Mutual Inductance and Capacitance Algorithm

Subroutine: mutual_cap_ind(seg,dist1). Returns mutual inductance, M, and mutual capacitance, C_m.

Purpose of Algorithm

To calculate mutual inductance and capacitance of coupled lines

Basic Description of Algorithm

The subroutine identifies the configurations and calculates the mutual inductance and capacitance between trace segments on a printed circuit board. Different algorithms are used depending on the signal current return paths.

(a) Strip lines (b) Microstrip lines

Figure 1. Configurations of coupled lines traces

Two configurations are considered when two signal current returns through a plane. Figure 1 illustrates coupled microstrip and strip lines. In both cases, algorithm begins by calculating the even mode capacitance, C_e, and odd mode capacitance, C_o, the mutual capacitance, C_m, and inductance, M, are given as [1]

$$C_m(\varepsilon_r) = \frac{1}{2}\left[C_o(\varepsilon_r) - C_e(\varepsilon_r)\right] \tag{1}$$

$$M = \frac{\mu_o \varepsilon_o}{2}\left(\frac{1}{C_e(\varepsilon_r = 1)} - \frac{1}{C_o(\varepsilon_r = 1)}\right) \tag{2}$$

Coupled microstrip lines

For a coupled microstrip line configuration, the components of the line capacitance are illustrated in Figure 2 [1].
For the even mode, the total capacitance C_e is given as [1, 2]

$$C_e(\varepsilon_r) = C_p + C_f + C_{f'} \tag{3}$$

where,

$$C_p = \frac{\varepsilon_o \varepsilon_r w}{h}$$

$$C_f = \frac{1}{2}\left[\frac{\sqrt{\varepsilon_r}}{cZ_o} - C_p\right]$$

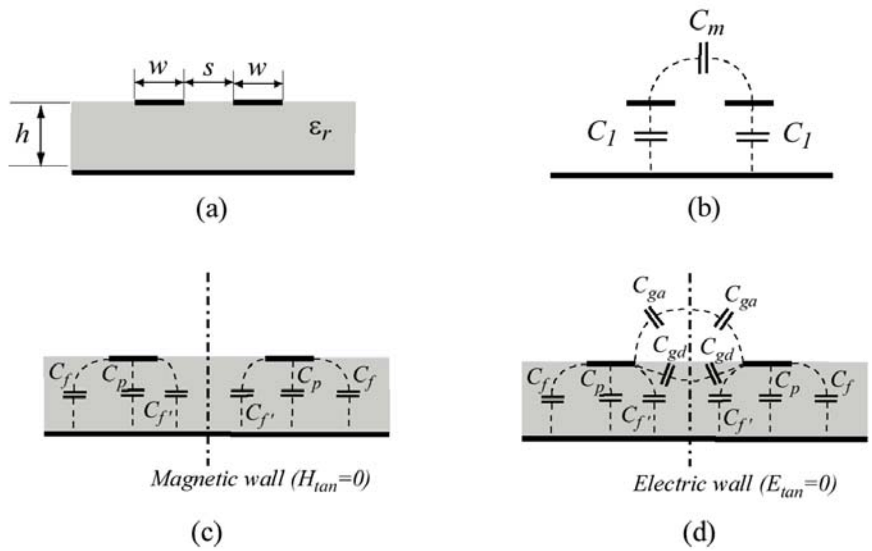

Figure 2. Configuration of coupled microstrip line (a), general equivalent circuit (b), and breakup of even mode (c) and odd mode (d) capacitance

$$C_{f'} = \frac{C_f \cdot \sqrt[4]{\varepsilon_o / \varepsilon_{re}}}{1 + A(h/s)\tanh(10s/h)}$$

$$A = \exp[-0.1\exp(2.33 - 1.5w/h)]$$

$$Z_o = \begin{cases} \frac{120\pi}{2\pi\sqrt{\varepsilon_{re}}}\ln(8h/w + 0.25w/h) & \text{if } w/h \le 1 \\ \frac{120\pi}{\sqrt{\varepsilon_{re}}}\{w/h + 1.393 + 0.667\ln(w/h + 1.444)\}^{-1} & \text{if } w/h \ge 1 \end{cases}$$

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\sqrt{1 + 10h/w}$$

C_p, C_f and C_f' represent the parallel plate capacitance and the fringing capacitances of the coupled microstrip line as shown in Figure 2. Z_o is the characteristic impedance of a microstrip line with the same ratio of w/h.

