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On the Evaluation of Antenna-Mode Currents Along Transmission Lines

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***Abstract*—In this paper, we derive an integral equation describing the antenna-mode currents along a two-wire transmission line (TL). We show that when the cross-sectional dimensions of the line are electrically small, the integral equation reduces to a pair of TL-like equations with equivalent line parameters (inductance and capacitance). The derived equations make it possible to compute the antenna-mode currents using any traditional TL coupling code with appropriate parameters. The derived equations are tested against numerical results obtained using numerical electromagnetics code (NEC), and reasonably good agreement is found.**

***Index Terms*—Antenna-mode current, transmission lines, transmission line-mode current.**

I. Introduction

TRANSMISSION line (TL) theory is a useful tool for the analysis of surge propagation along TLs. The basic assumptions of the TL theory are that [1]–[3]

1) the cross-sectional dimensions of the line are electrically small, so that the propagation occurs only along the line axis;
2) the response of the line is quasi-transverse electromagnetic (quasi-TEM); and
3) the sum of the line currents at any cross section of the line is zero.

For multiconductor lines above a reference ground, the reference conductor is the return path for the currents in the n overhead conductors.

If the sum of all the currents crossing a plane perpendicular to the direction of the line can be assumed to be zero, one can consider the "transmission line mode" currents only and neglect the so-called "antenna-mode" currents [1]–[4]. For a line consisting of a conductor above the ground, quasi-symmetry due to the presence of the ground plane results in a very small contribution of the antenna-mode currents and, consequently, the predominant mode on the line will be the TL mode [1].

For a two- (or multi-) conductor line, however, even for electrically small line cross sections, the presence of the antenna-mode currents implies that the sum of the currents in a cross section is not necessarily equal to zero [1], [2]. If we wish to compute only the load responses of the line, consideration of only the TL-model current is adequate, since the antenna-mode current response is small near the ends of the line. On the other hand, if we want to evaluate the current *along* the line, the presence of the antenna-mode currents needs to be taken into account, even for electrically small line cross sections. It has been shown in [1]–[5] that the amplitude of these antenna-mode currents may even be much larger than that of the TL-mode currents. Therefore, to compute the fields radiated by power line communication signals from indoor low-voltage power lines or radiated emissions from printed circuit boards, one must take into account the contribution of the antenna-mode currents as these are often the predominant sources of radiations [5]–[8].

To evaluate the antenna-mode currents, one has to apply general scattering theory. Lee [4] derived Telegrapher's equations for lines consisting of two parallel wires of unequal cross-sectional areas. In his development, Lee included the antenna-mode current that is the solution of the scattering problem involving two conductors excited by two symmetric incident waves [4]. This scattering problem can be approximated by that of one conductor with an equivalent radius, as in the case of a folded dipole antenna [4].

Some recent studies have proposed methods to include high frequency effects into the classical TL theory [9]–[14]. In this paper, we derive an integral equation describing the antenna-mode currents along a two-wire TL along the z-axis. We show that when the line cross-sectional dimensions are electrically small, the integral equation reduces to a pair of TL-like equations with equivalent line parameters (inductance and capacitance per unit length). The derived equations make it possible to compute the antenna-mode currents using a traditional TL code with appropriate parameters.

II. Derivation of the Integral Equation

In this section, following a mathematical development similar to the one used in [9], we will derive an integral equation that will be later particularized for both TL-mode and antenna-mode currents. Consider a two-wire line of length L in free space, as shown in Fig. 1. The two conductors are separated by a distance d. The line is illuminated by an external electromagnetic field. The currents along the two conductors $I_1(z)$ and $I_2(z)$ can be decomposed as follows:

$$I_1(z) = I_a(z) + I_{\mathrm{tl}}(z) \tag{1}$$

$$I_2(z) = I_a(z) - I_{\mathrm{tl}}(z) \tag{2}$$

where $I_a(z)$ and $I_{\mathrm{tl}}(z)$ represent the antenna-mode and TL-mode currents, respectively.

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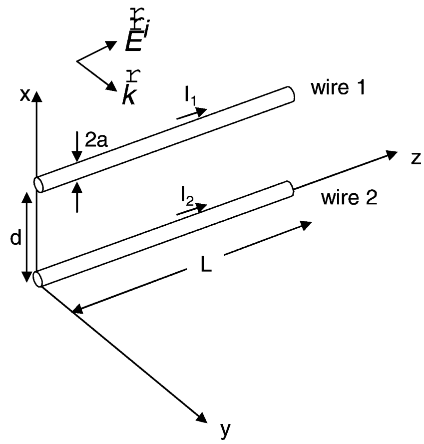

Fig. 1. Geometry of the problem.

Since the wires are assumed to be perfectly conducting, on the wire's surface, the total tangential electric field should be equal to zero. In the case of the thin wire approximation, this condition can be expressed as

$$\vec{e}_z \cdot \vec{E} = \vec{e}_z \cdot (\vec{E}^{\,i} + \vec{E}^{\,s}) = 0 \tag{3}$$

on the surface of the wires. In (3), $\vec{E}^{\,i}$ is the incident electric field in the absence of the two wires, and $\vec{E}^{\,s}$ is the scattered electric field that represents the reaction of the wires to the incident field.

