles. For a wire above ground, this separation represents a height above ground of .5cm. Although cable heights above ground can vary considerably, this was chosen to represent typical cables which are clamped to vehicle frames. The wire radius was chosen to be .1mm. This corresponds to #32 gauge wire which is somewhat typical of signal lines.

The frequency range was chosen to be from 100kHz to 1GHz. This provided an investigation of the line for frequencies where the line length was electrically short and also when it was electrically long. The line length is one wavelength at 300 MHz. The wire separation is $(1/100)\lambda$ at 300 MHz and $(1/30)\lambda$ at 1GHz.

The terminal impedances were chosen to be resistive and equal ($Z_0 = Z_{\mathcal{L}} = R$). Three values of R were investigated: $R = 50\Omega$, $R = R_C$, $R = 10k\Omega$ where R_C is the characteristic resistance of the line. These chosen impedances are somewhat representative of typical loads. It was felt that rather than investigating only matched loads (noncoaxial cables are rarely matched in practice) we should also investigate results for cases where the terminal impedances were much lower and much higher than the characteristic resistance, R_C, in order to obtain a realistic appraisal of the line response.

As to the excitation parameters - angle of incidence and polarization of the incident plane wave - three cases were investigated as shown in Fig. 4.

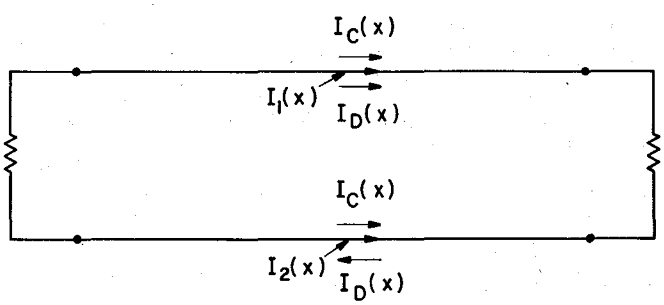
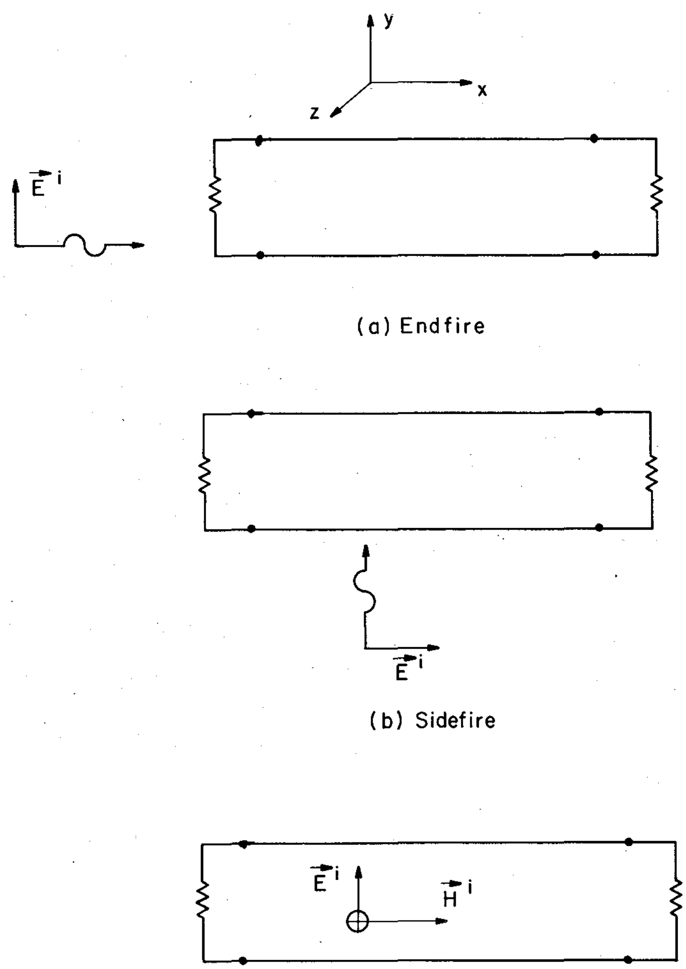