For the odd mode, the capacitance C_o is [1]

$$C_o(\varepsilon_r) = C_p + C_f + C_f' + C_{ga} + C_{gd} = 0.5C_{os} + C_{cps} \tag{4}$$

where,

$$C_{cps} = \varepsilon_o \frac{K(k')}{K(k)}$$

$$C_{os} = 4\varepsilon_o\varepsilon_r \frac{K(k_o)}{K(k_o')}$$

$$k = \frac{s}{s + 2w}, \; k' = \sqrt{1 - k_o^2}$$

$$k_o = \tanh\left(\frac{\pi w}{4h}\right)\coth\left[\frac{\pi}{4}\left(\frac{w+s}{h}\right)\right],\ k_o' = \sqrt{1-k_o^2}$$

C_{os} and C_{cps} represent the odd mode capacitance of the coupled strip line and the capacitance of the coplanar strip line with the same width and spacing. The function $K(k)$ and $K(k')$ are the complete elliptic function and its complement and their ratio is given by

$$\frac{K(k')}{K(k)} = \begin{cases} \frac{1}{\pi}\ln\left(2\frac{1+\sqrt{k'}}{1-\sqrt{k'}}\right) & \text{if } 0 \le k < \frac{1}{\sqrt{2}} \\ \pi \Big/ \ln\left(2\frac{1+\sqrt{k}}{1-\sqrt{k}}\right) & \text{if } \frac{1}{\sqrt{2}} \le k < 1 \end{cases}$$

Coupled strip lines

If the coupled segments are in a strip line configuration, the following set of equations are used

$$C_o(\varepsilon_r) = \frac{\sqrt{\varepsilon_r}}{cZ_o} \tag{5}$$

$$C_e(\varepsilon_r) = \frac{\sqrt{\varepsilon_r}}{cZ_e} \tag{6}$$

where,

$c = 3\times10^8$ m/s

$$Z_o = \frac{30\pi}{\sqrt{\varepsilon_r}}\frac{K(k_o^{'})}{K(k_o)}$$

$$Z_e = \frac{30\pi}{\sqrt{\varepsilon_r}}\frac{K(k_e^{'})}{K(k_e)}$$

$$k_o = \tanh\left(\frac{\pi w}{4h}\right)\coth\left[\frac{\pi}{4}\left(\frac{w+s}{h}\right)\right],\ k_o' = \sqrt{1-k_o^2}$$

$$k_e = \tanh\left(\frac{\pi w}{4h}\right)\tanh\left[\frac{\pi}{4}\left(\frac{w+s}{h}\right)\right],\ k_e' = \sqrt{1-k_e^2}$$

Assumptions

- All the signals propagate in quasi-TEM mode.
- The geometry of coupled line is symmetric. This means that the two traces have the same width and are on the same layer.
- The thickness of trace is negligible.
- The return plane is much wider than the traces.
- There are no effects due to fields at the terminations.

Implementation Details

The formulas used in this algorithm are approximations. The equations for the microstrip line provide reasonably accurate values in following range,

$$0.1 \le \frac{w}{h} \le 10, \quad 0.1 \le \frac{s}{h} \le 4, \quad 2 \le \varepsilon_r \le 18 \tag{7}$$

The equations for the strip lines are reasonably accurate provided,

$$0.35 \le \frac{w}{h} \tag{8}$$

Beyond these ranges, the calculated mutual capacitance and inductance may be inaccurate or even negative. The equations described above are applied only when the geometric constraints are satisfied. The algorithm returns values of zero (i.e. no coupling) if the calculated inductance or capacitance is negative.

If the geometry of traces does not satisfy the constraints in (7) or (8) or if the configuration has no return planes, mutual capacitance and mutual inductance are calculated using following equations.

$$C_m = \frac{\pi \, \varepsilon_o \varepsilon_r}{\ln\left(4(w+s)/w\right)}$$

$$M = \frac{\mu_o}{\pi} \cdot \frac{w}{w+s}$$

References

[1] K. C. Gupta, Ramesh Garg, Inder Bahl, and Prakash Bhartia, *Microstrip Lines and Slotlines*, 2nd edition, Artech House, Norwood, MA, 1996.

[2] Theodore Zeeff, et al., "Microstrip Coupling Algorithm Validation and Modification Based on Measurements and Numerical Modeling," *Proc. of the 1999 IEEE International Symposium on Electromagnetic Compatibility*, Seattle, WA, Aug. 1999, pp. 323-327.