The scattered electric field, produced by the line charge and current densities ρ and $\vec{J}$, can be expressed in terms of retarded scalar and vector potentials[1]

$$\vec{E}^{\,s} = -j\omega\vec{A} - \nabla\Phi \tag{4}$$

with

$$\vec{A}(\vec{r}) = \vec{A}_1(\vec{r}) + \vec{A}_2(\vec{r}) = \frac{\mu}{4\pi}\int_0^L I_1(\vec{r}\,'_1)\vec{e}_z g_1(\vec{r},\vec{r}\,'_1)dz' + \frac{\mu}{4\pi}\int_0^L I_2(\vec{r}\,'_2)\vec{e}_z g_2(\vec{r},\vec{r}\,'_2)dz' \tag{5}$$

and

$$\Phi(\vec{r}) = \Phi_1(\vec{r}) + \Phi_2(\vec{r}) = \frac{1}{4\pi\epsilon}\int_0^L \rho_1(\vec{r}\,'_1)g_1(\vec{r},\vec{r}\,'_1)dz' + \frac{1}{4\pi\epsilon}\int_0^L \rho_2(\vec{r}\,'_2)g_2(\vec{r},\vec{r}\,'_2)dz' \tag{6}$$

where $w = 2\pi f$, f is the frequency of the excitation, z' is the length variable along the wire axis, $\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$ is the position vector of the field point measured from the origin to the observation point, $\vec{r}\,'_{1,2}$ is the position vector of the source point from the origin to the position of the source (in the thin wire approximation, we assume that currents are concentrated in the wire axis for the first as well as for the second wire), and $I_1(z), I_2(z), \rho_1(z), \rho_2(z)$ are currents and charge densities along the two wires. $g_{1,2}(\vec{r},\vec{r}\,')$ are Green's functions given by

$$g_1(\vec{r},\vec{r}\,'_1) = \frac{\exp\left(-jk\left|\vec{r}-\vec{r}\,'_1\right|\right)}{\left|\vec{r}-\vec{r}\,'_1\right|} \tag{7}$$

$$g_2(\vec{r},\vec{r}\,'_2) = \frac{\exp\left(-jk\left|\vec{r}-\vec{r}\,'_2\right|\right)}{\left|\vec{r}-\vec{r}\,'_2\right|} \tag{8}$$

where k is the wave number in free space given by

$$k = \frac{\omega}{c}. \tag{9}$$

When the observation point is on the surface of wire 1, the expressions for the Green's functions become

$$g_1(z,z') = \frac{\exp\left(-jk\sqrt{(z-z')^2 + r_w^2}\right)}{\sqrt{(z-z')^2 + r_w^2}}. \tag{10}$$

$$g_2(z,z') = \frac{\exp\left(-jk\sqrt{(z-z')^2 + d^2}\right)}{\sqrt{(z-z')^2 + d^2}}. \tag{11}$$

The vector potentials for a point on the surface of wire 1 become

$$\vec{A}_1(d,0,z) = \frac{\mu}{4\pi}\int_0^L I_1(z')\vec{e}_z g_1(z,z')dz' \tag{12}$$

$$\vec{A}_2(d,0,z) = \frac{\mu}{4\pi}\int_0^L I_2(z')\vec{e}_z g_2(z,z')\,dz'. \tag{13}$$

The current and charge density along the wire are related by the continuity equation

$$\rho_{1,2} = -\frac{1}{j\omega}\nabla\cdot\vec{J}_{1,2} = -\frac{1}{j\omega}\frac{dI_{1,2}}{dz} \tag{14}$$

Introducing (14) into the equations for the scalar potentials [the first and second integrals on the right-hand side in (6)] and still considering an observation point on the surface of wire 1, we obtain

$$\Phi_1(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\int_0^L \frac{dI_1(z')}{dz'}g_1(z,z')\,dz' \tag{15}$$

$$\Phi_2(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\int_0^L \frac{dI_2(z')}{dz'}g_2(z,z')\,dz'. \tag{16}$$

Integrating (15) and (16) by parts, we obtain

$$\Phi_1(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\left[I_1(L)g_1(z,L) - I_1(0)g_1(z,0) - \int_0^L I_1(z')\frac{\partial g_1(z,z')}{\partial z'}\,dz'\right] \tag{17}$$

$$\Phi_2(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\left[I_2(L)g_2(z,L) - I_2(0)g_2(z,0) - \int_0^L I_2(z')\frac{\partial g_2(z,z')}{\partial z'}\,dz'\right]. \tag{18}$$

[1]All quantities are time-harmonic.

Since the line is open-circuited at both ends, we have

$$I_{1,2}(0) = I_{1,2}(L) = 0. \tag{19}$$

Due to the symmetry property of the Green's functions, we can derive from (10) and (11)

$$\frac{\partial}{\partial z'} g_{1,2}(z,z') = -\frac{\partial}{\partial z} g_{1,2}(z,z') \tag{20}$$

Using (19) and (20), the expressions for the retarded scalar potential (17) and (18) become

$$\Phi_1(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_1(z') g_1(z,z')\, dz' \tag{21}$$

$$\Phi_2(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_2(z') g_2(z,z')\, dz'. \tag{22}$$