Fig. 3



(a) Endfire

(b) Sidefire

(c) Broadside

Fig. 4

In the ENDFIRE case the wave is propagating in the direction of the line axis with the E field parallel to the terminations. The H field is perpendicular to the plane of the wires. Equations (2) show that, from the TL model standpoint, both E and H provide an excitation. In the SIDEFIRE case, the wave is propagating in the y direction with the E field in the x direction parallel to the wires and the H field in the -z direction, once again perpendicular to the plane of the wires. From (2), this E field does not provide an excitation for the TL model whereas the H field does. In the BROADSIDE case, the wave is propagating in the -z direction (perpendicular to the plane of the wires) with the E field parallel to the terminations and the H field parallel to the wires. From (2), this H field does not provide an excitation for the TL model whereas the E field does.

We would expect the common mode induced currents along the line to be much smaller than the differential mode currents for the ENDFIRE and BROADSIDE excitations. For the SIDEFIRE case, we would expect the common mode or antenna mode currents to be larger than for the other two cases since the E field is parallel to the wires. These expectations proved to be true in the computed MOM results. However, as far as the terminal currents are concerned, the common mode currents at the terminations were always much less than the differential mode currents and therefore the TL model provided adequate predictions of the total terminal currents. For currents along the line this was not true in the SIDEFIRE case although it held true in the ENDFIRE and BROADSIDE cases, as anticipated.

IV. Prediction Results

The predictions of the induced, differential mode currents at the left end of the line, I(0), via the TL model and the OSMOM code will be compared.

The results for ENDFIRE excitation and the three load impedances are shown in Figs. 5, 6, and 7. The OSMOM code predictions are shown as "0". Note that above 10MHz, the predictions of the TL model and the OSMOM code are virtually identical. For R = 50Ω the predictions are in error by only a few percent immediately prior to the high frequency resonances at which the line length is some multiple of λ/2 (150 MHz, 300 MHz, 450 MHz, etc.). This prediction error did not occur for $R = R_C$ and R = 10kΩ. The reason was thought to be re-radiation from the terminal segments which are modeled as a physical part of the structure with the MOM solution but not in the TL model. Additional investigation seems to confirm this [8].

Note that below 10 MHz, the predictions of the OSMOM code becomes erratic and ceases to correlate with those of the TL model. In this region, the wire separation is on the order of $\frac{1}{3000}$ λ and numerical errors introduced by the computer computations are apparently causing the MOM results to be unreliable. This was also pointed out in [11]. The TL predictions approach a 20dB/decade or linear variation with frequency as the frequency is reduced; this is theoretically correct.