Now, following mathematical developments similar to that in (15)–(22), we obtain the following expressions for the scalar and vector potentials for an observation point located on the surface of wire 2:

$$\Phi_1(0,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_1(z') g_2(z,z')\, dz' \tag{23}$$

$$\Phi_2(0,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_2(z') g_1(z,z') dz' \tag{24}$$

$$\vec{A}(0,0,z) = \frac{\mu}{4\pi}\int_0^L I_1(z')\vec{e}_z g_2(z,z') dz' \tag{25}$$

$$\vec{A}_2(0,0,z) = \frac{\mu}{4\pi}\int_0^L I_2(z')\vec{e}_z g_1(z,z')\, dz'. \tag{26}$$

Introducing (4) into (3), we obtain

$$\vec{e}_z \cdot \vec{E} = \vec{e}_z \cdot (\vec{E}^{\,i} + \vec{E}^{\,s}) = \vec{e}_z \cdot (\vec{E}^{\,i} - j\omega\vec{A} - \nabla\Phi) = 0 \tag{27}$$

which is equivalent to

$$E_z^i(x,y,z) - j\omega A_z(x,y,z) - \frac{d\Phi(x,y,z)}{dz} = 0. \tag{28}$$

For an observation point on the surface of wire 1, the above expression becomes

$$E_z^i(d,0,z) - j\omega A_z(d,0,z) - \frac{d\Phi(d,0,z)}{dz} = 0 \tag{29}$$

and similarly, for an observation point on wire 2, we have

$$E_z^i(0,0,z) - j\omega A_z(0,0,z) - \frac{d\Phi(0,0,z)}{dz} = 0. \tag{30}$$

Introducing the expressions for the scalar potentials [(21)–(24)] and those for the vector potentials [(12), (13), (25), and (26)] into (29) and (30) yields the desired integral equation, the solution of which is the induced currents $I_1(z)$ and $I_2(z)$. In the next two sections, we will particularize the integral equation for both the TL-mode and the antenna-mode currents.

III. Particularization for the TL-Mode Current

Taking the difference between (29) and (30) yields

$$E_z^i(d,0,z) - E_z^i(0,0,z) - j\omega(A_z(d,0,z) - A_z(0,0,z)) - \frac{d(\Phi(d,0,z) - \Phi(0,0,z))}{dz} = 0. \tag{31}$$

Now, let us consider the scattered voltage as defined in the standard TL theory [15]

$$V^s(z) = -\int_0^d E_x^s(x,0,z) dx. \tag{32}$$

Since the vector potential $\vec{A}(\vec{r})$ has only a z-component [see (5)], the x-component of the electric field can be written from (4) as

$$E_x^s(x,y,z) = -\frac{\partial}{\partial x}\Phi(x,y,z). \tag{33}$$

Introducing (33) into (32), the scattered voltage becomes

$$V^s(z) = \Phi(d,0,z) - \Phi(0,0,z). \tag{34}$$

Introducing (21)–(24) into (34), we obtain[2]

$$V^s(z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L (I_1(z') - I_2(z')) \times (g_1(z,z') - g_2(z,z'))\, dz'. \tag{35}$$

Now, introducing (12), (13), (25), and (26) into (31), we obtain[3]

$$\frac{dV^s(z)}{dz} + j\omega\frac{\mu}{4\pi}\int_0^L (I_1(z') - I_2(z'))(g_1(z,z') - g_2(z,z'))\, dz' = E_z^i(d,0,z) - E_z^i(0,0,z). \tag{36}$$

[2]Note that from (21) and (22)

$$\Phi(d,0,z) = \Phi_1(d,0,z) + \Phi_2(d,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L (I_1(z')g_1(z,z') + I_2(z')g_2(z,z'))\, dz'$$

and from (23) and (24)

$$\Phi(0,0,z) = \Phi_1(0,0,z) + \Phi_2(0,0,z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L (I_1(z')g_2(z,z') + I_2(z')g_1(z,z'))\, dz'.$$

[3]Taking into account (34), (31) reads

$$E_z^i(d,0,z) - E_z^i(0,0,z) - j\omega(A_z(d,0,z) - A_z(0,0,z)) - \frac{dV^s(z)}{dz} = 0.$$

The vector potential at $(d,0,z)$ is obtained by (12) and (13):

$$A_z(d,0,z) = A_{1z}(d,0,z) + A_{2z}(d,0,z) = \frac{\mu}{4\pi}\int_0^L (I_1(z')g_1(z,z') + I_2(z')g_2(z,z'))\, dz'$$

and the vector potential at $(0,0,z)$ is obtained from (25) and (26):

$$A_z(0,0,z) = A_{1z}(0,0,z) + A_{2z}(0,0,z) = \frac{\mu}{4\pi}\int_0^L (I_1(z')g_2(z,z') + I_2(z')g_1(z,z'))\, dz'.$$

Now, we have

$$I_1(z) - I_2(z) = 2I_{\mathrm{tl}}(z). \quad (37)$$

Therefore, (35) and (36) become, respectively

$$V^s(z) = -\frac{1}{2\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_{\mathrm{tl}}(z')(g_1(z,z') - g_2(z,z'))\,dz' \quad (38)$$

and

$$\frac{dV^s(z)}{dz} + j\omega\frac{\mu}{2\pi}\int_0^L I_{\mathrm{tl}}(z')(g_1(z,z') - g_2(z,z'))\,dz' = E_z^i(d,z) - E_z^i(0,z). \quad (39)$$

The above equations are indeed equivalent to the pair of equations derived by Tkatchenko *et al.* [9] for the case of a wire above a perfectly conducting ground.

It is shown in [9] that under the TL approximation, the integral

$$\int_0^L I_{\mathrm{tl}}(z')(g_1(z,z') - g_2(z,z'))\,dz' \quad (40)$$

reduces to

$$I_{\mathrm{tl}}(z')\,2\,\ln\left(\frac{d}{r_w}\right) \quad (41)$$

and therefore, (38) and (39) reduce to the well-known [15] field-to-TL equations.

IV. Particularization for the Antenna-Mode Current

We will now use (29) and (30) again, as in Section III, but this time, we will consider the sum of the two equations instead of their difference:

$$E_z^i(d,0,z) + E_z^i(0,0,z) - j\omega(A_z(d,0,z) + A_z(0,0,z)) - \frac{d(\Phi(d,0,z) + \Phi(0,0,z))}{dz} = 0. \quad (42)$$

Let us now define the antenna-mode scattered "voltage" as follows:

$$V_a^s(z) = \frac{\Phi(d,0,z) + \Phi(0,0,z)}{2}. \quad (43)$$

Note that instead of the scattered differential-mode voltage (32), which is defined as the integral of the electric field in the plane perpendicular to the wire, and which does not depend on the path of integration, the antenna-mode "voltage" is only the averaged potential for the two wires, which depends on the choice of the reference point.

Introducing (21)–(24) into (43), we obtain

$$V_a^s(z) = -\frac{1}{8\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L (I_1(z') + I_2(z')) \times (g_1(z,z') + g_2(z,z'))\,dz'. \quad (44)$$

Substituting (21)–(26) and (43) into (42), we obtain

$$\frac{dV_a^s(z)}{dz} + j\omega\frac{\mu}{8\pi}\int_0^L (I_1(z') + I_2(z'))(g_1(z,z') + g_2(z,z'))\,dz' = \frac{E_z^i(d,z) + E_z^i(0,z)}{2}. \quad (45)$$

Taking into account the fact that

$$I_1(z) + I_2(z) = 2I_a(z) \quad (46)$$

(44) and (45), respectively, become

$$V_a^s(z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_a(z')(g_1(z,z') + g_2(z,z'))\,dz' \quad (47)$$

and

$$\frac{dV_a^s(z)}{dz} + j\omega\frac{\mu}{4\pi}\int_0^L I_a(z')(g_1(z,z') + g_2(z,z'))\,dz' = \frac{E_z^i(d,z) + E_z^i(0,z)}{2}. \quad (48)$$

The antenna-mode current $I_a(z)$ is the solution of the pair (47) and (48). These two equations are similar in their forms to those describing the TL-mode current [(38) and (39)]. However, they differ in two points[4]: 1) The source term for the antenna-mode current is the *sum* of the tangential electric fields along the two conductors, whereas, for the TL-mode current, it is given by their difference, and 2) the Green's function for the antenna-mode current is given by $g_1(z,z') + g_2(z,z')$, whereas for the TL-mode current, it is given by $g_1(z,z') - g_2(z,z')$.

V. Solution of the Equations for the Antenna-Mode Current for Lines with Electrically Short Cross Sections

In this section, we will simplify (47) and (48) for the lines with electrically short cross sections.

First, let us rewrite (47) as

$$V_a^s(z) = -\frac{1}{4\pi\varepsilon j\omega}\frac{d}{dz}\int_0^L I_a(z')(g_1(z,z') + g_2(z,z'))\,dz' = -\frac{1}{4\pi\varepsilon j\omega}\int_0^L I_a(z')\frac{\partial}{\partial z}(g_1(z,z') + g_2(z,z'))\,dz'. \quad (49)$$

Considering the fact that

$$\frac{\partial}{\partial z}(g_1(z,z') + g_2(z,z')) = -\frac{\partial}{\partial z'}(g_1(z,z') + g_2(z,z')) \quad (50)$$

[4] Actually, a third difference between these two systems is connected with the gauge dependence of the obtained equations. The initial integro-differential equations for the current and potential, which are described at the end of Section II, are dependent on the used gauge. The form is different if we use the Lorenz condition, as it is usual in applied electrodynamics, from the form obtained using the Coulomb gauge [13]. In the low-frequency case ($2kd \ll 1$), the gauge dependence for the system (38)–(39) disappears (and we come to the gauge-independent classical transmission system), but this dependence remains for the system (47) and (48).

we can rewrite (49) as follows:

$$V_a^s(z) = \frac{1}{4\pi\varepsilon j\omega}\int_0^L I_a(z')\frac{\partial}{\partial z'}(g_1(z,z') + g_2(z,z'))\,dz'. \tag{51}$$

Now, integrating (51) by parts yields

$$V_a^s(z) = \frac{1}{4\pi\varepsilon j\omega}\left[I_a(z')(g_1(z,z') + g_2(z,z'))\right]_0^L - \frac{1}{4\pi\varepsilon j\omega}\int_0^L \frac{dI_a(z')}{dz'}(g_1(z,z')+g_2(z,z'))\,dz'. \tag{52}$$

The antenna-mode current being zero at the line extremities, the first term of the right-hand side of (52) is simply zero. Thus, (52) becomes

$$V_a^s(z) = -\frac{1}{4\pi\varepsilon j\omega}\int_0^L \frac{dI_a(z')}{dz'}(g_1(z,z') + g_2(z,z'))\,dz'. \tag{53}$$