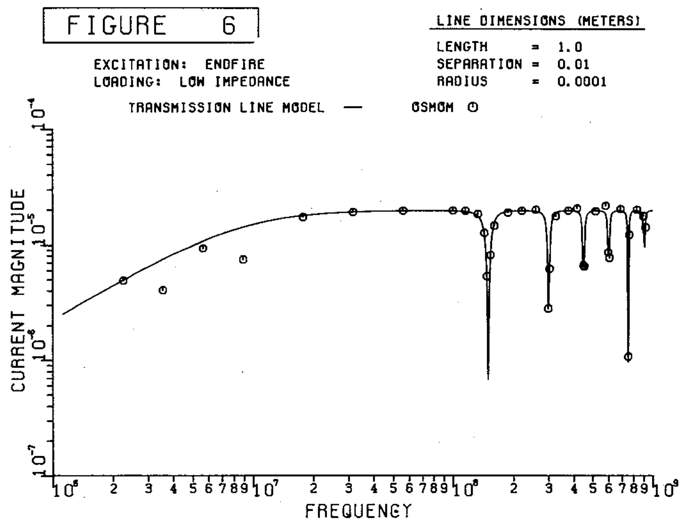


Fig. 6

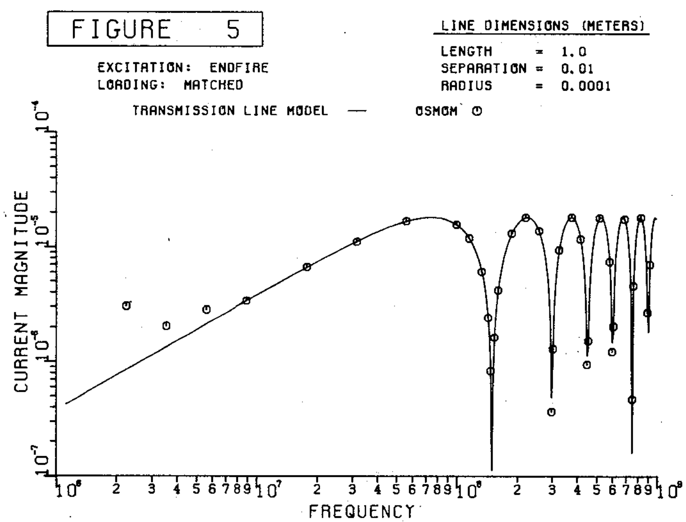


Fig. 5

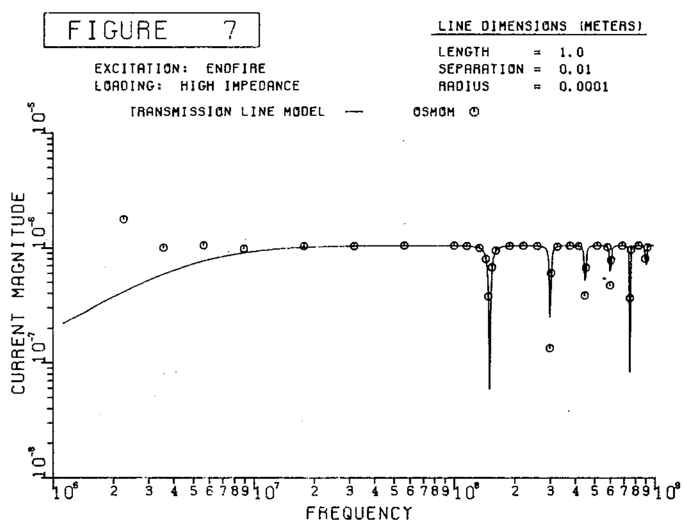


Fig. 7

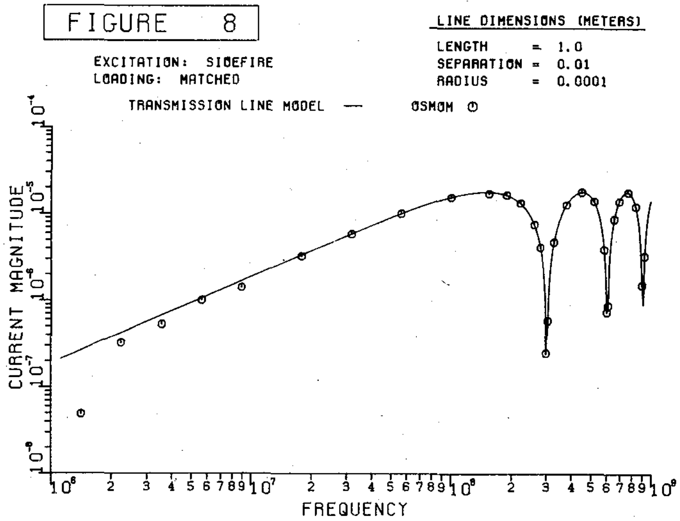


Fig. 8

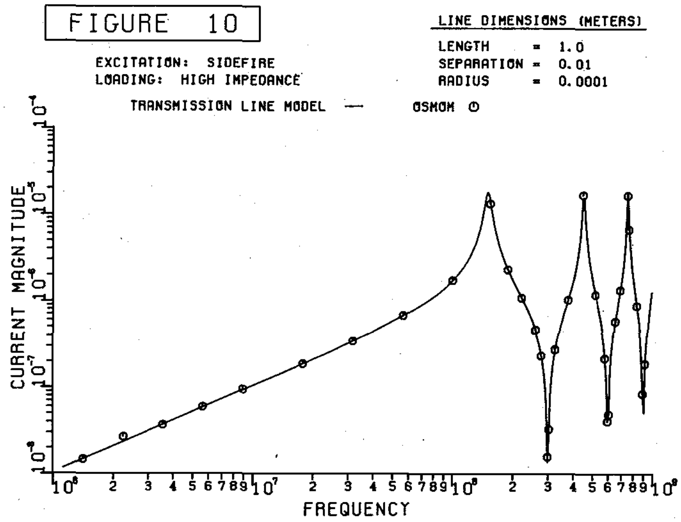


Fig. 10

The corresponding results for SIDEFIRE excitation are shown in Figs. 8, 9, and 10 and for BROADSIDE excitation in Figs. 11, 12, and 13. The conclusions obtained from the ENDFIRE case apply to these cases also. In general, the predictions of the OSMOM code provide remarkable correlation with the predictions of the TL model so long as the electrical separation of the wires is not too small; that is, above 10 MHz where $d = \frac{1}{3000} \lambda$.