Considering the expressions for the Green's functions, the integral terms in (48) and (53) can be, respectively, written as

$$\int_0^L I_a(z')(g_1(z,z') + g_2(z,z'))dz' = \int_0^L I_a(z')\left(\frac{e^{-jk\sqrt{(z-z')^2+r_w^2}}}{\sqrt{(z-z')^2+r_w^2}} + \frac{e^{-jk\sqrt{(z-z')^2+d^2}}}{\sqrt{(z-z')^2+d^2}}\right)dz' \tag{54}$$

$$\int_0^L \frac{dI_a(z')}{dz'}(g_1(z,z') + g_2(z,z'))dz' = \int_0^L \frac{dI_a(z')}{dz'}\left(\frac{\exp\left(-jk\sqrt{(z-z')^2+r_w^2}\right)}{\sqrt{(z-z')^2+r_w^2}} + \frac{\exp\left(-jk\sqrt{(z-z')^2+d^2}\right)}{\sqrt{(z-z')^2+d^2}}\right)dz'. \tag{55}$$

Now, when $kd \ll 1$ and $L \gg d$, and considering that the Green's function decays rapidly[5] as a function of $|z-z'|$, (54) and (55) can be, respectively, approximated as

$$\int_0^L I_a(z')(g_1(z,z') + g_2(z,z'))dz' \cong I_a(z)\int_0^L \left(\frac{1}{\sqrt{(z-z')^2+r_w^2}} + \frac{1}{\sqrt{(z-z')^2+d^2}}\right)dz' \tag{56}$$

$$\int_0^L \frac{dI_a(z')}{dz'}(g_1(z,z') + g_2(z,z'))dz' \cong \frac{dI_a(z)}{dz}\int_0^L \left(\frac{1}{\sqrt{(z-z')^2+r_w^2}} + \frac{1}{\sqrt{(z-z')^2+d^2}}\right)dz'. \tag{57}$$

[5]Note that the decay is not as rapid as in the case of the TL-mode current, for which the Green's function is given by the difference of the two terms $g_1 - g_2$ [10].

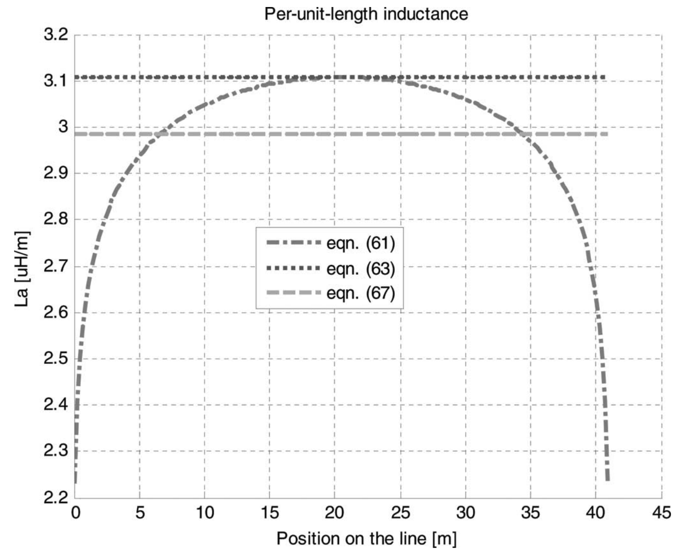

Fig. 2. Antenna-mode line per unit length inductance. Line parameters: $L = 41$ m, $d = 20$ cm, $r_w = 1.5$ mm.

The integral in (56) and (57) can be solved analytically to yield

$$\int_0^L \left(\frac{1}{\sqrt{(z-z')^2+r_w^2}} + \frac{1}{\sqrt{(z-z')^2+d^2}}\right)dz' = \ln\left\{\frac{\left[L-z+\sqrt{(L-z)^2+r_w^2}\right]\left[z+\sqrt{z^2+r_w^2}\right]}{r_w^2}\frac{\left[L-z+\sqrt{(L-z)^2+d^2}\right]\left[z+\sqrt{z^2+d^2}\right]}{d^2}\right\}. \tag{58}$$

Introducing (58) into (48) and (57) yields

$$\frac{dV_a^s(z)}{dz} + j\omega L_a' I_a(z) = \frac{E_z^i(d,0,z) + E_z^i(0,0,z)}{2} \tag{59}$$

$$\frac{dI_a(z)}{dz} + j\omega C_a' V_s{}^a(z) = 0 \tag{60}$$

where L_a' and C_a' are the equivalent per unit length inductance and capacitance for the antenna-mode current, respectively, given in (61) and (62), shown at the bottom of the next page. The antenna-mode current $I_a(z)$ is the solution of the pair of TL-like equations (59) and (60). Note that the equivalent antenna-mode per unit length inductance and capacitance are dependent on z. Figs. 2 and 3 present L_a' and C_a' as a function of z along the line for a 41-m long line ($d = 20$ cm and $r_w = 1.5$ mm). It can be seen that the inductance is minimum at the two line ends and reaches its maximum at the line center. Conversely, the capacitance reaches its maximum value at the line ends and its minimum at the center.

Considering now the observation points along the line far from the line extremities, it is straightforward to show that the per unit length inductance and capacitance expressions reduce

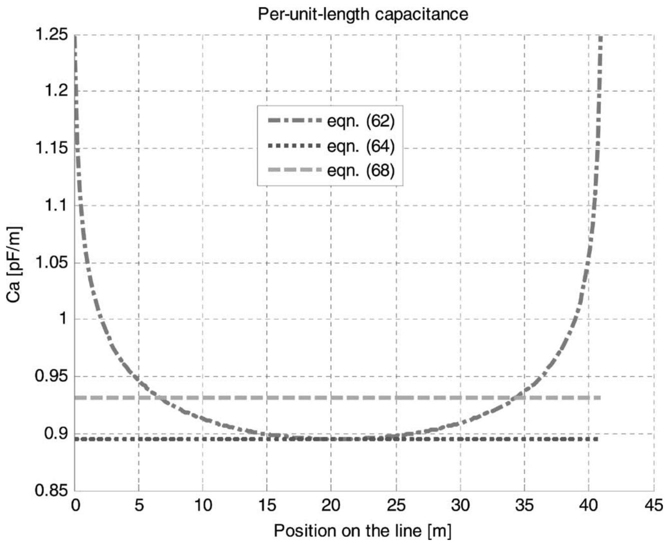

Fig. 3. Antenna-mode line per unit length capacitance. Line parameters: $L = 41$ m, $d = 20$ cm, $r_w = 1.5$ mm.

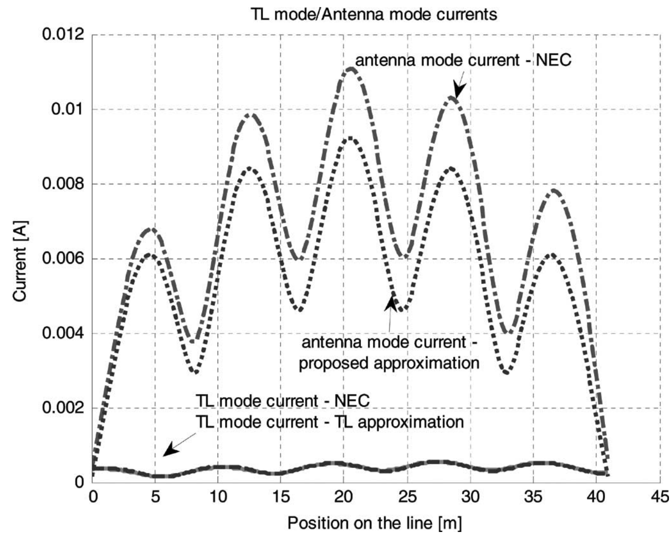

Fig. 4. Computed results for the TL-mode and antenna-mode current magnitudes along the 41-m long line for an angle of incidence $\psi = 45°$ and $f = 20$ MHz.

to

$$\hat{L}'_a = \frac{\mu_0}{2\pi} \ln \frac{L^4}{r_w^2 d^2} \tag{63}$$

$$\hat{C}'_a = \frac{2\pi\varepsilon_o}{\ln(L^4/r_w^2 d^2)}. \tag{64}$$

The average values of the inductance and the capacitance can be obtained, respectively, by

$$\tilde{L}'_a = \frac{1}{L}\int_0^L L'_a(z)\,dz \tag{65}$$

$$\tilde{C}'_a = \frac{1}{L}\int_0^L C'_a(z)\,dz. \tag{66}$$

After straightforward mathematical manipulations, for $r \ll L$ and $d \ll L$, we obtain

$$\tilde{L}'_a = \frac{\mu_0}{2\pi} \ln \frac{0.29L^4}{r_w^2 d^2} \tag{67}$$

$$\tilde{C}'_a = \frac{2\pi\varepsilon_o}{\ln(0.29L^4/r_w^2 d^2)}. \tag{68}$$

The above values are also represented in Figs. 2 and 3.

VI. Validation of the Proposed Equations: Comparison with the Numerical Electromagnetics Code (NEC)

The proposed pair of TL-like equations (59) and (60) for the computation of the antenna-mode currents are tested here versus numerical results obtained using the NEC [16]. The derived equations (59) and (60) are solved using a conventional TL code (such as the frequency-domain approach presented in [1]). The per unit length inductance and capacitance associated with the antenna-mode currents are determined using the approximate expressions (67) and (68).

We will consider a two-conductor line (as in Fig. 1) characterized by $L = 41$ m, $d = 0.2$ m, and $r_w = 1.5$ mm. The wires are assumed to be perfectly conducting, and the load impedances at the $z = 0$ and $z = L$ ends are taken to be $Z_1 = Z_2 = 293\,\Omega$, which is approximately equal to one half of the characteristic impedance of the line. This structure is excited by an incident plane wave that propagates in the plane of the line and impinges on it with an angle of incidence ψ.