In the modeling of the line via the MOM codes, the wires and the terminations were broken into segments. For the above case each of the top and bottom wires were represented as 24 segments. Each segment was therefore 4.17cm in length. Each termination was represented as one segment with length equal to the wire separation, 1cm. Note that there is a change in adjacent segment lengths between the last segment of each wire and the terminal segments of about 4 to 1. In many cases, large changes in adjacent segment lengths, particularly at corners, can result in accuracy problems with some MOM codes. This is often related to the choice of expansion and testing functions. The above choice of segmentation was chosen to avoid large computation times with the MOM codes. With the MOM technique, the solution at each frequency requires the inversion of an N x N matrix where N equals the total number of segments on the structure. For the above segmentation, $N = 1 + 24 + 1 + 24 = 50$. In order to keep all segments on the structure equal in length and remove the abrupt change in segment length would have resulted in N = 202 and the computation times would be rather large. For N = 50, the OSMOM code requires approximately 80 seconds for computation at 10 frequencies in single precision on an IBM 370/165 computer. The TL model requires only 2 s for 200 frequencies! Computation time for N = 202 at the required number of frequencies via the MOM codes could not be justified.

It was expected that the large 4:1 change in segment length would cause problems with the WRSMOM code due to its algorithm [5]. However, it was not expected