Fig. 4 presents the computed results for the TL-mode and antenna-mode current magnitudes along the line, for a frequency $f = 20$ MHz and for an angle of incidence $\psi = 45°$. The calculations are performed using the constant average expressions for the equivalent inductance and capacitance given, respectively, by (67) and (68). It can be seen that the computed results using the derived equations are in reasonably good agreement with the numerical results obtained using NEC. In the same figures, we have also represented the TL-mode current magnitudes calculated using both NEC and the TL approximation. In this case,

$$L'_a = \frac{\mu_0}{4\pi} \ln \left\{ \frac{\left[L - z + \sqrt{(L-z)^2 + r_w^2}\right]\left[z + \sqrt{z^2 + r_w^2}\right]}{r_w^2} \frac{\left[L - z + \sqrt{(L-z)^2 + d^2}\right]\left[z + \sqrt{z^2 + d^2}\right]}{d^2} \right\} \tag{61}$$

and

$$Ca' = \frac{4\pi\varepsilon_o}{\ln\left\{ \frac{\left[L-z+\sqrt{(L-z)^2+r_w^2}\right]\left[z+\sqrt{z^2+r_w^2}\right]}{r_w^2} \frac{\left[L-z+\sqrt{(L-z)^2+d^2}\right]\left[z+\sqrt{z^2+d^2}\right]}{d^2} \right\}}. \tag{62}$$

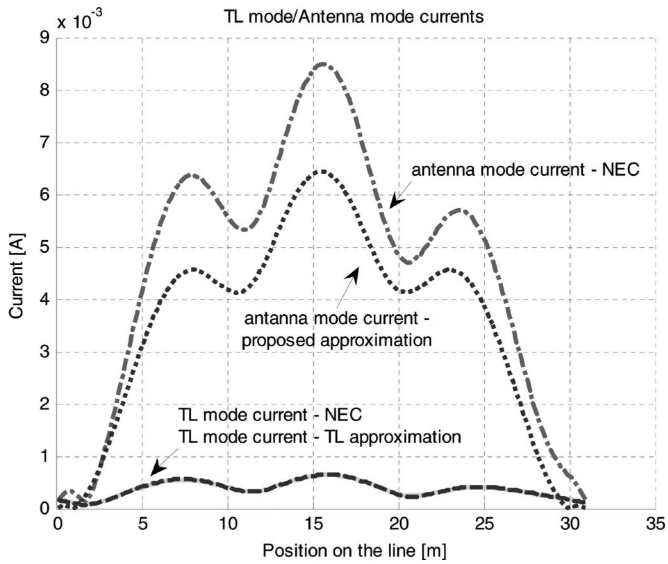

Fig. 5. Computed results for the TL-mode and antenna-mode current magnitudes along the 31-m long line for an angle of incidence $\psi = 60^\circ$ and $f = 20$ MHz.

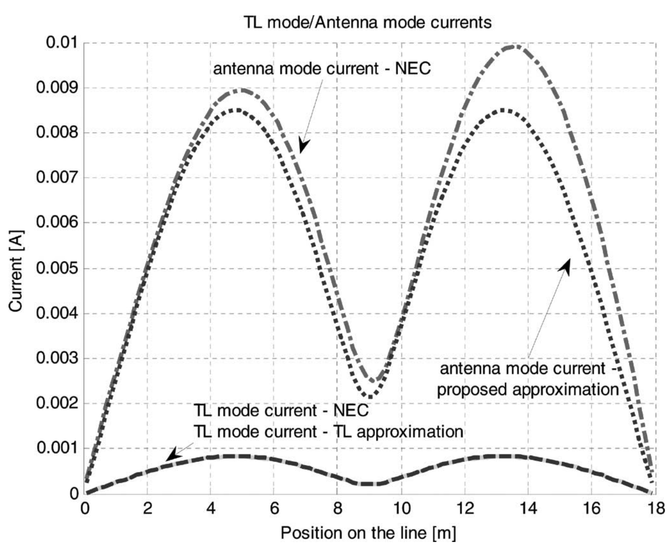

Fig. 6. Computed results for the TL-mode and antenna-mode current magnitudes along the 18-m long line for an angle of incidence $\psi = 60^\circ$ and $f = 20$ MHz. In this case, line was left opened at both the ends.

as expected, the two results are in excellent agreement to the point of being indistinguishable.

It is worth noting that even for the considered low frequencies, the antenna-mode currents can reach much larger values than do the TL-mode currents. In Figs. 5 and 6, we present similar results for different line lengths, namely 31 m (Fig. 5) and 18 m (Fig. 6). It can be seen that the computed results using the derived equations are again in reasonably good agreement with the numerical results obtained using NEC.

The observed differences between the proposed approach and the NEC results can be essentially attributed to the simplified procedure adopted for the determination of the equivalent inductance and capacitance parameters described in Section V.[6] More elaborate techniques such as those described in [12], [13], [17]–[20] could be used in this respect for the considered problem.

VII. Conclusion

For a TL, even if the line cross section is electrically small, the presence of the antenna-mode currents causes the sum of the currents at a cross section to be, in the general case, different from zero. Since the antenna-mode current response is small near the ends of the line, it is appropriate to consider the TL-model current only if all that is needed is the response of the line at the load. On the other hand, if we wish to evaluate the current along the line, the presence of the antenna-mode currents needs to be taken into account, even for electrically small line cross sections.

We have derived an integral equation describing the antenna-mode currents along a two-wire TL.

We have further shown that when the line cross-sectional dimensions are electrically small, the integral equation reduces to a pair of TL-like equations with equivalent line parameters (per unit length inductance and capacitance). The derived equations make it possible to compute the antenna-mode currents using a traditional TL code with appropriate parameters.

The derived equations were tested versus numerical results obtained using NEC and reasonably good agreement was found.