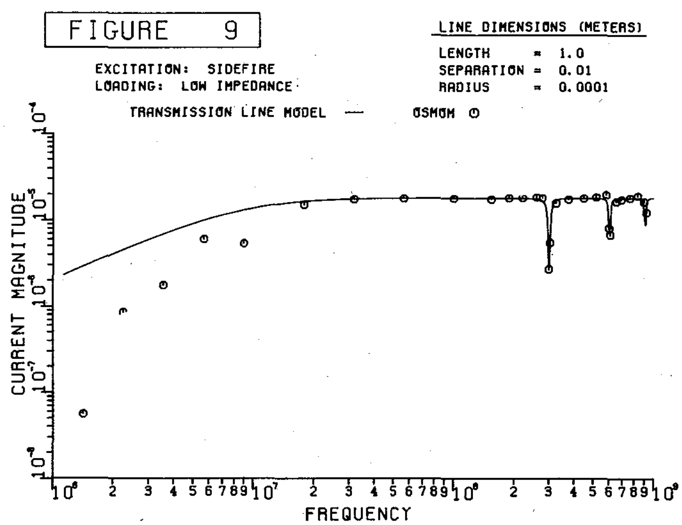


Fig. 9

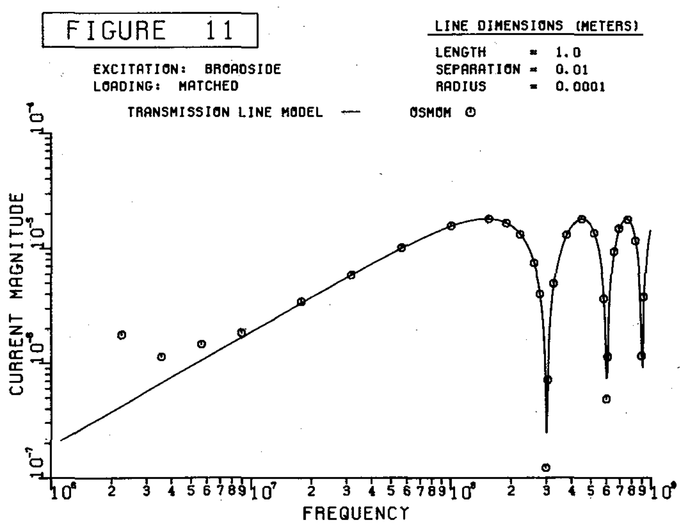


Fig. 11

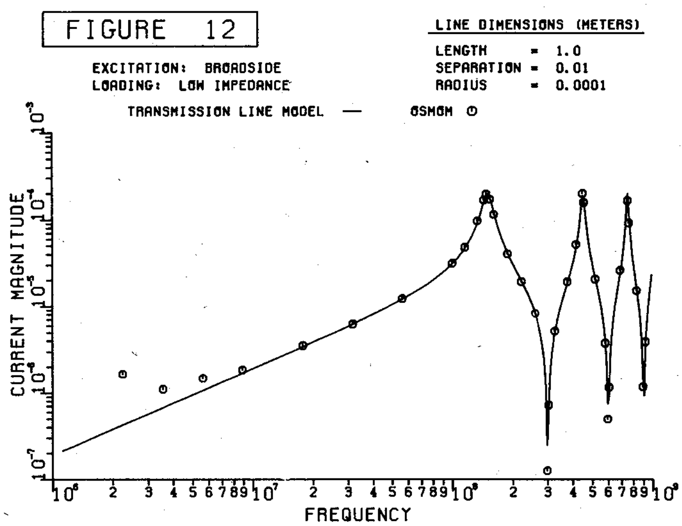


Fig. 12

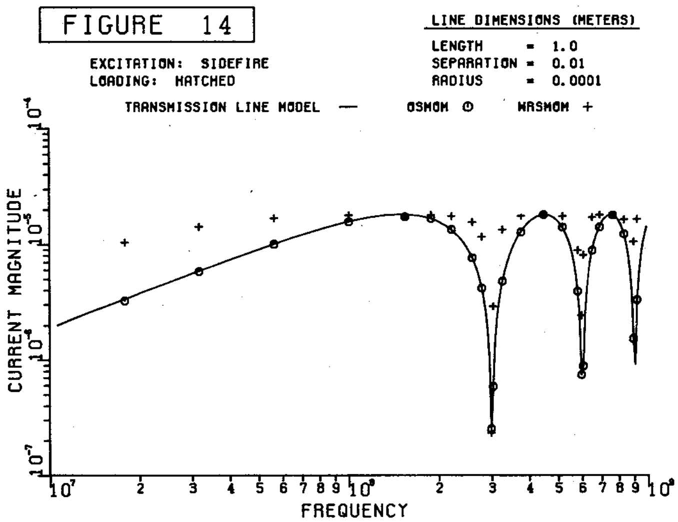


Fig. 14

that OSMOM would provide the good correlation with the TL model even with this large 4:1 change in segment size at the corners. To illustrate the problems that the large segment size change at the corners produces in the WRSMOM code, we have shown the differential mode terminal current predictions obtained with WRSMOM for the .01 separation, $R = R_C$ and SIDEFIRE excitation in Fig. 14. The predictions of WRSMOM are shown as "+". To show the effect of choosing segments so that there is no large change in adjacent segment lengths, we increased the wire separation to 5cm. The top and bottom wires were divided into 20 segments each and the terminations were each represented by one segment. In this case all segments on the structure were identical in length (5cm). The results are shown in Fig. 15. Note that WRSMOM and OSMOM provide similar results and both correlate well with the TL predictions. The WRSMOM results begin to show error above 600 MHz. This is felt to be partially a result of the inadequate sampling of the E field by the point matching technique in and near the terminal segment [8]. The OSMOM solution should be quite accurate for an additional reason. In OSMOM, the elements of the impedance matrix are calculated using the rigorous, closed-form impedance expressions in terms of exponential integrals. This calculation method provides a much more accurate (but expensive) solution than the numerical integration techniques (such as Simpson's rule for integration) which are normally used.

V. Summary and Conclusions

It is reasonable to believe that the predictions of the OSMOM code would correlate well with experimental results and therefore represent accurate, baseline data. With this assumption, the TL model is seen to provide highly accurate predictions of the terminal currents induced on a two-wire transmission line by an incident, electromagnetic field.

There are, of course, certain restrictions on the applicability of the TL model. It is generally

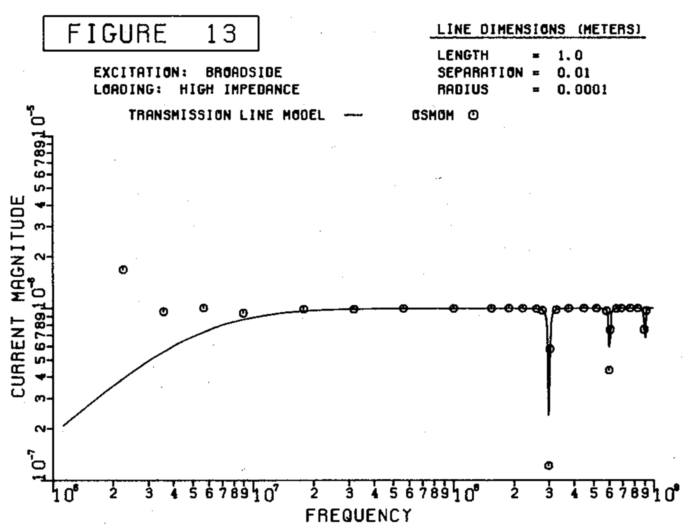


Fig. 13

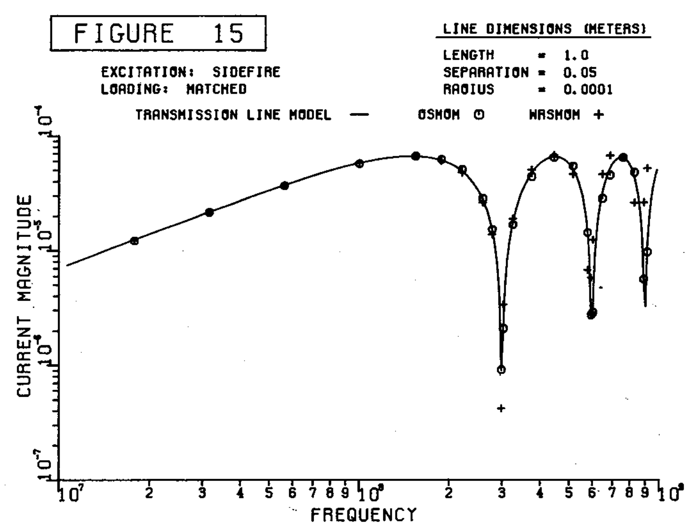


Fig. 15

expected that the TL model will cease to provide accurate predictions when the cross-sectional dimensions of the line, i.e., wire separation, become electrically large. For the highest frequency investigated, 1GHz, and the smallest separation, .01m, the wire separation is $1/30\lambda$. Accurate predictions were obtained with the TL model up to this frequency. When the wire separation was increased to .05m, the electrical separation was increased to approximately $1/6\lambda$ at 1 GHz. Even in this case, the TL model provided accurate predictions.

On the other hand, the MOM code predictions began to break down below 10 MHz. At 10 MHz, the wire separation of .01m is $\frac{1}{3000}\lambda$ which is very small, electrically. This indicates that the MOM codes would be of little use for a large range of frequencies for which the prediction of the induced currents would be desired in many practical cases. For these frequencies the TL model serves a useful role and would be the only feasible model.

A complete "validation" of the TL model is, of course, impossible. This study is intended to increase our knowledge of conditions under which the TL model provides accurate predictions. It should be pointed out that obtaining accurate data to compare the TL model predictions against is very difficult. Accurate experimental data are especially difficult to obtain. We chose to provide baseline data with two MOM codes. Even then, we found that one must be very careful in choosing and implementing these MOM codes to obtain those data. The point matching, pulse expansion method in the WRSMOM code must be applied with considerable care. The piecewise sinusoid, Galerkin technique in the OSMOM code proved to be very accurate even when applied in situations which are commonly thought to be risky as was the case here. Thus one should perhaps use several MOM codes, each of which employ different expansion functions and testing functions in order to be assured that the MOM baseline data are in fact accurate. Even though the MOM techniques have proved to be quite accurate in modeling many structures, one must be very careful in applying these to a particular problem. Differences in expansion functions and testing functions appear to have considerable impact on the care one must exercise in applying the codes.

It should also be pointed out that the distributed sources, $V_s(x)$ and $I_s(x)$, in the TL model must be modified when the two wires are physically very close together [4]. It appears from [4] that so long as the ratio of separation to wire radius, d/r, is larger than 5 to 10, the sources for the TL model used in this work are adequate. For the chosen configuration $d = .01$m and $r = .1$mm so that $d/r = 100$.

Acknowledgements

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