Work is currently underway to improve the accuracy of the model, specifically with regard to the determination of the line parameters and evaluation of currents at resonant frequencies, and inclusion of multiconductor lines.

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References

[1] F. M. Tesche, M. V. Ianoz, and T. Karlsson, *EMC Analysis Methods and Computational Models*. New York: Wiley, 1997.

[2] C. R. Paul, *Analysis of Multiconductor Transmission Lines*. New York: Wiley, 1994.

[3] S. Frankel, *Quasi-TEM Transmission Line Theory*, Interaction Note 135, Nov. 1972.

[4] K. S. H. Lee, "Two parallel terminated conductors in external fields," *IEEE Trans. Electromagn. Compat.*, vol. EMC-20, no. 2, May 1978.

[5] C. R. Paul, "A comparison of the contributions of common-mode and differential-mode currents in radiated emissions," *IEEE Trans. Electromagn. Compat.*, vol. 31, no. 2, pp. 189–193, May 1989.

[6] R. Razafferson, L. Koné, B. Demoulin, A. Zeddam, and F. Gauthier, "Effects of the loads and wiring size on PLC electromagnetic pollution," in *Proc. 2003 IEEE Int. Symp. Electromagn. Compat.*, vol. 1, Istanbul, Turkey, pp. 170–173.

[7] N. Korovkin, E. Marthe, F. Rachidi, and E. Selina, "Mitigation of electromagnetic field radiated by PLC systems in indoor environment," *Int. J. Commun. Syst.*, vol. 16, pp. 417–426, May 2003.

[6]It is worth noting that the use of (63) and (64) instead of the average values (67) and (68) used in the simulations does not significantly improve the agreement.

[8] U. Reggiani, A. Massarini, L. Sandrolini, M. Ciccotti, D. W. P. Thomas, and C. Christopoulos, "Experimental verification of predicted electromagnetic fields radiated by straight interconnect cables carrying high-frequency currents," in *Proc. IEEE Power Tech. Conf.*, vol. 1, Bologna, Italy, 2003, pp. 23–26.

[9] S. Tkatchenko, F. Rachidi, and M. Ianoz, "Electromagnetic field coupling to a line of finite length: Theory and fast iterative solutions in frequency and time domains," *IEEE Trans. Electromagn. Compat.*, vol. 37, no. 4, pp. 509–518, Nov. 1995.

[10] S. Tkatchenko, F. Rachidi, and M. Ianoz, "High-frequency electromagnetic field coupling to long terminated lines," *IEEE Trans. Electromagn. Compat.*, vol. 43, no. 2, pp. 117–129, May 2001.

[11] F. Delfino, R. Procopio, M. Rossi, and P. Girdinio, "Technique for computing the response of a line of finite length excited by high frequency electromagnetic field," *Proc. Inst. Electr. Eng. Sci. Meas. Technol.*, vol. 149, no. 5, pp. 289–292, Sep. 2002.

[12] H. Haase, J. Nitsch, and T. Steinmetz, "Transmission-line super theory: A new approach to an effective calculation of electromagnetic interactions," *URSI Radio Sci. Bull.*, no. 307, pp. 33–60, Dec. 2003.

[13] J. Nitsch and S. Tkachenko, "Complex-valued transmission-line parameters and their relation to the radiation resistance," *IEEE Trans. Electromagn. Compat.*, vol. 47, no. 3, pp. 477–487, Aug. 2004.

[14] A. Maffucci, G. Miano, and F. Villone, "An enhanced transmission line model for conducting wires," *IEEE Trans. Electromagn. Compat.*, vol. 46, no. 4, pp. 512–528, Nov. 2004.

[15] A. K. Agrawal, H. J. Price, and S. H. Gurbaxani, "Transient response of a multiconductor transmission line excited by a nonuniform electromagnetic field," *IEEE Trans. Electromagn. Compat.*, vol. EMC-22, no. 2, pp. 119–129, May 1980.

[16] G. J. Burke and A. J. Poggio, *Numerical Electromagnetic Code (NEC): Description Theory*. Livermore, CA: Lawrence Livermore Lab., 1981.

[17] M. Ianoz, N. V. Korovkin, S. V. Kochetov, E. E. Selina, S. V. Tkachenko, and G. V. Vodopianov, "A finite length model using the equivalent circuits for the analysis of electromagnetic radiation," in *Proc. Int. Symp. Electromagn. Compat.*, Rome, Italy, 1998, pp. 632–636.

[18] N. V. Korovkin, S. V. Kochetov, E. E. Selina, S. V. Tkachenko, and M. Ianoz, "A model for a finite length transmission line considering skin and radiation effects," in *Proc. 14th Int. Zurich Symp. Electromagn. Compat.*, Zurich, Switzerland, Feb. 18–20, 2001, p. 44H4.

[19] H. Haase and J. Nitsch, "Full-wave transmission line theory (FWTLT) for the analysis of three-dimensional wire-like structures," in *Proc. 14th Int. Zurich Symp. Tech. Exhib. Electromagn. Compat.*, Zurich, Switzerland, Feb. 2001, pp. 235–240.

[20] H. Haase and J. Nitsch, "Investigation of nonuniform transmission line structures by a generalized transmission line theory," in *Proc. 15th Int. Zurich. Symp. Tech. Exhib. Electromagn. Compat.*, Zurich, Switzerland, Feb. 2003, pp. 597–602.

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